Approximate Symmetries of Point Patterns

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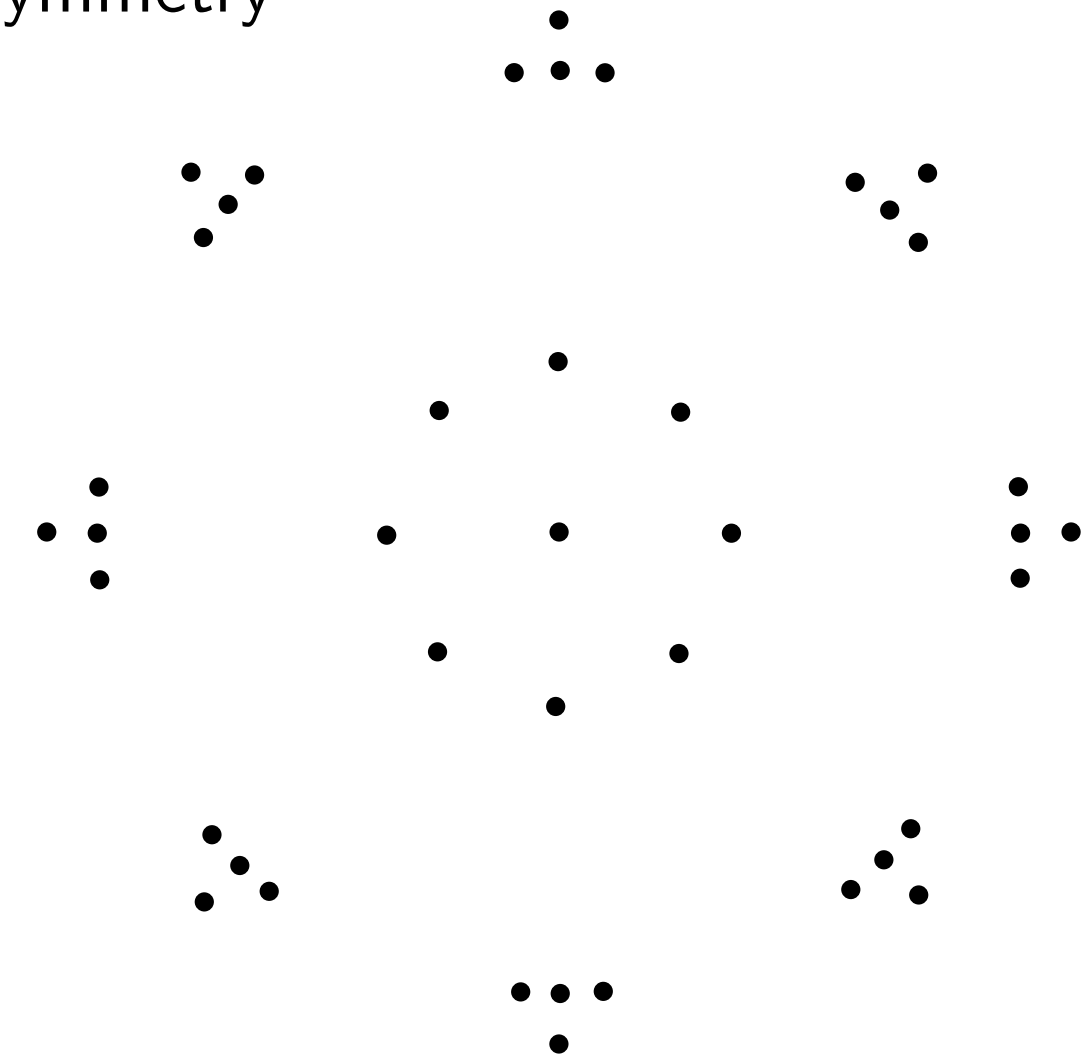
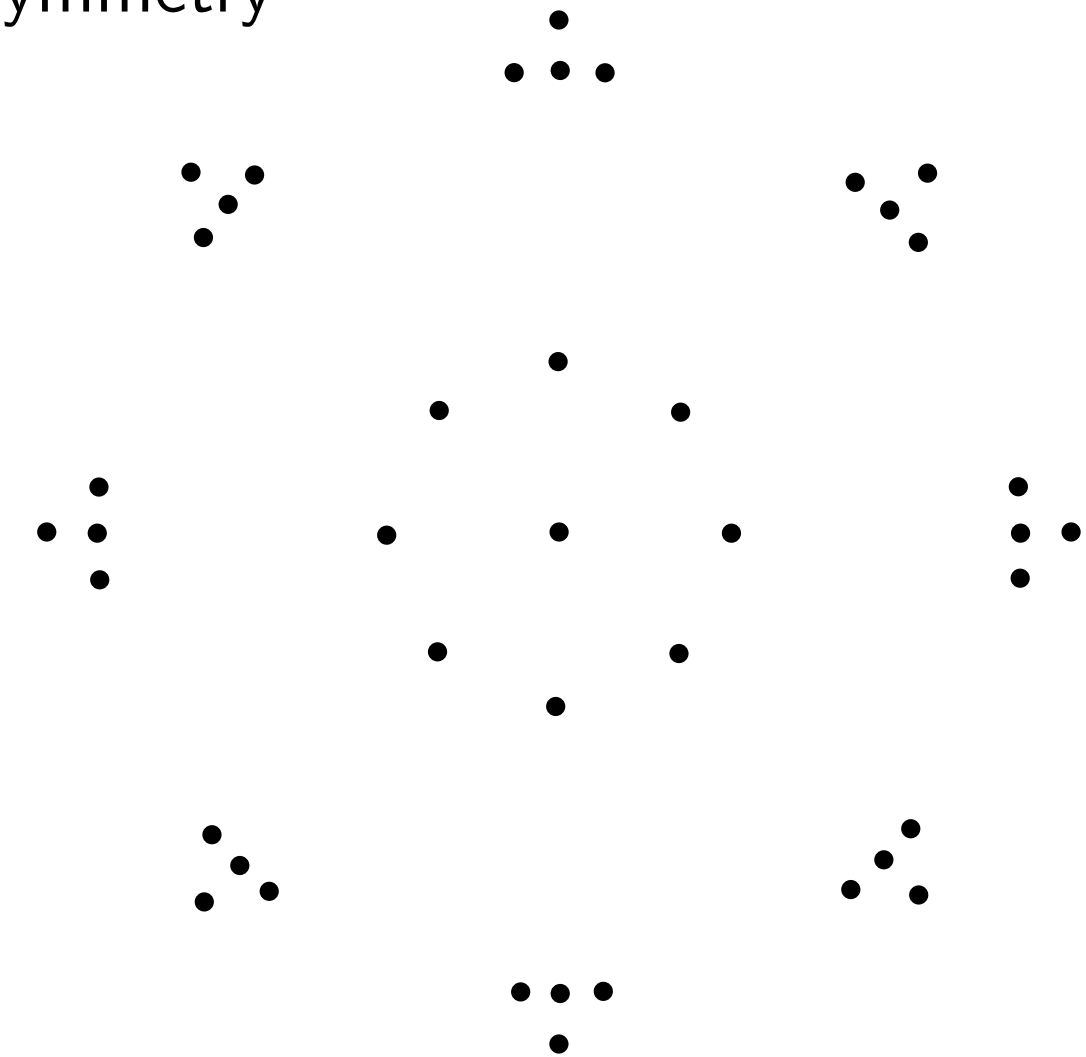
perfect symmetry



