Random sampling of a cylinder yields a not so nasty Delaunay triangulation

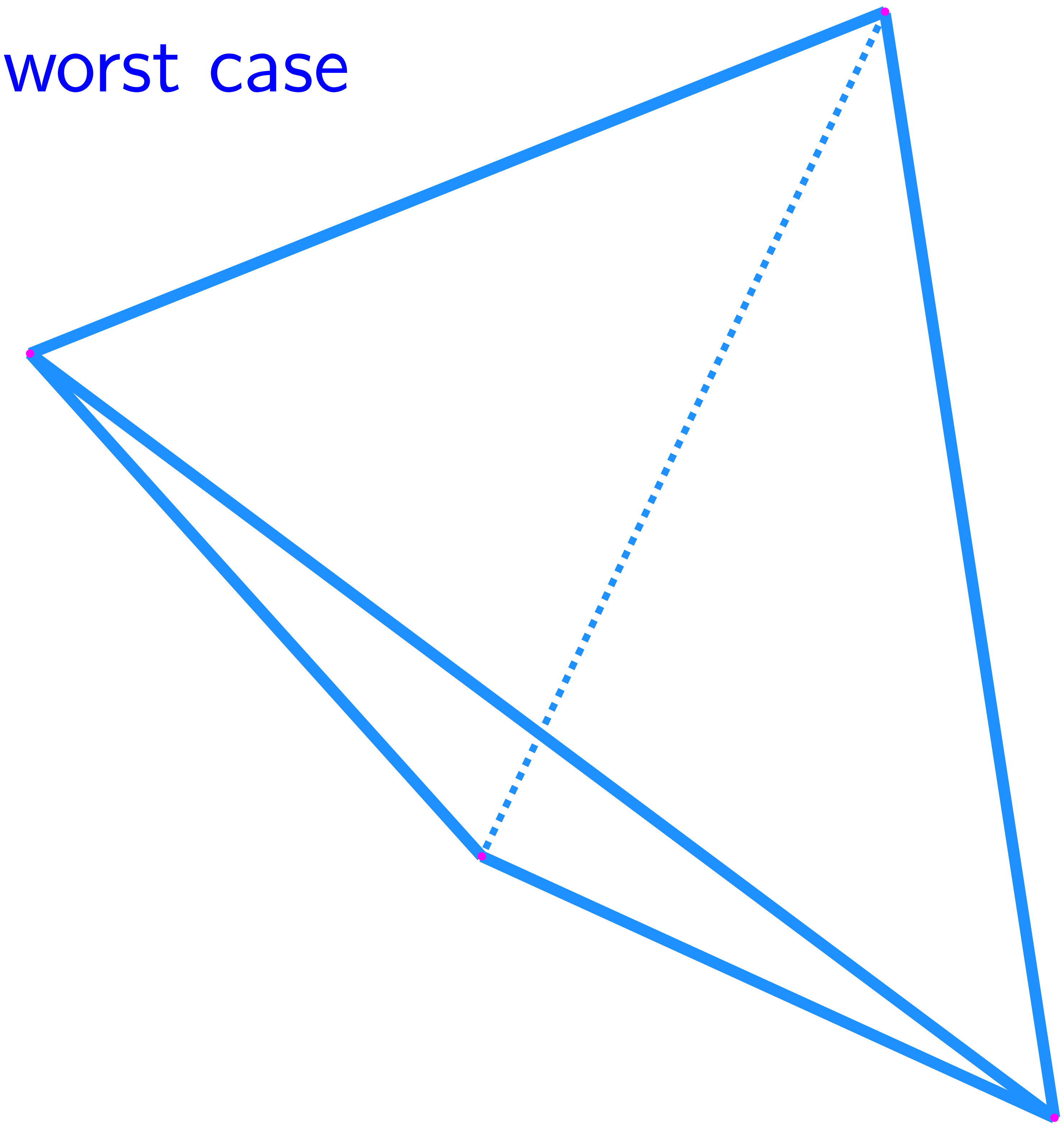
Olivier Devillers

Xavier Goaoc

INRIA

3D Delaunay triangulation

Quadratic in the worst case



3D Delaunay triangulation

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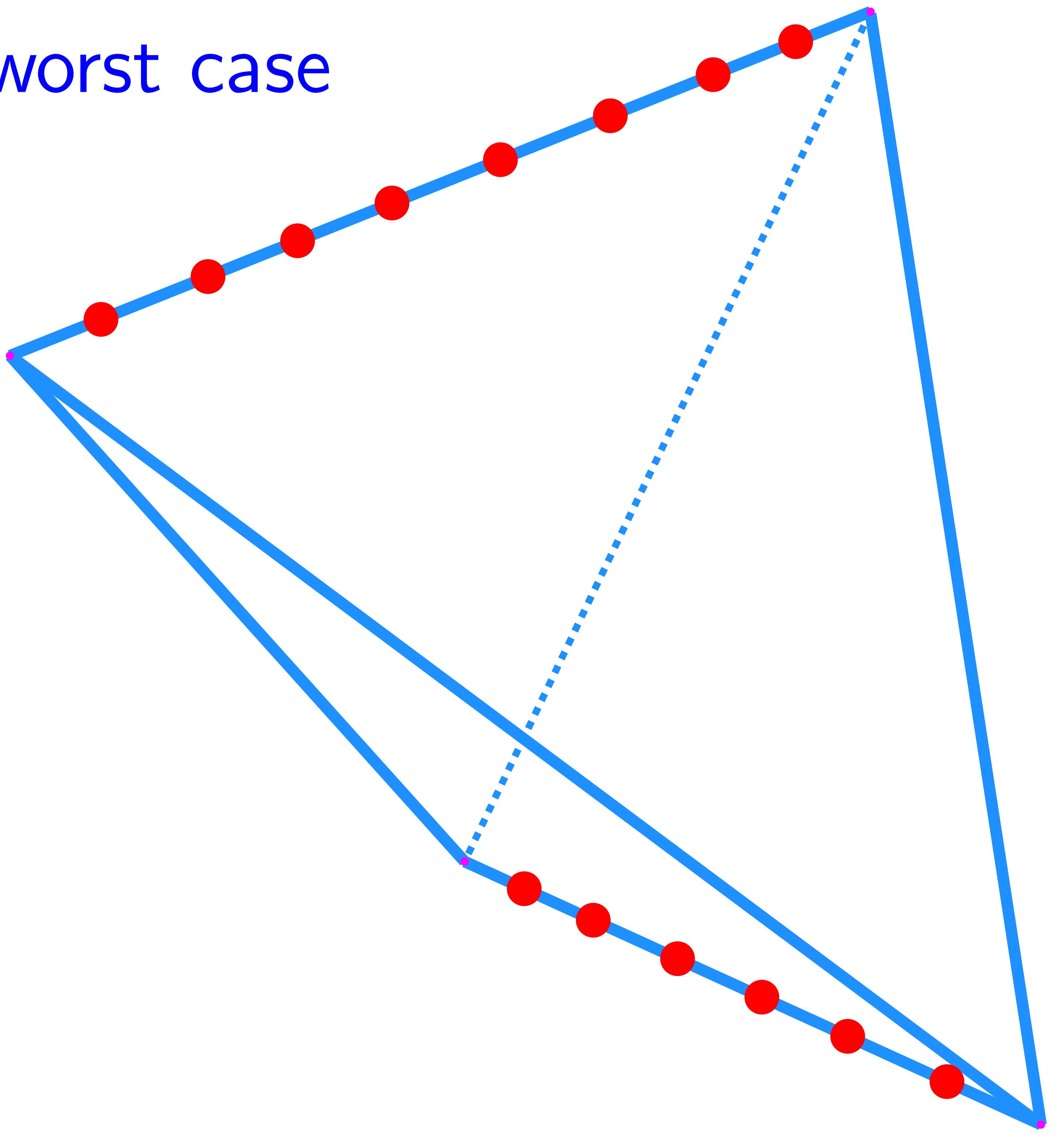
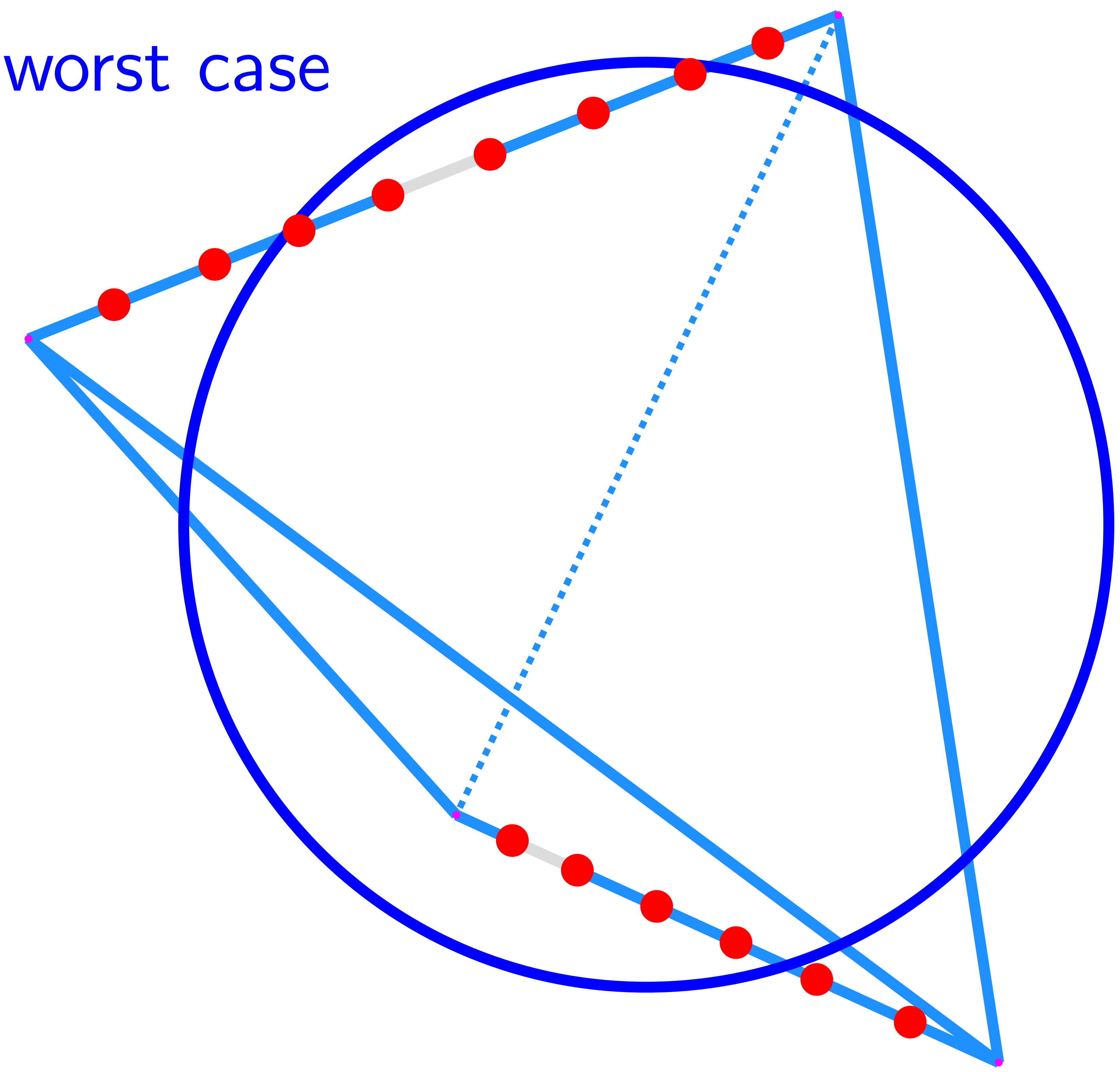


3D Delaunay triangulation

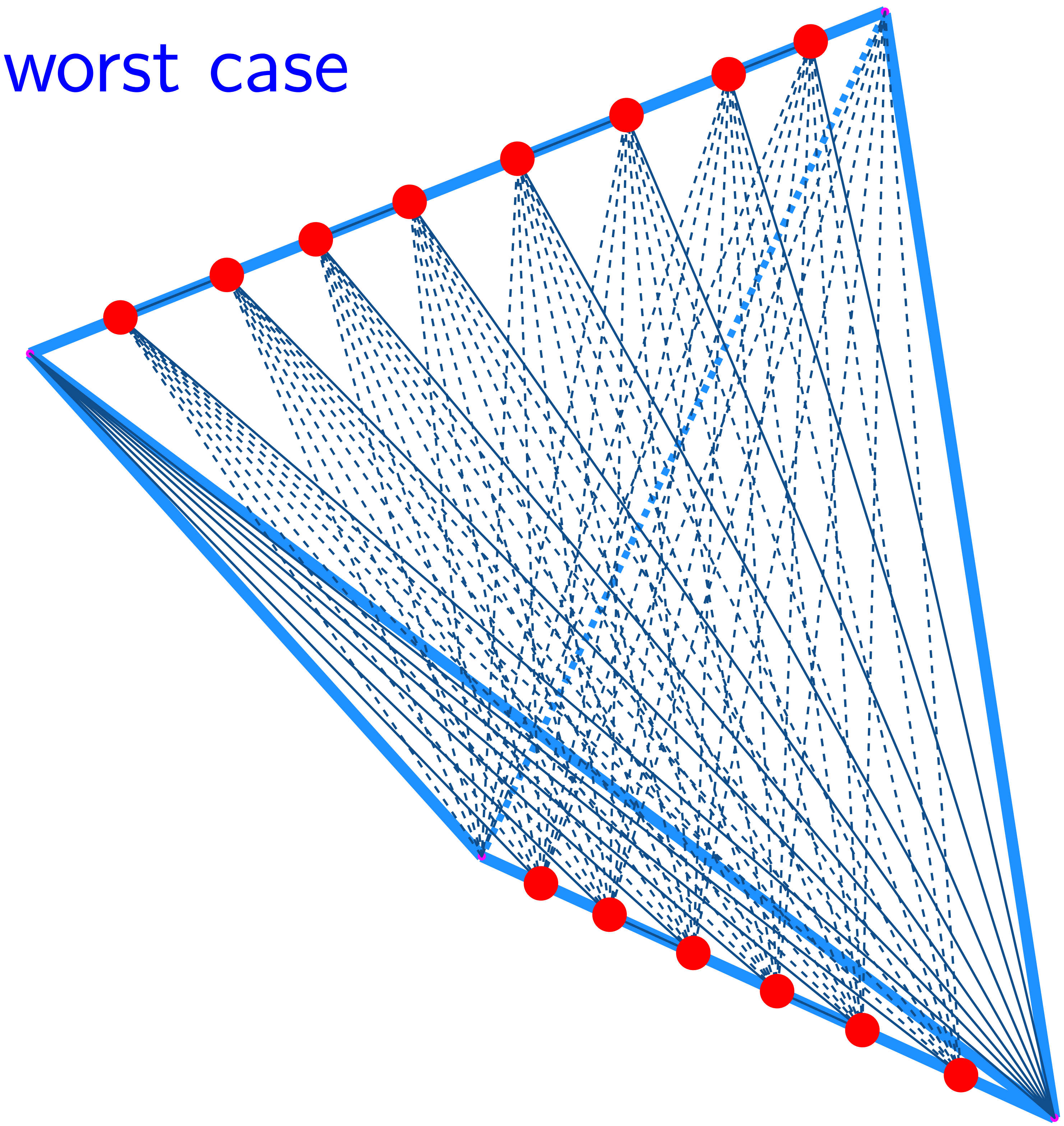
Quadratic in the worst case



3D Delaunay triangulation

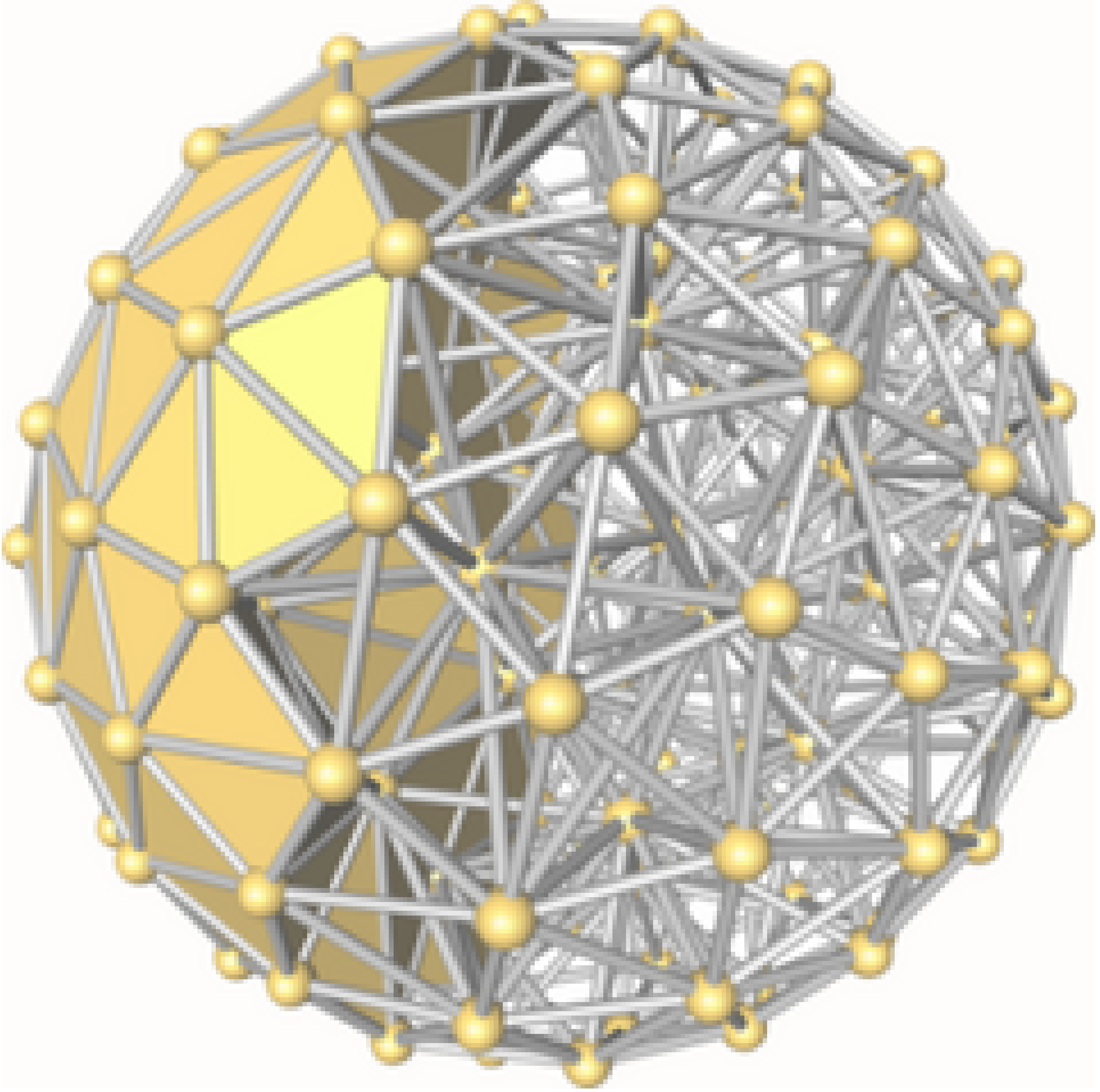
Quadratic in the worst case

$$\Omega(n^2)$$



Linear in random case

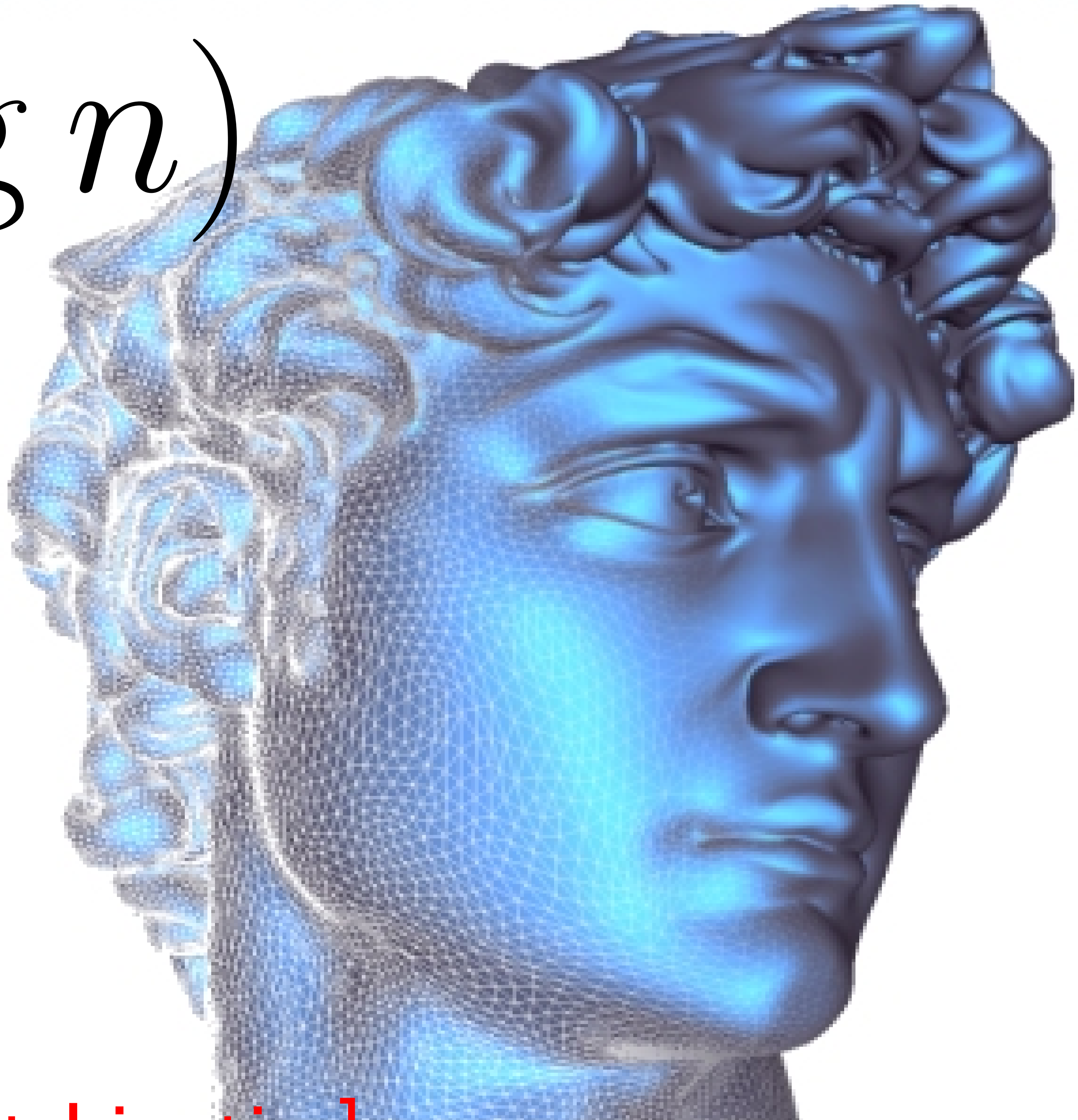
$$O(n)$$



[Dwyer]

