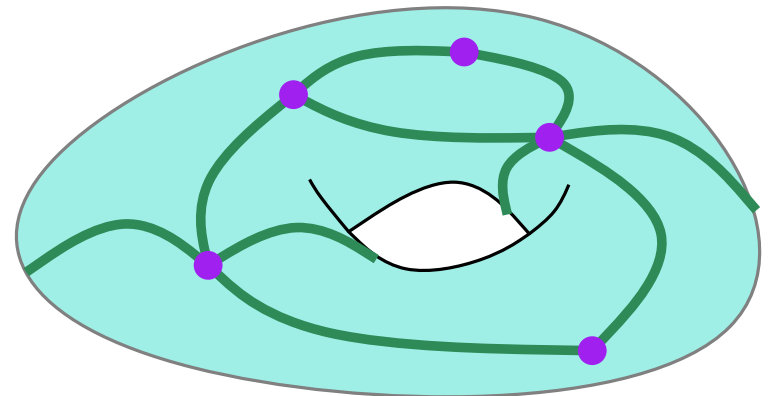


Straight line drawing of a graph on the flat torus

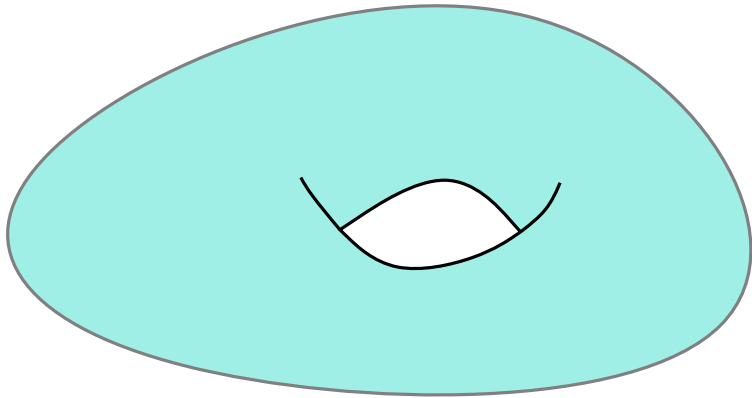
Luca Castelli Aleardi, LIX

Olivier Devillers, INRIA

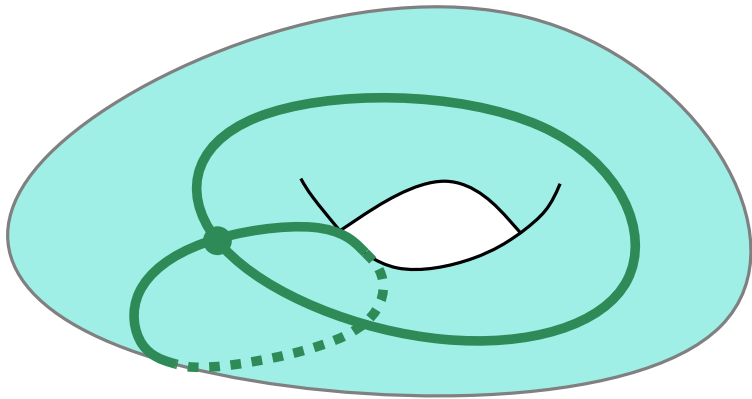
Éric Fusy, LIX



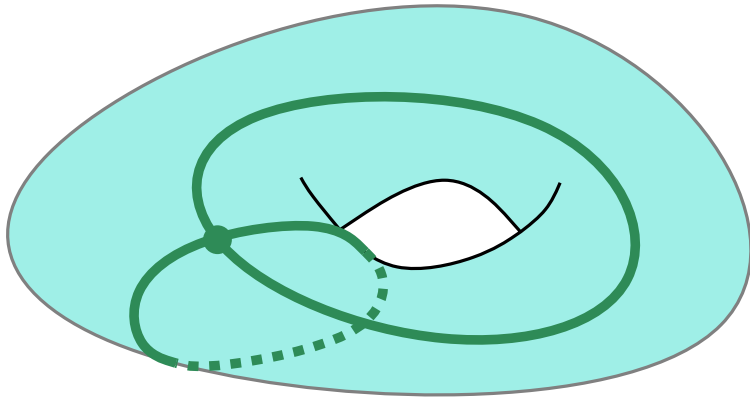
Torus



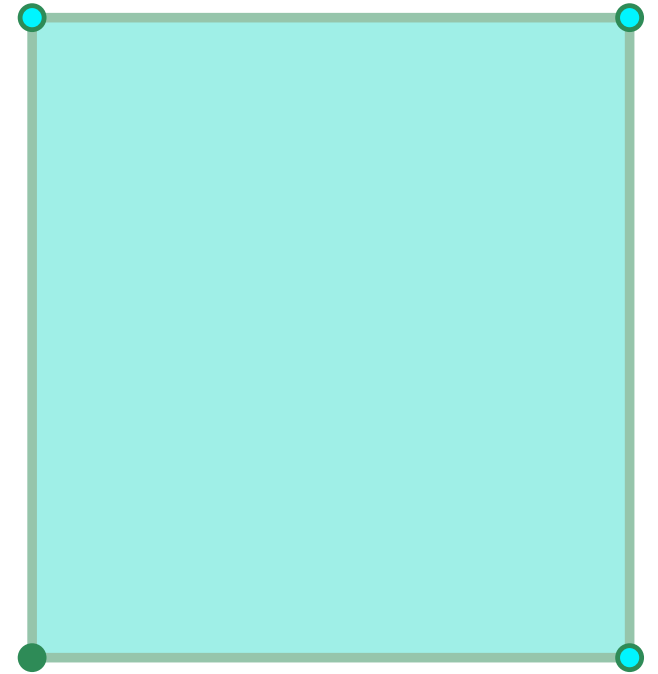
Torus



Torus

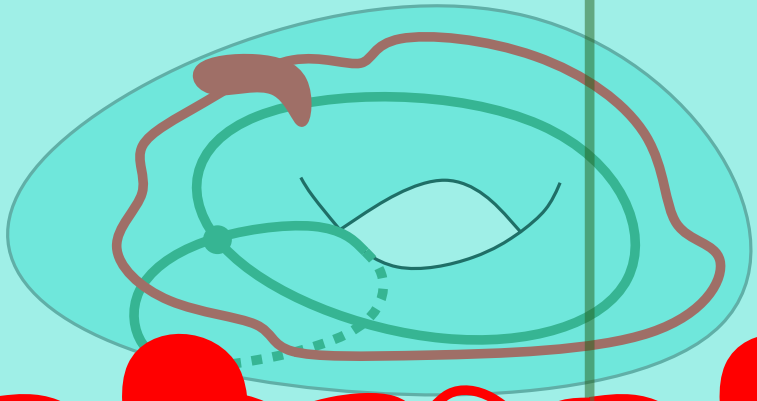


Flat torus

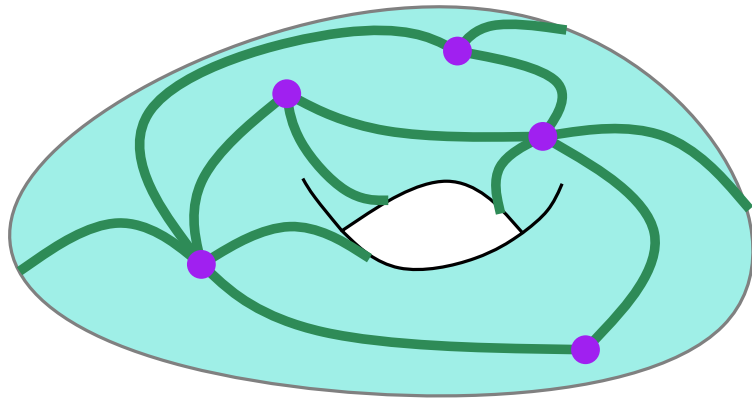


Torus

Flat Torus

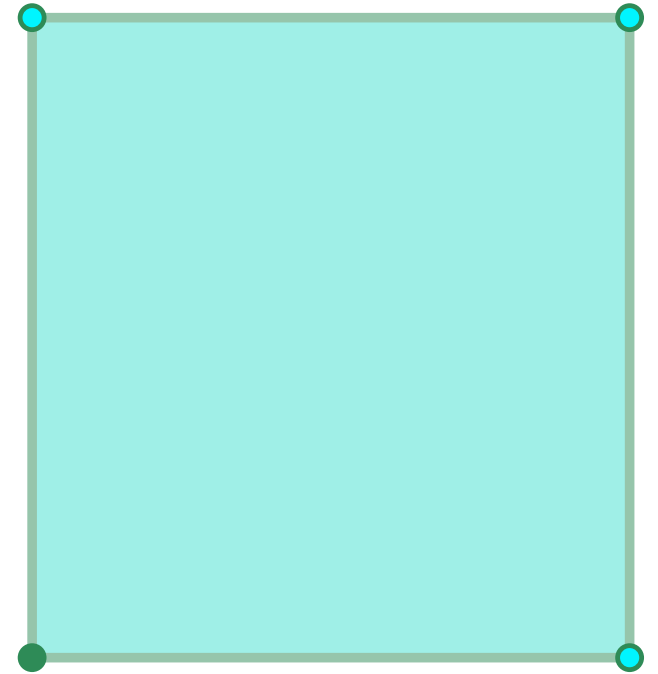


Torus



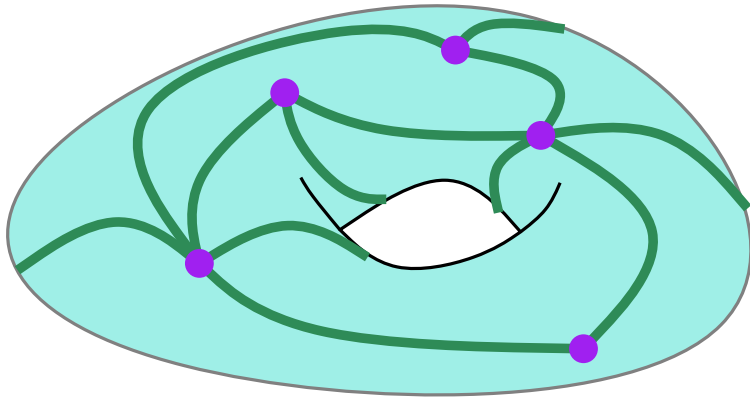
Graph

Flat torus



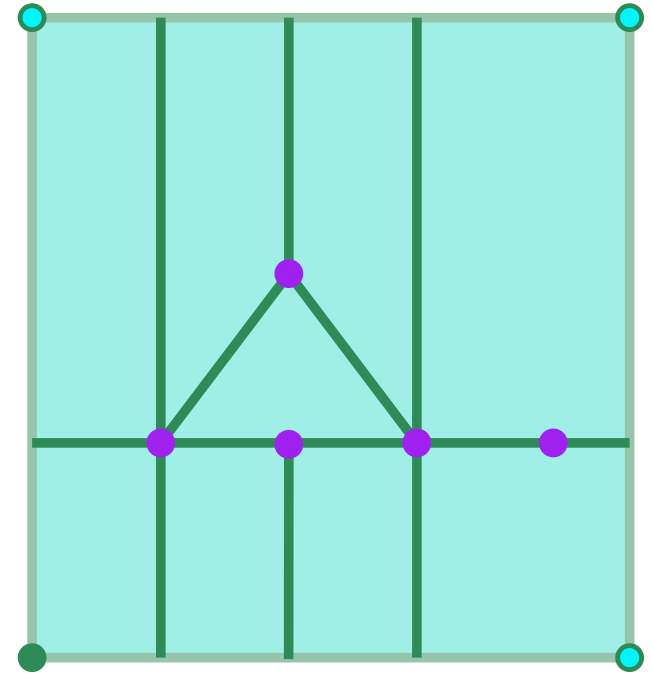
Problem statement

Torus



Graph

Flat torus



Convex straight line drawing

Result

Given a map on a torus

Result

Given a map on a torus
(essentially 3-connected)

Get a

Convex straight line drawing
in rectangle $O(n) \times O(n^{\frac{3}{2}})$

[De Fraysseix, Pach, & Pollack]

Given a triangulation

Get a

Planar straight line drawing
in rectangle $O(n) \times O(n)$

Algorithm, global view

triangulation of a cylinder, no chords, no 2-cycles

Vertices ordering

Incremental drawing (boundary characteristics)

triangulation of a cylinder, with chords, 2-cycles & loops

Split triangulation in pieces

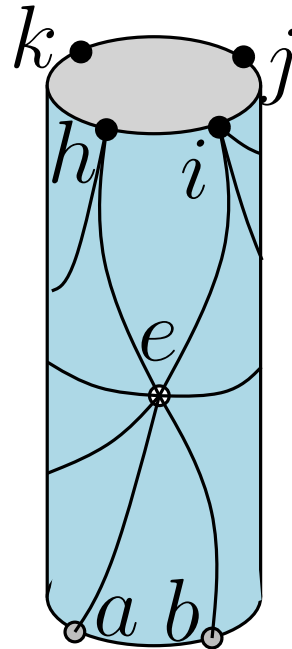
maps of a cylinder

Different boundary characteristics

triangulation and maps of a torus

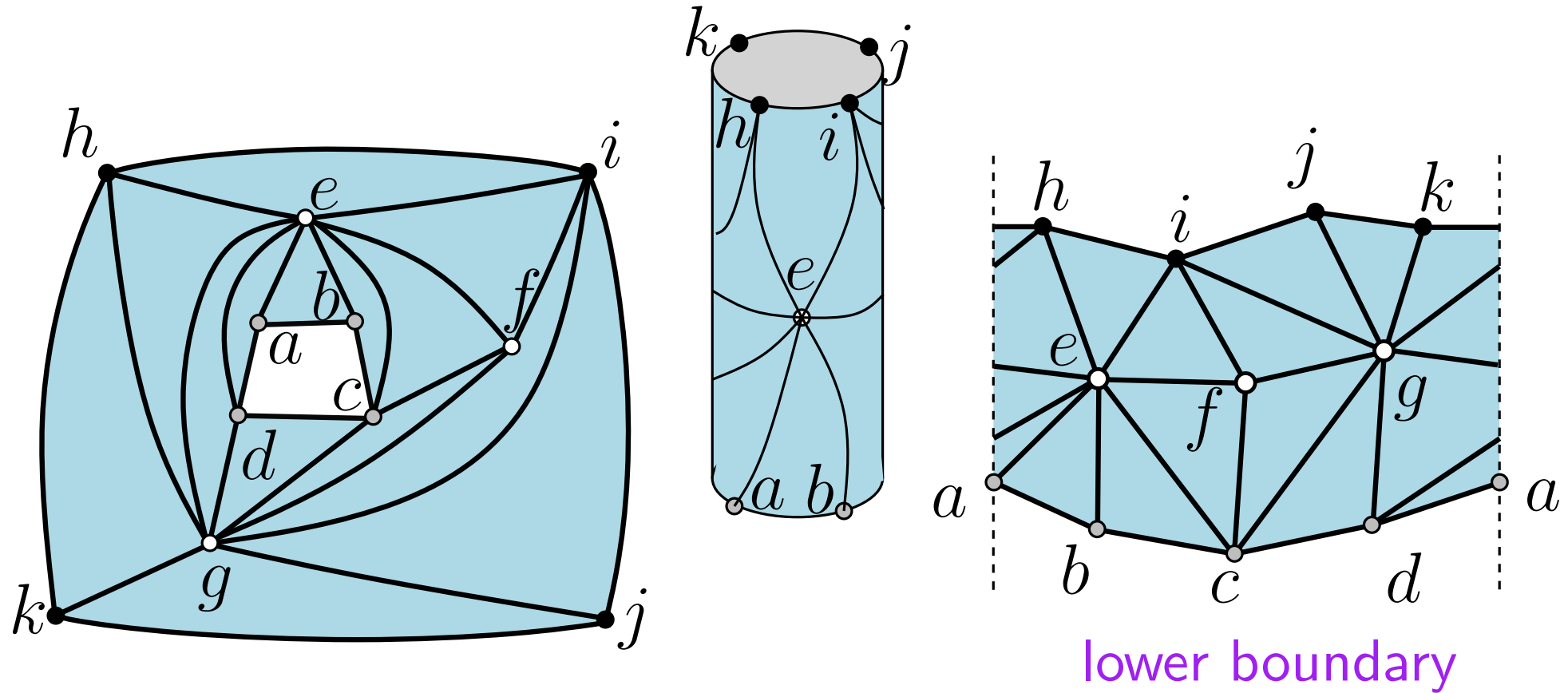
Cut the torus \rightarrow cylinder

Algorithm, vertices ordering
triangulation of a cylinder, no chords, no 2-cycles



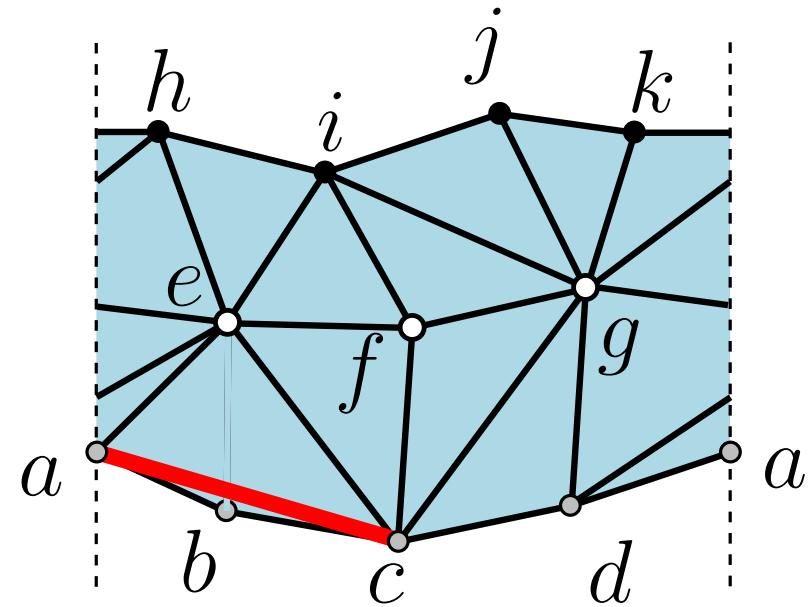
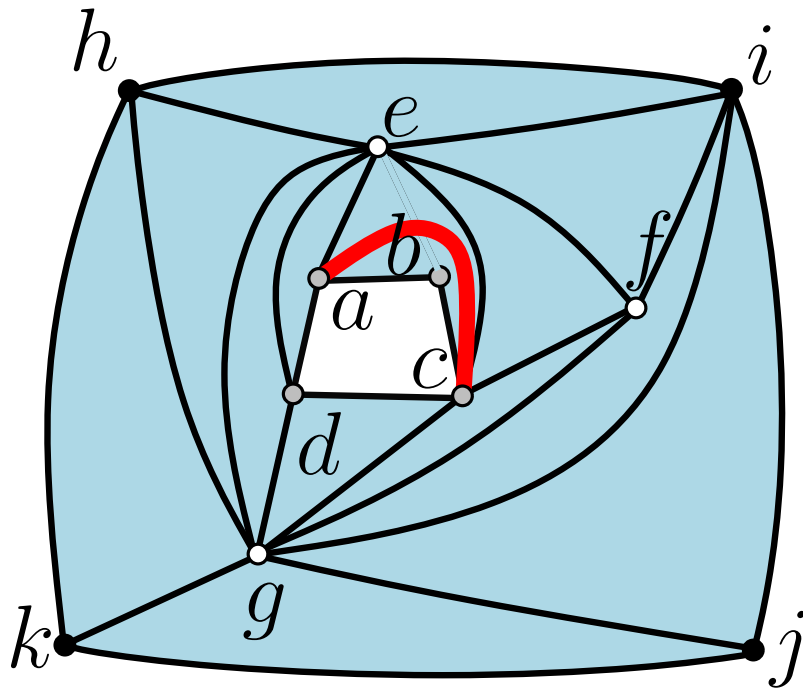
Algorithm, vertices ordering
triangulation of a cylinder, no chords, no 2-cycles