perfect symmetry

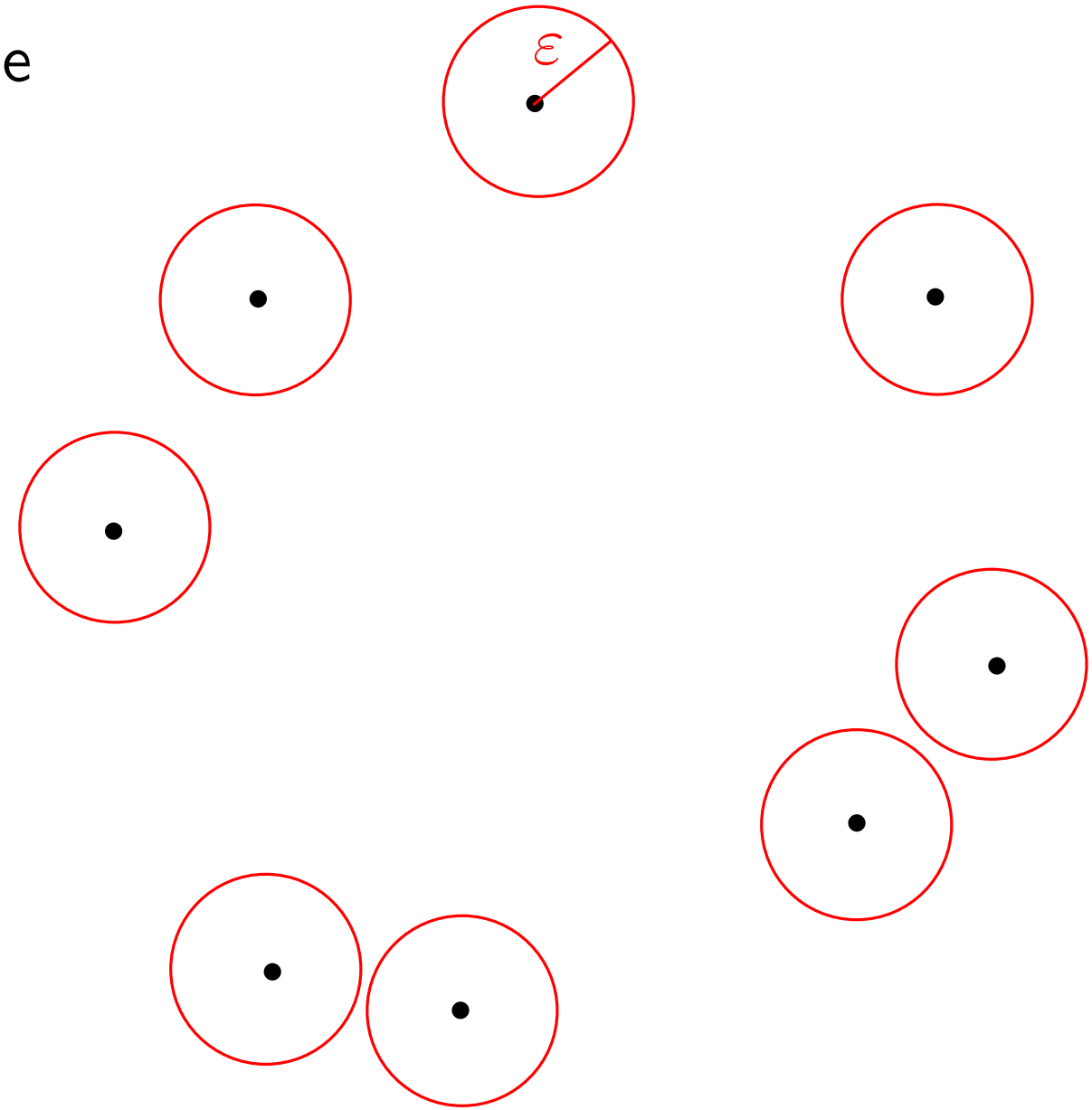


can easily be determined

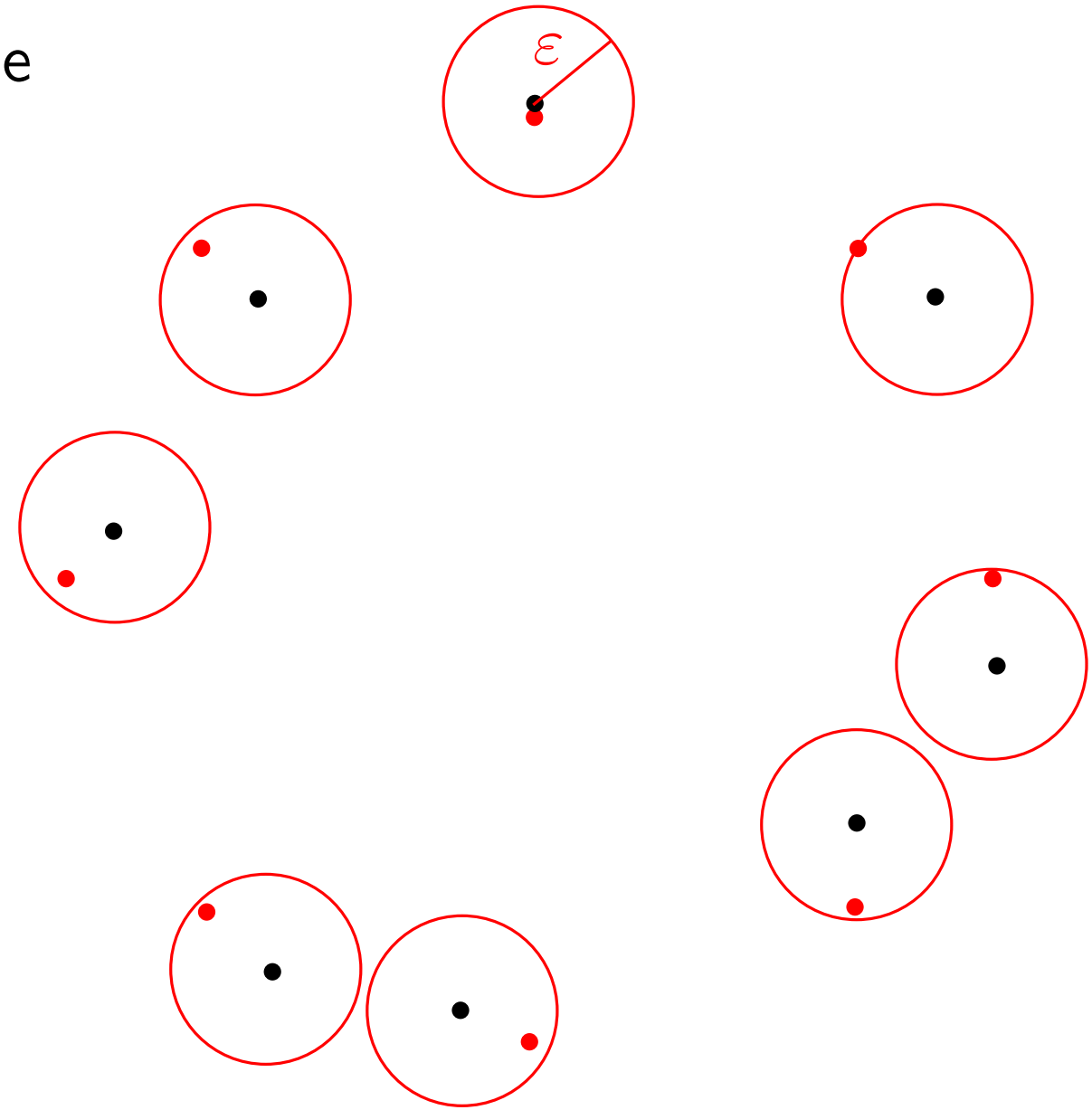
Atallah, Wolter 1985

approximate
symmetry

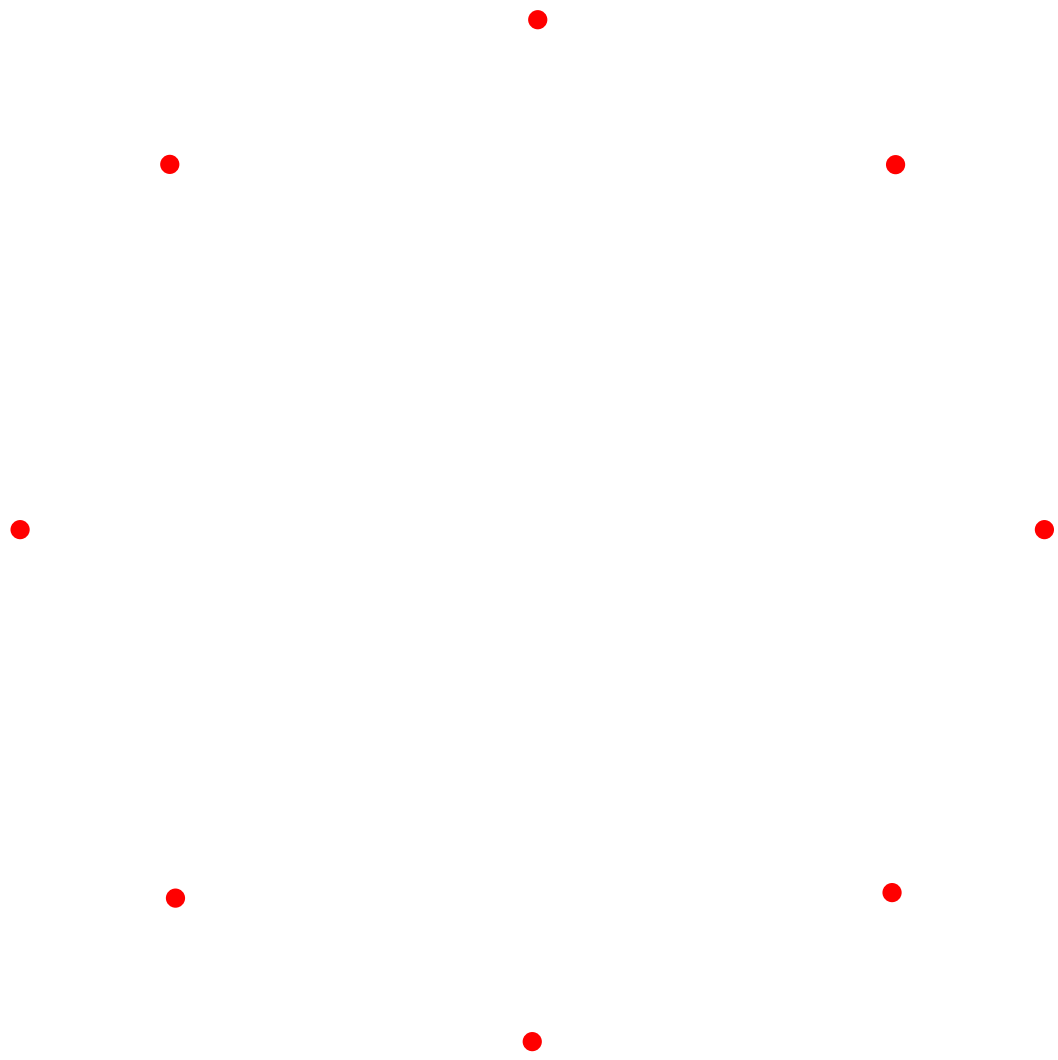
ε-
approximate
symmetry