Almost linear for "well" sampled generic surface

$$O(n \log n)$$

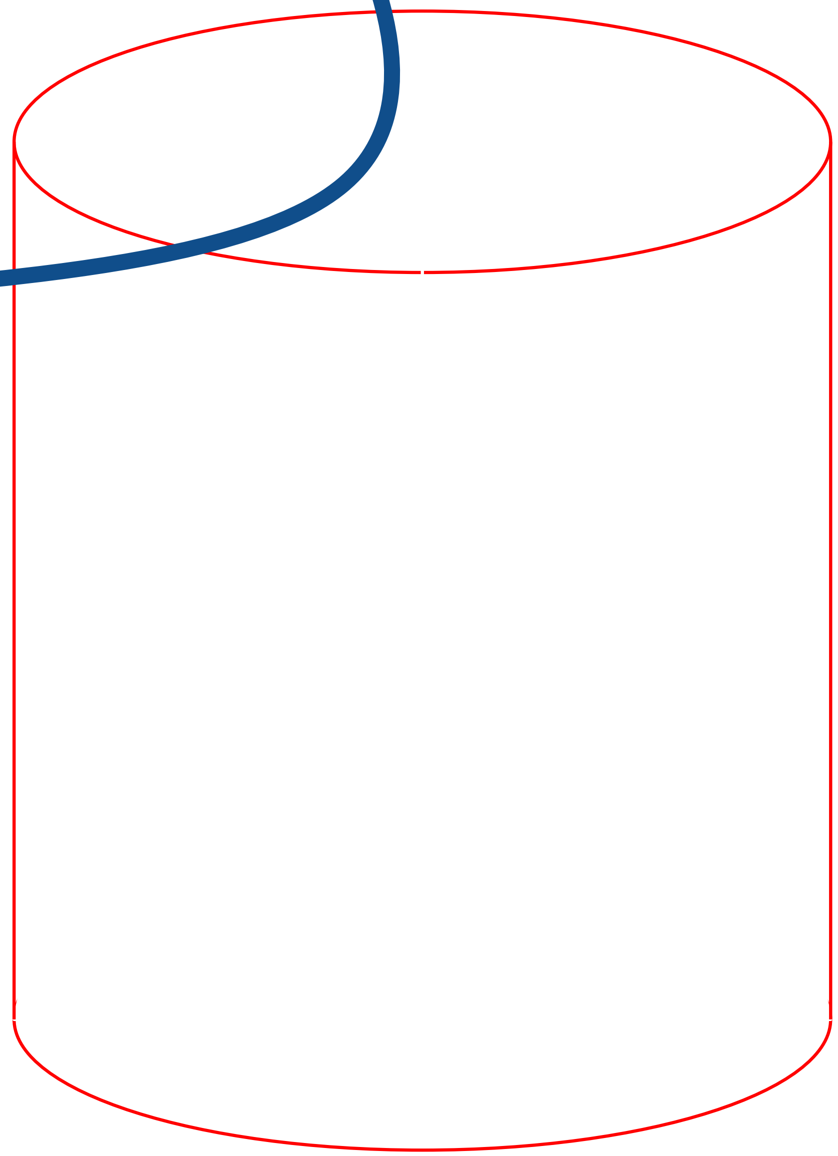


[Attali Boissonnat Lieutier]

Almost linear for "well" sampled generic surface

$$O(n \log n)$$

no 1D skeleton shapes



[Attali Boissonnat Lieutier]

Almost linear for "well" sampled generic surface

$$O(n \log n)$$

evenly distributed points

$$O(n \log^3 n)$$

[Attali Boissonnat Lieutier]

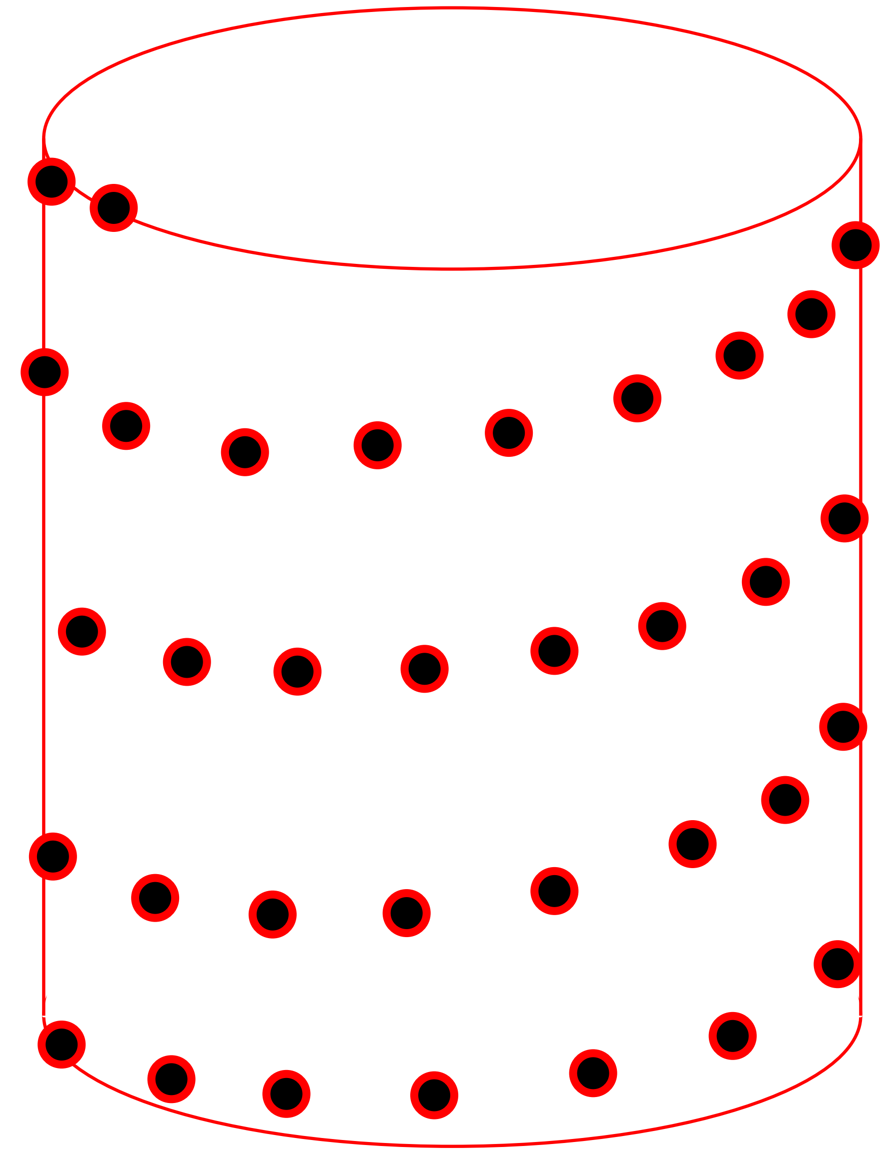
Spread dependant

$$O(n\Delta)$$

[Erickson]

Spread dependant

$$O(n\Delta)$$



$$O(n\sqrt{n})$$

[Erickson]

Spread dependant

$$O(n\Delta)$$

evenly distributed points

$$O(n\sqrt{n \log n})$$

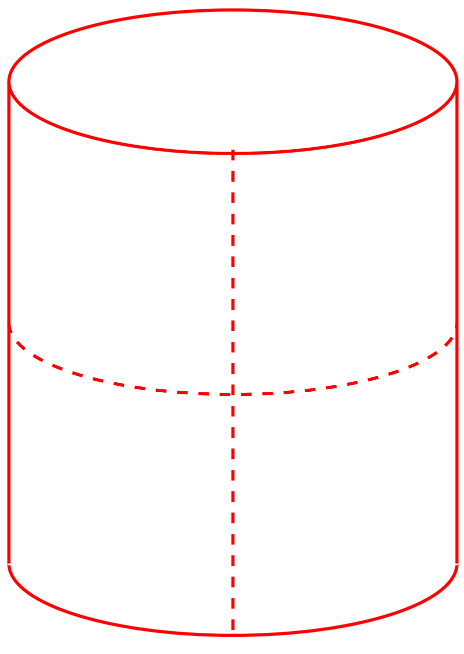
[Erickson]

This paper

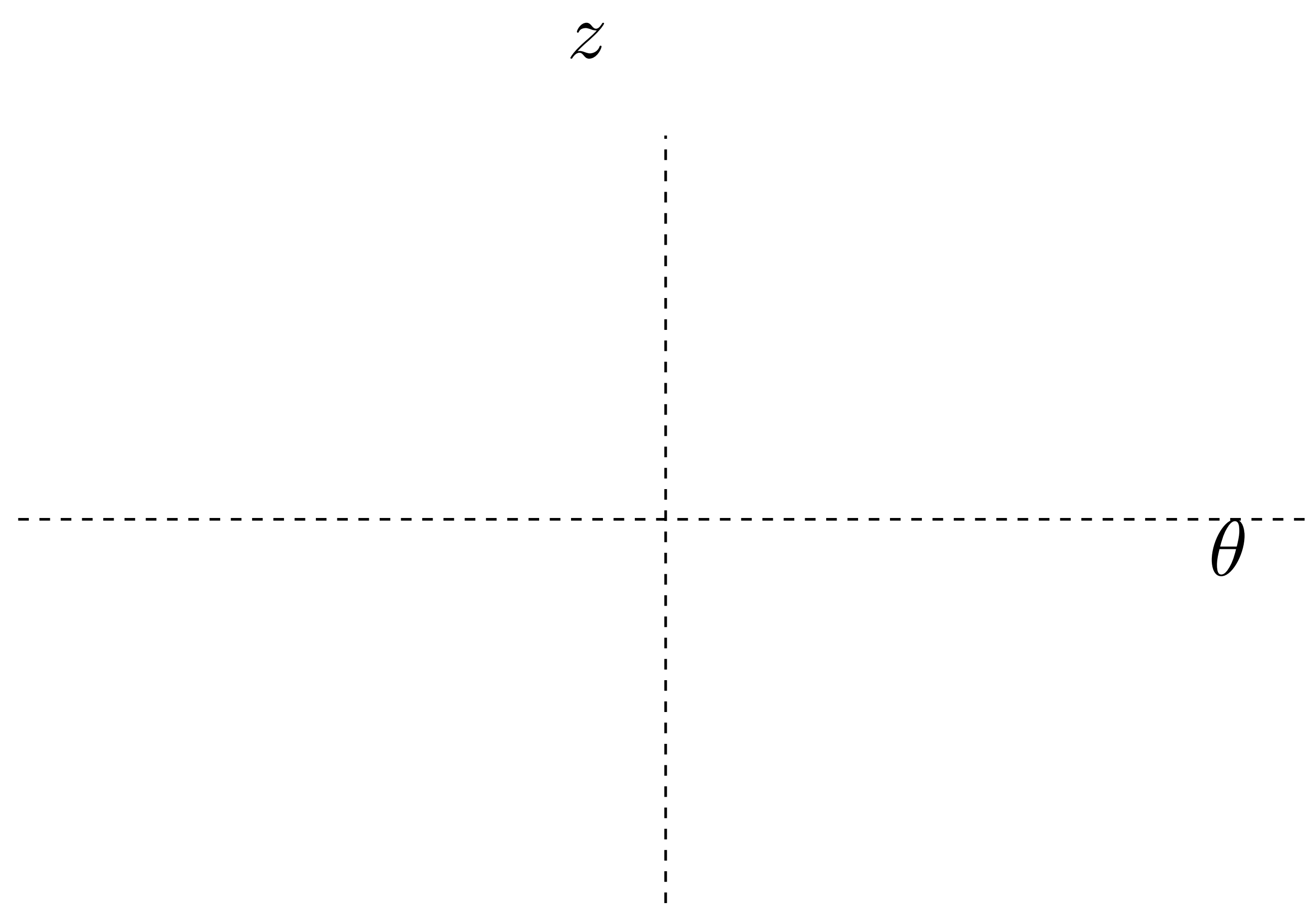
evenly distributed points on a cylinder

$$\Theta(n \log n)$$

Intersection sphere/cylinder



$$r = 0.9$$



Intersection sphere/cylinder

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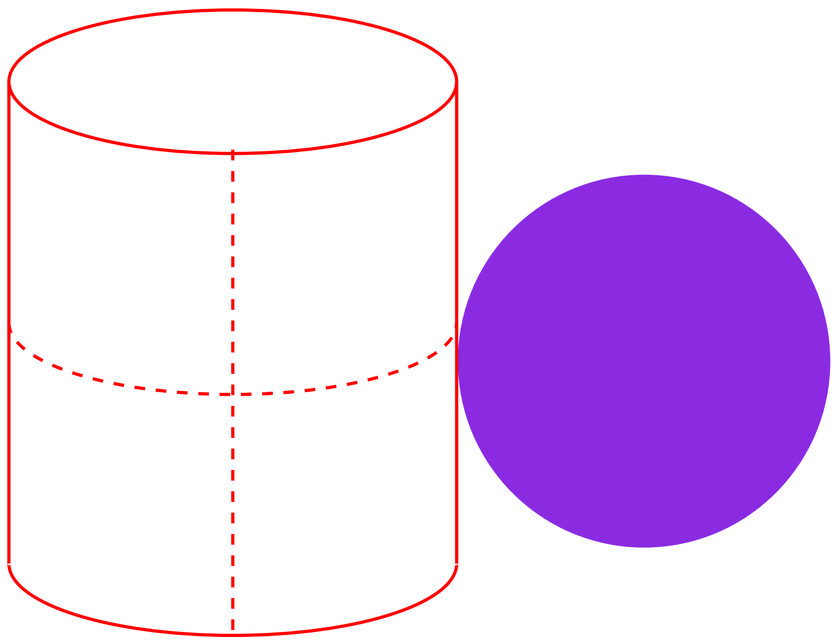
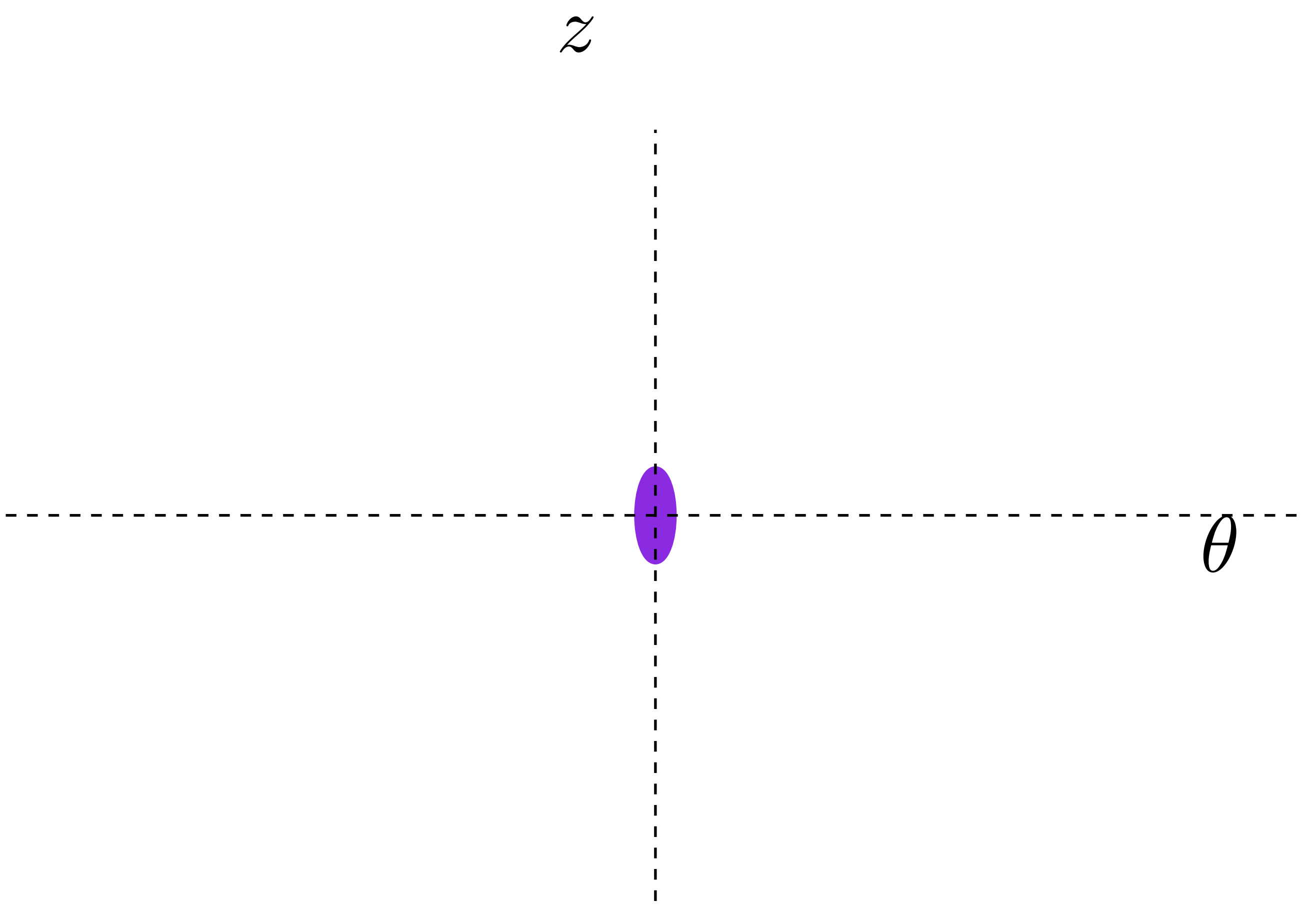
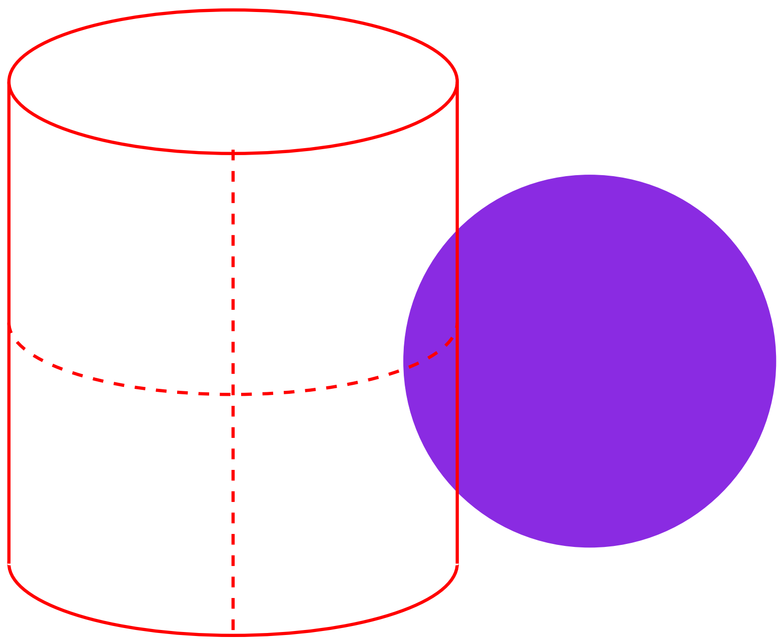