Annular view / periodic view



Algorithm, vertices ordering
triangulation of a cylinder, no chords, no 2-cycles

Annular view / periodic view

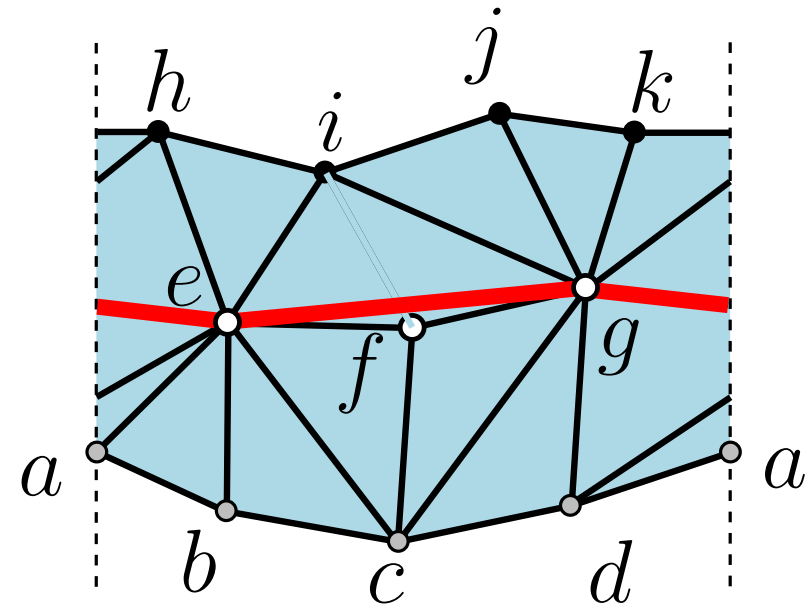
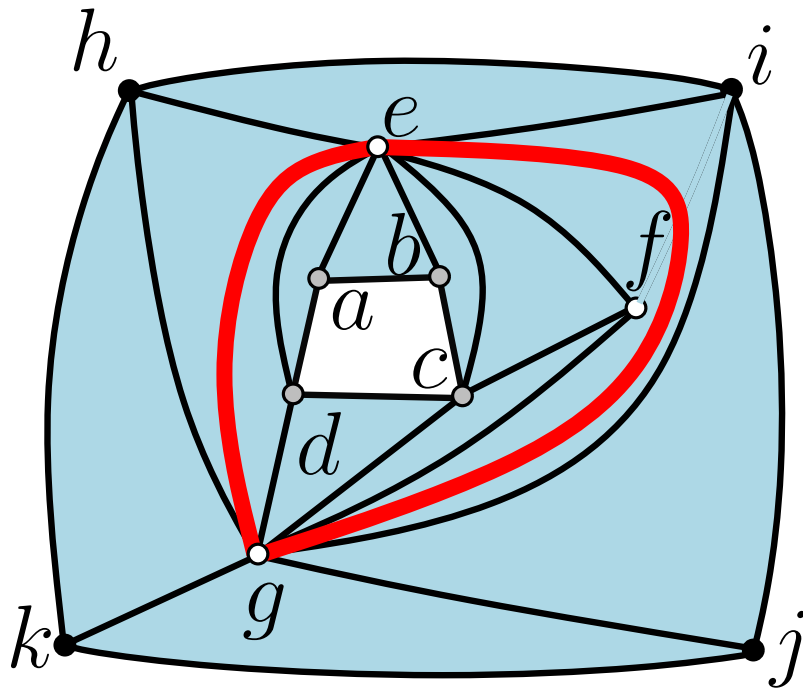


lower boundary

no chords at lower boundary

Algorithm, vertices ordering
triangulation of a cylinder, no chords, no 2-cycles

Annular view / periodic view

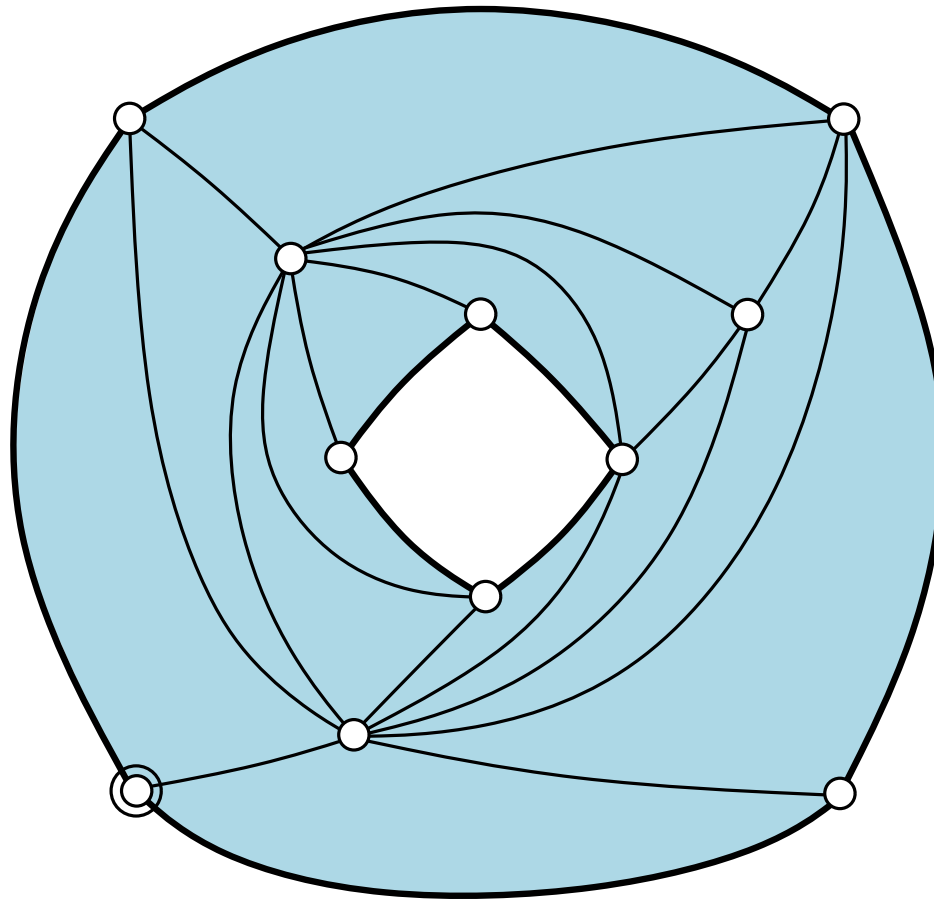


lower boundary

no 2-cycles

Algorithm, vertices ordering
triangulation of a cylinder, no chords, no 2-cycles

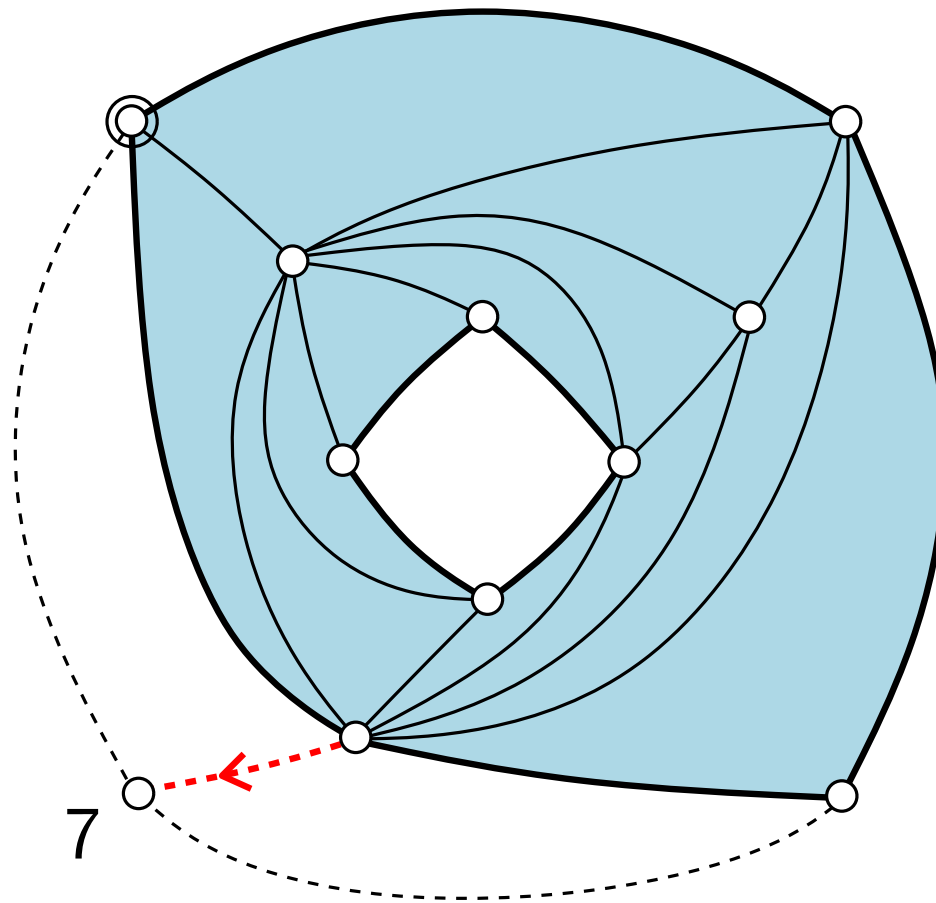
Shelling order



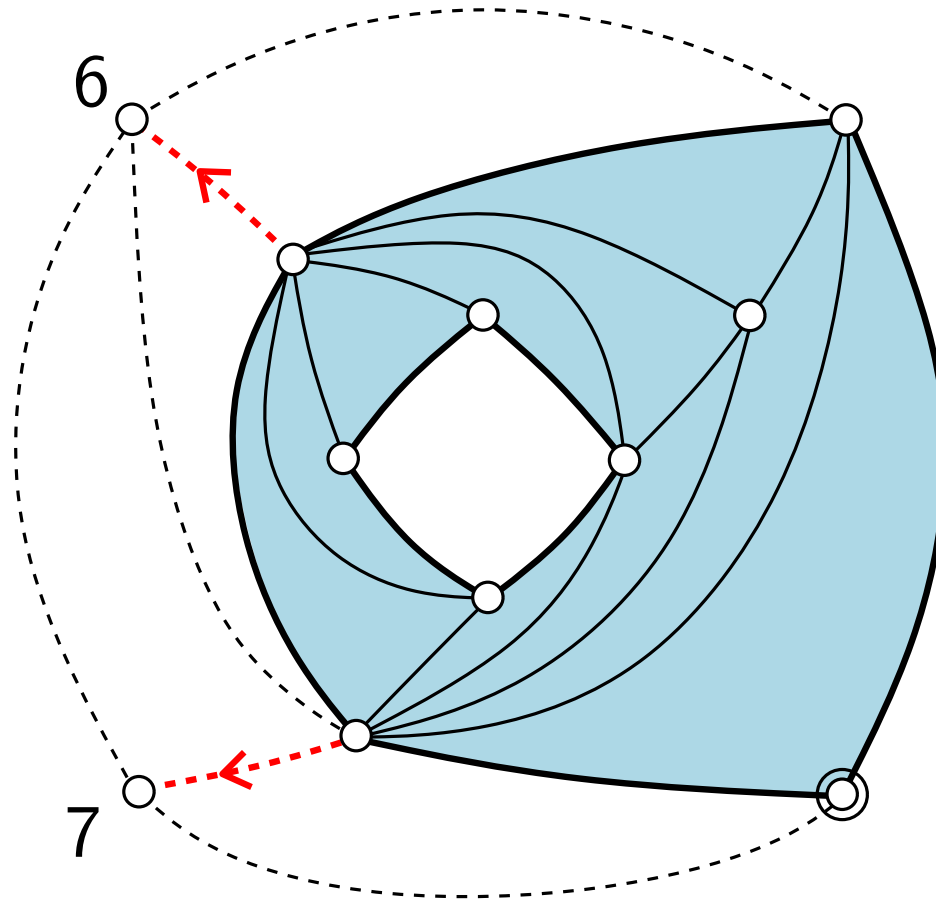
Algorithm, vertices ordering

triangulation of a cylinder, no chords, no 2-cycles

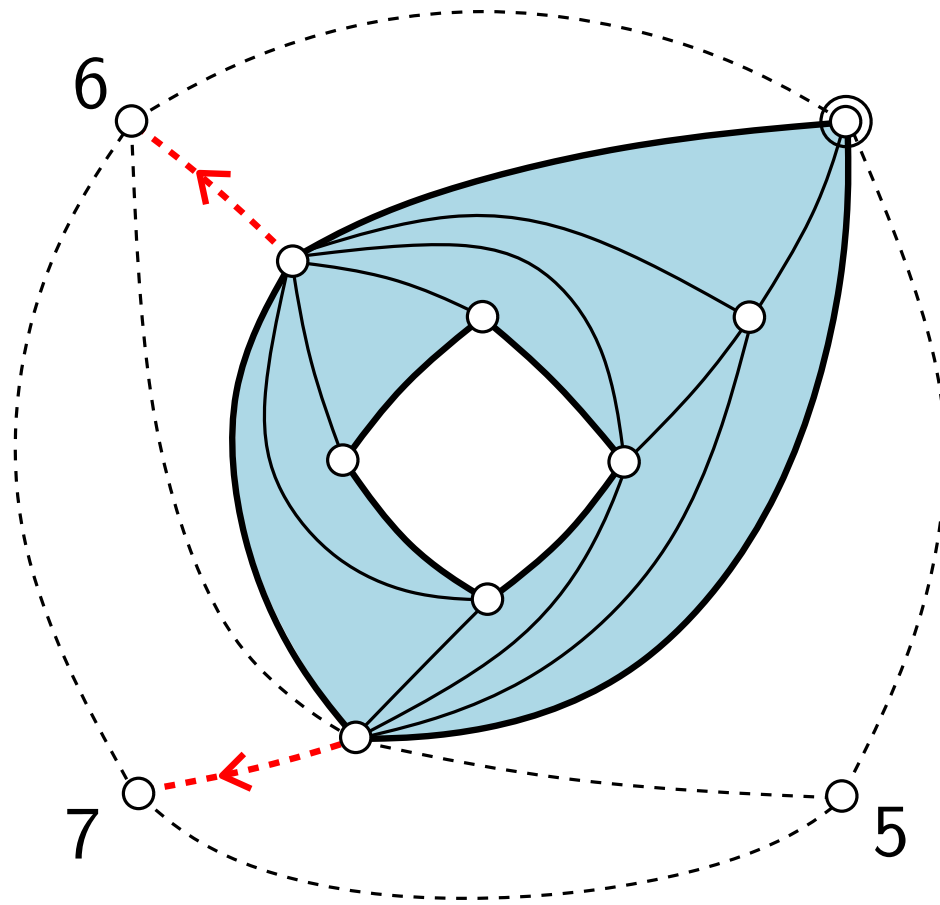
Remove vertices, so that the remaining part is an annulus



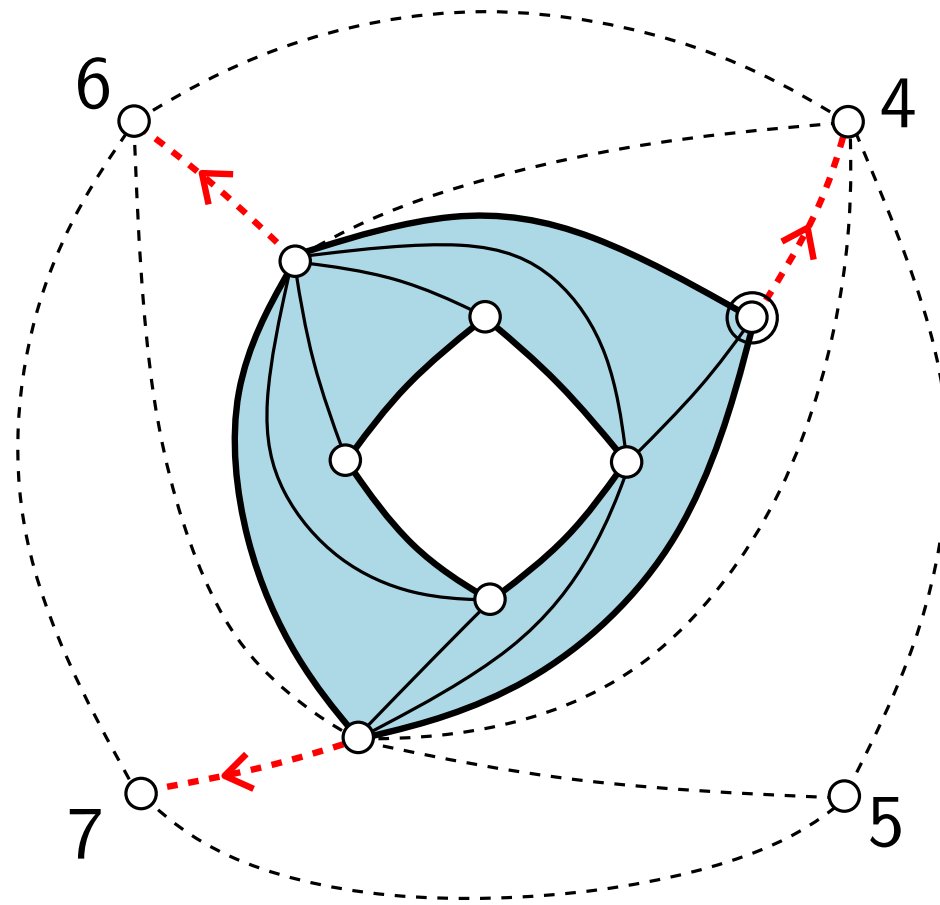
Algorithm, vertices ordering
triangulation of a cylinder, no chords, no 2-cycles