ε-
approximate
symmetry



ε-
approximate
symmetry



Deciding approximate symmetry of a finite point set with given tolerance ε is NP-hard for symmetry groups $C_3, C_4, ...$ and $D_2, D_3, ...$

Iwanowski 1990

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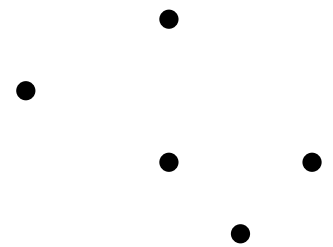
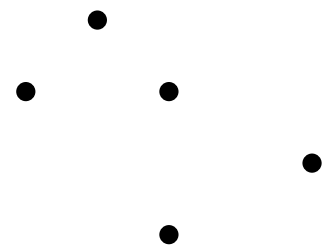
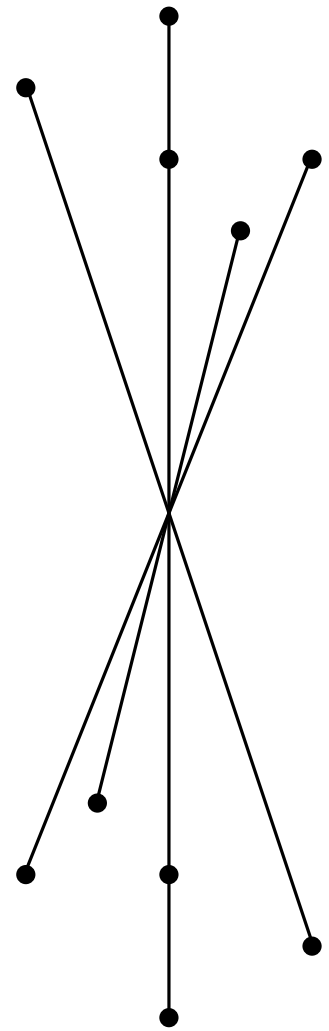
which implies:
finding the smallest ε for which an approximate symmetry exists is NP-hard.