Intersection sphere/cylinder



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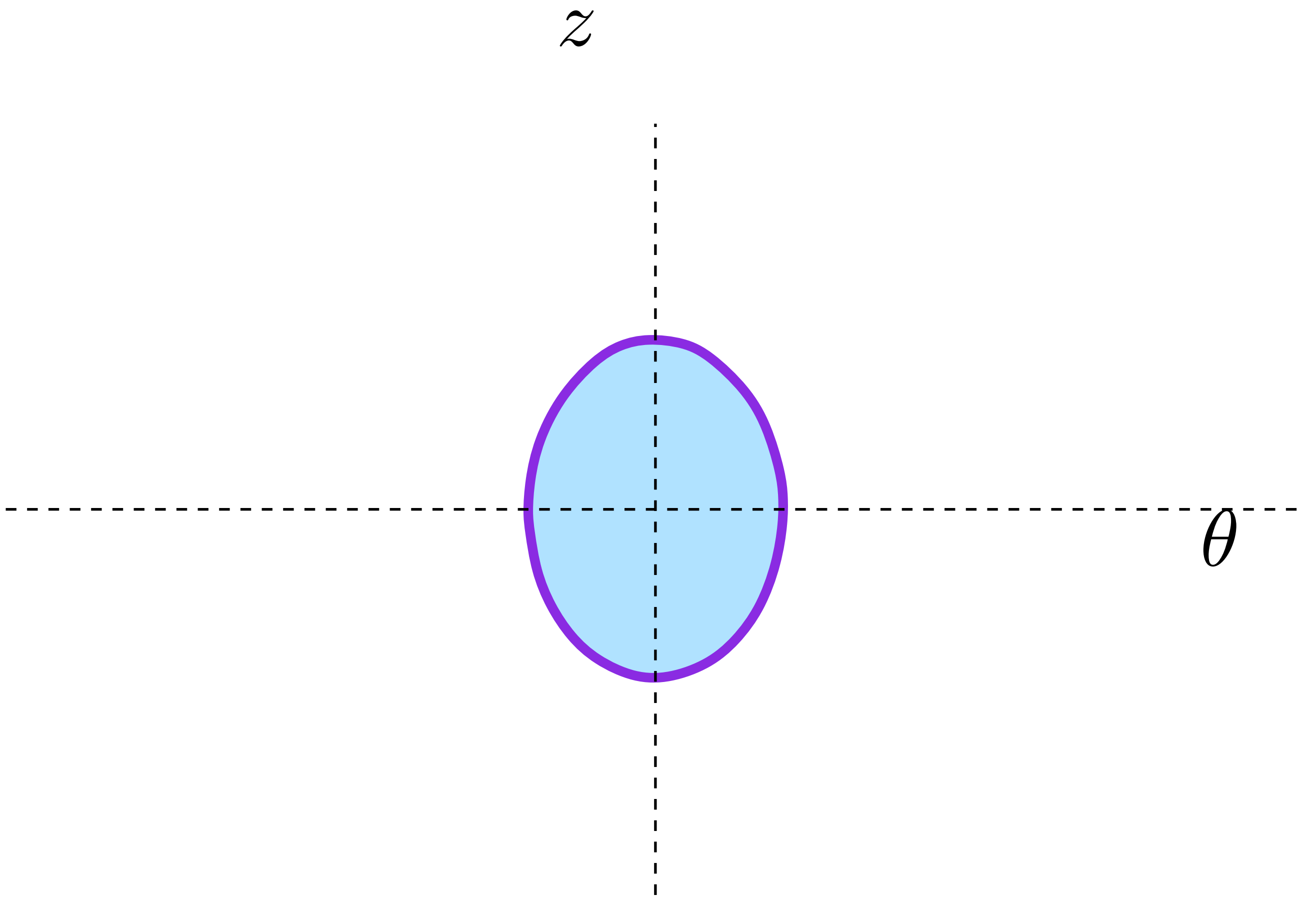
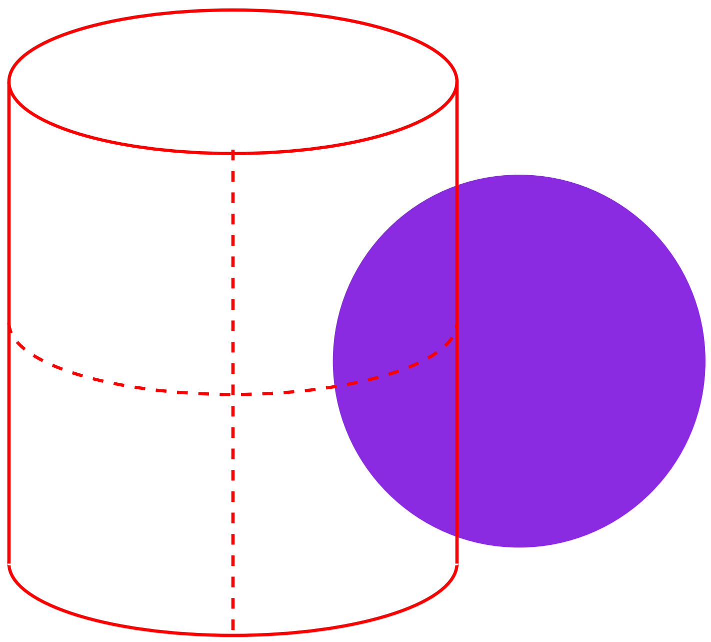
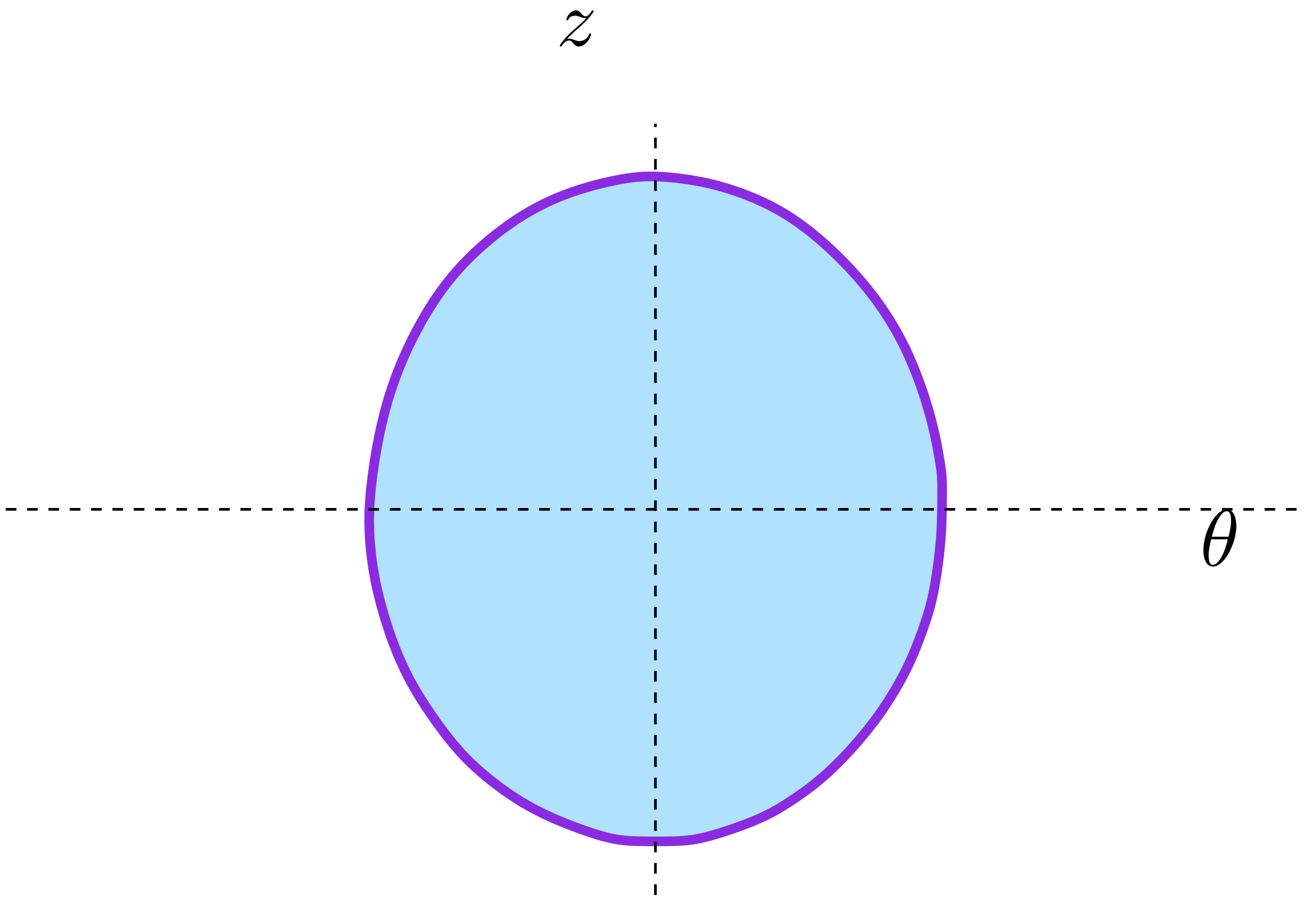


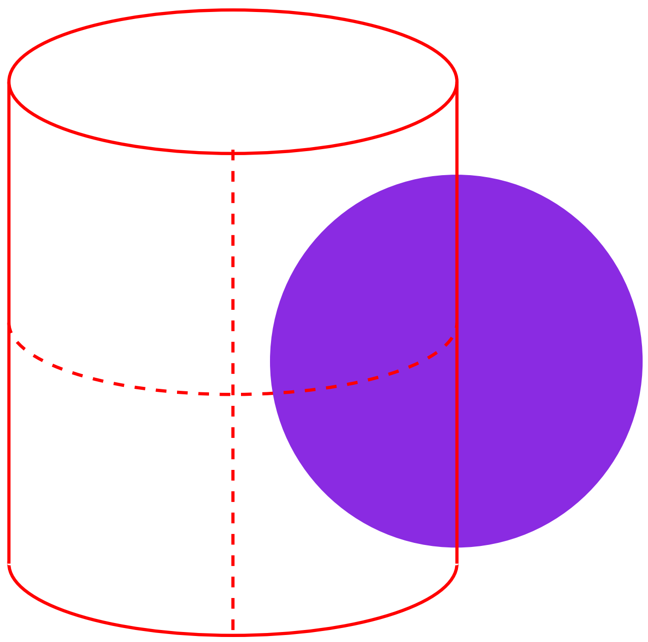
Intersection sphere/cylinder



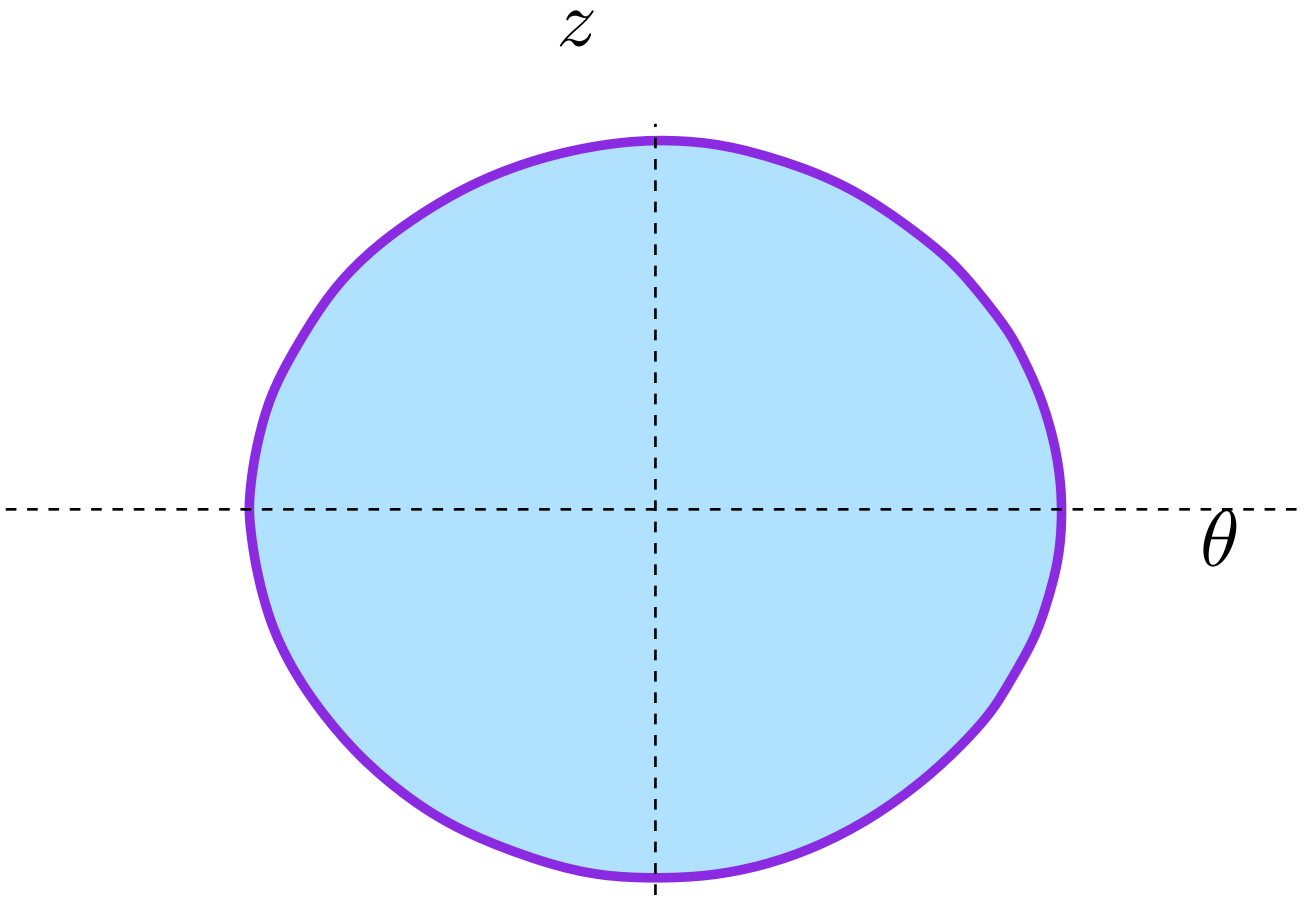
$$r = 0.9$$



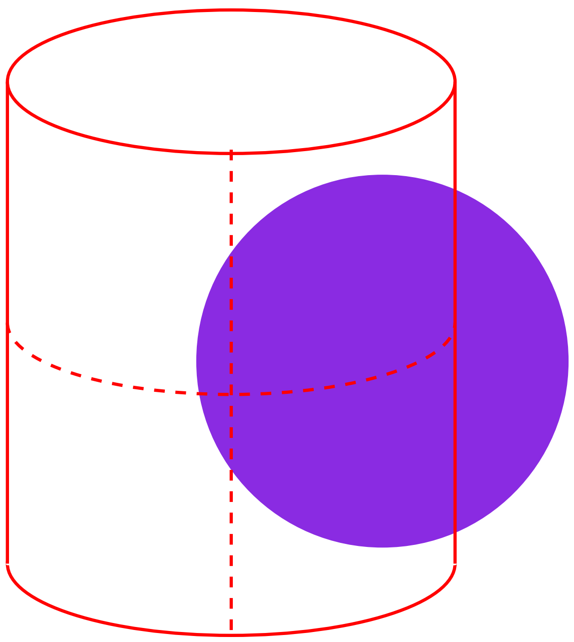
Intersection sphere/cylinder



$r = 0.9$



Intersection sphere/cylinder



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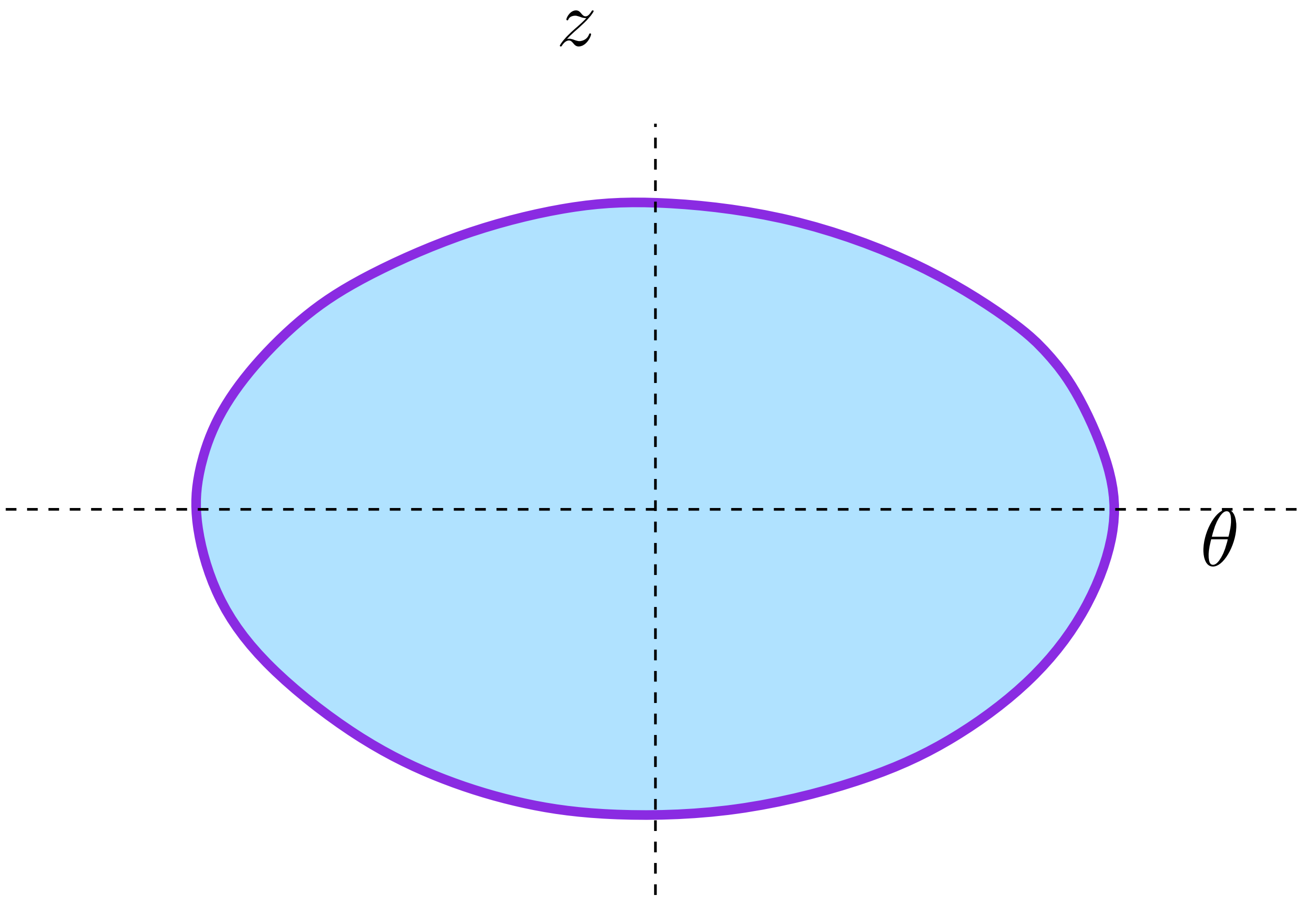
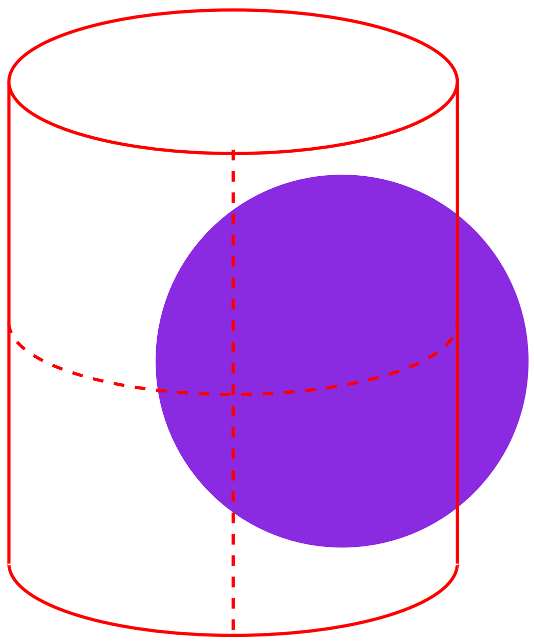