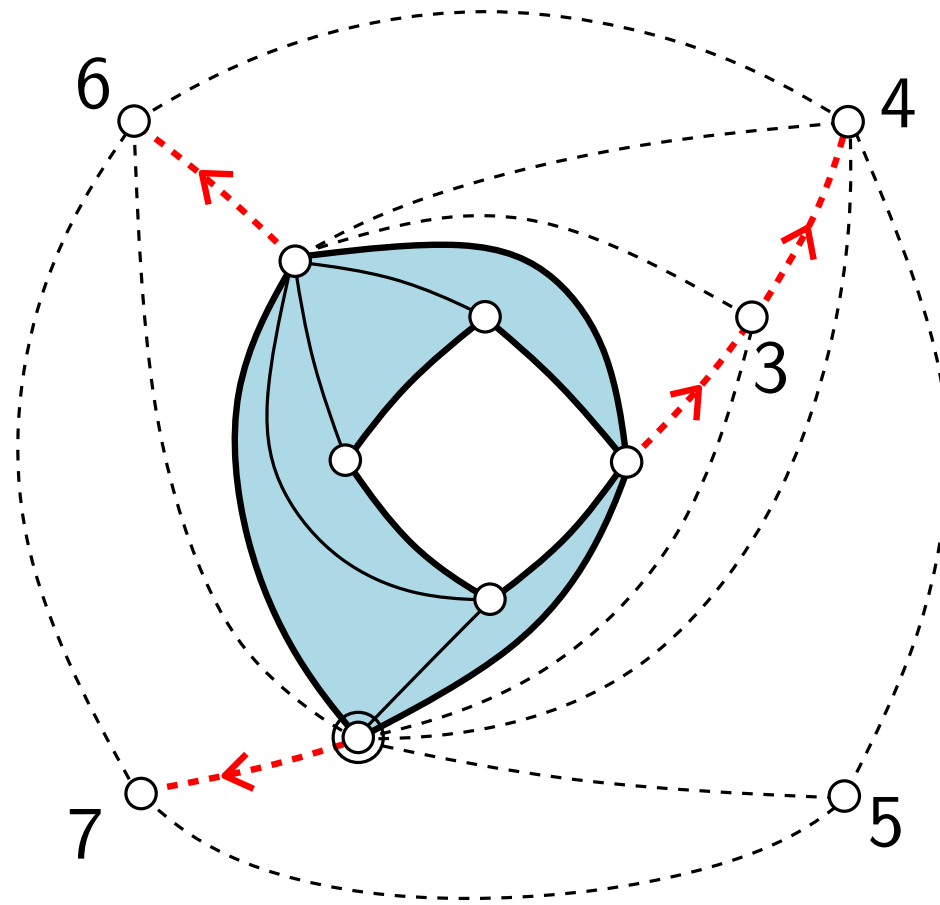
Algorithm, vertices ordering
triangulation of a cylinder, no chords, no 2-cycles



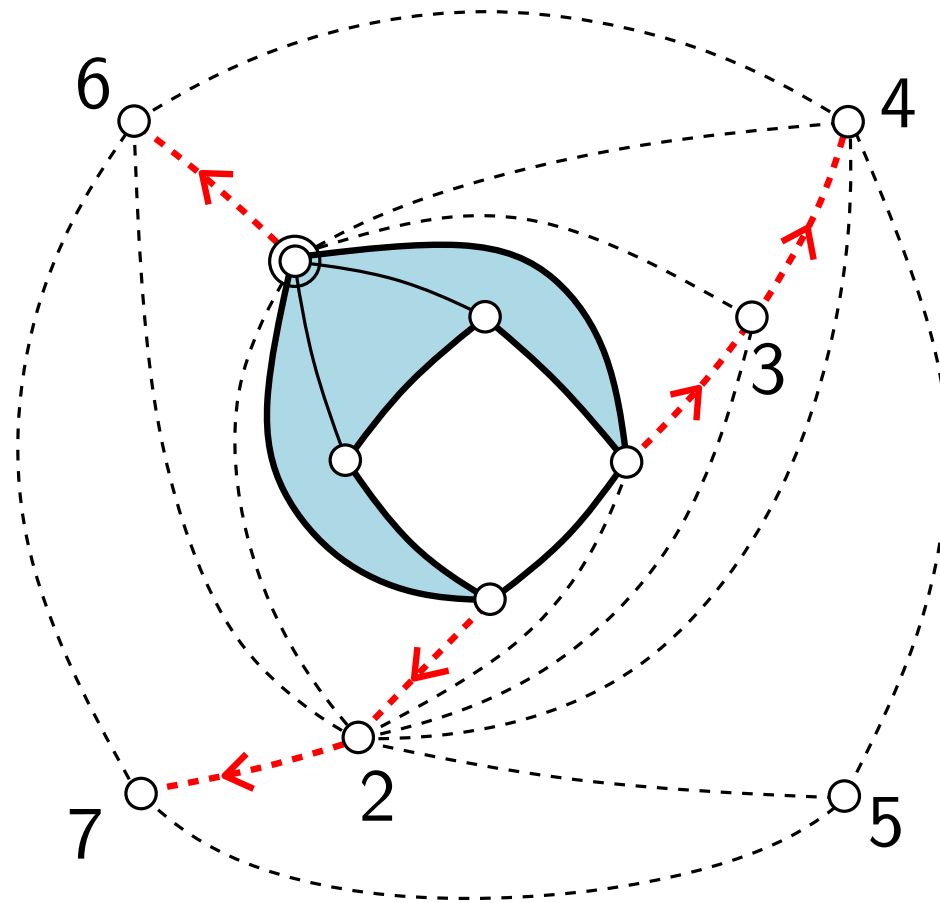
Algorithm, vertices ordering
triangulation of a cylinder, no chords, no 2-cycles



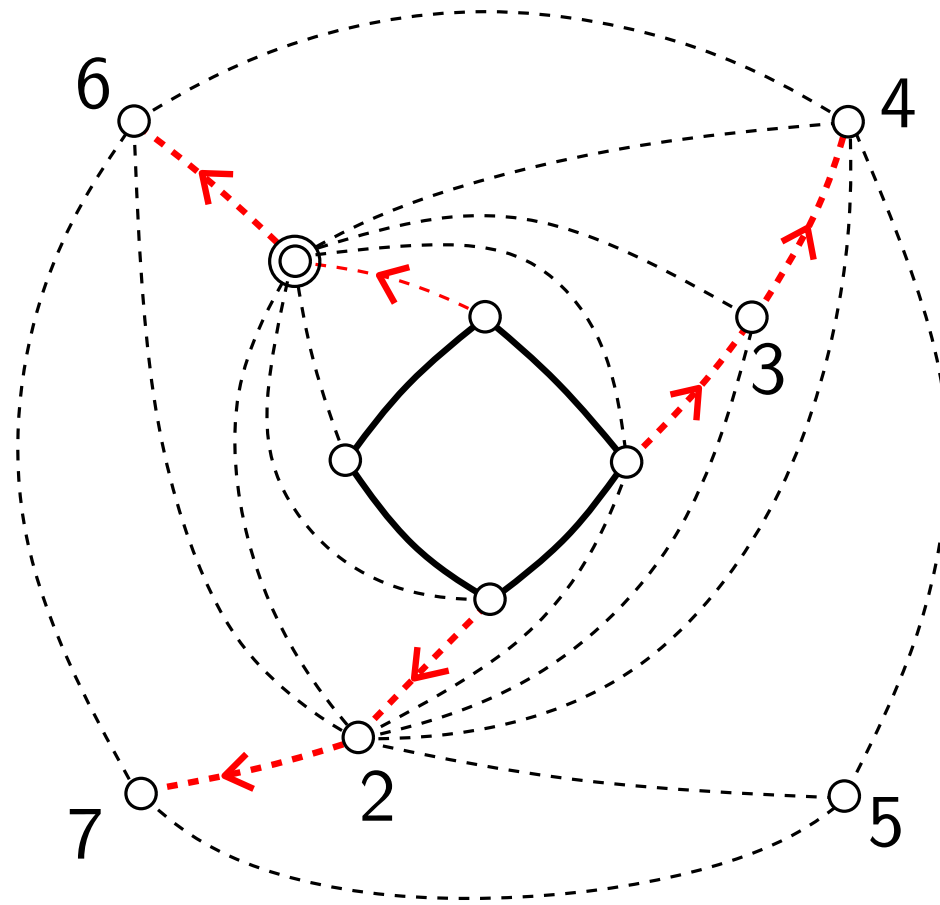
Algorithm, vertices ordering
triangulation of a cylinder, no chords, no 2-cycles