C_2-Symmetry

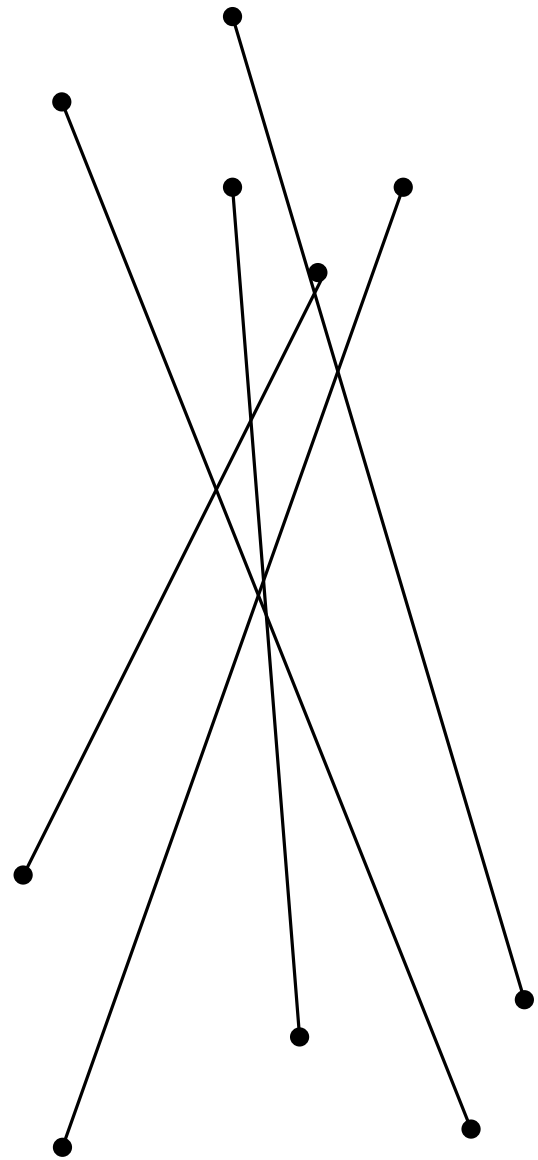




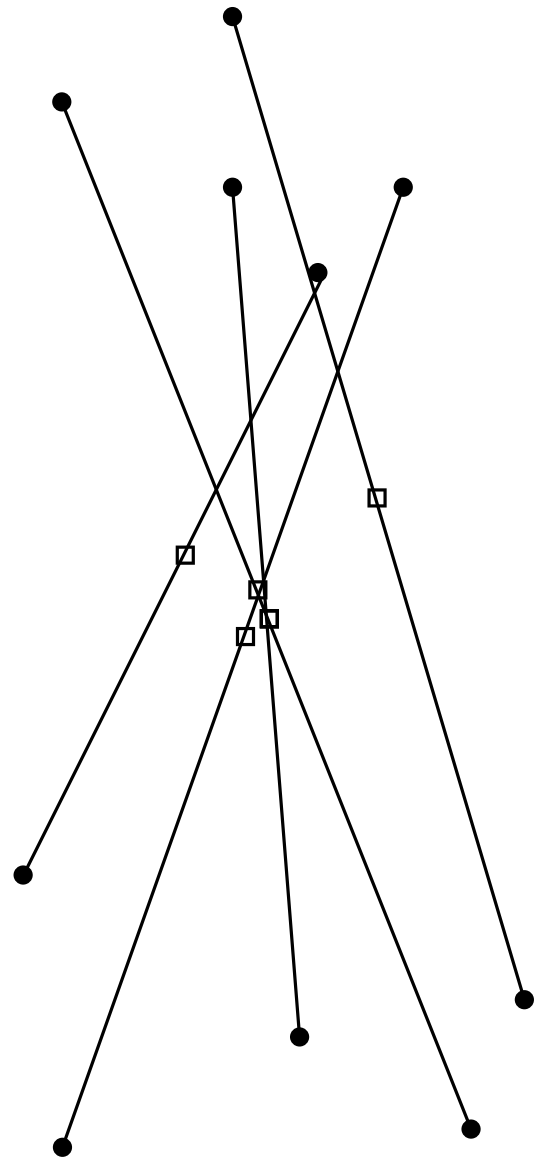
C_2-Symmetry



approximate
C_2-symmetry,
partition known



approximate
C_2-symmetry,
partition known



approximate
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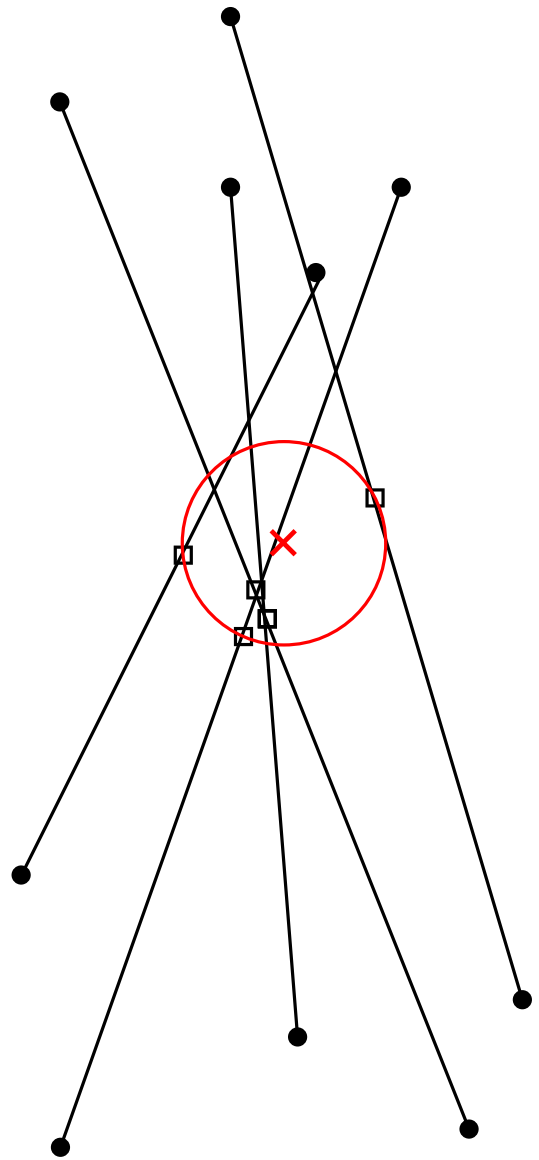

Approximate C_2-symmetry, partition unknown

1. Determine the set Q of all bisecting points between any pair of points in P.
2. Determine the set C of all enclosing circles through any triple and pair of points in Q.