Intersection sphere/cylinder



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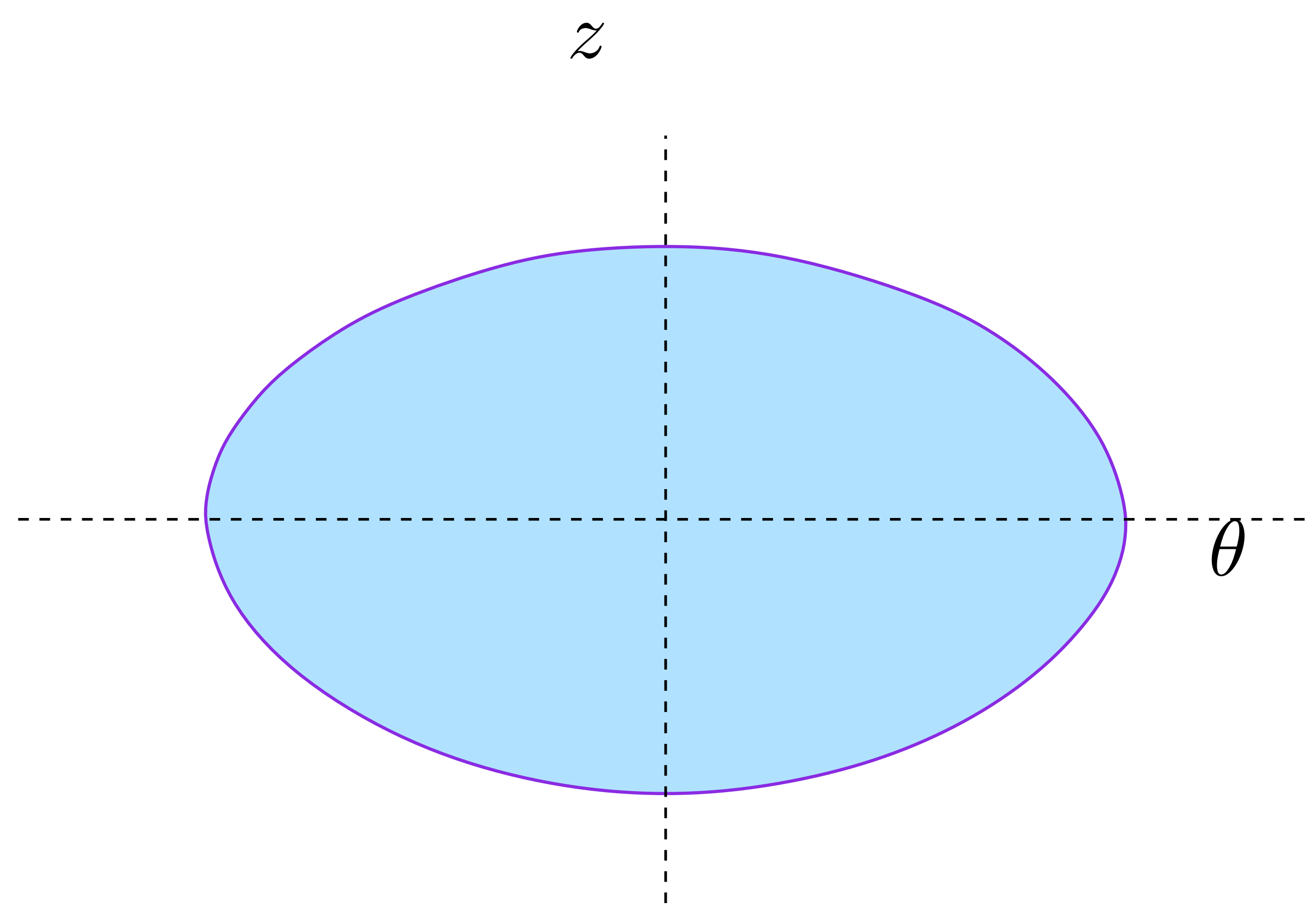
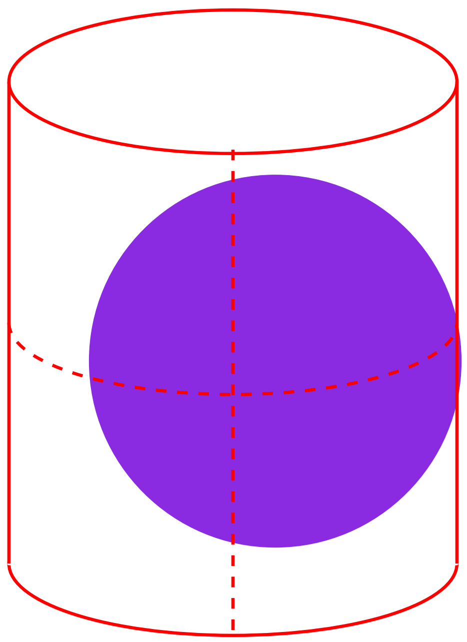
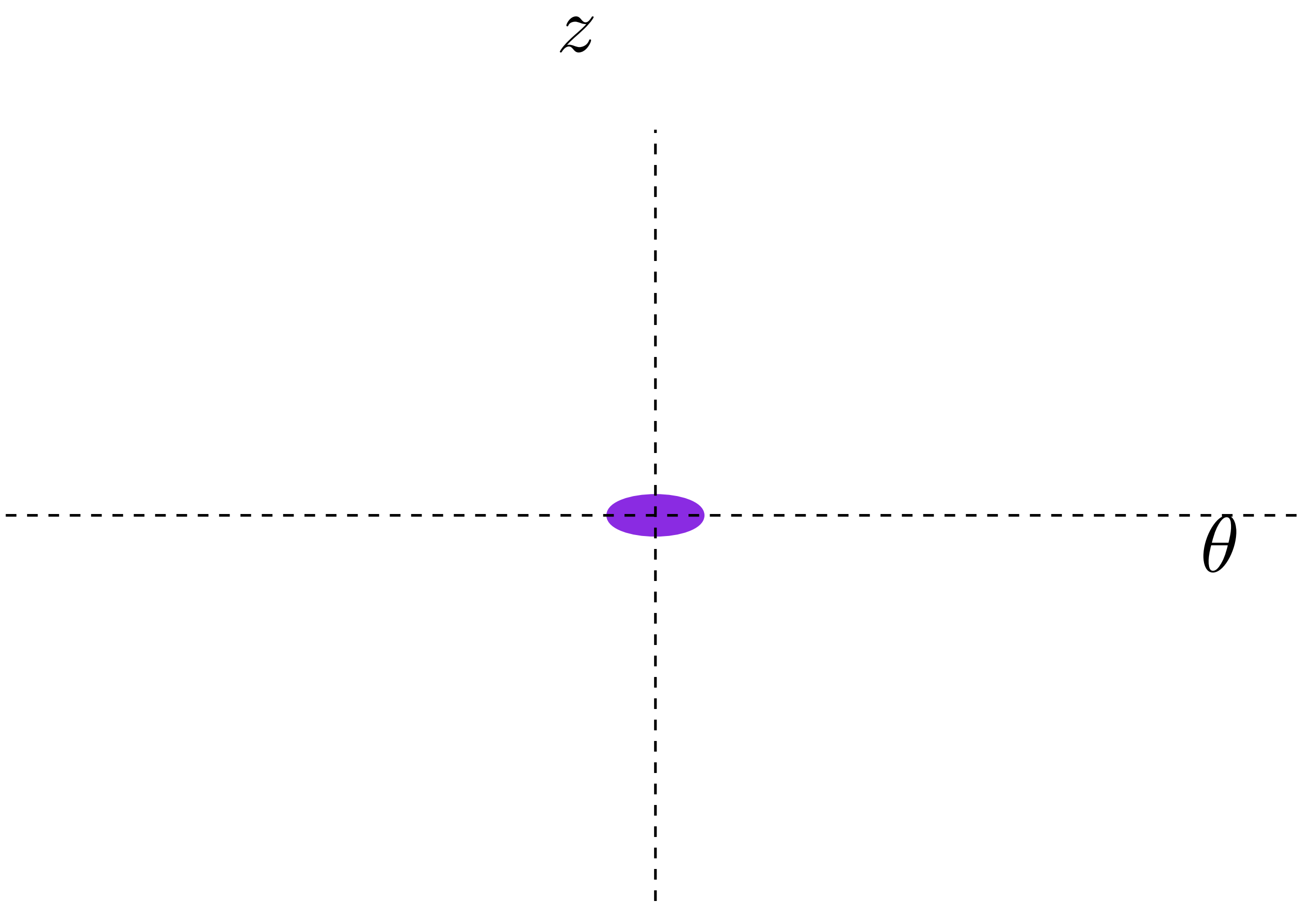