Algorithm, vertices ordering
triangulation of a cylinder, no chords, no 2-cycles



Algorithm, vertices ordering
triangulation of a cylinder, no chords, no 2-cycles

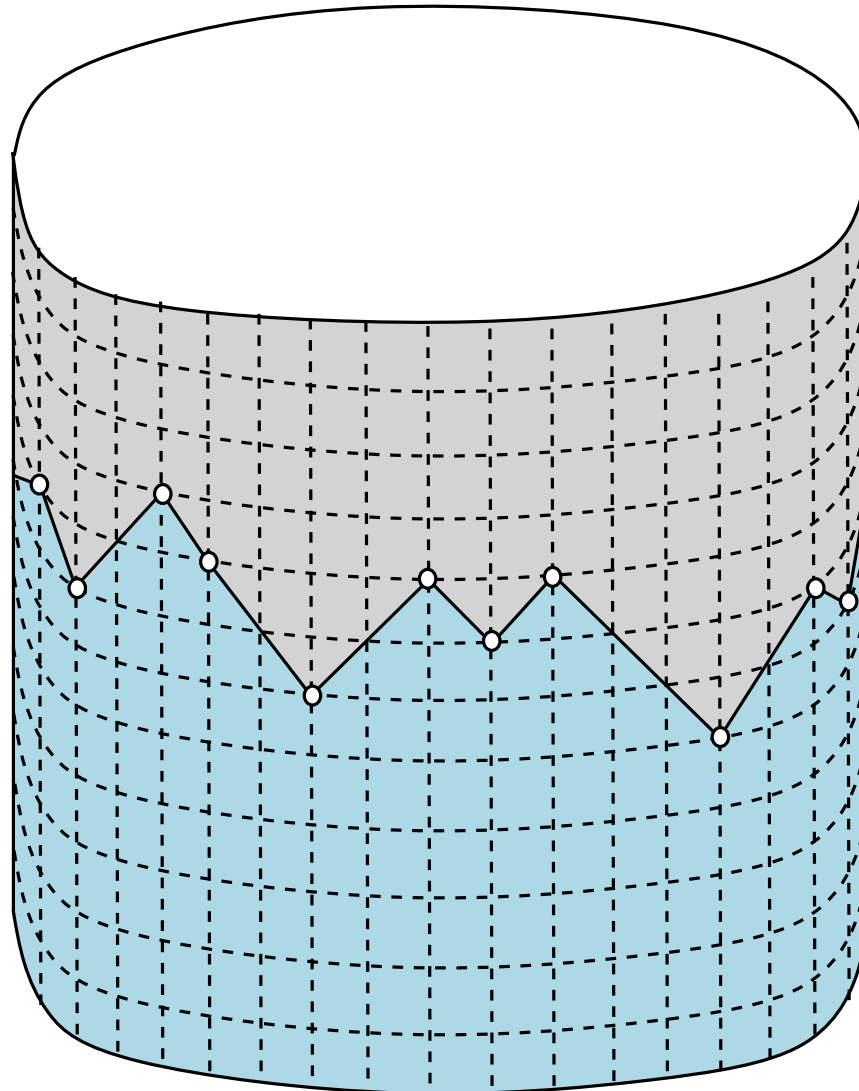


Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles
incremental drawing

Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles

incremental drawing

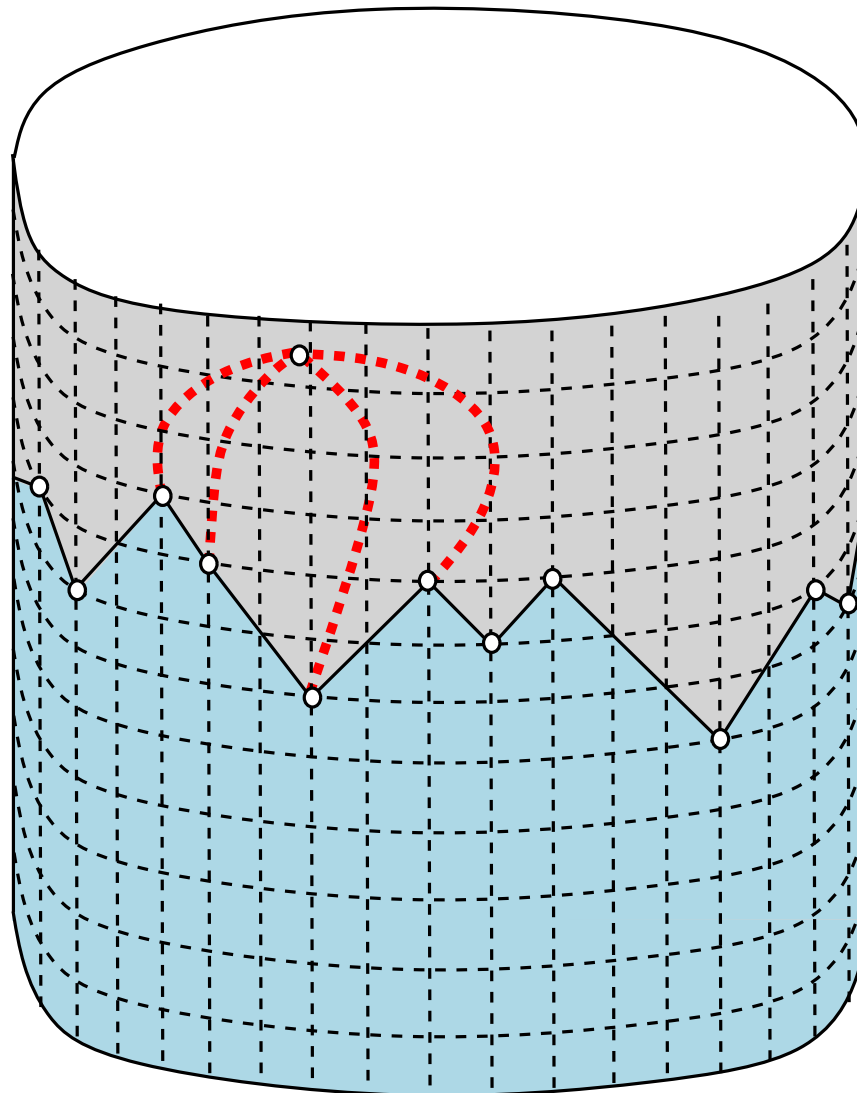
slopes $+1$ or -1



Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles

incremental drawing

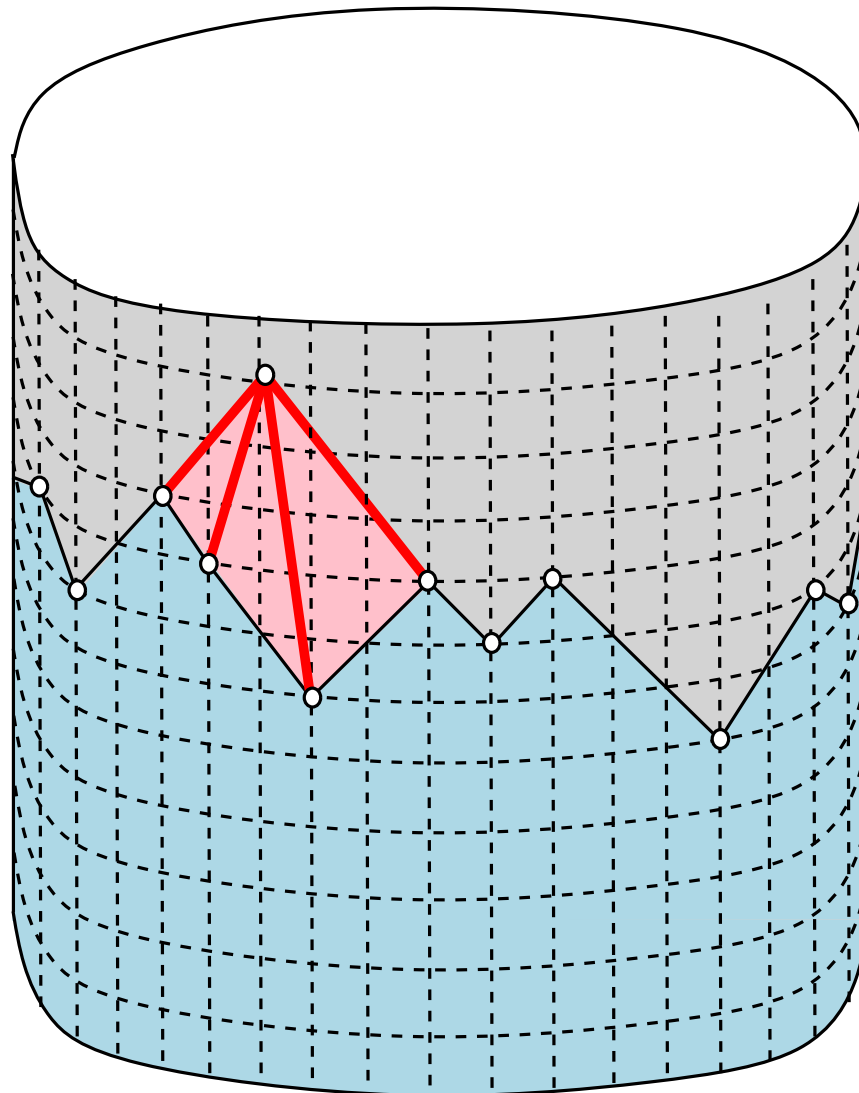
slopes $+1$ or -1



Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles

incremental drawing

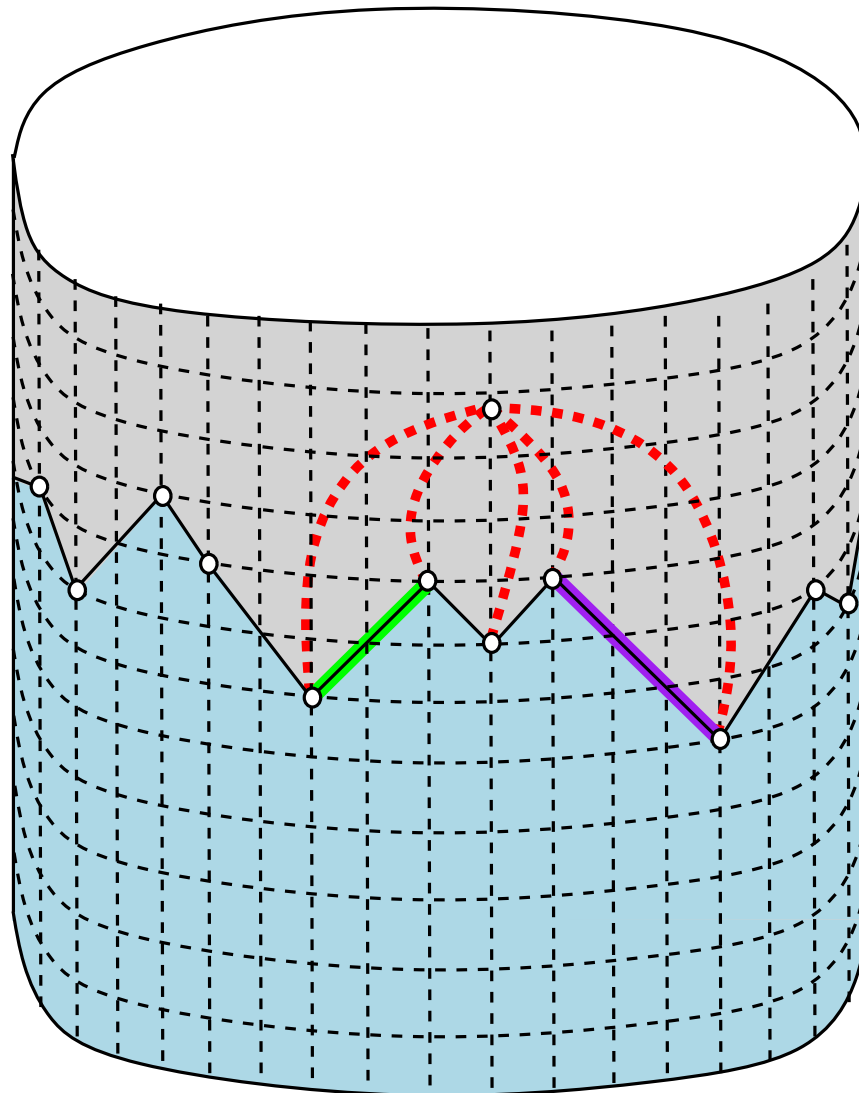
slopes $+1$ or -1



Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles

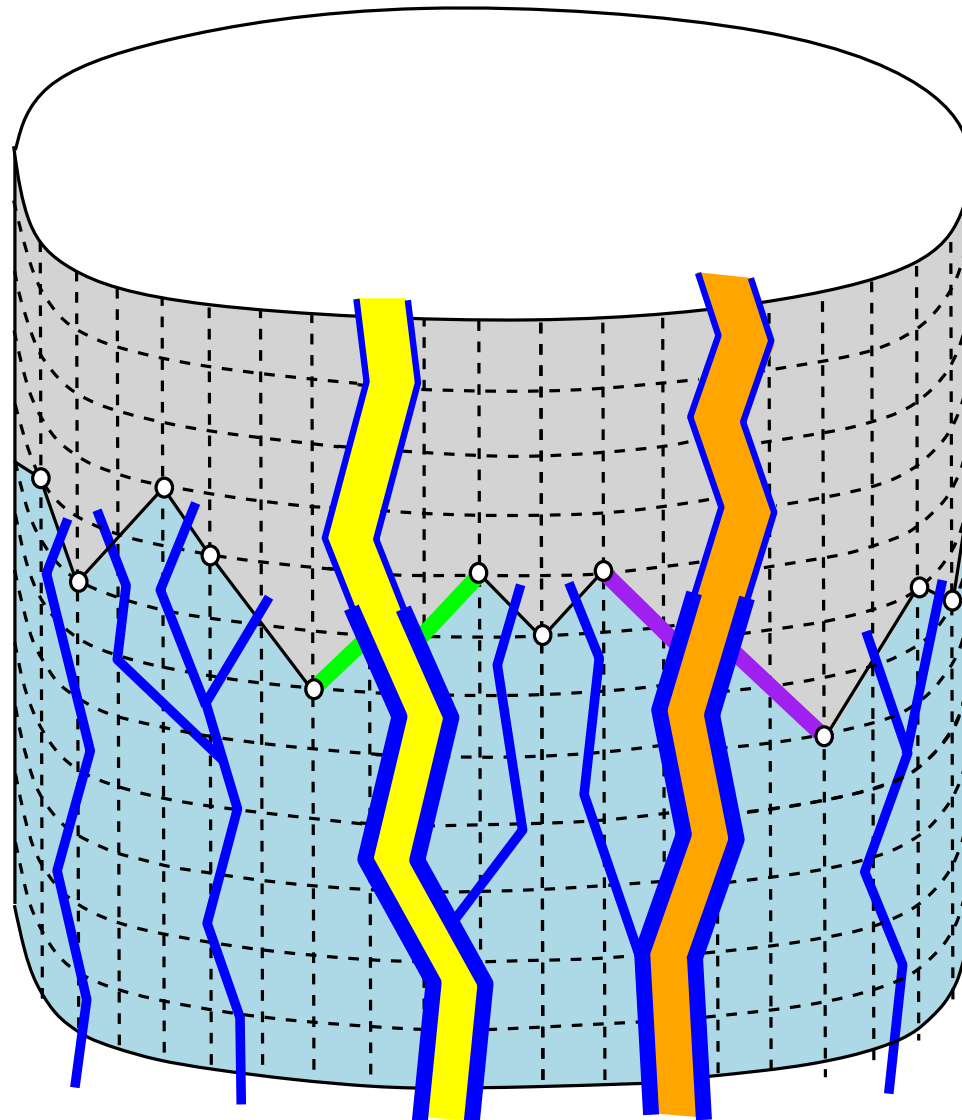
incremental drawing

slopes $+1$ or -1



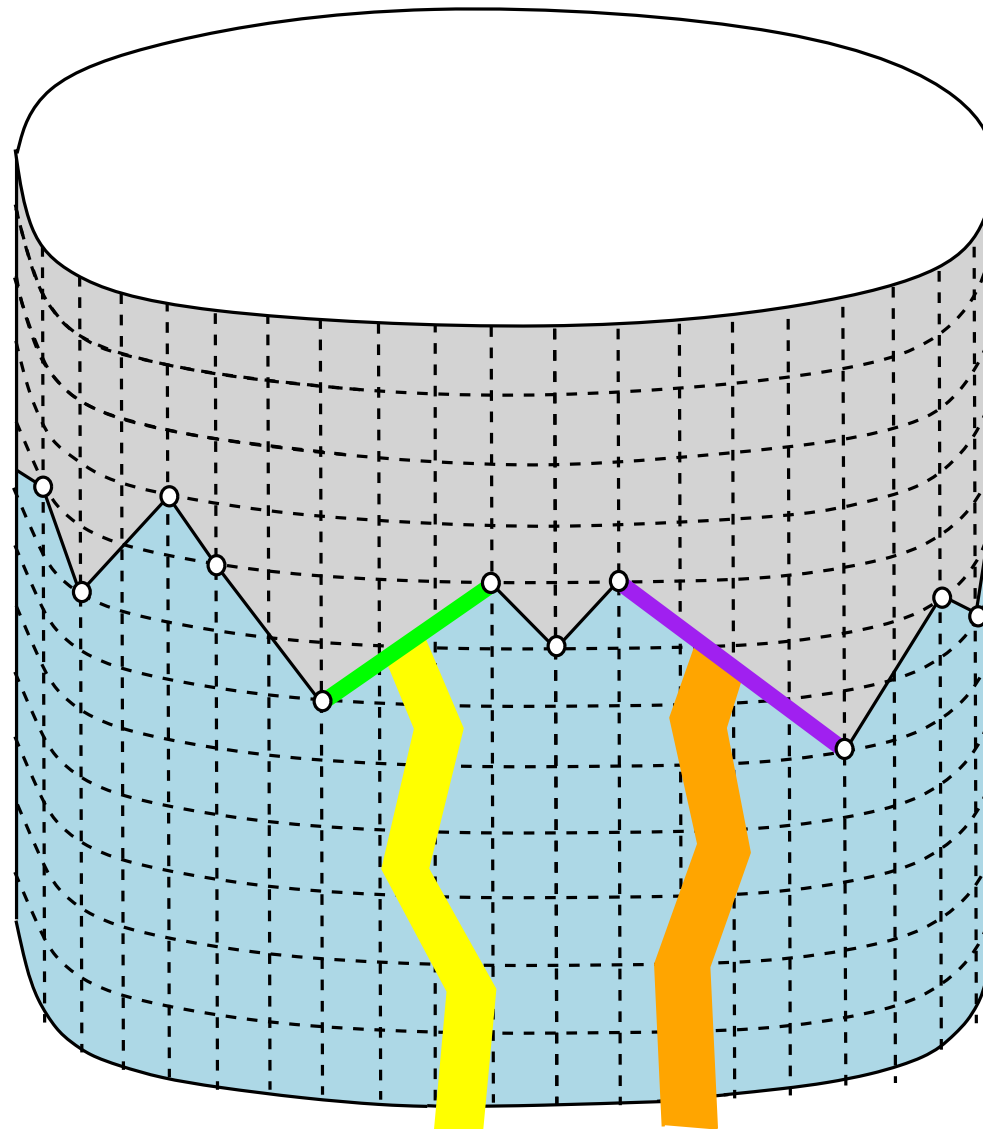
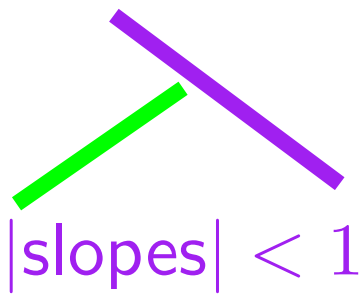
Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles

incremental drawing



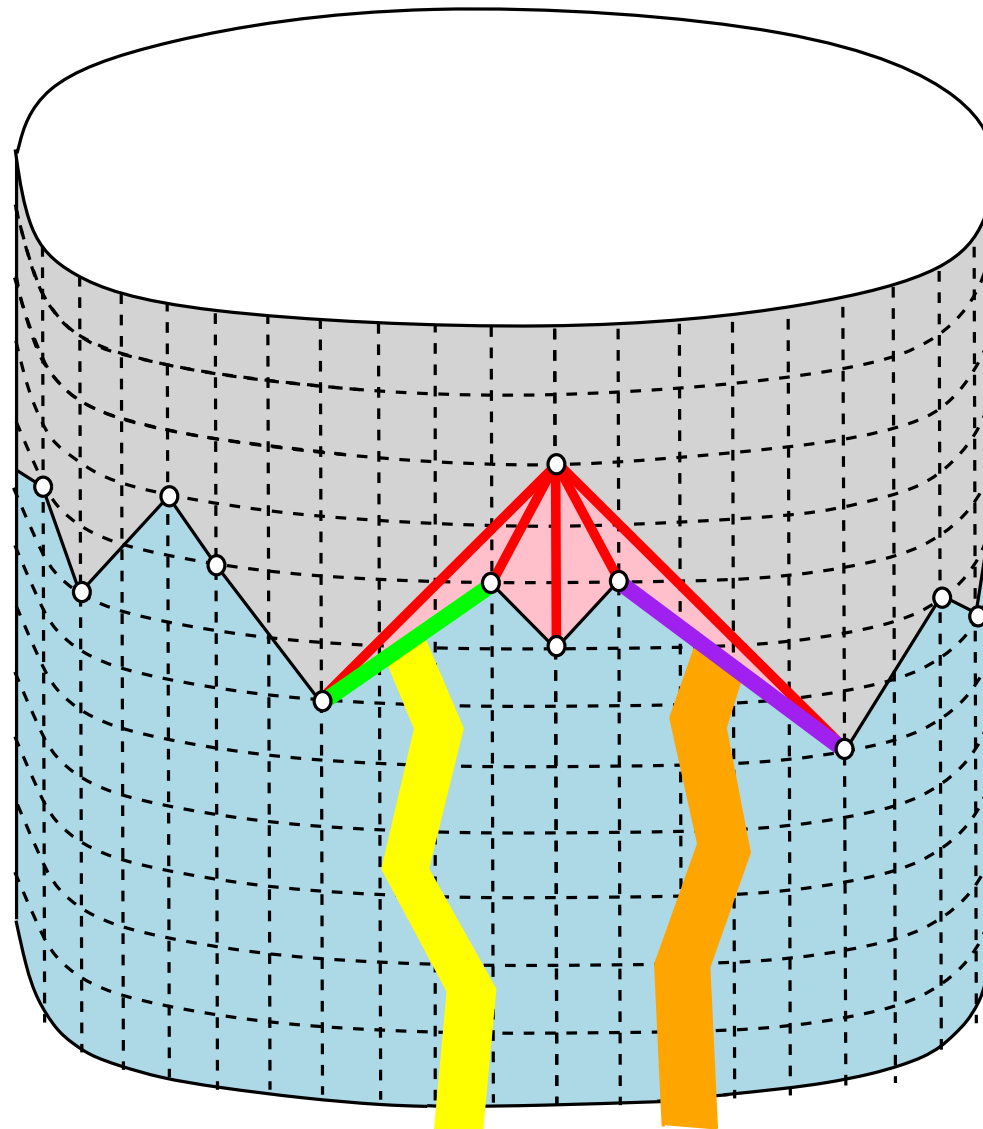
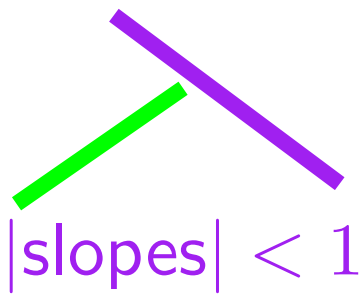
Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles

incremental drawing



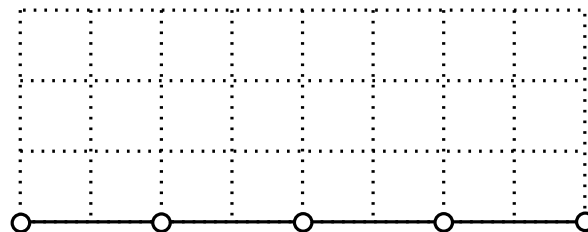
Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles

incremental drawing

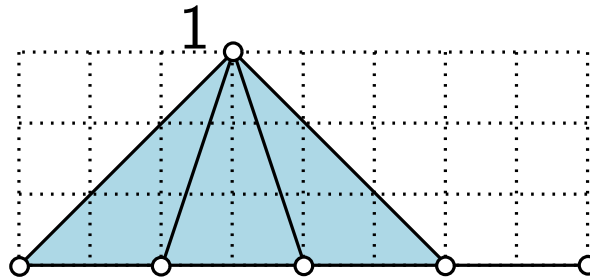


Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles

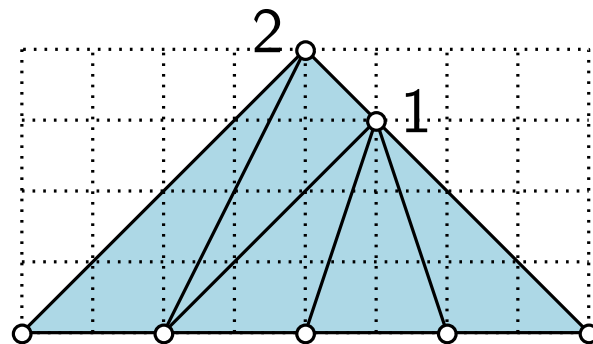
Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles



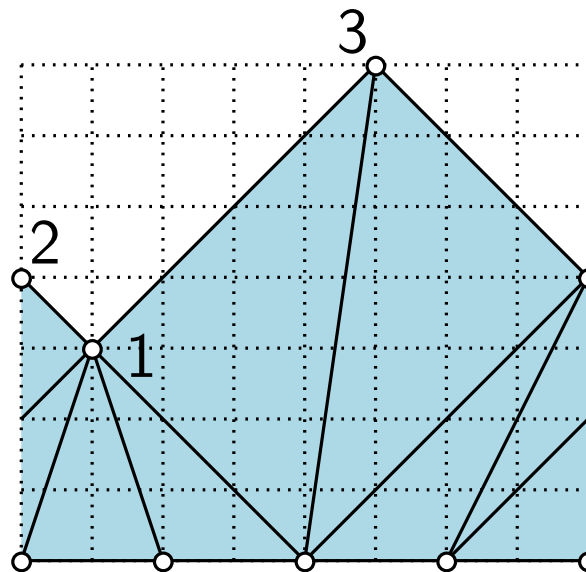
Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles



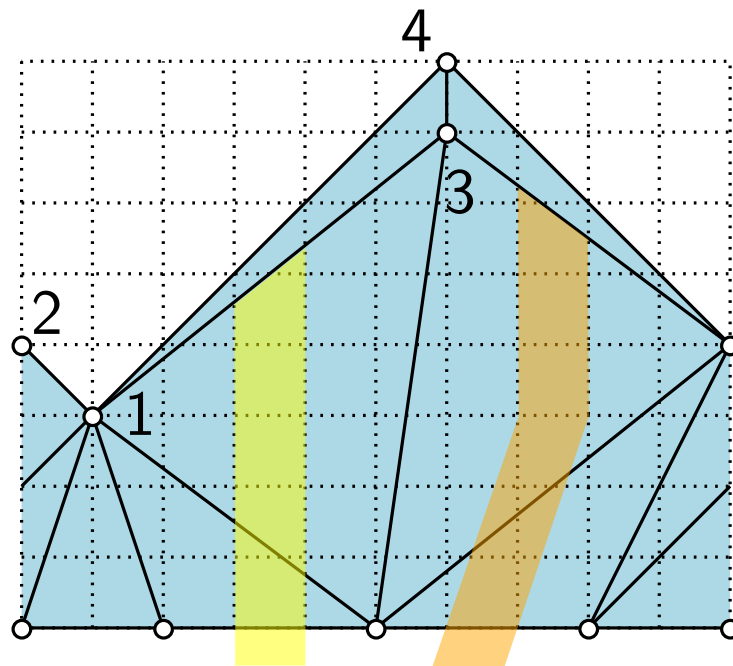
Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles



