ADT CGALmesh

Pierre Alliez (INRIA Geometrica)
Mariette Yvinec (INRIA Geometrica)
presentation by Monique Teillaud

Journées nationales des ARC, ADT et Actions Exploratoires
Novembre-Décembre 2010
ADT CGALmesh

March 2009 - March 2011

People
From EPI Geometrica:
   Researchers: Pierre Alliez, Jean-Daniel Boissonnat, Mariette Yvinec
   Post-doc: Dobrina Boltcheva
   Phds: Jane Tournois, Nader Salman
Dream ingeneer (2 years funded by the ADT): Stéphane Tayeb
From Geometry Factory: Laurent Rineau
External partners: IRCAD, BRGM, CSTB, Distène.

Goal
Develop an open platform for mesh generation based on the CGAL library http://www.cgal.org, aiming at generation of simplicial meshes: 2D, 3D and surface meshes. Preferred targeted applications in medical imaging, geological reconstruction
Motivations

Good simplicial meshes

- Well shaped elements
- Appropriate sizing of elements
- Accurate approximation of boundaries and internal surfaces.
- Control over the complexity of the mesh

are required by many applications:
- Visualisation, modelisation (cultural heritage, medical imaging)
- Scientific computation, finite element PDE solving
- Simulation (crash test, fluid dynamics)
Context

Geometrica research around mesh generation

- Triangulations
  - Delaunay, weighted Delaunay (in CGAL and Matlab) [Boissonnat-Devillers-Pion-Teillaud-Yvinec]
  - Periodic triangulations [Caroli-Teillaud]
  - Triangulations dD [Boissonnat-Devillers-Hornus]

- Mesh generation
  - Isotropic surface meshes [Boissonnat-Oudot]
  - Isotropic 2D and 3D meshes [Rineau-Yvinec]
  - Anisotropic meshes [Boissonnat-Wormser-Yvinec]
  - Mesh optimization [Alliez-Tournois]
  - Meshes from medical imaging [Boissonnat-Boltcheva-Yvinec]

- Surface reconstruction
  - From point sets: [Alliez-Samozino-Yvinec], [Alliez-Cohen-Steiner]
  - From cross-sections (parallel or not): [Boissonnat-Memari]
  - From stereovision [Salman-Yvinec]
Context

Geometrica publications around mesh generation

Mesh Generation
Phd S. Oudot 05, Phd L. Rineau 07, SGP03, SMA04, GMod05, SoCG06, IPMI07, CGTA07, 2xIMR07, SGP09, MICCAI09, SGP10.

Mesh Optimization
SIGGRAPH04, SIGGRAPH05, IMR07, SIGGRAPH09, IMR09, Phd J. Tournois 09.

Anisotropic Meshes
TCS08, SoCG08, Phd C. Wormser 08.

Surface Parametrization and Remeshing
EG02, SIGGRAPH02, IMR03, SMA03, SIGGRAPH06, SGP06, TOG06, SGP08

Surface Reconstruction
EG01, CGTA02, SGP06, SoCG07, SGP07, Phd M. Samozino 07, SGP08, ACCV09, Phd N. Salman 2010.

Mesh-based Image Segmentation
SGP06, CVPR07
The open source CGAL project [http://www.cgal.org](http://www.cgal.org) started in 1996, involving 8 European research institutes and was supported by two successive European projects.

The goal of CGAL is to make the large body of geometric algorithms developed in the field of computational geometry available for research and industrial applications.

- CGAL has currently no challenger.
- CGAL is well-known for its carefull handling of robustness issues.
Why Another Mesh Generator

The assets of CGALmesh

CGALmesh uses the approach:
Delaunay refinement + mesh optimization,
based on Delaunay restricted triangulations,

which yields the following assets:

▶ Curved and planar surfaces are meshed the same way.
▶ Bounding surfaces and volumes or multi-volumes are meshed in a single process.
▶ A strict decoupling between the meshing engine and the domain oracle (queries on bounding surfaces and domain).
This makes the mesh generator highly flexible on input domain representation:
polyhedral domains, 3D grey-level or segmented 3D images, implicit surfaces are handled.
Restricted Delaunay triangulation