Intersection sphere/cylinder

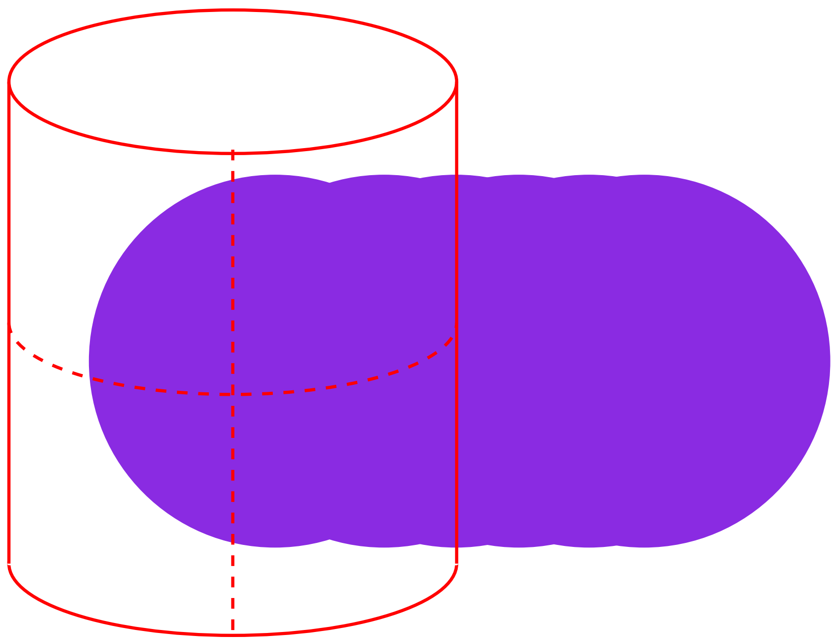


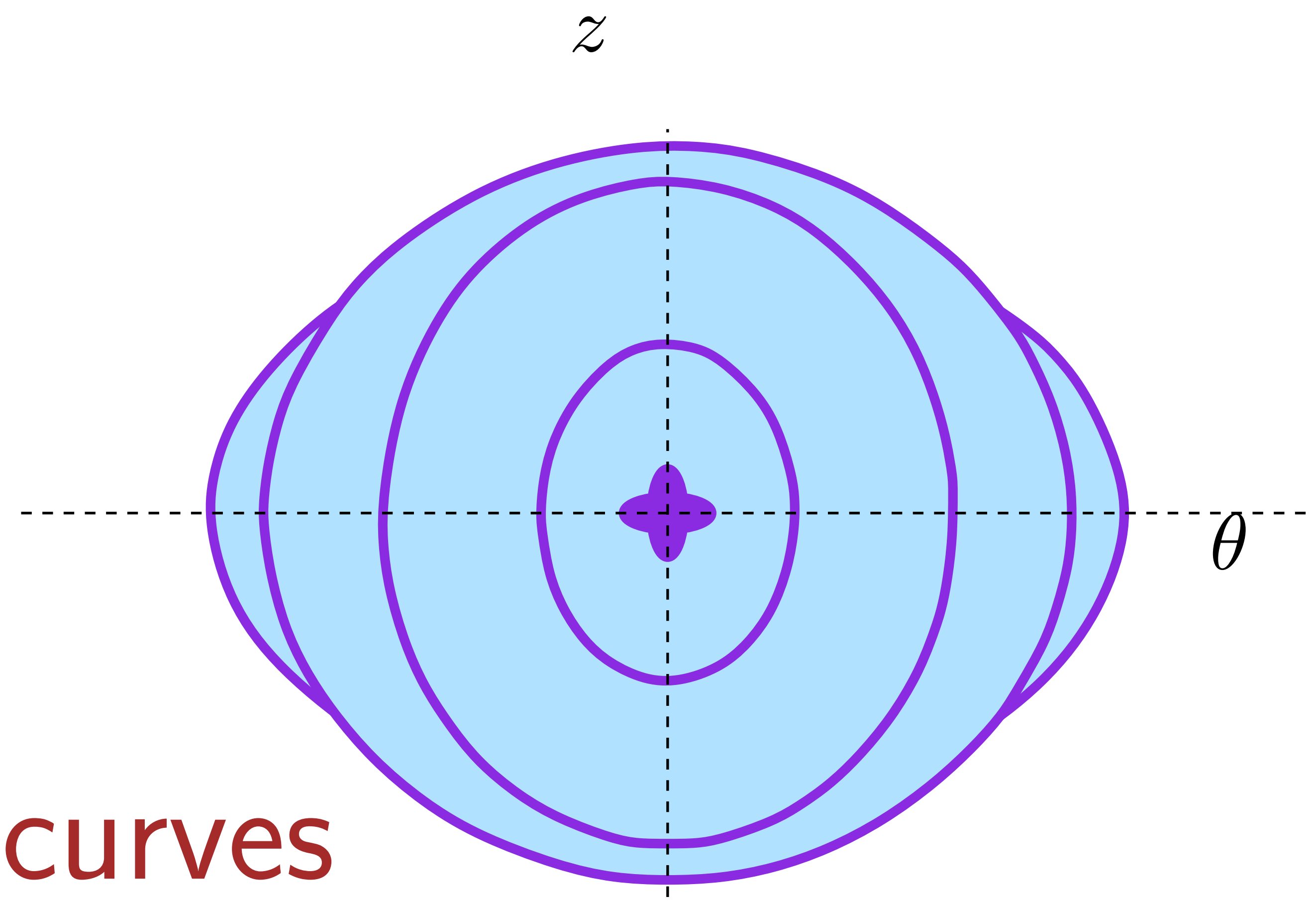
small radius $r = 0.9$



Intersection sphere/cylinder

small radius $r = 0.9$

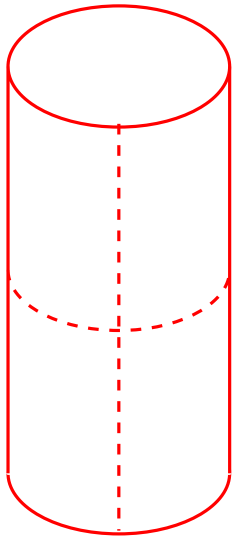


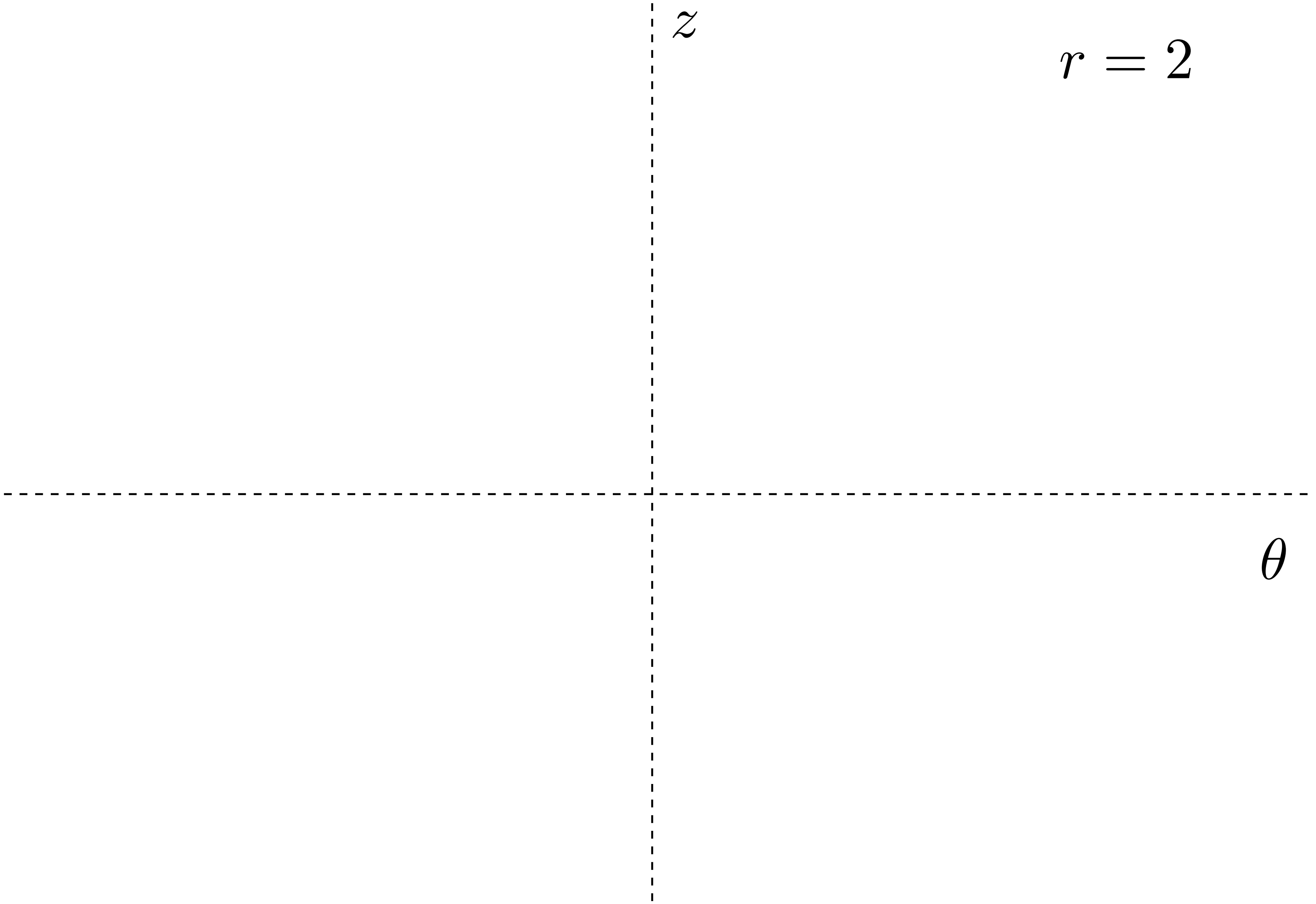


convex closed curves

Intersection sphere/cylinder

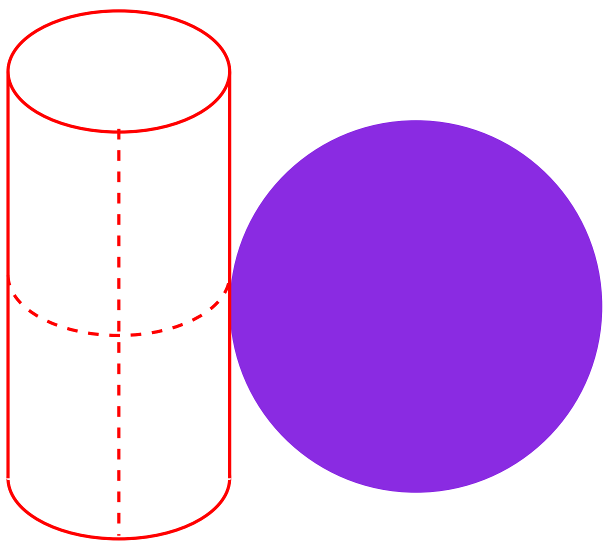
big radius

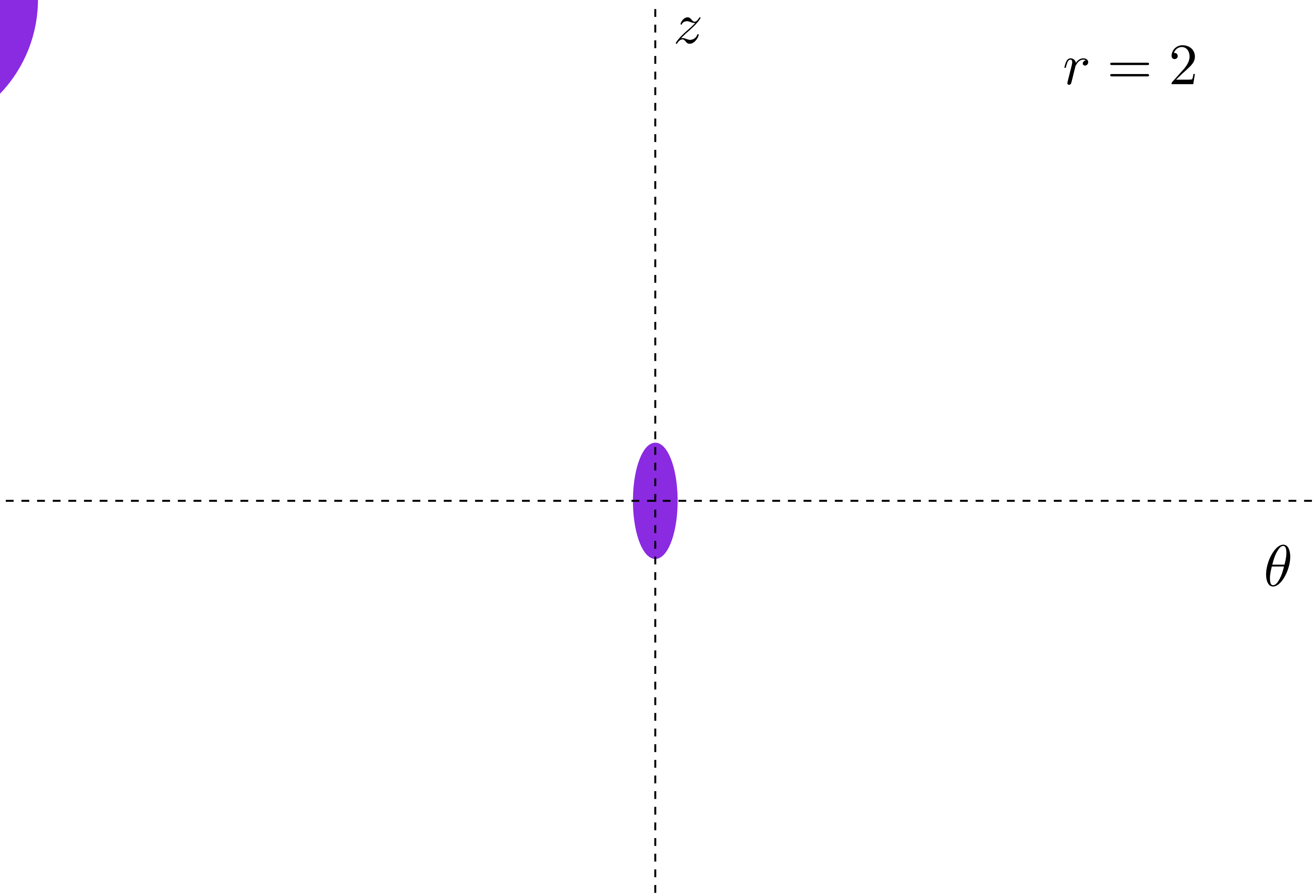




Intersection sphere/cylinder

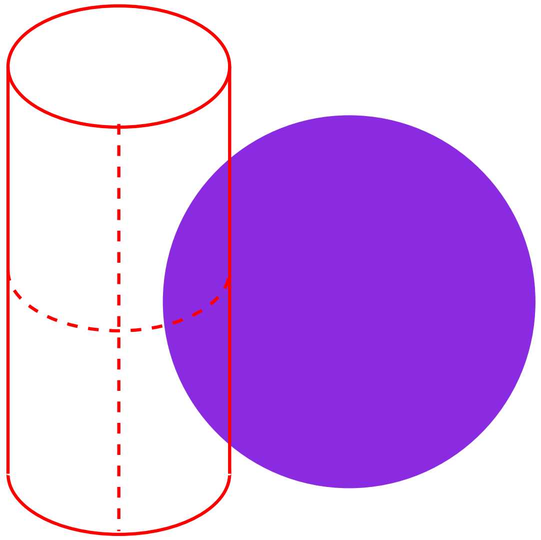
big radius

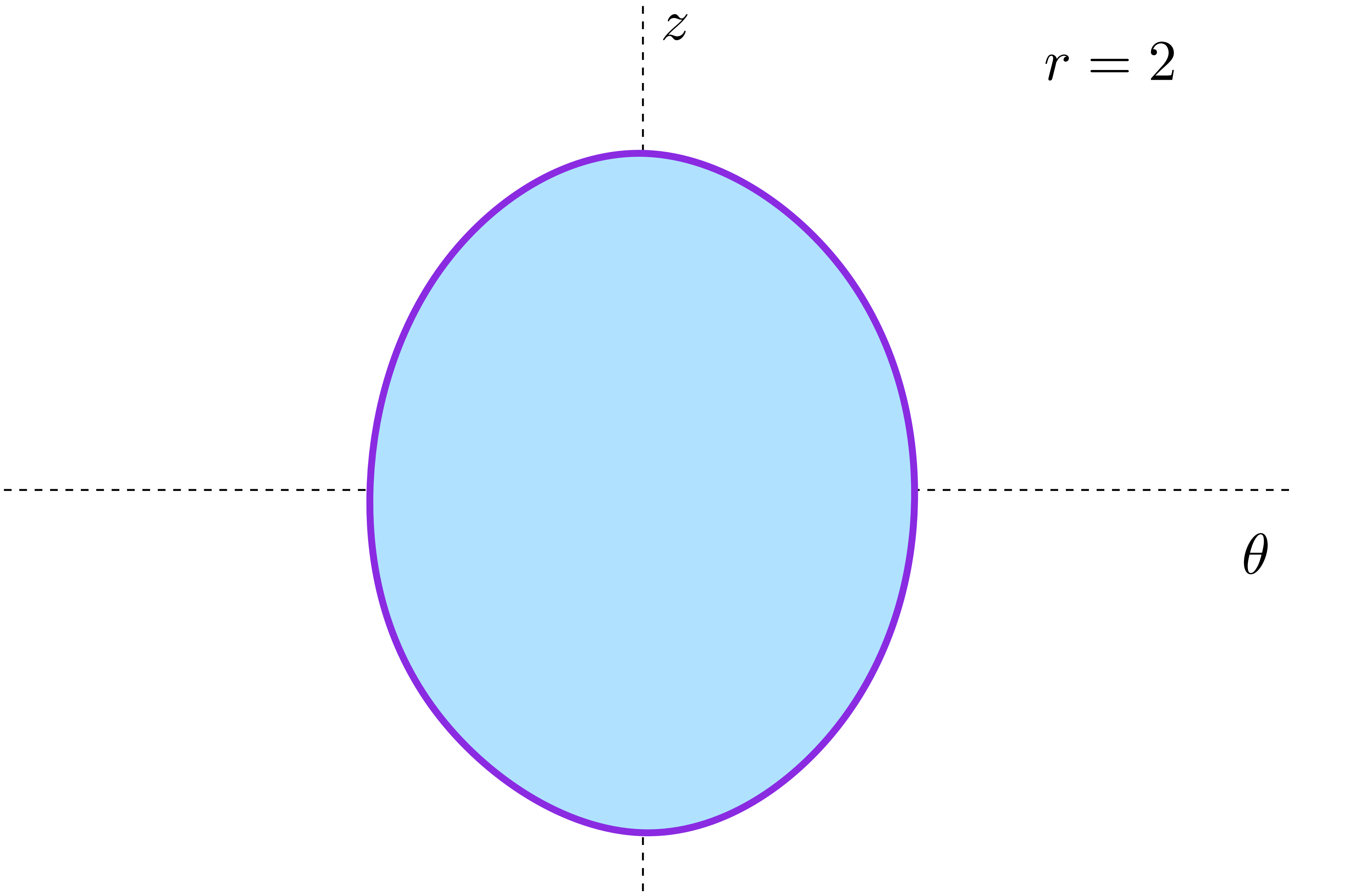




Intersection sphere/cylinder

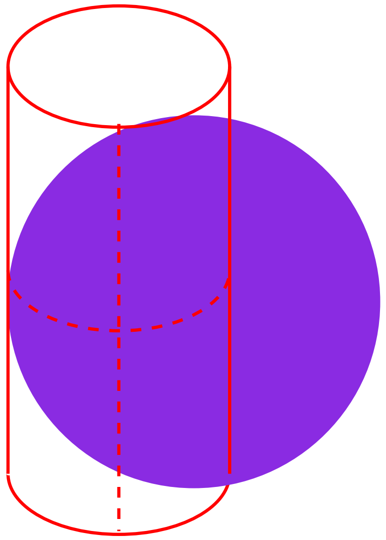
big radius

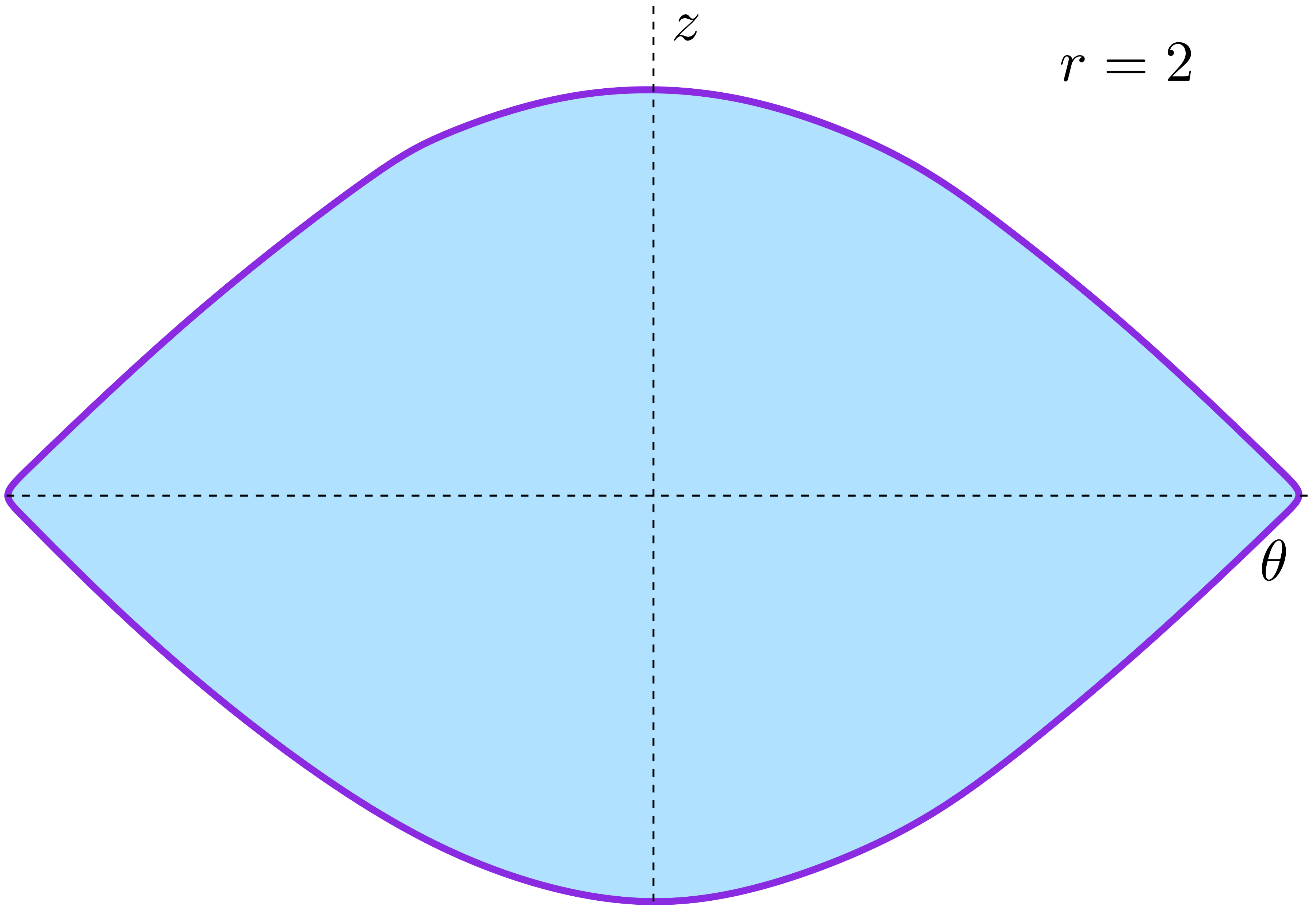




Intersection sphere/cylinder

big radius





Intersection sphere/cylinder

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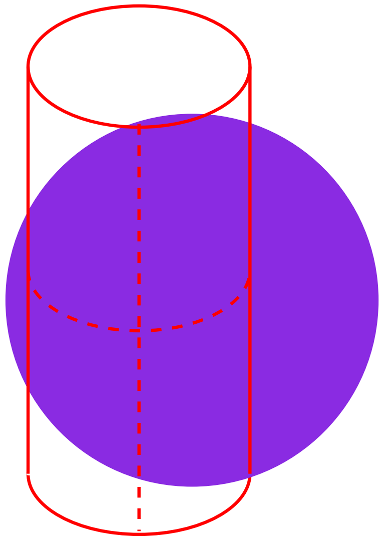

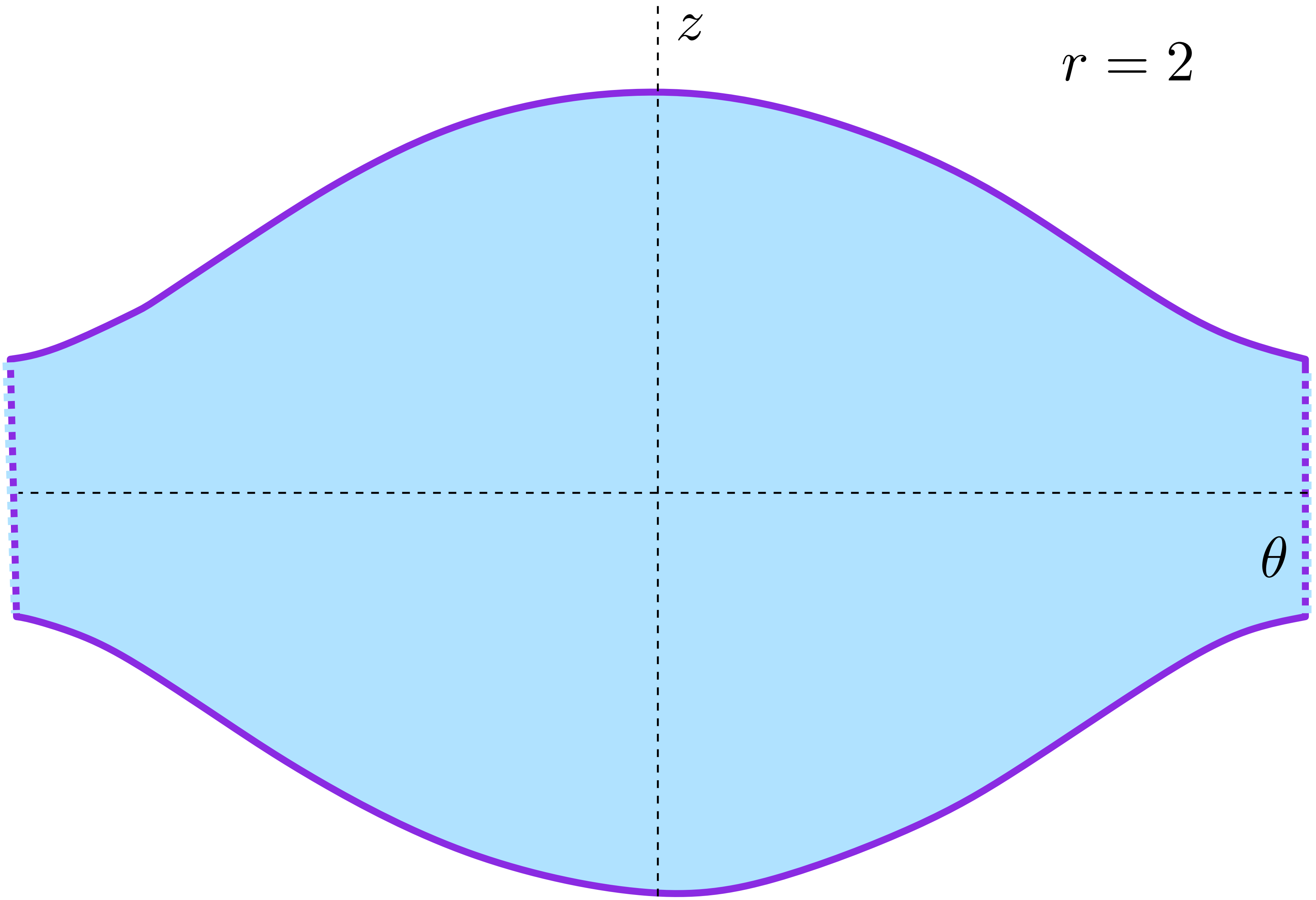

Intersection sphere/cylinder

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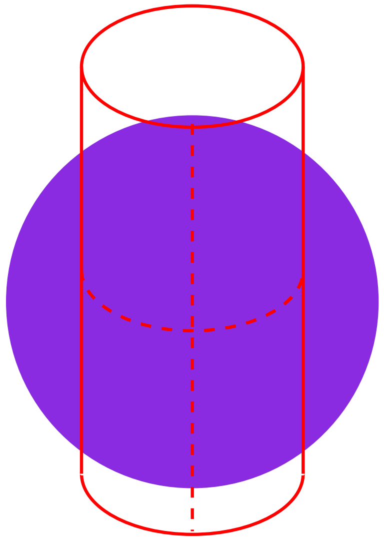

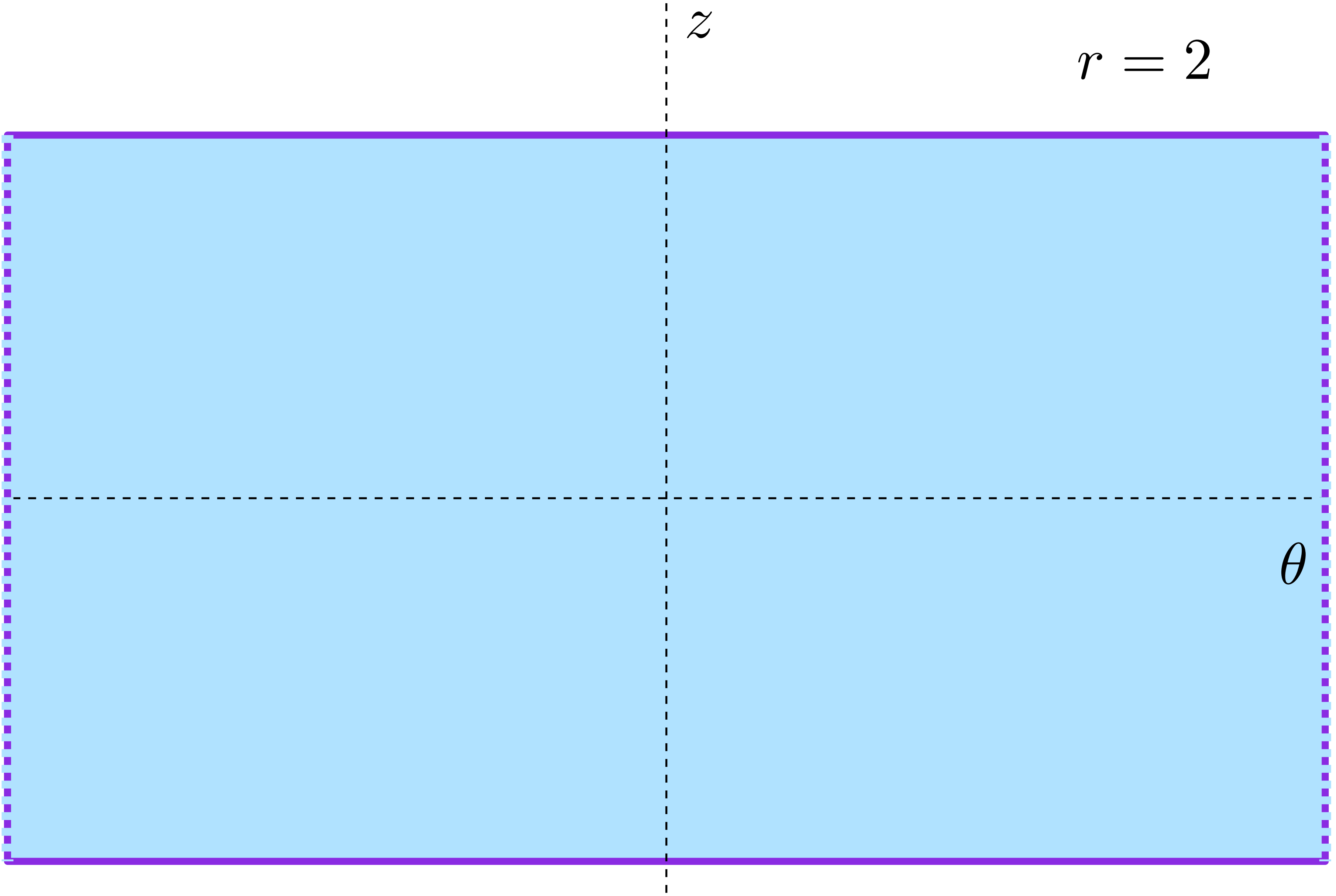

Intersection sphere/cylinder

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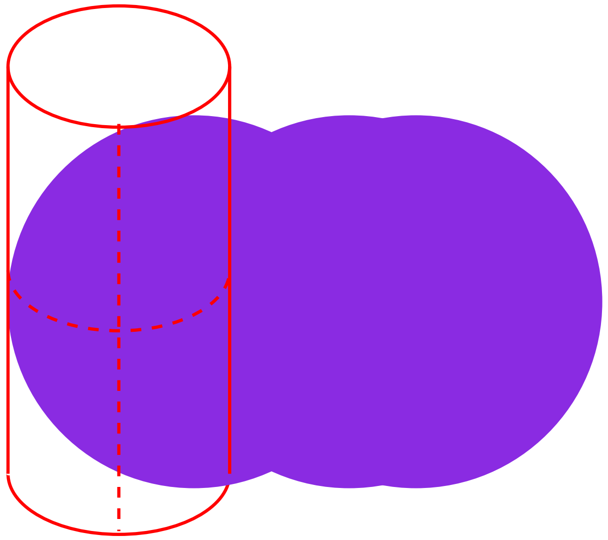
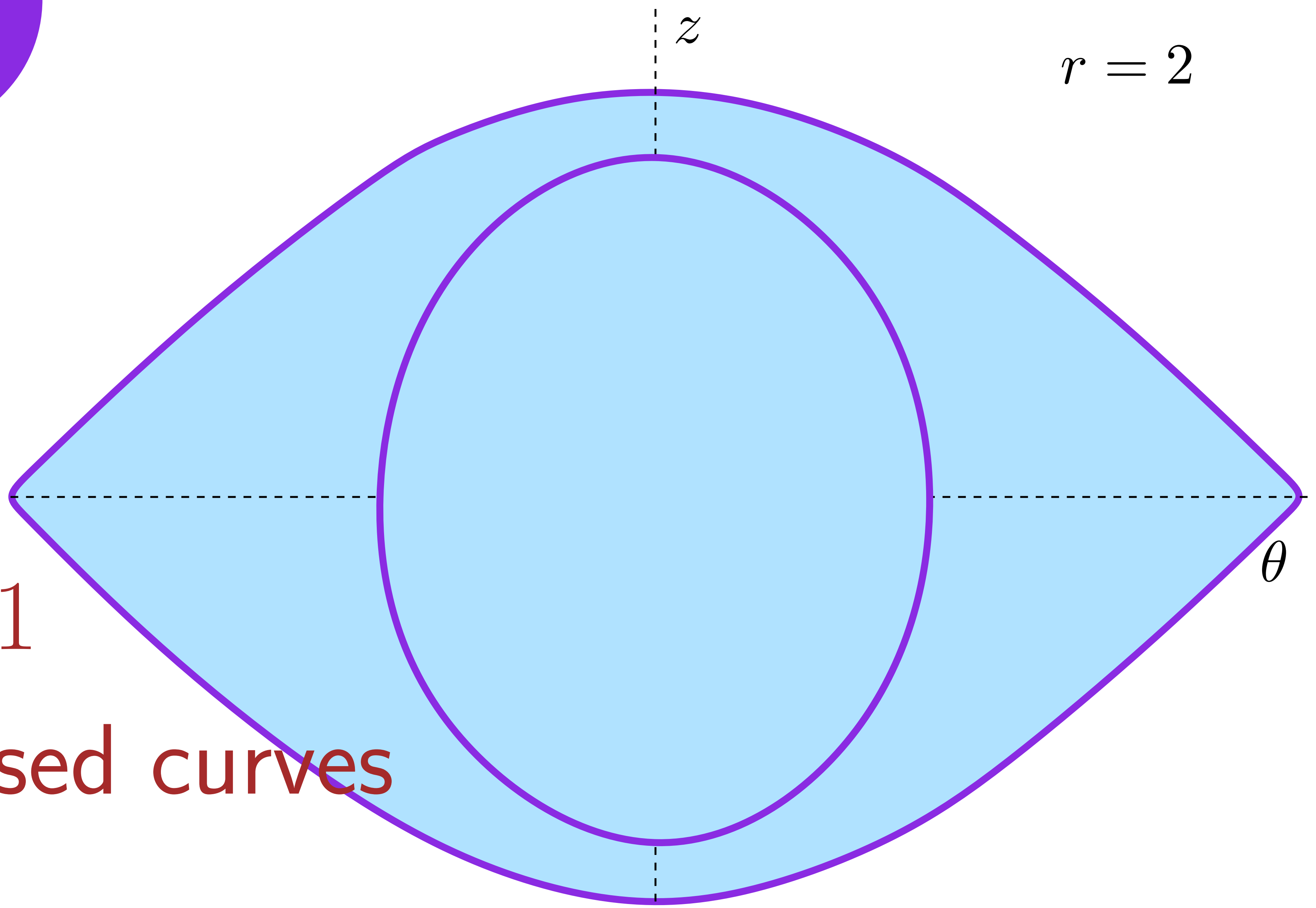


