The Betweenness Centrality on 1-Dimensional Periodic Graphs

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1 Introduction and Preliminaries

Periodic graphs are infinite graphs obtained by connecting the same unit finite graphs periodically. They have been used to model various repetitive structures, including crystals. As mentioned in [1], average shortest path length of the finite subgraphs of the periodic graph affect some properties of crystal lattices. We know from [2] that the average length can be significantly increased by removing a node with a large *betweenness centrality*, a ratio of shortest paths passing though that node. As we aim to find a node that is critical for the crystal properties, those facts motivate us to extend the definition of betweenness centrality to the periodic graph. In this talk, as a preliminary result towards the 2-dimensional case, we propose definitions of betweenness centrality on the 1-dimensional periodic graphs, and algorithms for computing them, using the theory of Gröbner bases, which is used also in [3].

Consider a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is a finite set and $\mathcal{E} \subseteq \{((i,j),g) : i,j \in \mathcal{V}, g \in \mathbb{Z}\}$ is an edge set with integral labels. The *1-dimensional periodic graph* $G = (V,E)$ *generated by* $\mathcal{G}$ is a directed infinite graph, such that $V = \mathcal{V} \times \mathbb{Z}$, $E = \{(((i,h),(j,h+g))) : h \in \mathbb{Z}, ((i,j),g) \in \mathcal{E}\}$.

Let $H = (U,W)$ be a finite graph and $s,t \in U$, we denote by σ^H_{st}, the number of all shortest paths connecting s and t in G, and by $\sigma^H_{st}(v)$, the number of paths containing v among those shortest paths. The betweenness centrality of a vertex v on a finite graph H, denoted by $g^H(v)$, is defined as $\sum_{s \neq v \neq t \in U} \frac{\sigma^H_{st}(v)}{\sigma^H_{st}}$.

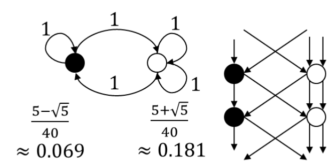

Figure 1: The graph with the betweenness centrality of each node (left), the periodic graph generated from that graph (right)

2 Our Contributions

The betweenness centrality defined in the previous section can go to infinity on periodic graphs. Thus, we define the betweenness centrality for a periodic graph G, which can be regarded as an extension of the finite case, as follows. Fix a vertex $v \in \mathcal{V}$. For $D \in \mathbb{N}$ (resp. $D \in \mathbb{N}$ and $x \in \mathbb{Z}$), let G^D (resp. G^D_x) be the subgraph induced by the set $V^D = \{(u,y) : d_G((v,0),(u,y)) \leq D\}$ (resp. $V^D_x = \{(u,y) : d_G((v,0),(u,y)) \leq D, y \geq x\}$) of vertices. Notice that G^D and G^D_x are finite graphs.

Definition 1 *For $x \in \mathbb{Z}$, the half infinite betweenness centrality* $\mathrm{hbc}^G_x(v)$ *of v with boundary x (resp. the double infinite betweenness centrality* $\mathrm{dbc}^G(v)$ *of v) is* $\lim_{D\to+\infty} \frac{1}{|V^D_x|} g^{G^D_x}((v,0))$ *(resp.* $\lim_{D\to+\infty} \frac{1}{|V^D|^2} g^{G^D}((v,0))$*.)*

The main results of our work are shown in the following theorem.

Theorem 1 $\mathrm{hbc}^G_x(v)$ *and* $\mathrm{dbc}^G(v)$ *converge, and there exist exponential-time algorithms for computing them. Particularly, when G is planar, those algorithms run in polynomial time.*

References

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