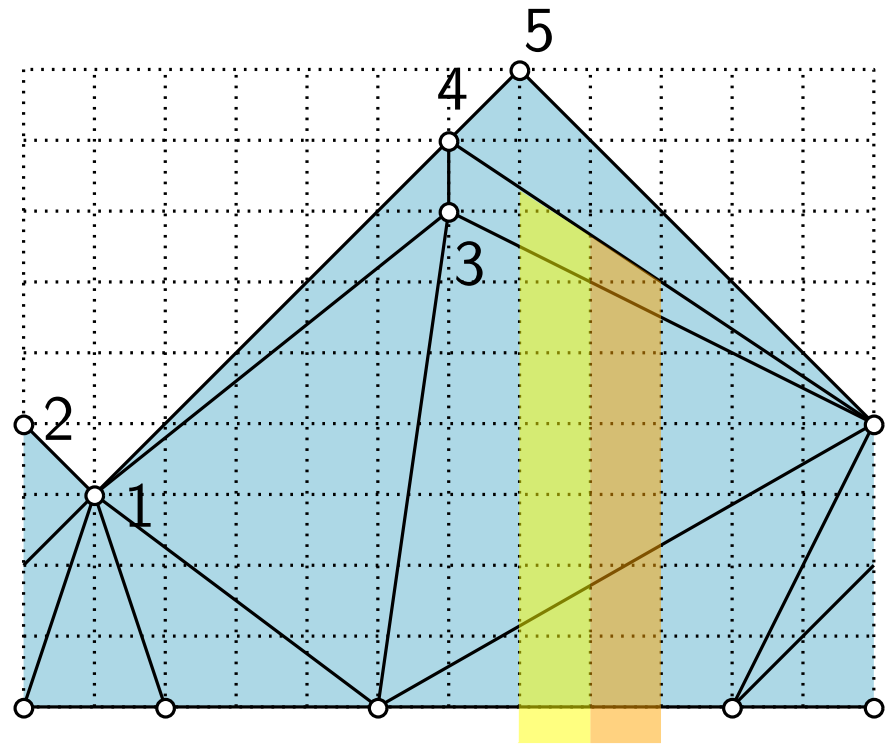
Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles



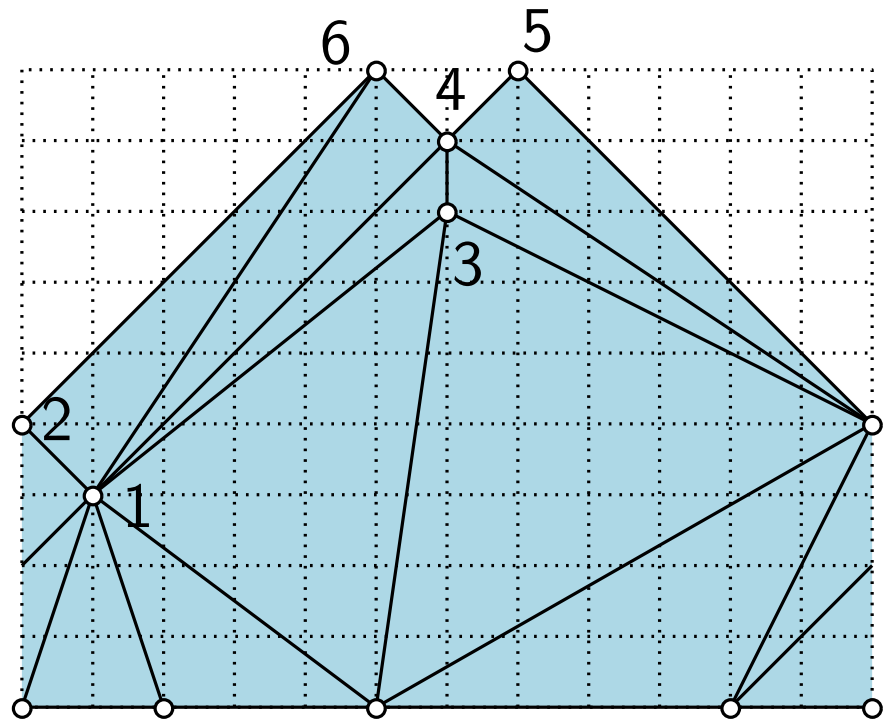
Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles



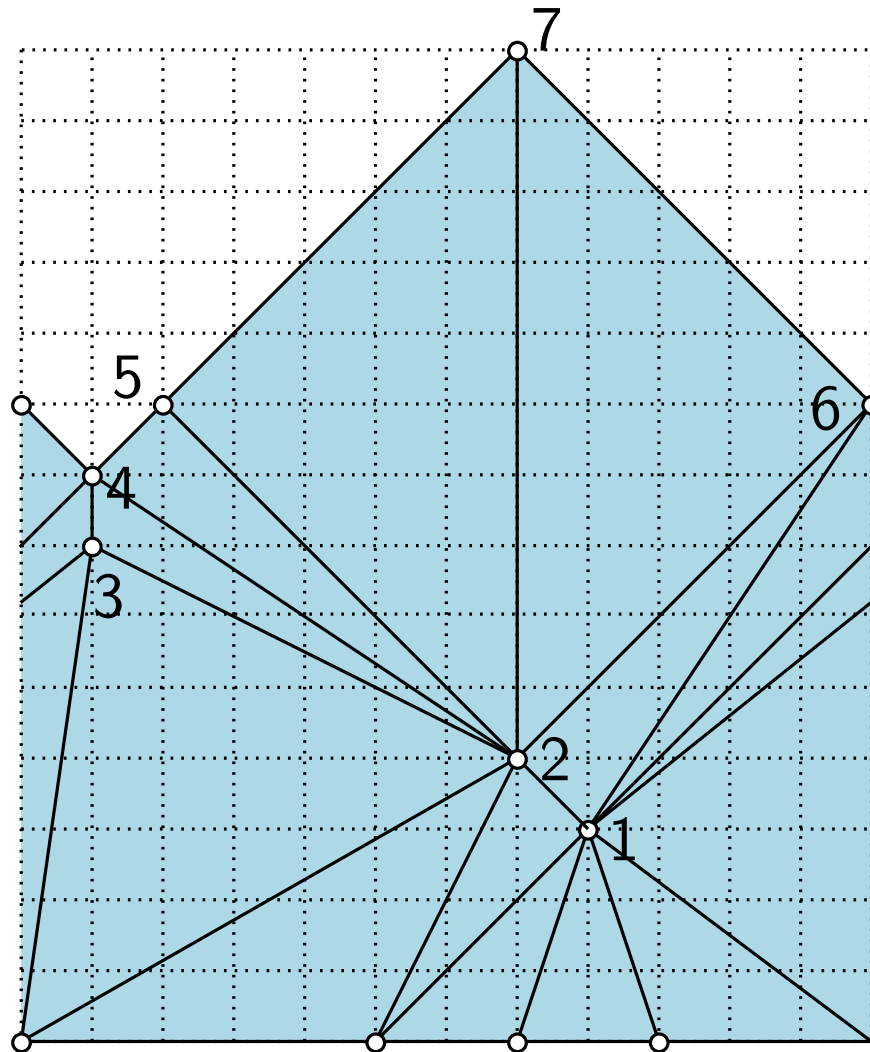
Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles



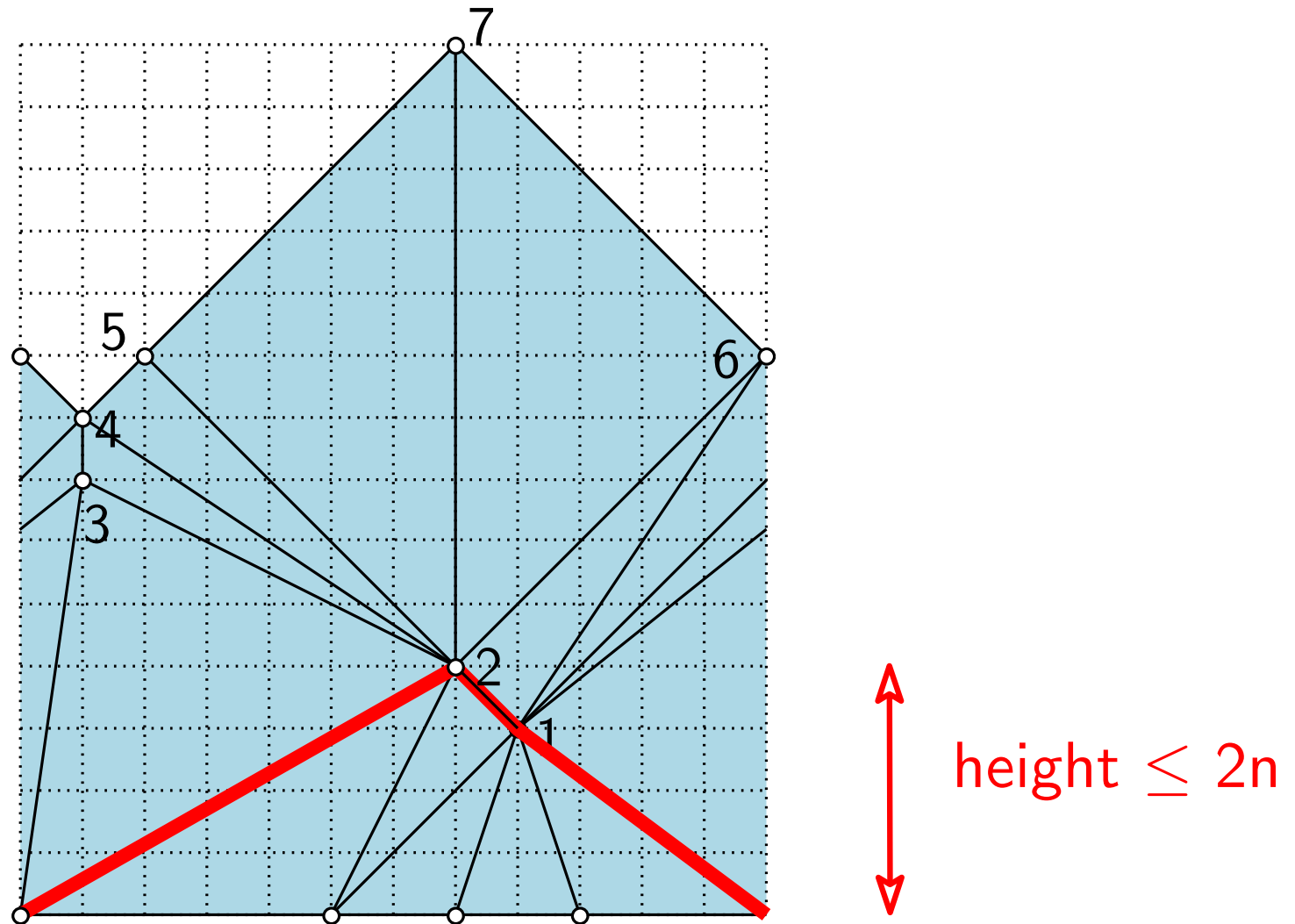
Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles



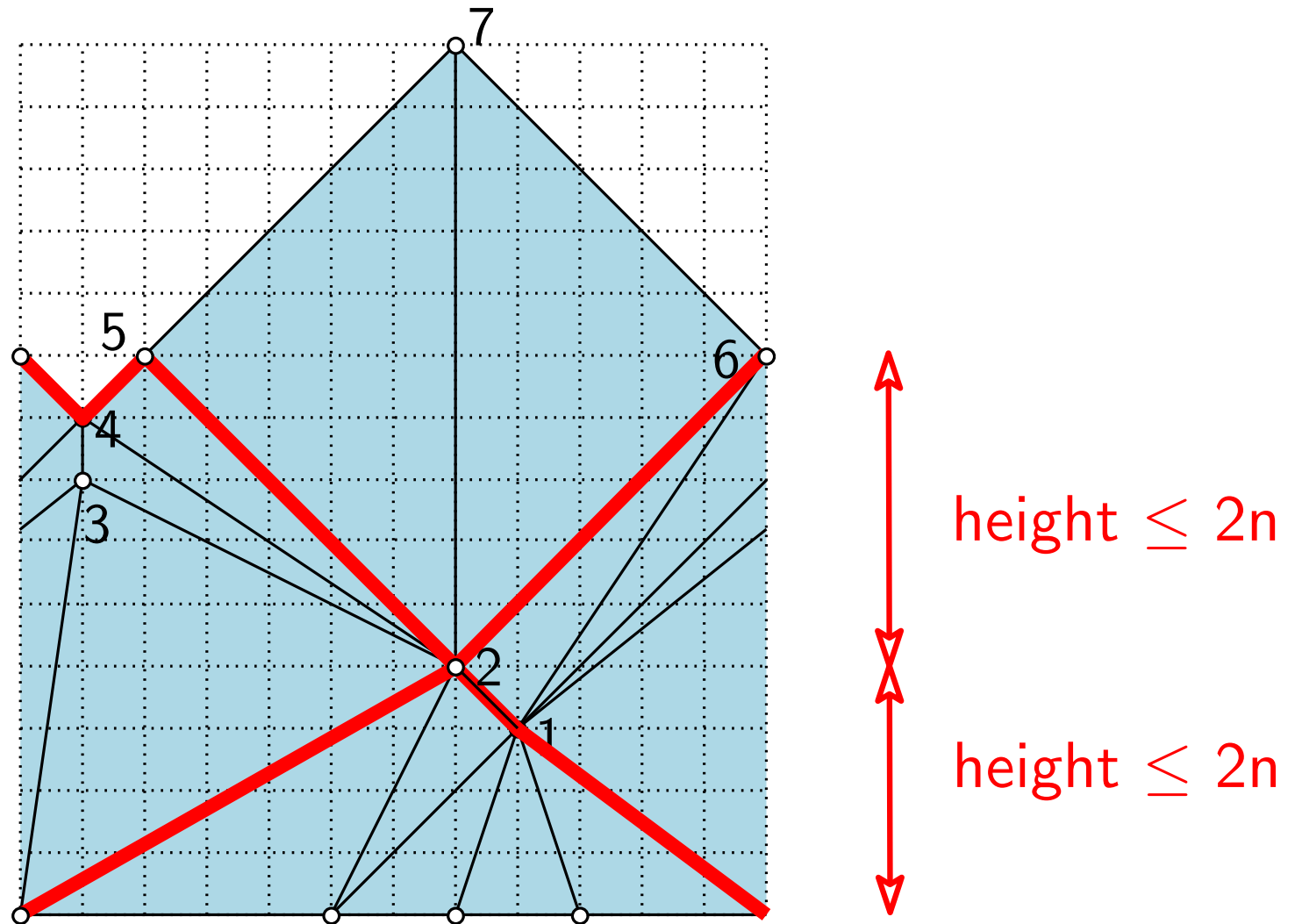
Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles



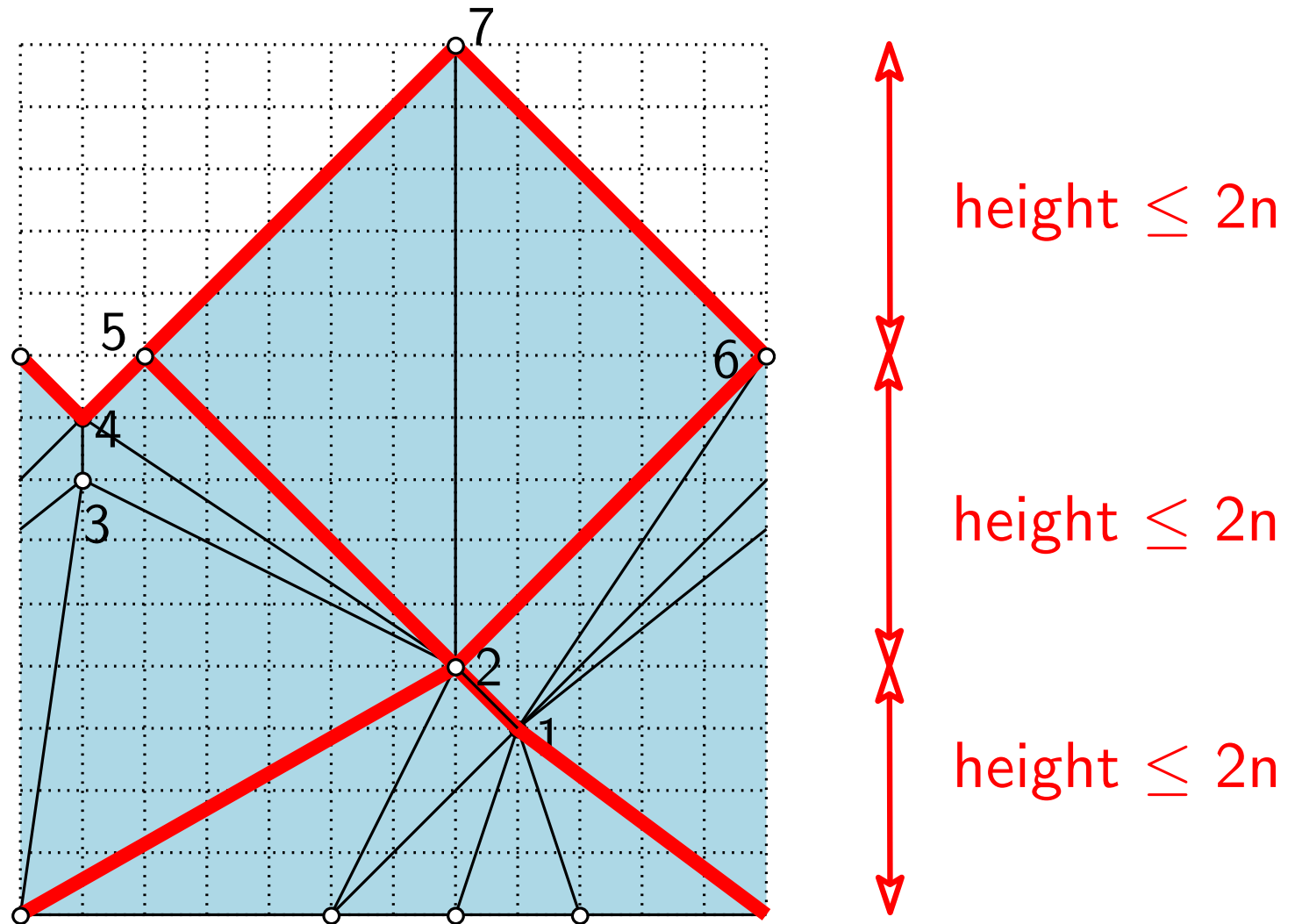
Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles