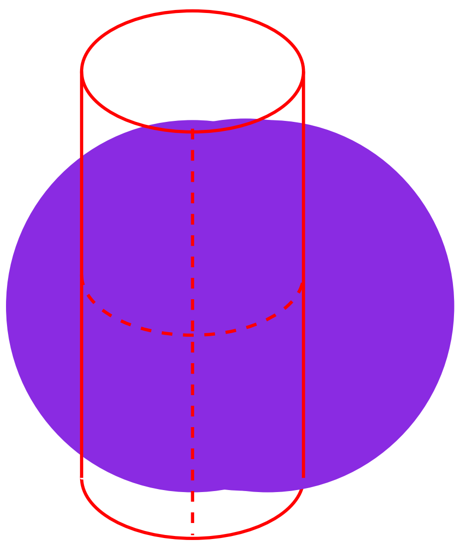


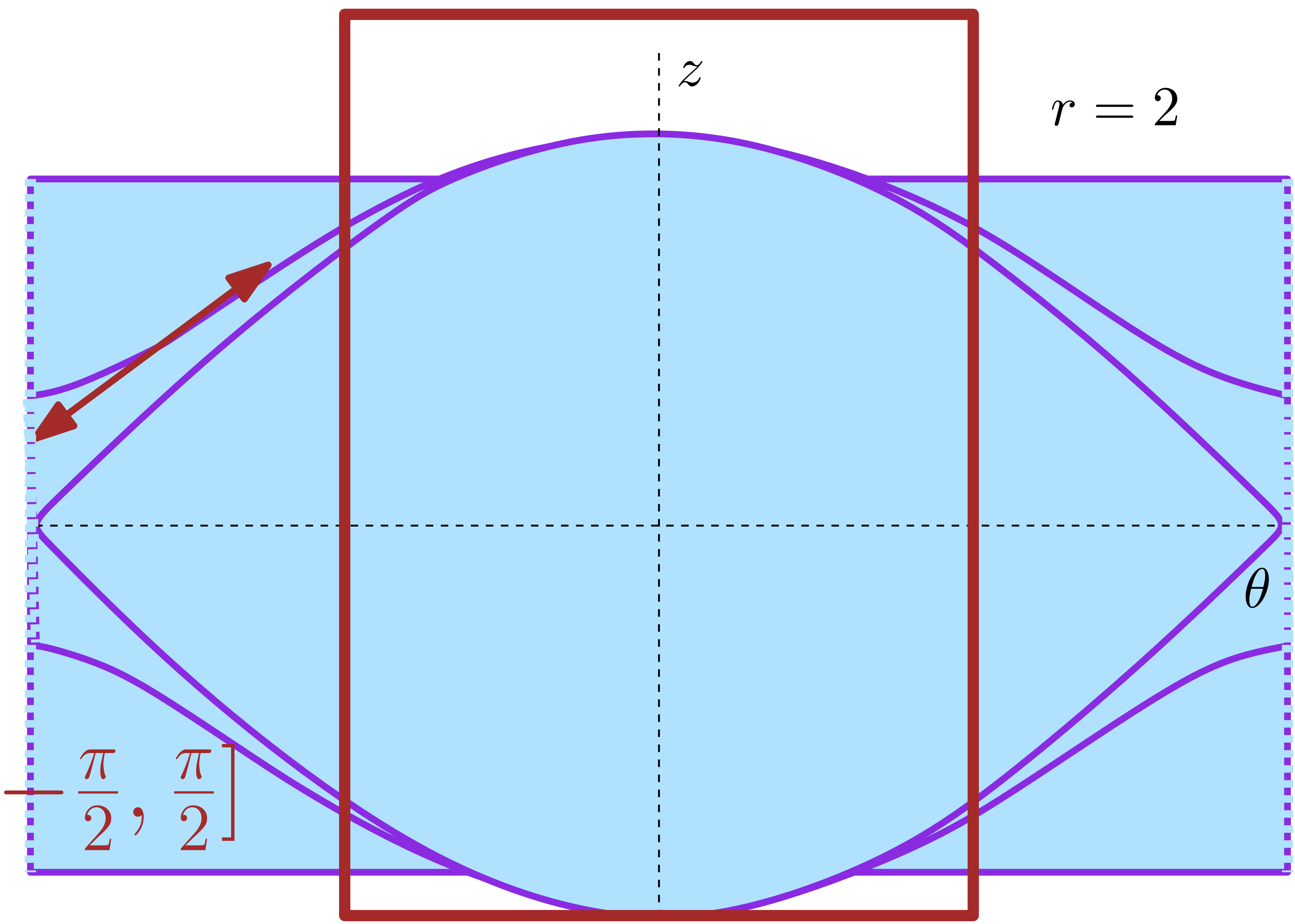
$r \leq R + 1$

convex closed curves

Intersection sphere/cylinder

big radius

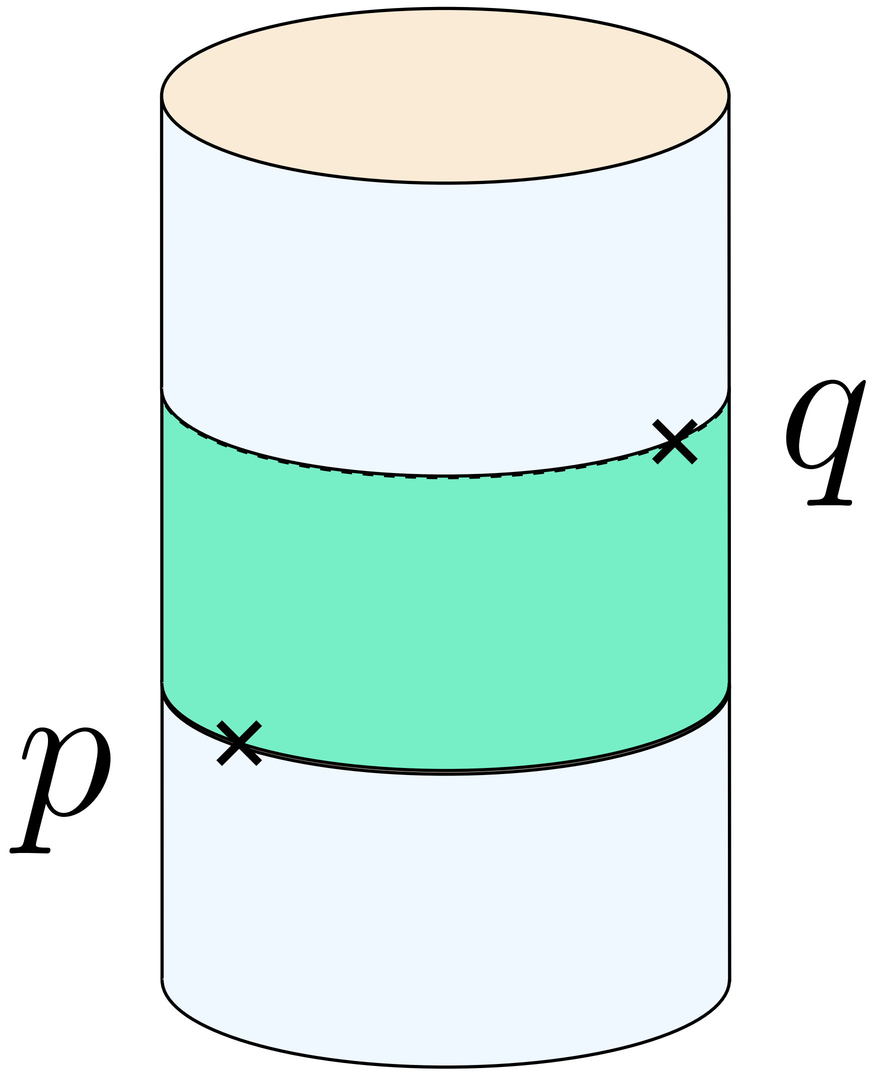




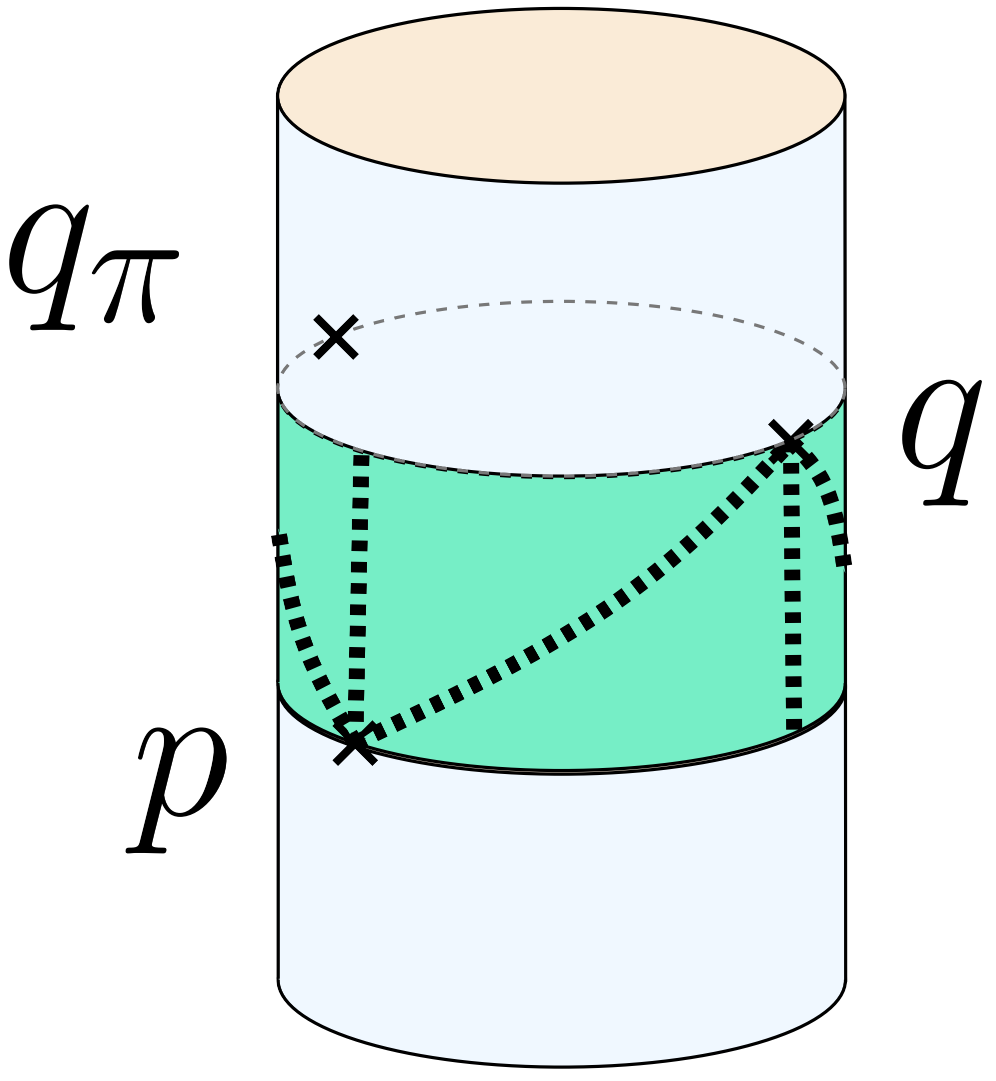
$r \geq R + 1$

convex on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

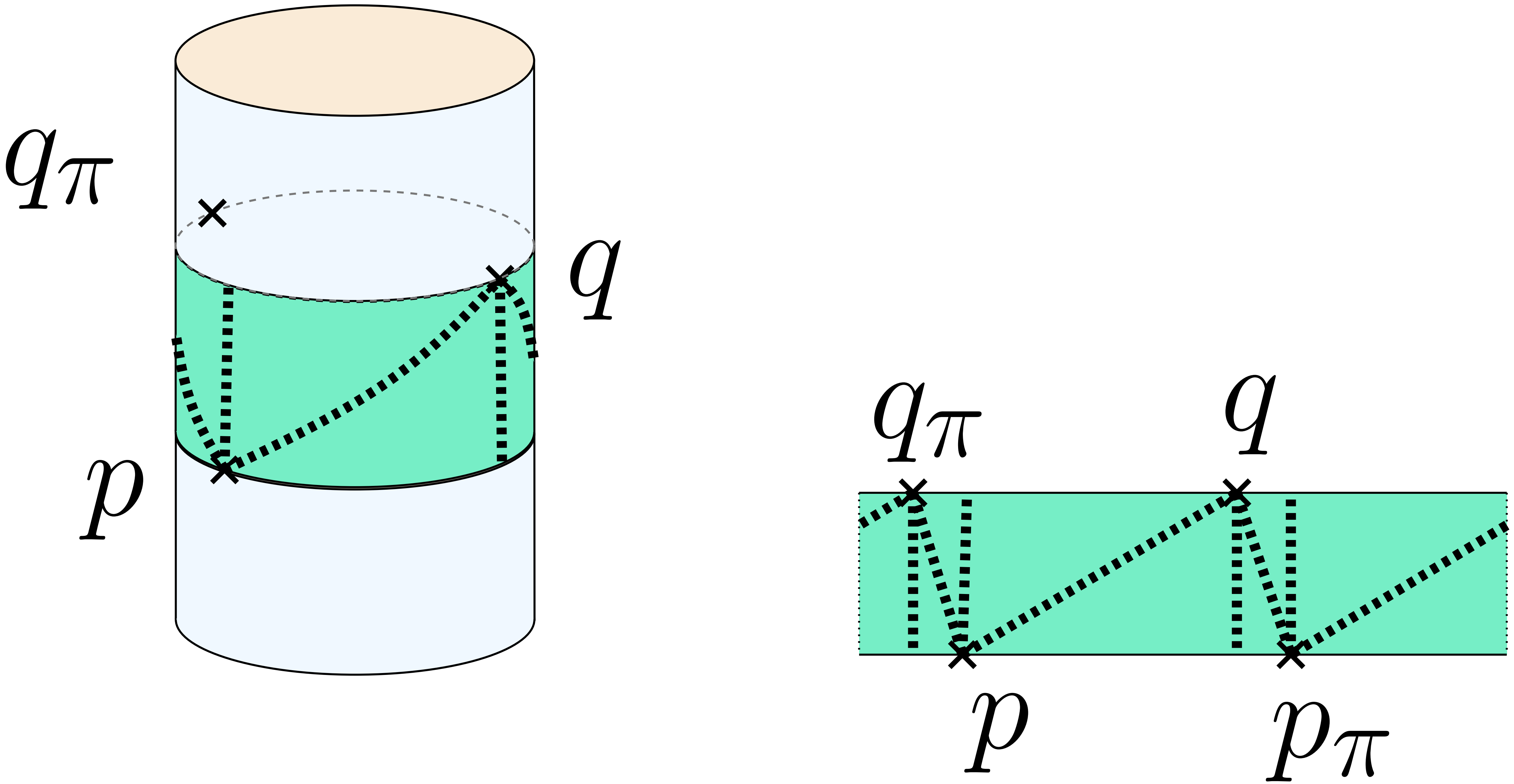
Triangulated slab graph



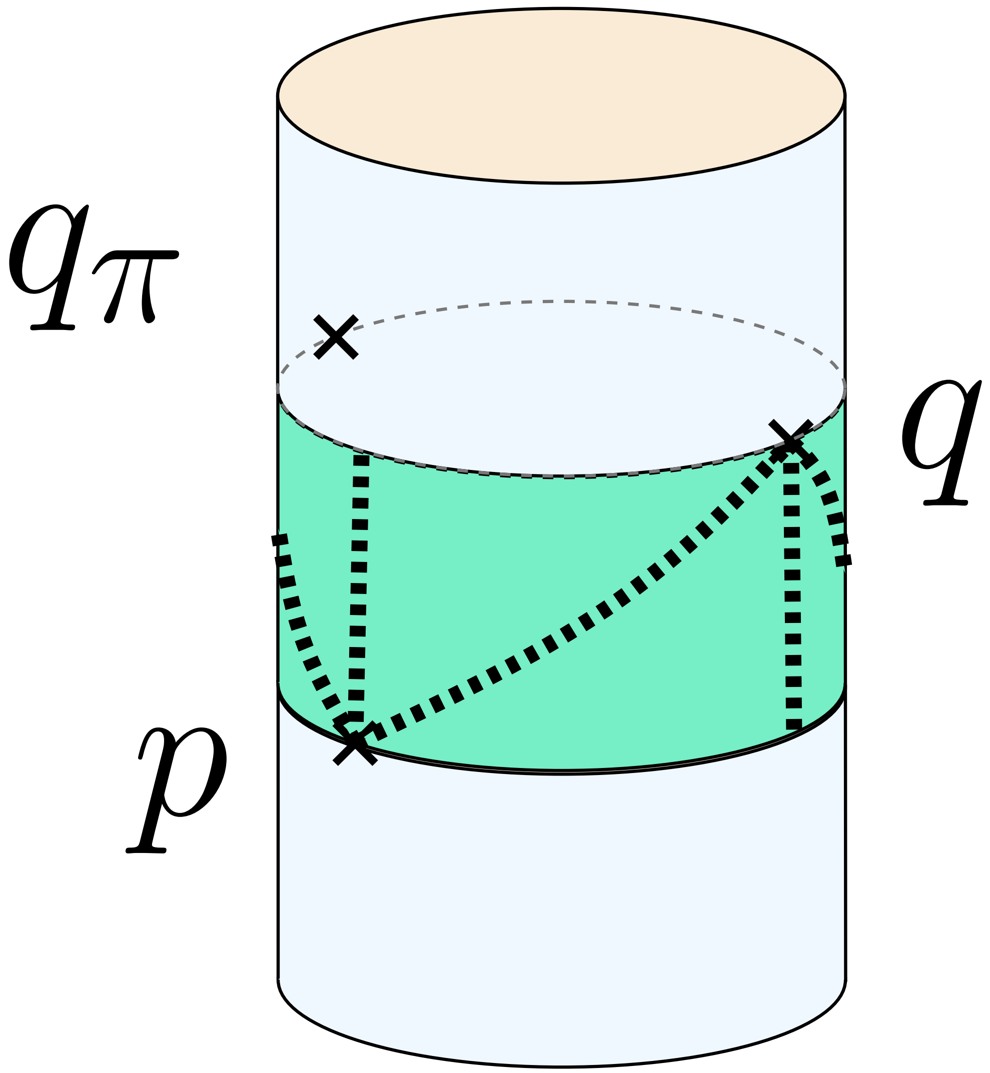
Triangulated slab graph

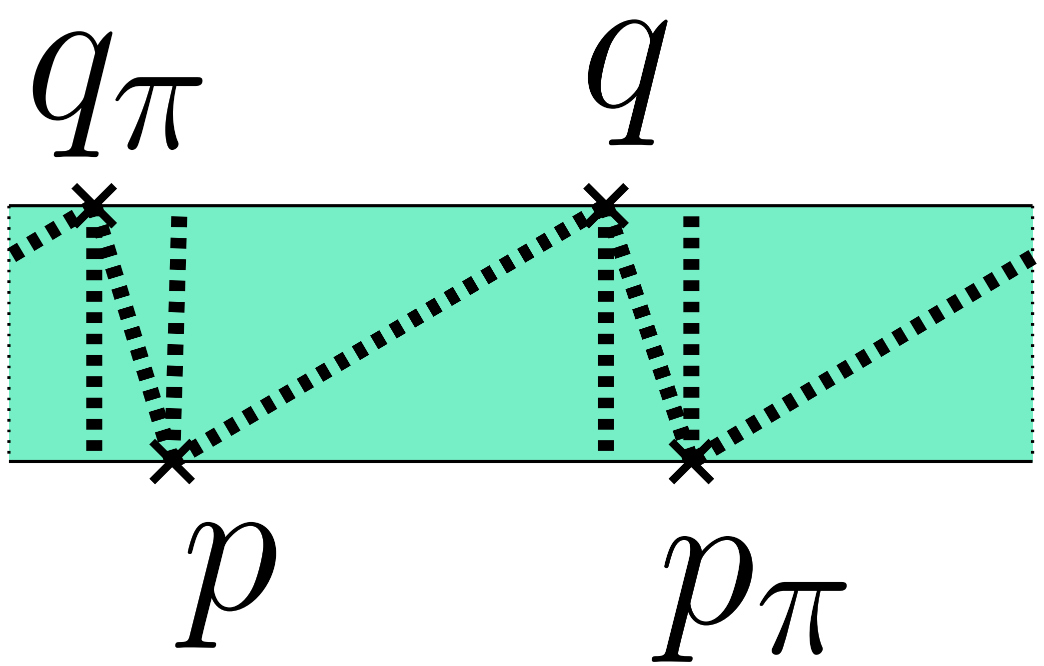


Triangulated slab graph



Triangulated slab graph





pq edge of TSG

if one of the 8 triangles empty

