Approximate Symmetries of Point Patterns

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For a given finite point set $P \subset \mathbb{R}^2$ one can ask if P is symmetric with respect to the finite symmetry groups C_k and D_k. Optimal algorithms solving this problem in $O(n \log n)$ time where presented by Highnam [3] for two-dimensional point sets and by Alt et al. [1] for three-dimensional point sets.

In practice, hardly one input can be considered symmetric due to the representation of the input or perturbations while generating the input data. Therefore, one wants to extend the definition of symmetry to inputs that are not perfectly symmetric themselves, but are "close" to a symmetric point set.
Given a point set P that is not symmetric one can ask for a symmetric point set Q which approximates P with parameter ε. Formally, we ask for a bijection $f : P \to Q$, where $|p - f(p)| \leq \varepsilon$, for all $p \in P$.

This problem is called *approximate symmetry detection* (ASD). It can be considered either as decision problem, where one asks if the input set P is approximate symmetric with given parameter ε, or as optimization problem, where the best value for ε is to be computed and serves as a measure of symmetry for P.

The ASD decision problem was shown to be in P for symmetry groups D_1 and C_2 and NP-complete for symmetry groups D_2 and D_k, C_k for $k \geq 3$ by Iwanowski [5] (see also Iwanowski [4]).

We investigated several restrictions of the ASD problem regarding, e.g., the symmetry group or the input.
We will present the following results (see Dieckmann [2]):

A polynomial time algorithm for the ASD optimization problem for symmetry group C_2 which is based on a simple geometric argument.

Polynomial time algorithms for the ASD decision problem even for symmetry groups $C_k, k \geq 3$ if the points in P do not lie too close together.

A polynomial time algorithm for the ASD decision problem for a point set P, $|P| = n$ and symmetry group D_n, so the size of the point set equals the order of the symmetry group. This is equivalent to the question if there exists a regular n-gon Q which approximates P with given parameter ε. In fact, the optimization version of this problem is LP-type if a mapping between the points in P and the vertices of Q is given.

A point set Q, $|Q| = mk$ with symmetry group C_k can be partitioned into m regular k-gons $Q_1, \ldots, Q_m$, all having the same rotation center.
We will present a polynomial time algorithm for the ASD decision problem for symmetry groups $C_k, k \geq 3$ and point sets P, where $|P| = mk$ and a partition of P into m subsets $P_1, \ldots, P_m$, each of size k is given, and P_i is approximated by $Q_i, 1 \leq i \leq m$.

References

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