3. For each circle in $c \in C$:
   (a) Determine the other points of Q contained in c and construct the corresponding graph G on P.
   (b) Check for a perfect matching in G.
4. Return the smallest radius of any circle with a positive answer in Step 3.

Approximate Regular n-gon

Given a set P of n points and $\varepsilon > 0$, decide whether the ε-neighborhoods of the points of P contain 1-1 the vertices of a regular n-gon Q.

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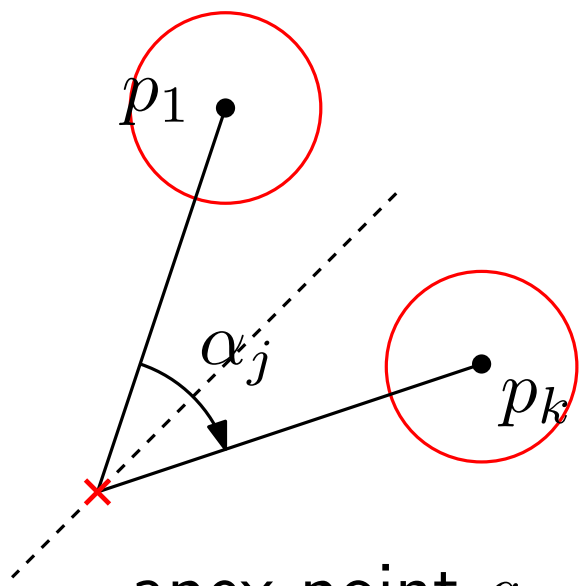