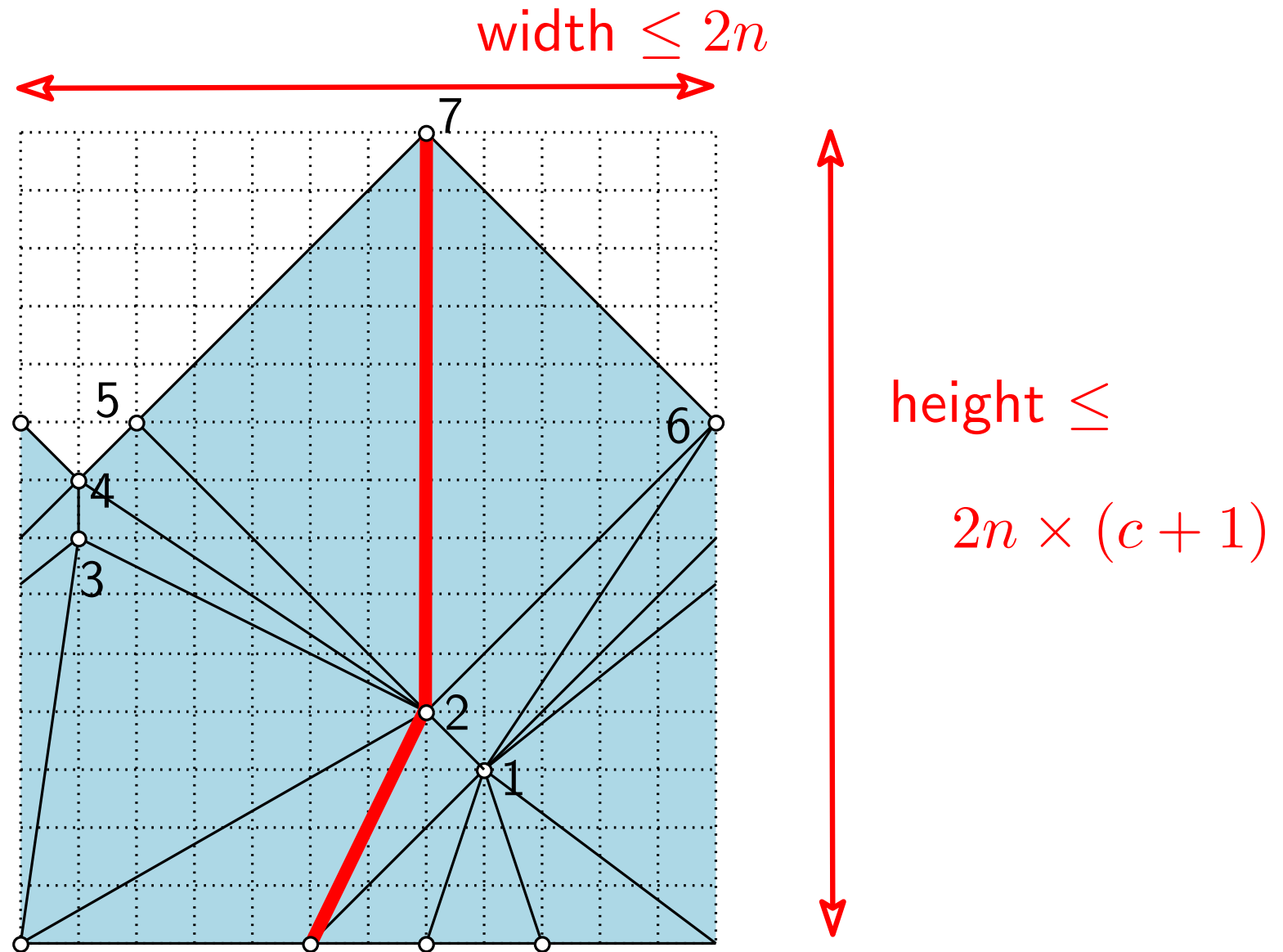
Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles



Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles

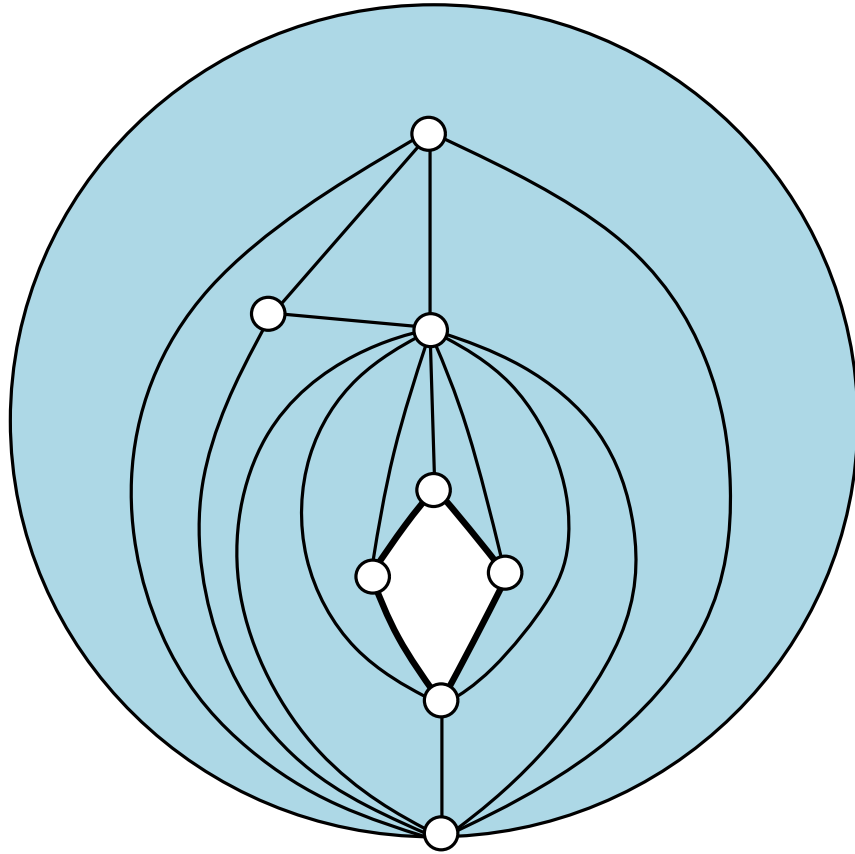


Algorithm, incremental drawing
triangulation of a cylinder, no chords, no 2-cycles

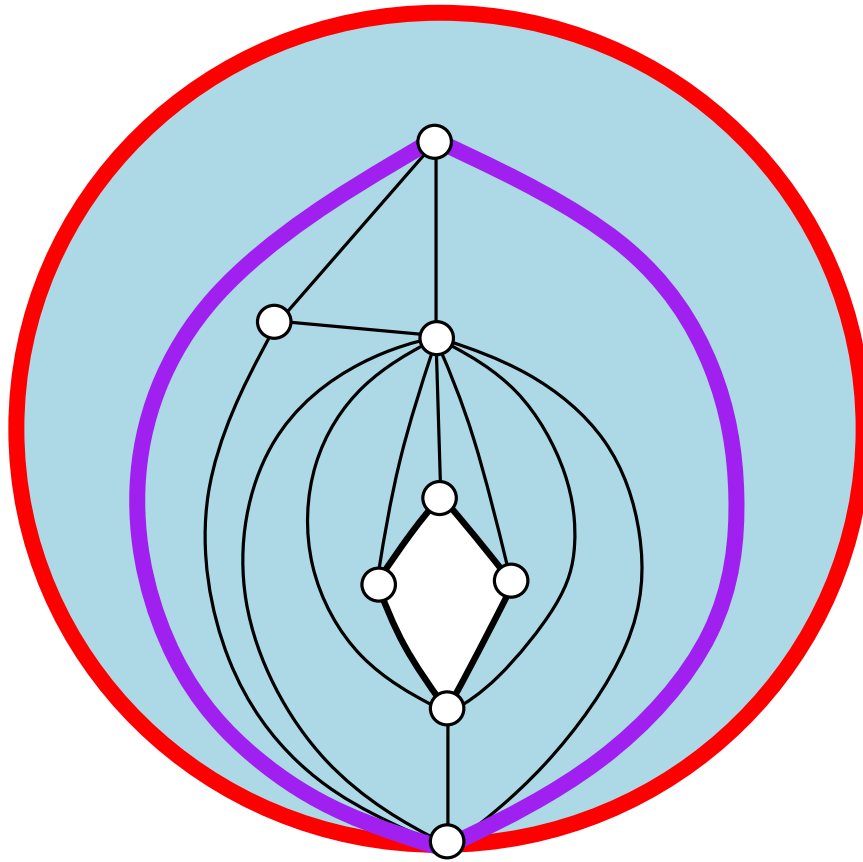


Algorithm, 2-cycles and loops

Algorithm, 2-cycles and loops



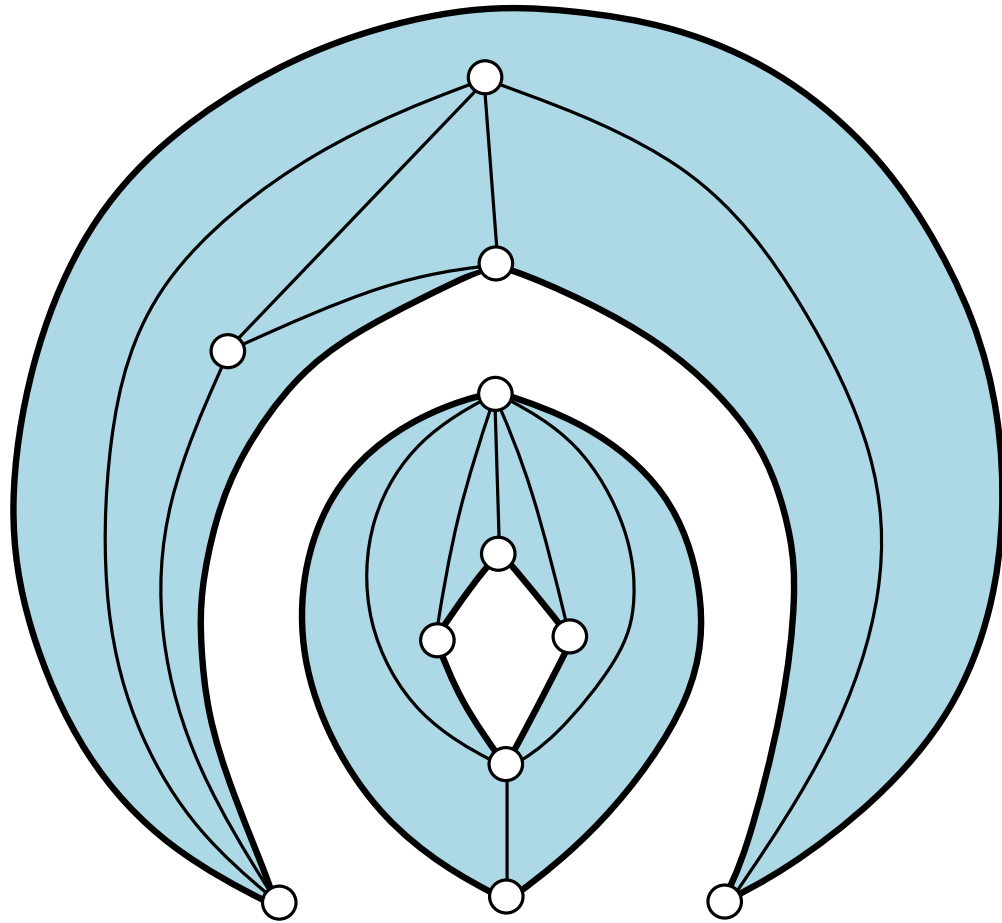
Algorithm, 2-cycles and loops



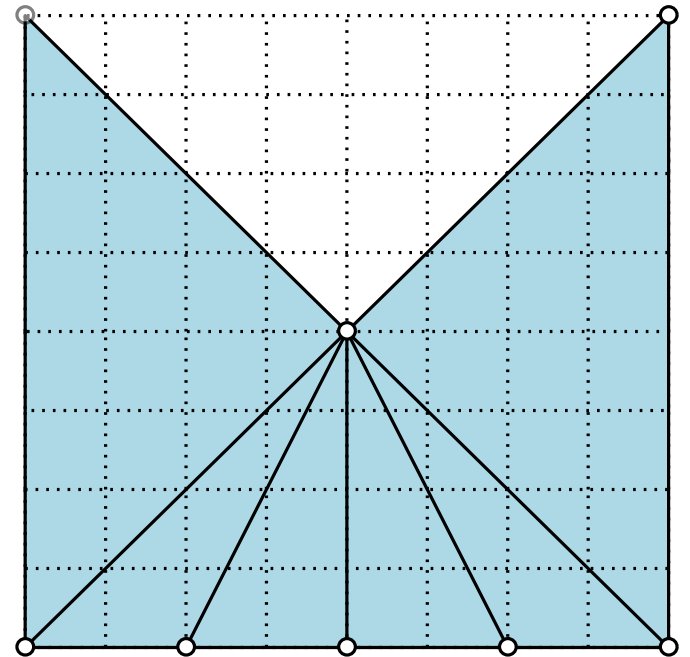
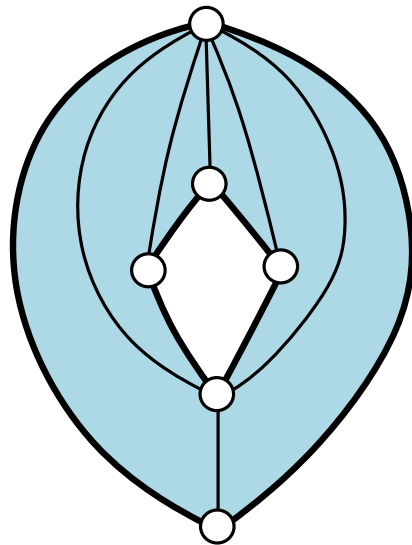
loop

2-cycle

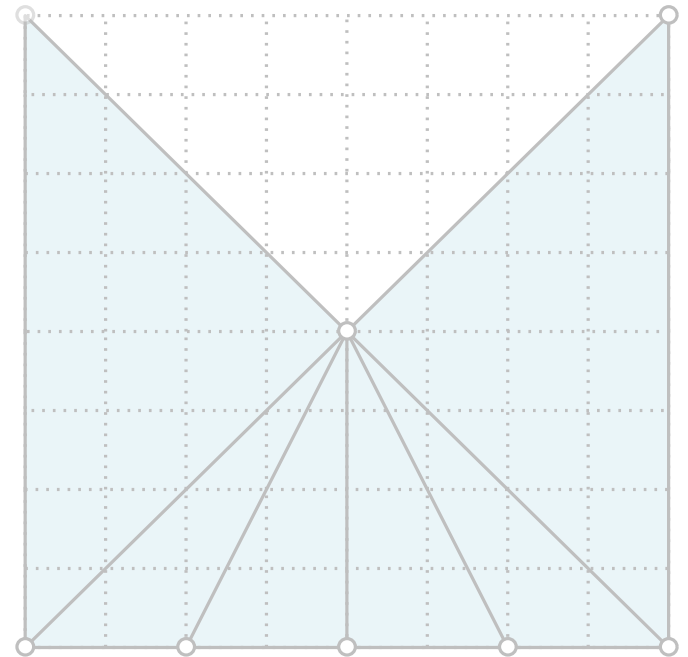
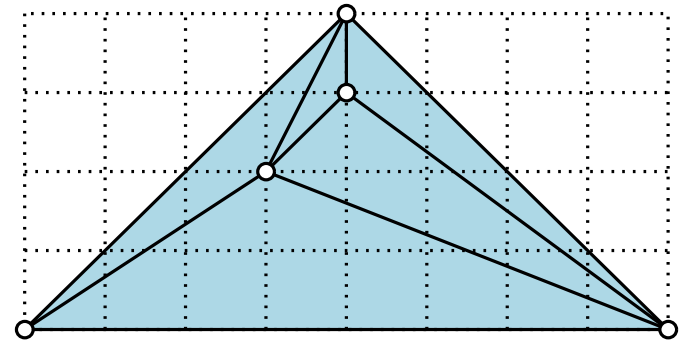
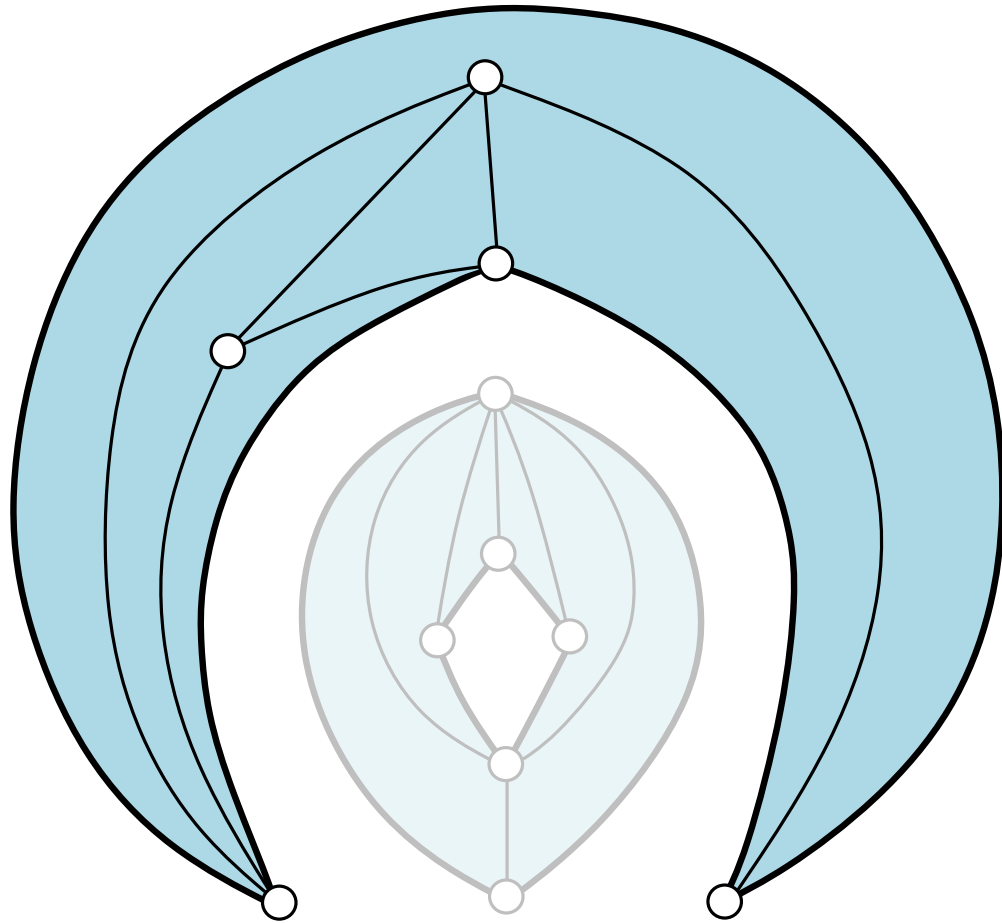
Algorithm, 2-cycles and loops