Delaunay $\subset$ TSG

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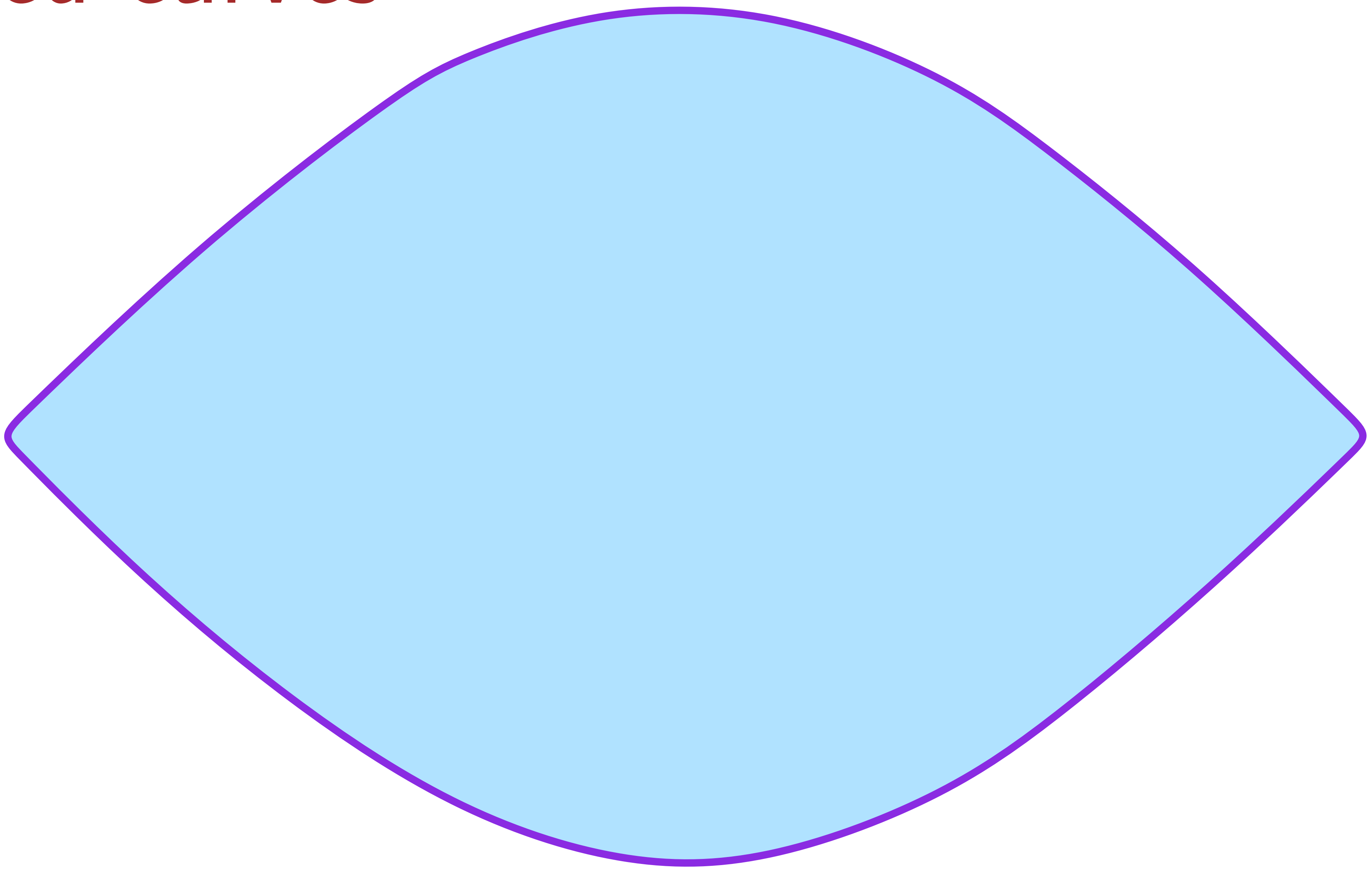
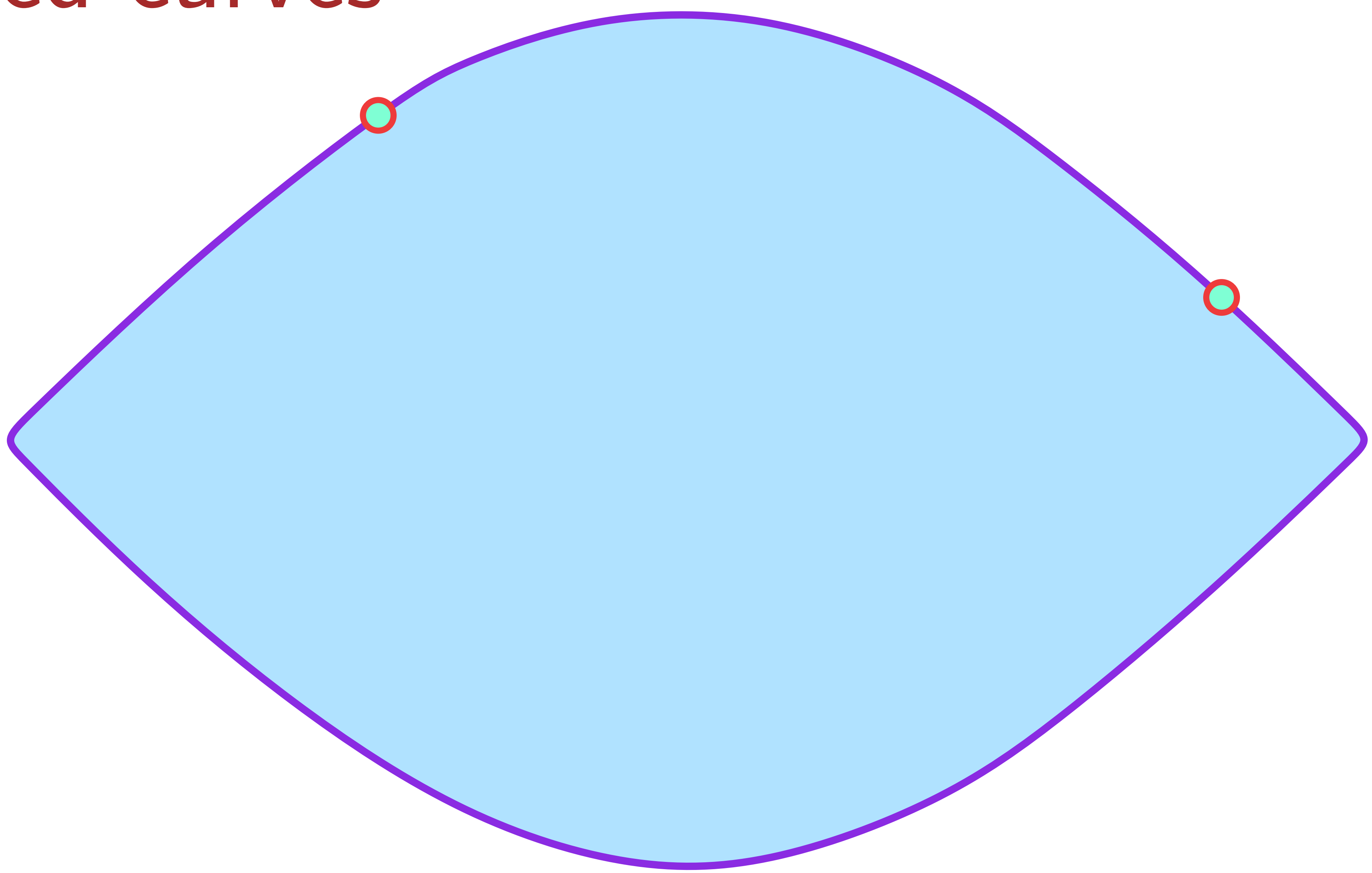
convex closed curves



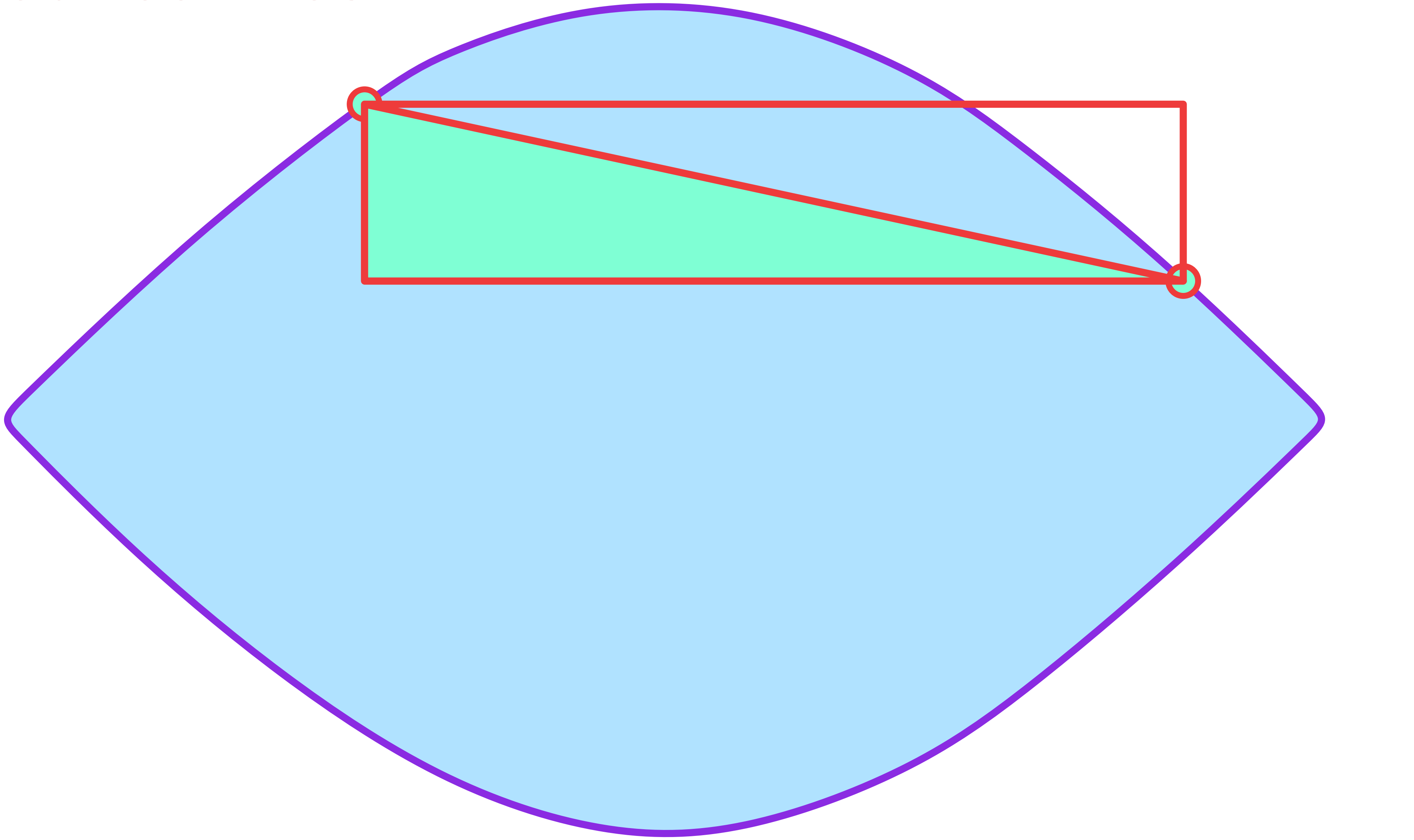
Delaunay $\subset$ TSG

convex closed curves



Delaunay $\subset$ TSG

convex closed curves



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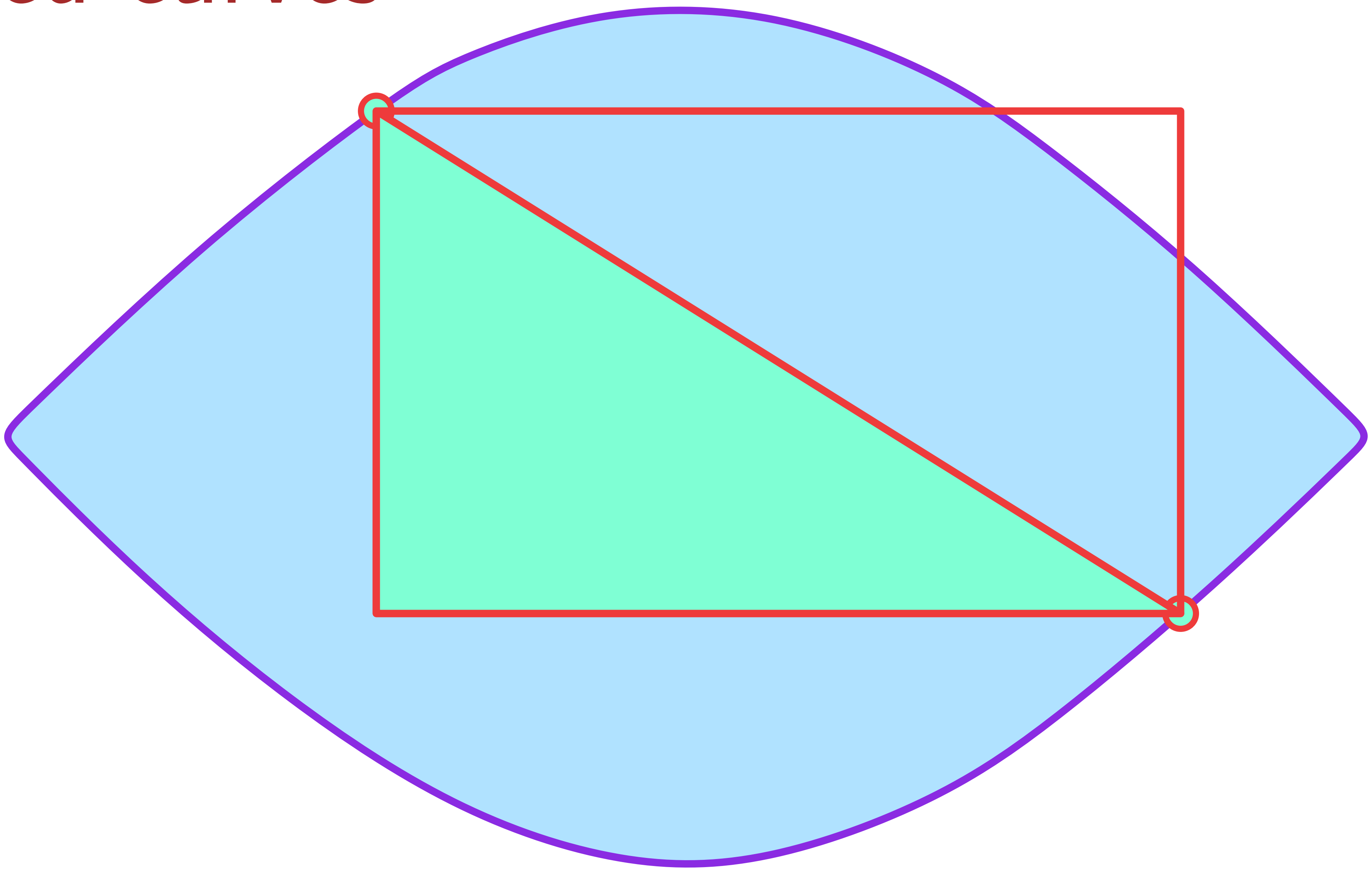
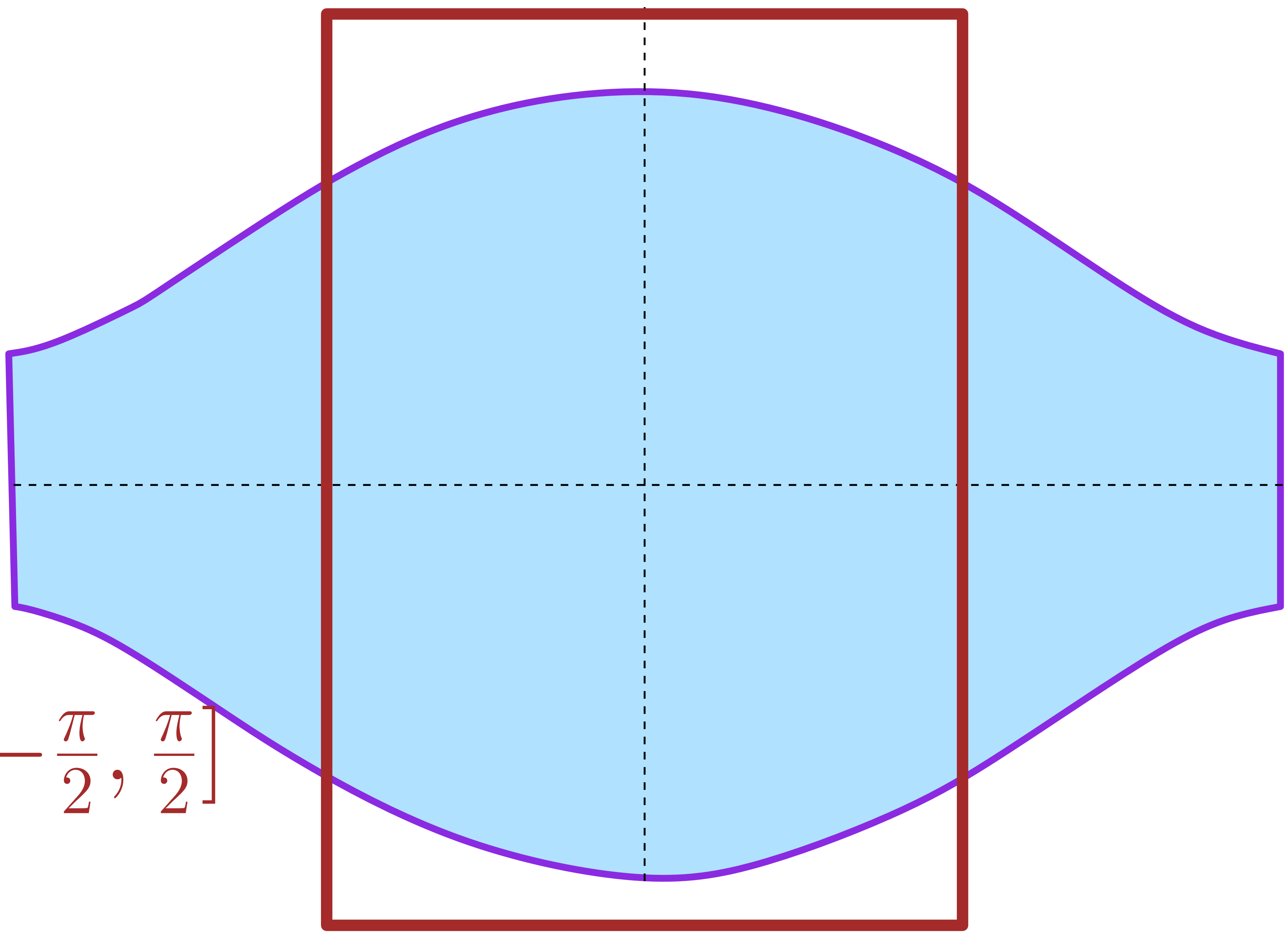


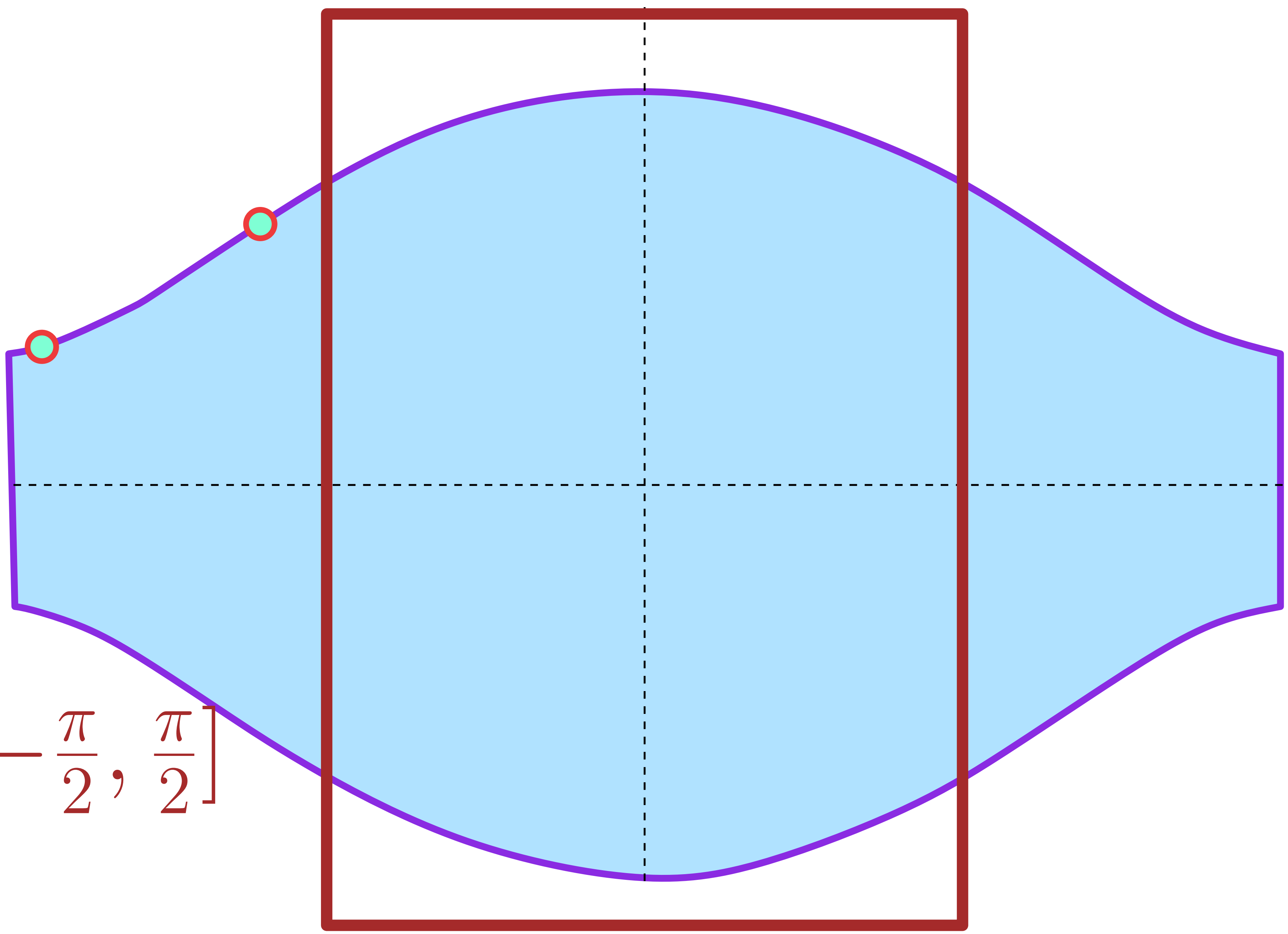
Delaunay $\subset$ TSG



$r \geq R + 1$

convex on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Delaunay $\subset$ TSG