apex point $a_{k,j}$

$$\alpha_j = j\,2\pi/n$$

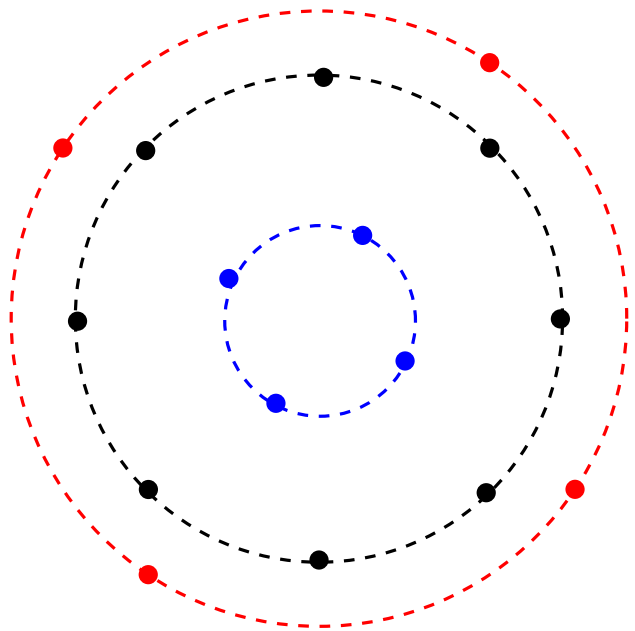
$$k = 2, ..., n$$
$$j = 1, ..., n-1$$

Approximate Regular n-gon

Given a set P of n points and $\varepsilon > 0$, decide whether the ε-neighborhoods of the points of P contain 1-1 the vertices of a regular n-gon Q.

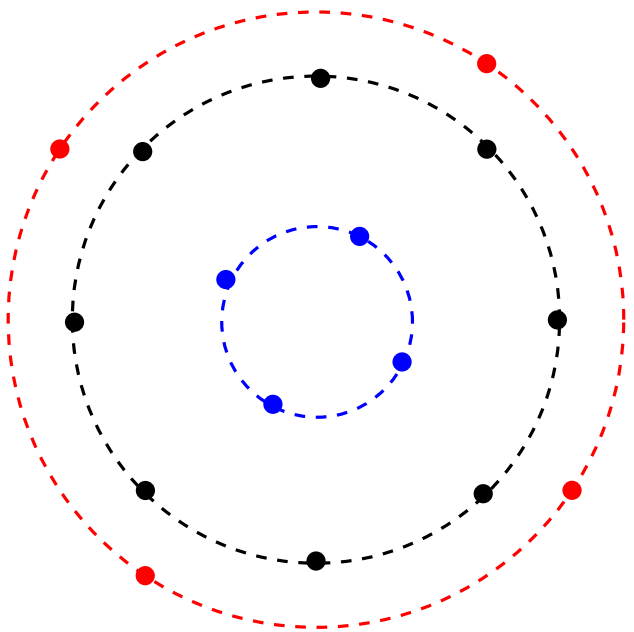
can be solved in $O(n^{10.5} \log n)$ using *bottleneck matching*

Given Partition



Given Partition

Given a set P of n points, $\varepsilon > 0$, $k \in \mathbf{N}$, and a partition $P = P_1 \cup ... \cup P_m$. Then it can be decided in time $O(n^4 k^{22.5})$ whether P is ε-approximately C_k-symmetric where the partition corresponds to the partition into orbits by the symmetries.



Sufficiently Separated Point Sets

Let $k \in \underline{\mathsf{N}}, \varepsilon > 0$, and $P \subset \underline{\mathsf{R}}^2, |P| = n$ such that $\|p - q\| \geq c\varepsilon$ for some constant $c > 4$ and any two distinct $p, q \in P$. Then, it can be decided in time $O(n^4 k^6)$ whether P is ε-approximately C_k-symmetric.

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($O(n \log n)$ if the center is given).