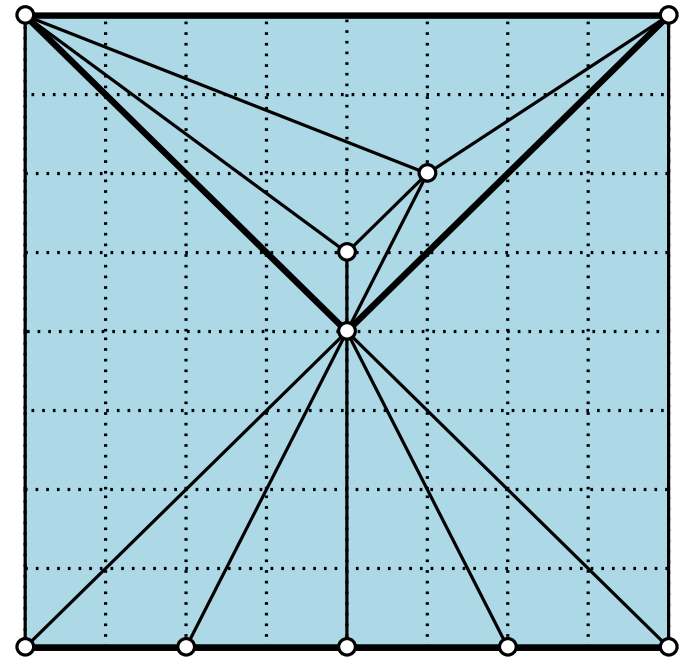
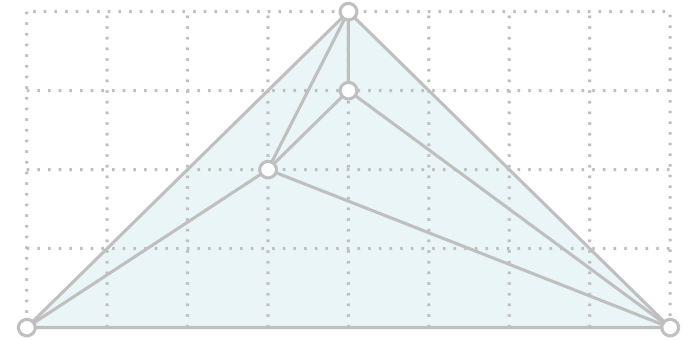
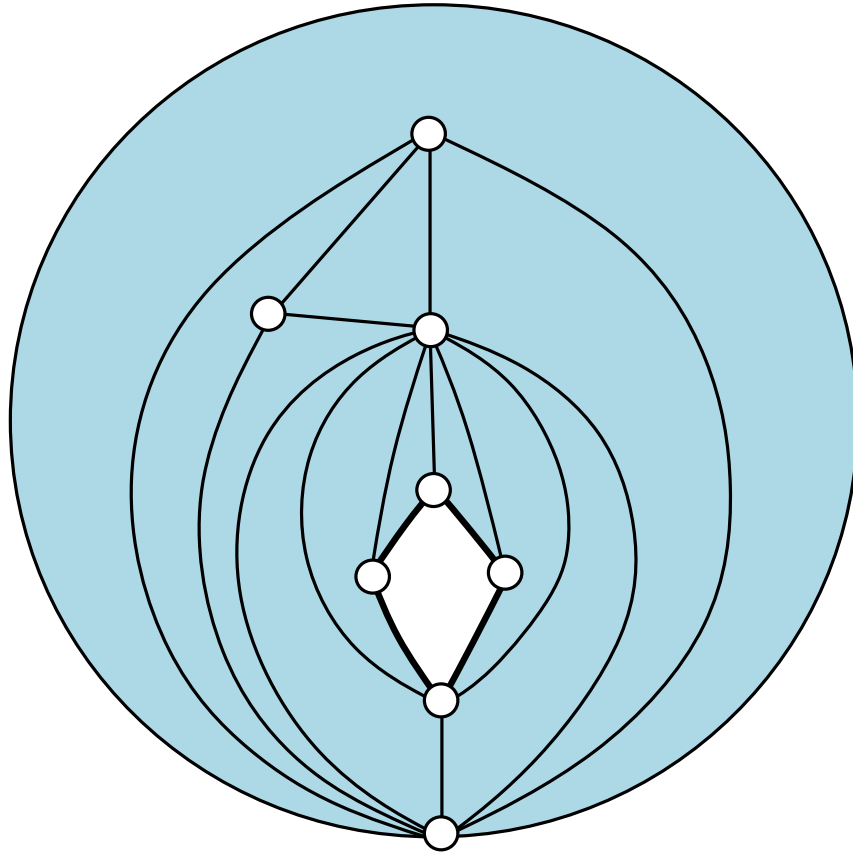
Algorithm, 2-cycles and loops



Algorithm, 2-cycles and loops

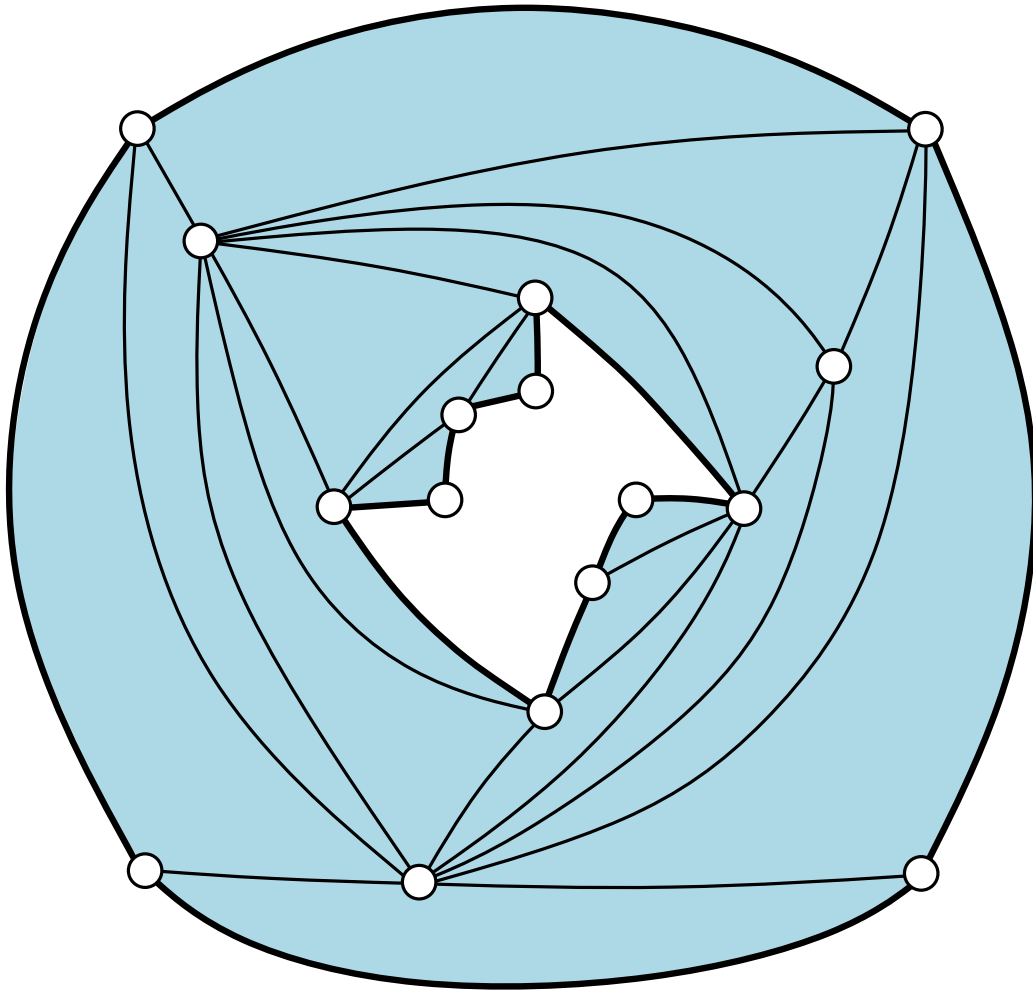


Algorithm, 2-cycles and loops

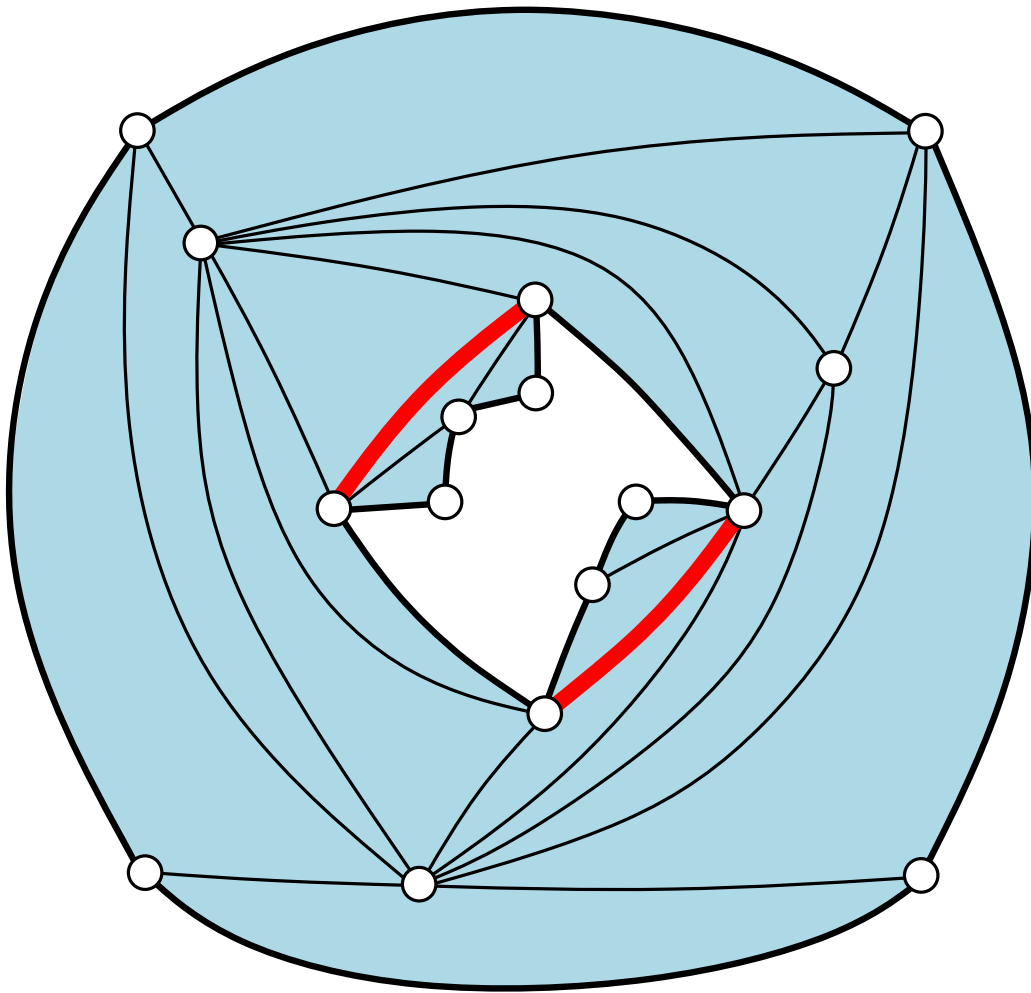


Algorithm, chords

Algorithm, chords

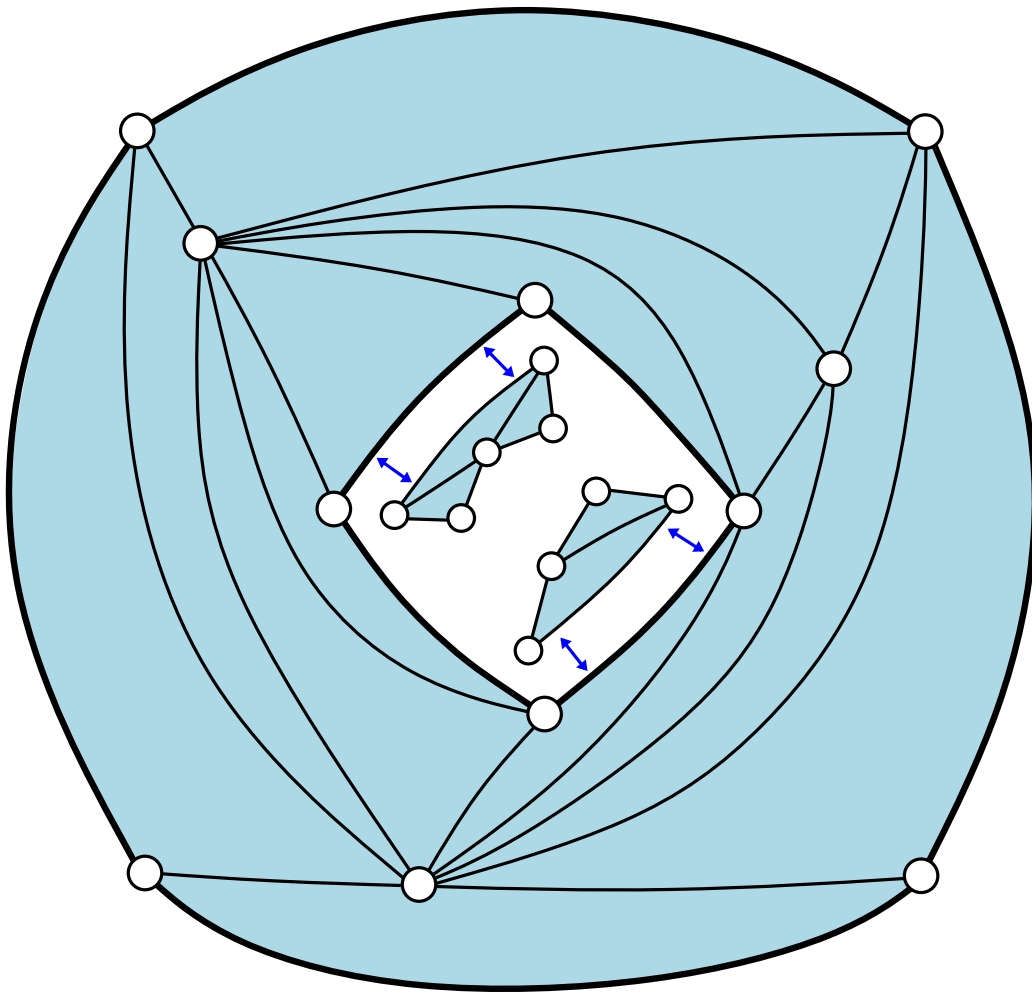


Algorithm, chords

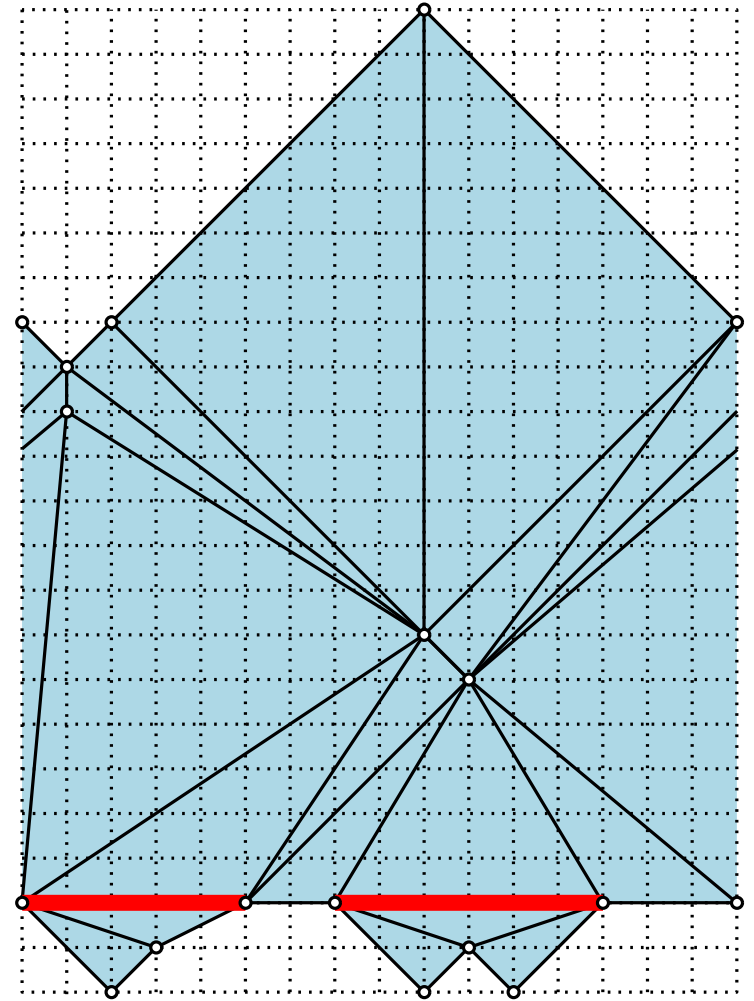
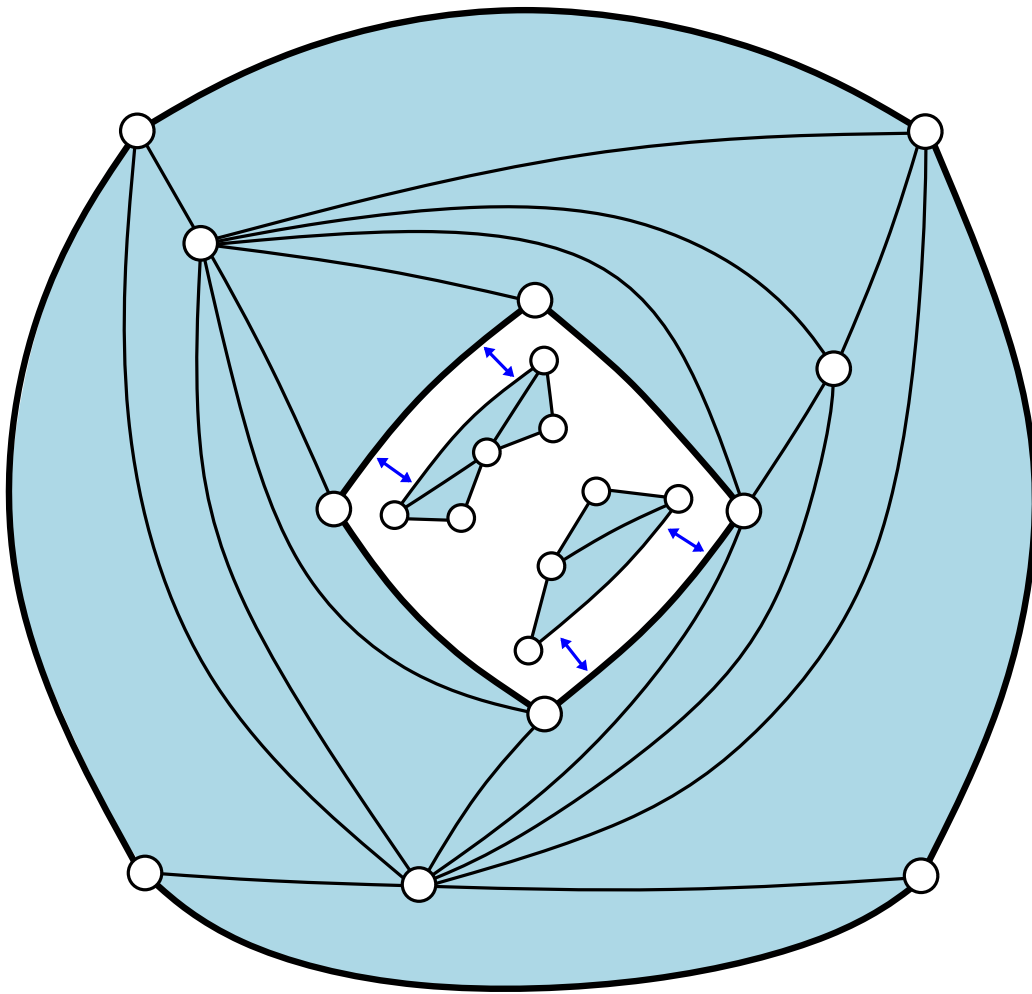