- Sampling $P$, surface $S$
- $\text{Del}(P)$ Delaunay triangulation
- $\text{Vor}(P)$ dual Voronoi diagram
  - its dual Voronoi diagram
- The Delaunay triangulation restricted to $S$
  - $\text{Del}_{|S}(P)$ subcomplex of $\text{Del}(P)$
    - formed by faces
    - whose dual intersect $S$
Restricted Delaunay triangulation

sampling $P$, surface $S$
- $\text{Del}(P)$ Delaunay triangulation
- $\text{Vor}(P)$ dual Voronoi diagram
  - its dual Voronoi diagram
- The Delaunay triangulation restricted to $S$
  $\text{Del}_{|S}(P)$ subcomplex of $\text{Del}(P)$
  - formed by faces
  - whose dual intersect $S$

[Edelsbrunner and Shah 97]
[Amenta and Bern 98]
[Boissonnat and Oudot 05]
If $P$ is dense enough on $S$

- $\text{Del}_{|S}(P)$ is homeomorphic to $S$
- $\text{Del}_{|S}(P)$ is a good approximation of $S$
  - in term of Hausdorff distance
  - normals, area and curvature estimation
Delaunay refinement meshing engine

The algorithms refines:

**Bad facets:** $f \in \text{Del}_S(P)$
- oversized (sizing field)
- badly shaped (min angle bound)
- inaccurate (distance bound)

**Bad Tetrahedra:** $t \in \text{Del}_O(P)$
- oversized (sizing field)
- badly-shaped (radius-edge ratio)

**Required oracle on domain to be meshed**
- point location in domain and subdomains
- intersection detection/computation between boundary surfaces and segments (Delaunay edges)
Meshing surfaces and multi-domains in a single process

CGALmesh provides a shortcut to standard pipeline
CGALmesh design

CGALmesh decouples meshing engine from domain oracle

```
while ( is_bad (simplex) )
    refine(simplex);
```
CGALmesh design

CGALmesh decouples meshing engine from domain oracle

User Data

Domain

Criteria

Query "oracle"

Predicate
- do_intersect_surface(Segment s)
- is_in_domain(Point p)

Construction
- get_intersection_point(Segment s)

Mesh generation Engine

while (is_bad(simplex))
    refine(simplex);

Mesh

Output

Guarantees
- Mesh matches criteria

CGAL Kernel

Criteria

is_bad

Answer
CGALmesh design

The core mesh generation algorithm is independent from input domain representation.
The story board of the ADT CGALmesh

the starting point
The story board of the ADT CGALmesh

- **ADT**
- **Mesh3**
- **Oracles**

Prototypes Specifications | smooth domains |
---|---|

- Optimization (quality)
- Demo
- Optimization (performances)
- Piecewise smooth domains
- Benchmark
- Benchmark on partner data

Prototype | Oracle: Surface reconstruction | Benchmark on partner data
---|---|---
Prototype | Oracle: Polyhedral surfaces | Improve intersections
The story board of the ADT CGALmesh

March 2009: arrival of the ADT ingeneer Stéphane Tayeb
October 2010: early departure of Stéphane Tayeb for a new job
Summary of CGALmesh Achievements

- Topology and geometry approximation guarantees
- Control of size and shape of tetrahedra possibly non uniform sizing field
- Meshing multi-domains
Summary of CGALmesh Achievements

- Topology and geometry approximation guarantees
- Control of size and shape of tetrahedra possibly non uniform sizing field
- Meshing multi-domains
Summary of CGALmesh Achievements

- Topology and geometry approximation guarantees
- Control of size and shape of tetrahedra possibly non uniform sizing field
- Meshing multi-domains

Visible human
CGALmesh flexibility

Oracles currently provided for

Oracles are currently provided for domains defined by:

- implicit functions
- 3D images
- polyhedral surfaces
- 3D images segmented in multi-domains
- point sets
- ...
Meshing 3D domains
Example: a polyhedral domains

- 28K vertices
- 94K tetrahedra
- 40s mesh generation
Reconstruction from point sets

- Point cloud (40K vertices)
- Interpolated surface
- Approximated surface (20K vertices)
- Approximated surface (85K vertices)
Reconstruction from 3D images
Meshing 3D multi-domains

Input from segmented 3D medical images [IRCAD]

