Ultrarigid periodic frameworks

Louis Theran

Abstract

I will present recent progress on algebraic and combinatorial characterizations of periodic frameworks obtained with Justin Malestein in [3].

Background

A *periodic framework* is an infinite structure in Euclidean d-space, made of fixed-length bars connected by universal joints and symmetric with respect to a lattice $\Gamma \cong \mathbb{Z}^d$. The allowed motions preserve the lengths and connectivity of the bars, and symmetry with respect to the action of Γ on the graph $\tilde{G}$ that has as its edges the bars (but not necessarily the geometric representation of Γ). A periodic framework is *periodically rigid* if the allowed motions all arise from applying a Euclidean isometry to the whole structure and *periodically flexible* otherwise.

For this setting there are good algebraic [1] and, in dimension 2, combinatorial [2] characterizations of rigidity and flexibility. These results (and other similar ones) make use of the fact that the configurations spaces of periodic frameworks are homeomorphic to finite-dimensional algebraic varieties. Simply dropping the symmetry constraints on the motions invalidates this assumption, so tools from [1, 2] don't apply.

Ultrarigidity

In this talk, I will discuss *periodically ultrarigid* periodic frameworks, which are periodically rigid and remain so when the symmetry constraint on the motions is relaxed to require only symmetry with respect to *any finite index sub-lattice* $\Lambda < \Gamma$.

In particular, I will describe:

- An *algebraic characterization* of infinitesimal ultrarigidity based on new rigidity matrices with entries in an appropriate group ring.
- A simple *algorithm* for checking whether a framework is periodically ultrarigid that uses new bounds on the minimal orders of torsion points in varieties.
- Polynomial-time checkable *combinatorial characterizations* of generically ultrarigid graphs in dimension $d = 2$ with n vertices and $m = 2n + 1$ edges.

References

[1] C. S. Borcea and I. Streinu. Periodic frameworks and flexibility. *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.*, 466(2121):2633–2649, 2010. ISSN 1364-5021. doi: 10.1098/rspa.2009.0676. URL `http://dx.doi.org/10.1098/rspa.2009.0676`.

[2] J. Malestein and L. Theran. Generic combinatorial rigidity of periodic frameworks. *Adv. Math.*, 233:291–331, 2013. ISSN 0001-8708. doi: 10.1016/j.aim.2012.10.007. URL `http://dx.doi.org/10.1016/j.aim.2012.10.007`.

[3] J. Malestein and L. Theran. Ultrarigid periodic frameworks. Preprint, 2014. URL `https://www.dropbox.com/s/g9ig4rkifgb4e0n/ur.pdf`.

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