

## Voronoi diagrams and Bolza surface

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A periodic Delaunay triangulations in a Euclidean space can be seen as delaunay triangulations in a closed Euclidean (aka. flat) manifold. The case of a manifold  $E^d/G$ , where  $G$  is a crystallographic group, was addressed in 2D and 3D [7, 9], and more recent work [3, 4] led to CGAL packages for the flat torus [5, 8].

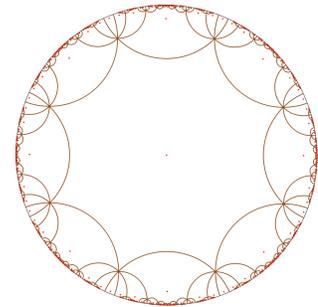
To the best of our knowledge, there were no known similar results in hyperbolic spaces.

A periodic triangulation in the hyperbolic plane is defined by an infinite point set that is the image of a finite point set by some (non commutative) discrete group generated by hyperbolic translations.

We focus here on the group defining the Bolza surface, homeomorphic to a torus having two handles. This setting is used very diverse fields [1, 6, 10].

The talk will show a few properties of Voronoi diagrams on the Bolza surface. Intuition is challenged there, in particular because hyperbolic translations do not commute in general.

Details and more general results can be found in [2].



### References

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