chords

Algorithm, chords

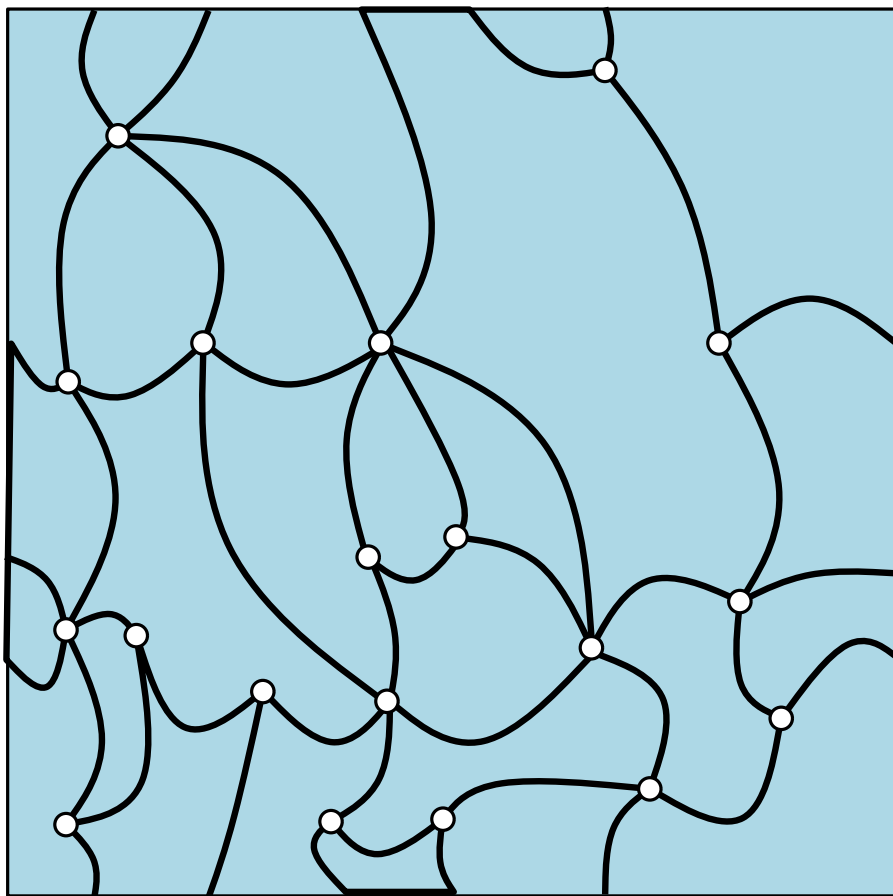


Algorithm, chords

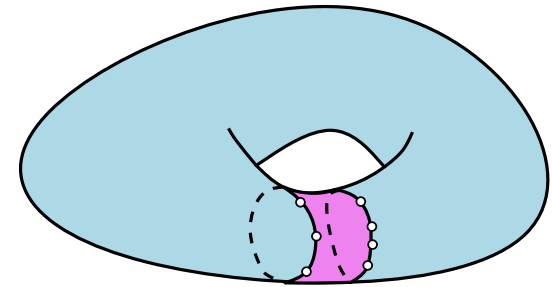
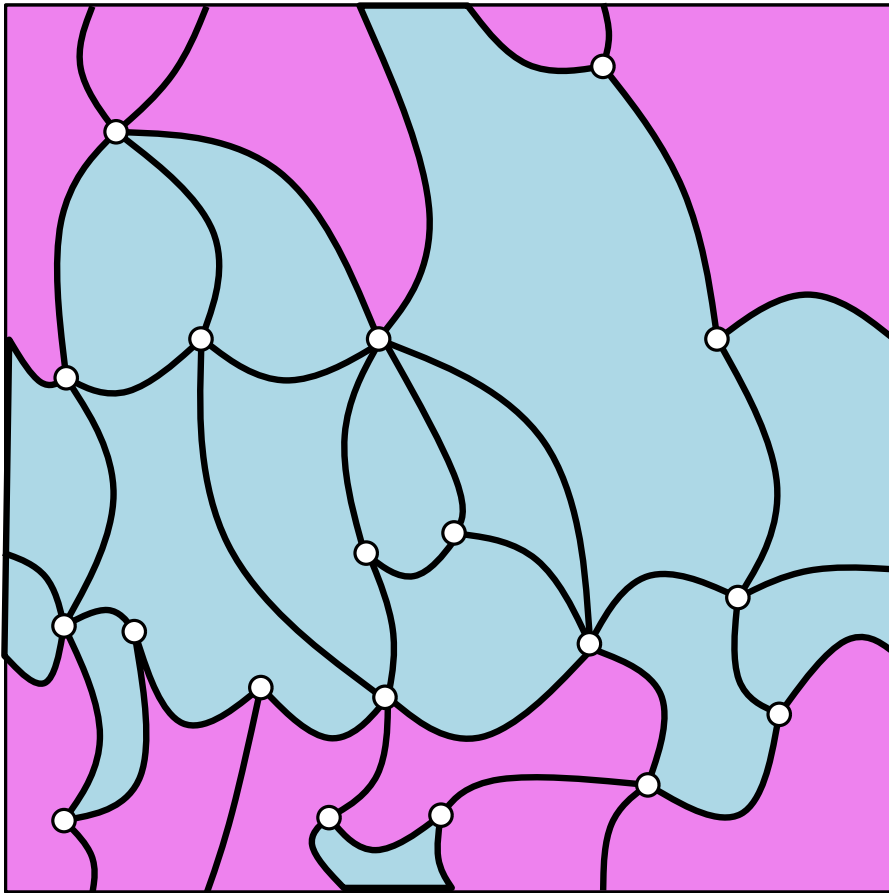


Algorithm, from cylinder to torus

Algorithm, from cylinder to torus

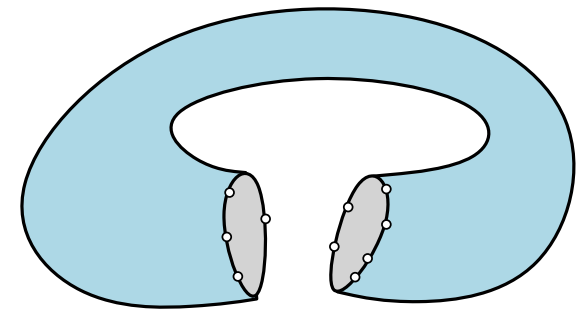
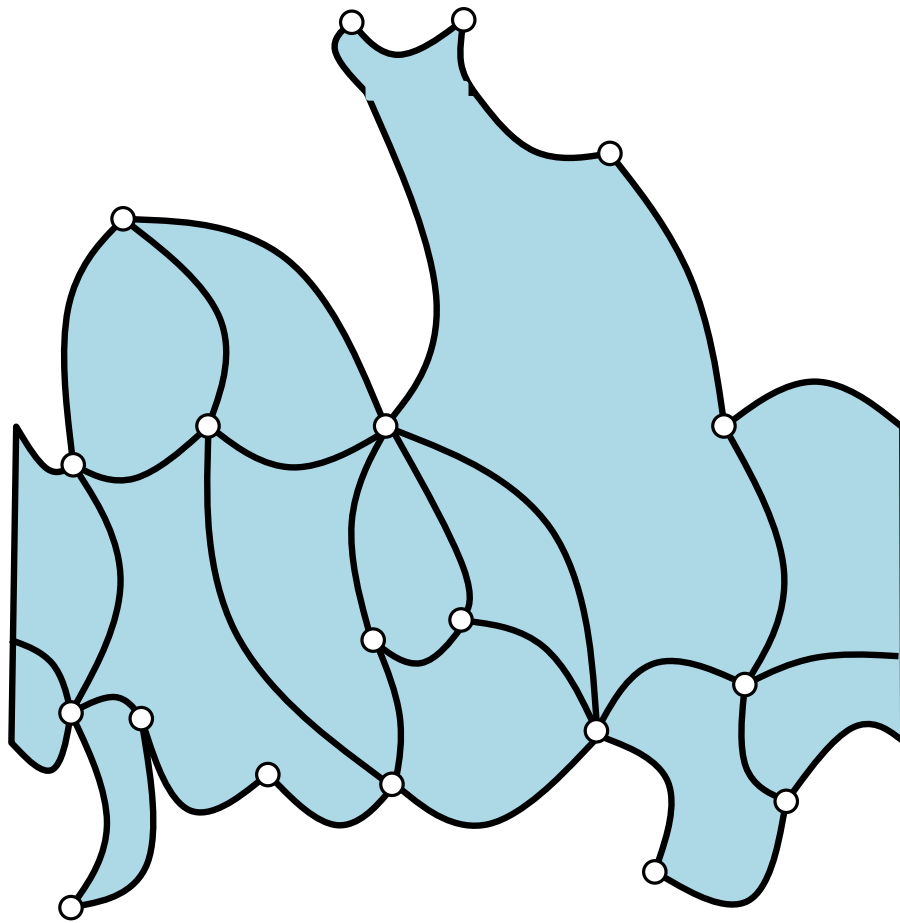


Algorithm, from cylinder to torus



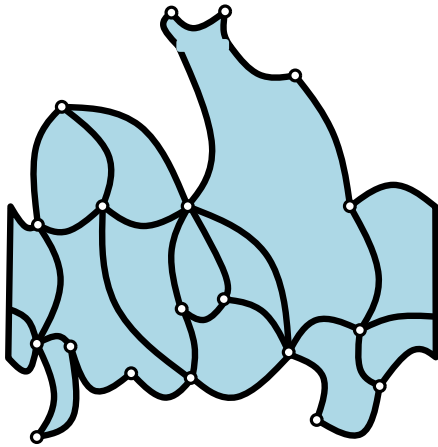
tambourine

Algorithm, from cylinder to torus

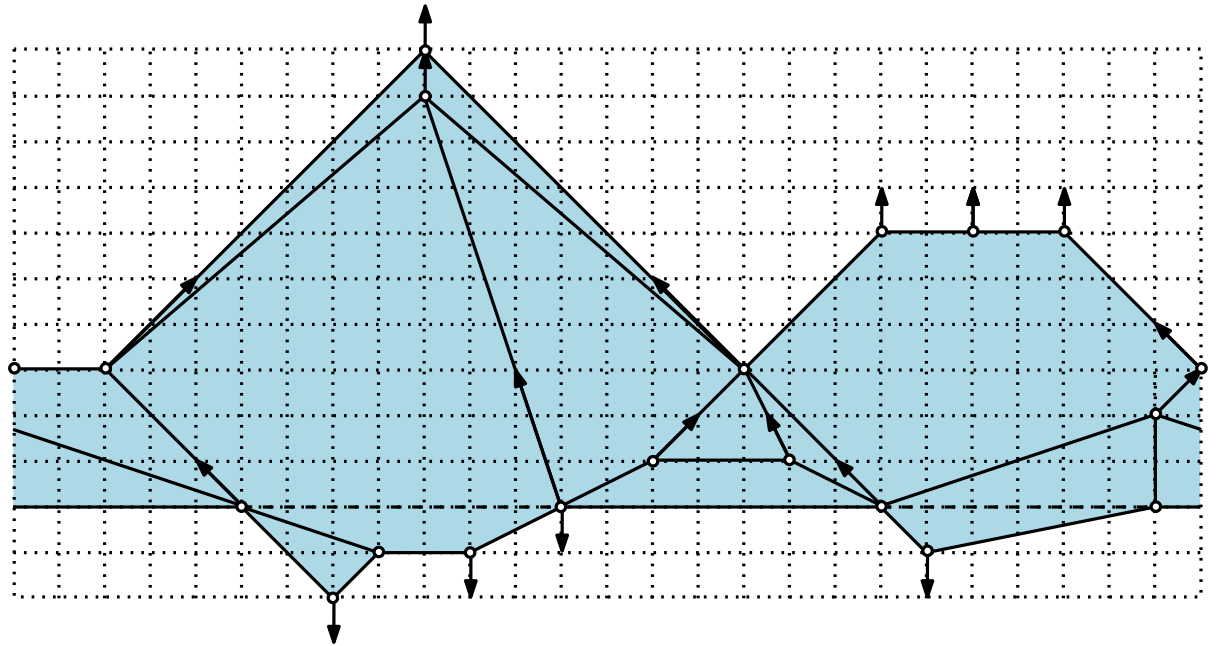


cylinder

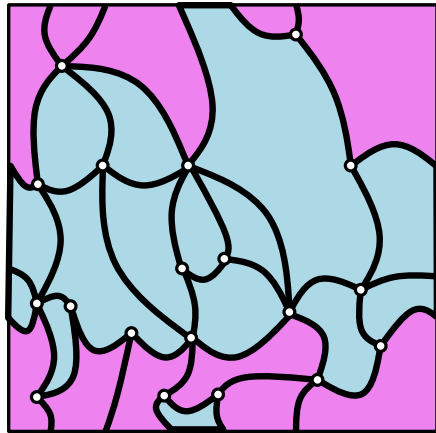
Algorithm, from cylinder to torus



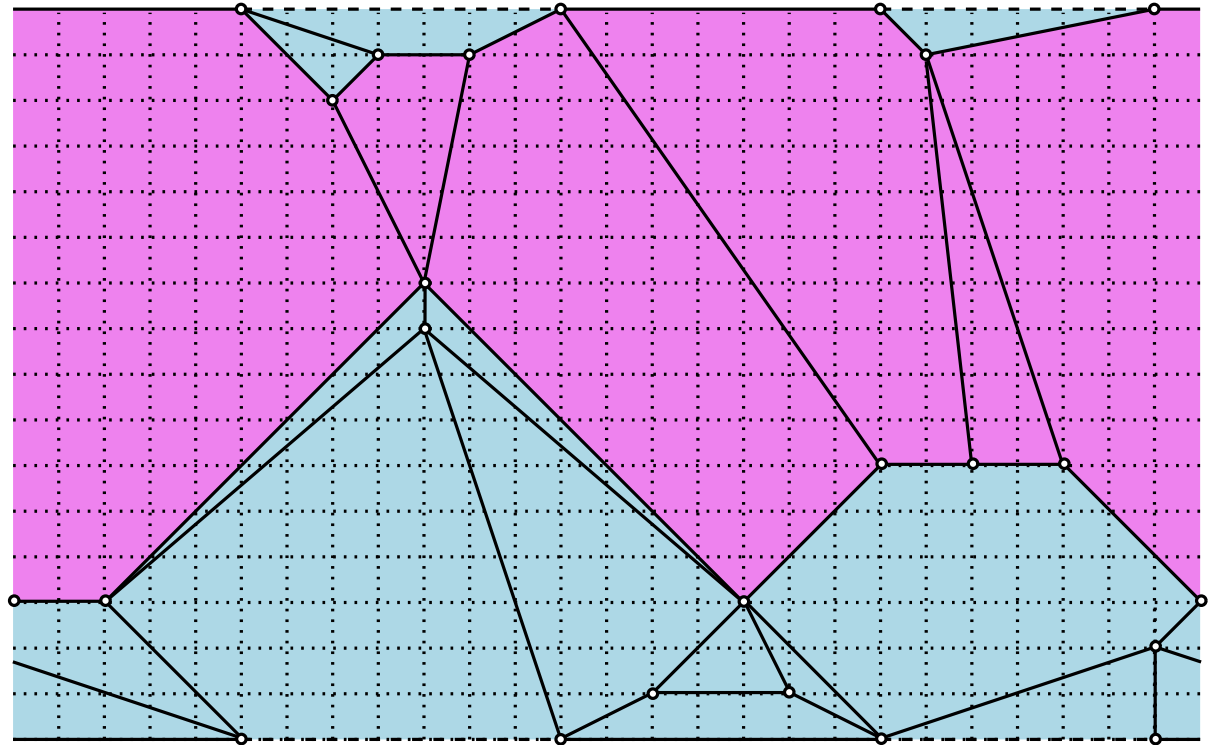
cylinder



Algorithm, from cylinder to torus

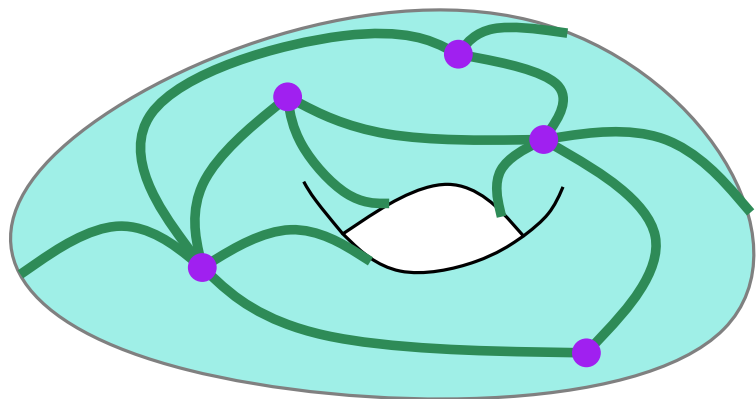


torus

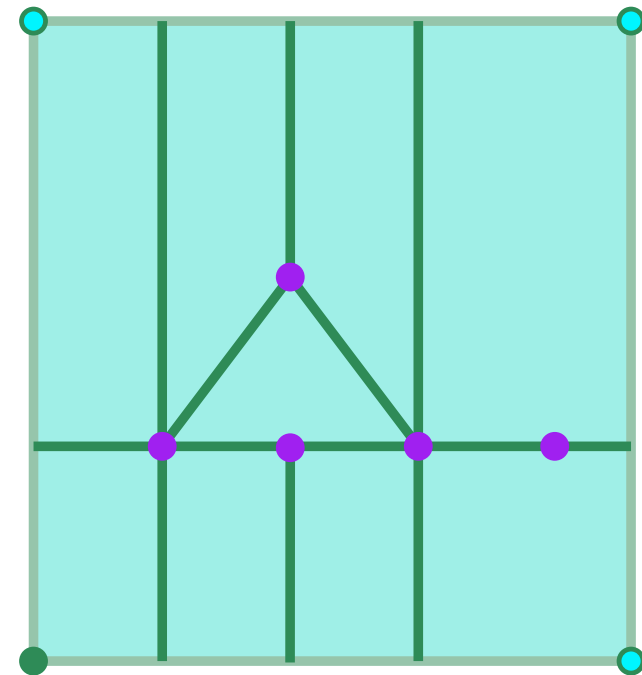


Given a map on a torus

Given a map on a torus
(essentially 3-connected)



Get a



Convex straight line drawing
on grid $2n \times (1 + 2n(c + 1))$

$$c \leq \sqrt{2n}$$