Embeddings of the square flat torus and smooth fractals*

Boris Thibert†

In the mid 1950s, J. Nash and N. Kuiper showed it was possible to isometrically embed a flat torus in three dimensional Euclidean space. This result was counter-intuitive since the Gauss curvature prevents those embeddings to be of class C^2. The maps of Nash and Kuiper are of class C^1, which means in particular the existence of a tangent space at each point of the embeddings.

In the 70-80s, M. Gromov, revisiting the results of Nash and others, extracted the underlying notion of their works: the h-principle. This principle states that the existence of many partial differential relations reduces to purely topological questions. He also proposed several methods for solving some of them, such as the Convex Integration Theory, that provides a quasi-constructive way to build sequences of embeddings converging towards isometric embeddings.

Within the Hévéa team [3], we have adapted the Convex Integration Theory to get an algorithm for the construction of isometric embeddings of the square flat torus [1, 2]. Our construction and the visualisation of the embeddings have revealed a paradoxal geometrical object: while the resulting surface is of class C^1, the normal vector exhibits a fractal structure. More precisely, one shows that the normal vector can be approximated by an infinite product of rotation matrices. This behavior is reminiscent in the one dimensional case of a Riesz product, that is known to have a fractal structure.

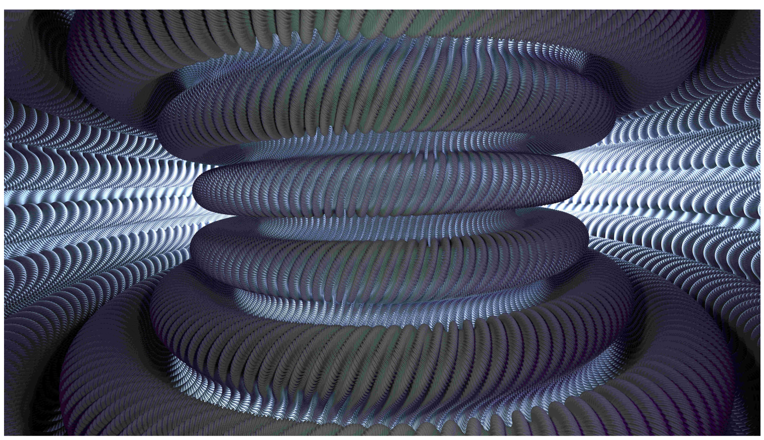

Figure 1: Flat torus from inside

References

[1] V. Borrelli, S. Jabrane, F. Lazarus, B. Thibert. Flat tori in three-dimensional space : an implementation of convex integration. *Proceedings of the National Academy of Sciences* 109 (19), 7218-7223, 2012.

[2] V. Borrelli, S. Jabrane, F. Lazarus, B. Thibert. Isometric embeddings of the square flat torus in ambient space. *Ensaios Matemáticos*, 24:1-91, 2013

[3] Hevea project. V. Borrelli, S. Jabrane, F. Lazarus, D. Rohmer, B. Thibert. http://hevea.imag.fr

*Research partially supported by Région Rhônes-Alpes (projet CIBLE) and CNRS (projet PEPS).

†Laboratoire Jean Kuntzmann, Université Grenoble Alpes

This is a short abstract of a presentation given at the Workshop on Geometric Structures with Symmetry and Periodicity, part of the CG Week 2014. It has been made public for the benefit of the community and should be considered a preprint rather than a formally reviewed paper. Thus, this work may appear in a conference with formal proceedings and/or in a journal.