<table>
<thead>
<tr>
<th>Size bound (mm)</th>
<th>vertices nb</th>
<th>facets nb</th>
<th>tetrahedra nb</th>
<th>CPU Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>3,743</td>
<td>3,735</td>
<td>19,886</td>
<td>0.880</td>
</tr>
<tr>
<td>8</td>
<td>27,459</td>
<td>19,109</td>
<td>159,120</td>
<td>6.97</td>
</tr>
<tr>
<td>4</td>
<td>199,328</td>
<td>76,341</td>
<td>1,209,720</td>
<td>54.1</td>
</tr>
<tr>
<td>2</td>
<td>1,533,660</td>
<td>311,420</td>
<td>9,542,295</td>
<td>431</td>
</tr>
</tbody>
</table>
Meshing 3D domains

Input from segmented 3D medical images
Meshing implicitly defined multi-domains

Boolean operations

- 38K vertices
- 168K tetrahedra
- 8s mesh generation time
Meshing implicitly defined multi-domains

Multi-domain defined by the distance to a polyhedron
Why is optimization required?

- Delaunay refinement provides control on:
  - Topology
  - Approximation accuracy: bounded surface-facets distance
  - Elements sizing: bound on facets and tets circumradii
  - Elements quality:
    - radius-edge ratio (i.e. min angle) of facets
    - radius-edge ratio of tetrahedra

- Delaunay refinement is blind to sliver
  → no bound on dihedral angles of tetrahedra

![Mesh Examples](image)
Mesh optimization

Optimization processes in CGALmesh

4 optimization processes can be sequentially combined.

- Global optimization processes
  - Lloyd relaxation
  - ODT (Optimal Delaunay Triangulation) relaxation

- Local optimization
  - Vertex perturbation
  - Sliver exudation

Parameters of optimization processes

to trade computation time for mesh quality

- targeted lowed bound on dihedral angle
- time limit
- number of iterations for Lloyd and ODT relaxation
- convergence bound for Lloyd and ODT relaxation
Mesh optimization

Original Mesh
(50k vertices, 290k tets, 10 seconds)

ODT smoothing
(global optimization, 110s)

ODT + Sliver perturbation
(local optimization, 40s)
Mesh optimization

Sliver removal efficiency

Worst dihedral angle (in deg.)

Mesh 1 2 3 4 5 6 7 8 9 10 11 12 12.6
Time (in s) 105.0 2.2 4.5 9.1 17.3 36.2 62.9 131.0 245.1 432.7 778.2 1358 2371

Dihedral angle distribution

Number of angles

Dihedral angle (in deg.)

67k vertices, 225k tets, 105 s
Handling sharp features

The method of protecting balls
[Cheng, Dey et al 07]

Cover sharp edges with protecting balls
- balls cover the edges
- balls do not include each other centers
- two balls centered on different edges do not intersect
- three balls do not intersect

Use a weighted Delaunay triangulation

Initialize mesh with protecting balls
Run weighted version of Delaunay refinement

Segments joining centers of consecutive protecting balls are guaranteed to be edges in the mesh
Meshing with sharp features
A polyhedral example
Meshing with sharp features

Example from medical imaging
Meshing with sharp features
Example from medical imaging
Meshing with sharp features
(Outdoor laser)
Licences and users

CGALmesh licenses

- Opensource GPL + LGPL (http://www.cgal.org/)
- Commercial licenses available from Geometry Factory
  http://www.geometryfactory.com/

CGALmesh already used or in test:

  INRIA  EPI Nachos, Athena, Asclepios, SOFA
  Academic CSTB, BRGM, ANR Kidpocket
  Industrial (confidential)
  Alumni More than 500 posts on the 3D mesh generator in CGAL’s users mailing list
Summary

1. **CGAL 3.8** the next public release of the library CGAL, whose outcome is scheduled in **march 2011** will offer a full fledge version of **CGALmesh** i.e. a 3D simplicial meshes generation package, handling:
   - curved surfaces
   - multi-domains
   - sharp edges
   - mesh optimization
   - various possible definition of domains to be mesh

2. Only tests and benchmarking on real data from potential users might suffer from some delay owing to the early departure of the engineer hired for the ADT.
Conclusion

Thanks for your attention

CGALmesh is available from http://www.cgal.org