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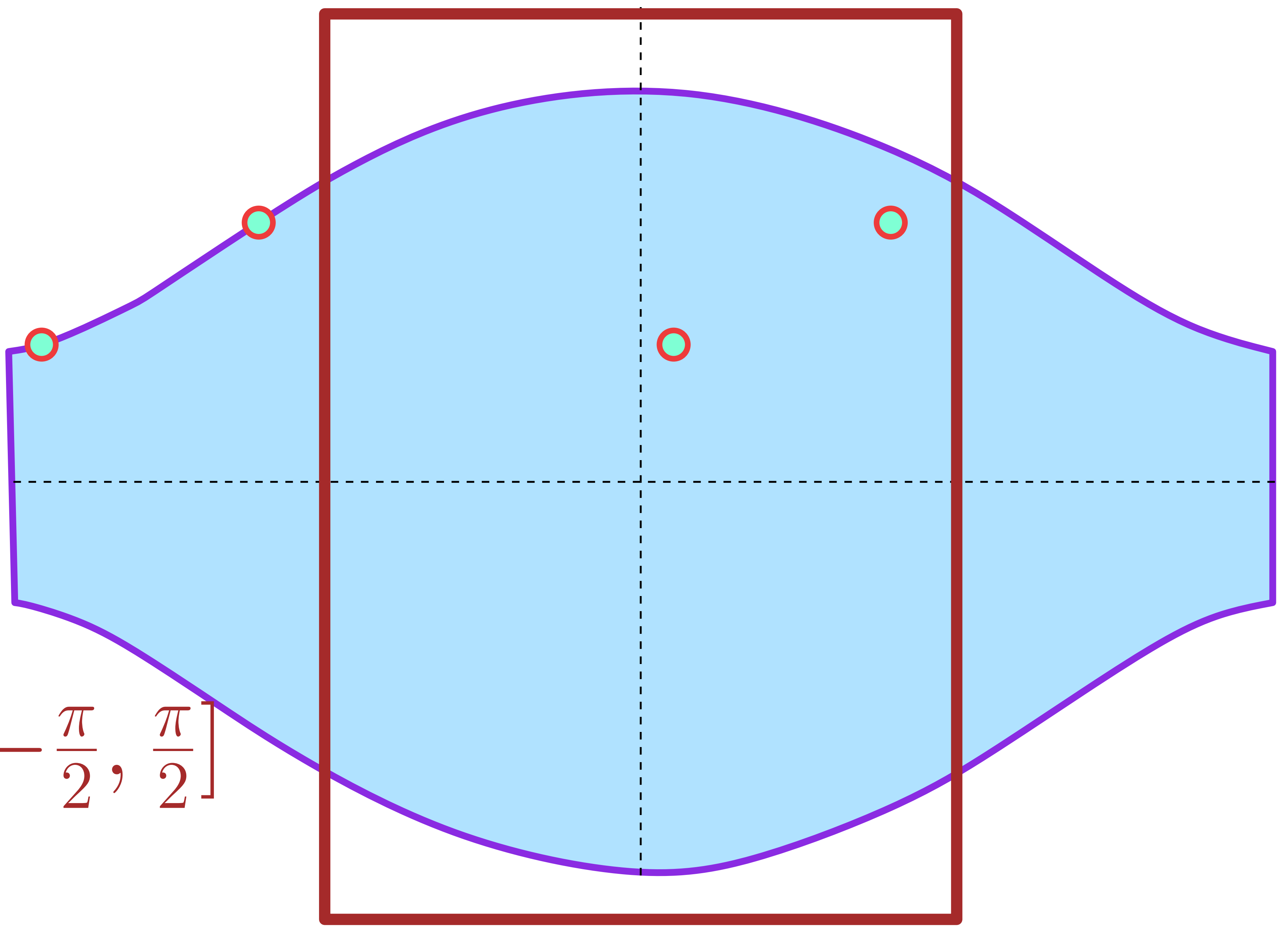
Delaunay ⊂ TSG



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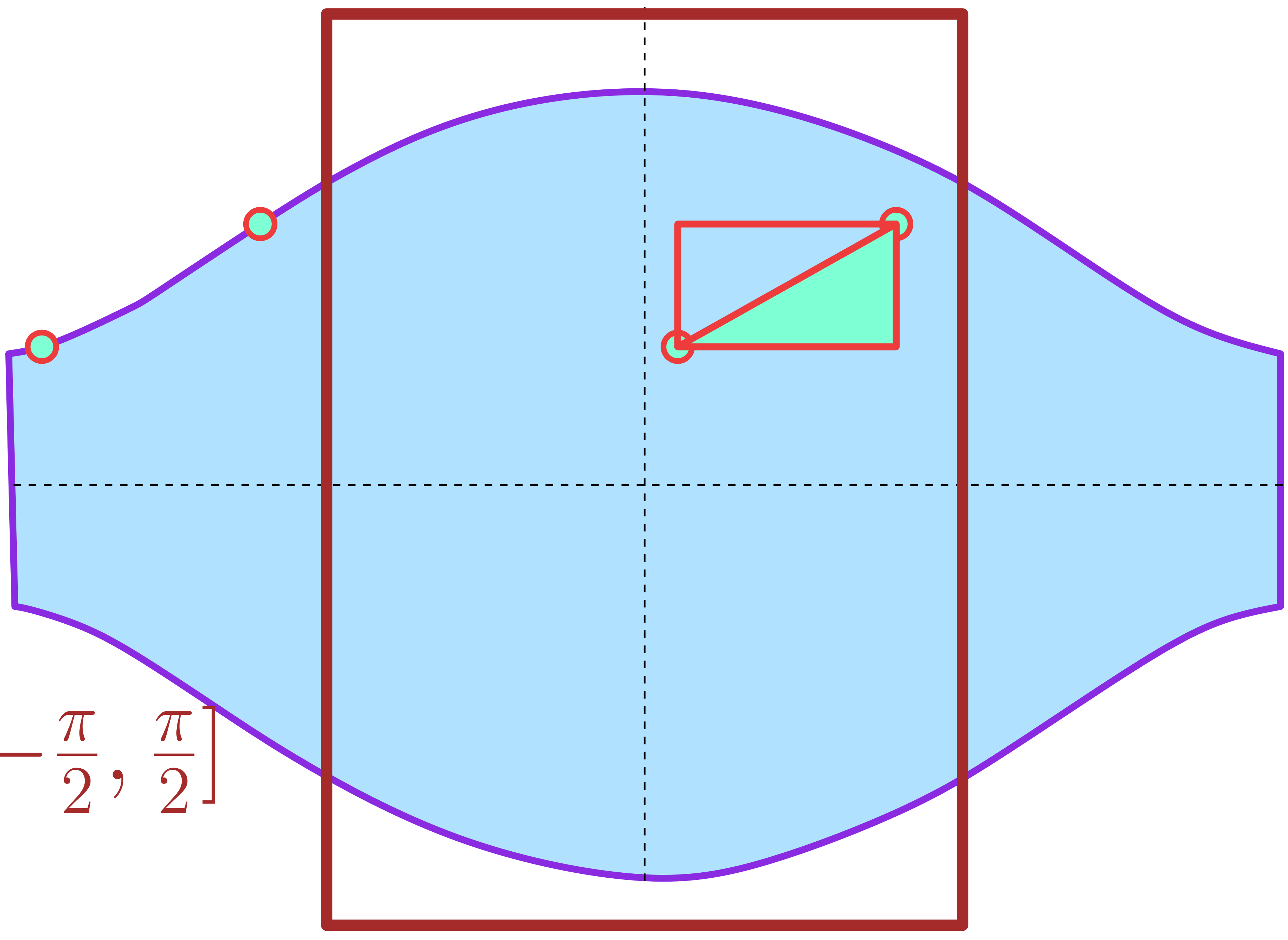

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$$|\mathsf{TSG}| = O(n \log n)$$

$Pr(pq \in \mathsf{TSG})$

$$|\mathsf{TSG}| = O(n \log n)$$

$$Pr(pq \in \mathsf{TSG}) = \int_{z_q} \int_{\theta_q} Pr(pq \in \mathsf{TSG}) d\theta dz$$

$$|\mathsf{TSG}| = O(n \log n)$$

$$
\begin{aligned}
&Pr(pq \in \mathsf{TSG}) \\
&= \int_{z_q} \int_{\theta_q} Pr(pq \in \mathsf{TSG})\, d\theta dz \\
&\leq O\left(\int \int Pr(\text{first of the 8 triangle is empty})\, d\theta dz \right)
\end{aligned}
$$

$|\mathsf{TSG}| = O(n \log n)$

$$
\begin{aligned}
&Pr(pq \in \mathsf{TSG}) \\
&= \int_{z_q}\int_{\theta_q} Pr(pq \in \mathsf{TSG})\, d\theta dz \\
&\leq O\left(\int\int Pr(\text{first of the 8 triangle is empty})\, d\theta dz\right) \\
&\leq O\left(\int\int (1 - z\theta)^{n-2}\, d\theta dz\right)
\end{aligned}
$$

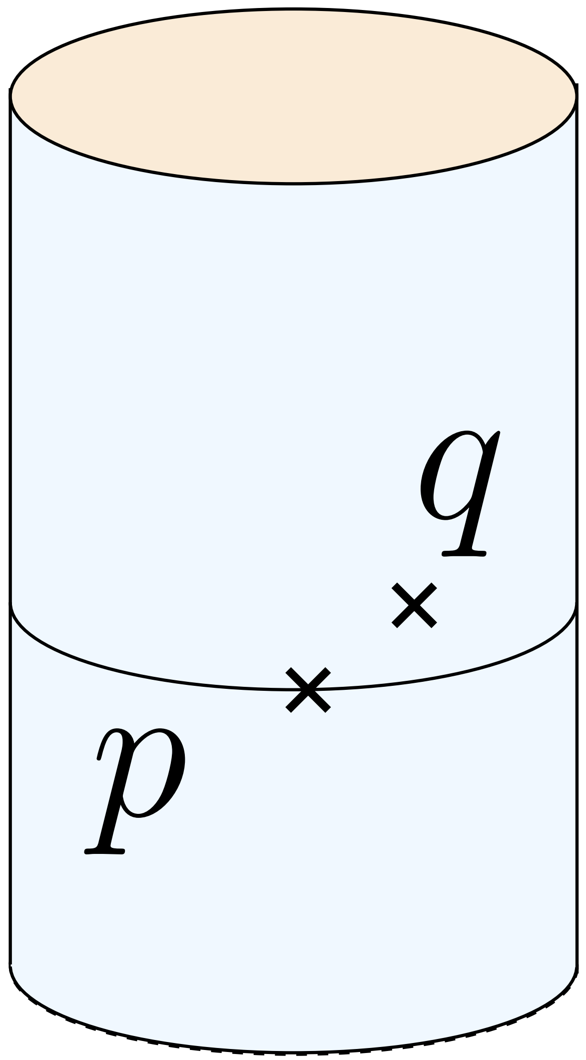
$$|\mathsf{TSG}| = O(n \log n)$$

$$
\begin{aligned}
&Pr(pq \in \mathsf{TSG}) \\
&= \int_{z_q} \int_{\theta_q} Pr(pq \in \mathsf{TSG}) d\theta dz \\
&\leq O\left(\int \int Pr(\text{first of the 8 triangle is empty}) d\theta dz\right) \\
&\leq O\left(\int \int (1 - z\theta)^{n-2} d\theta dz\right) \\
&\leq O\left(\int \int e^{nz\theta} d\theta dz\right)
\end{aligned}
$$

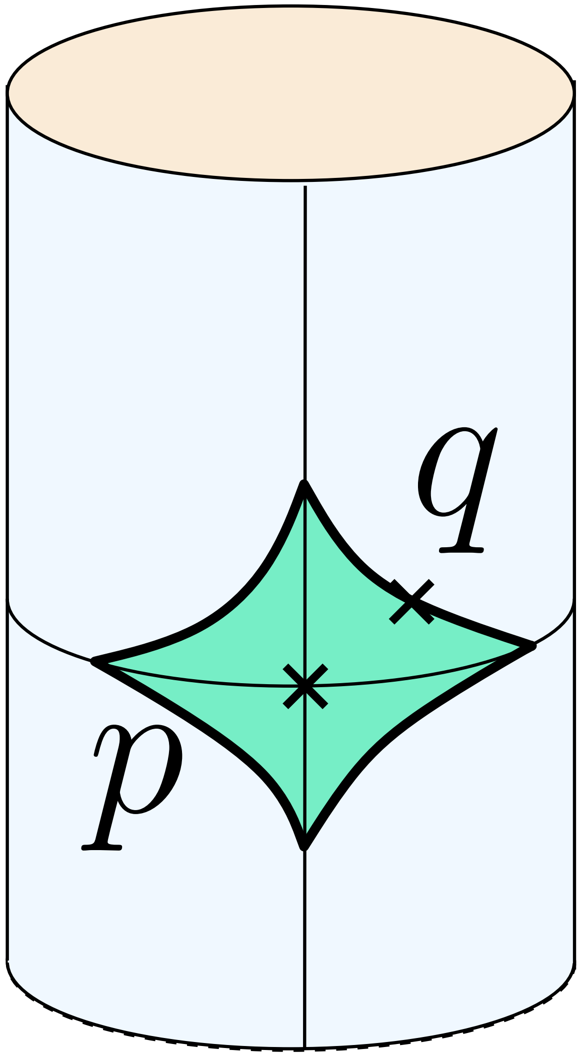
$$|\mathsf{TSG}| = O(n \log n)$$

$$\begin{aligned}
&Pr(pq \in \mathsf{TSG}) \\
&= \int_{z_q}\int_{\theta_q} Pr(pq \in \mathsf{TSG})\,d\theta dz \\
&\leq O\left(\int\int Pr(\text{first of the 8 triangle is empty})\,d\theta dz\right) \\
&\leq O\left(\int\int (1 - z\theta)^{n-2}\,d\theta dz\right) \\
&\leq O\left(\int\int e^{nz\theta}\,d\theta dz\right) \\
&\leq O\left(n \log n\right)
\end{aligned}$$

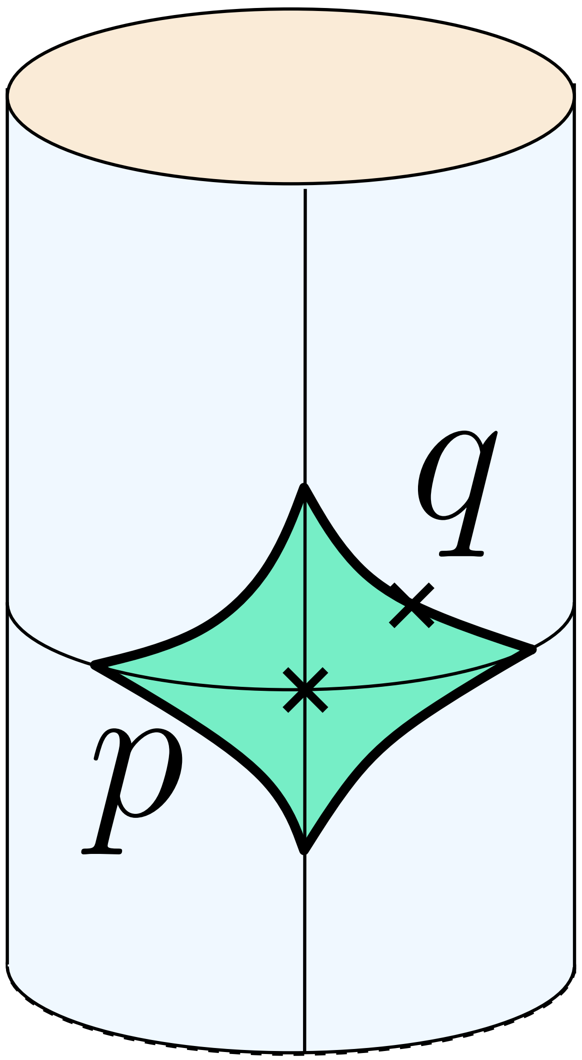
Rhombus graph

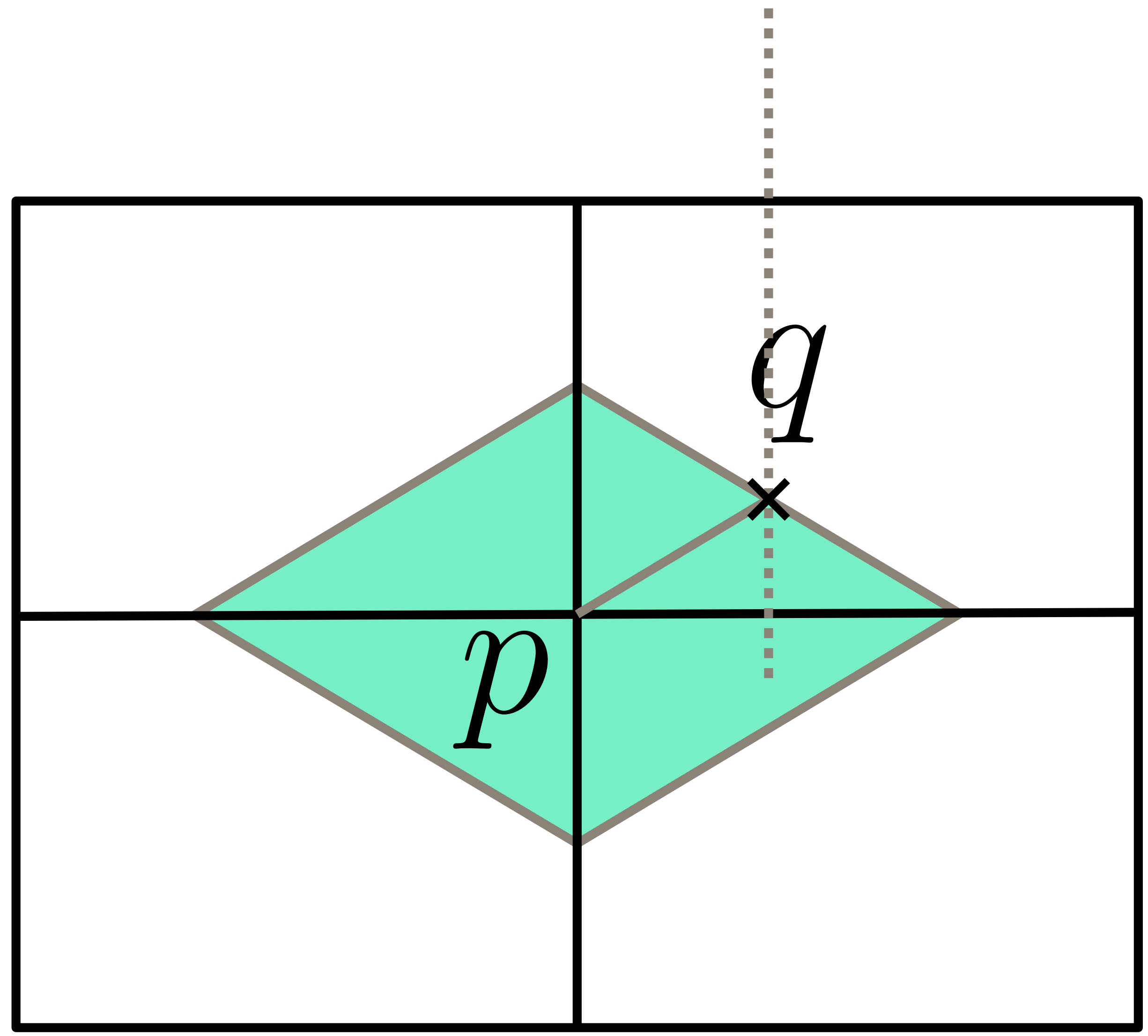


Rhombus graph

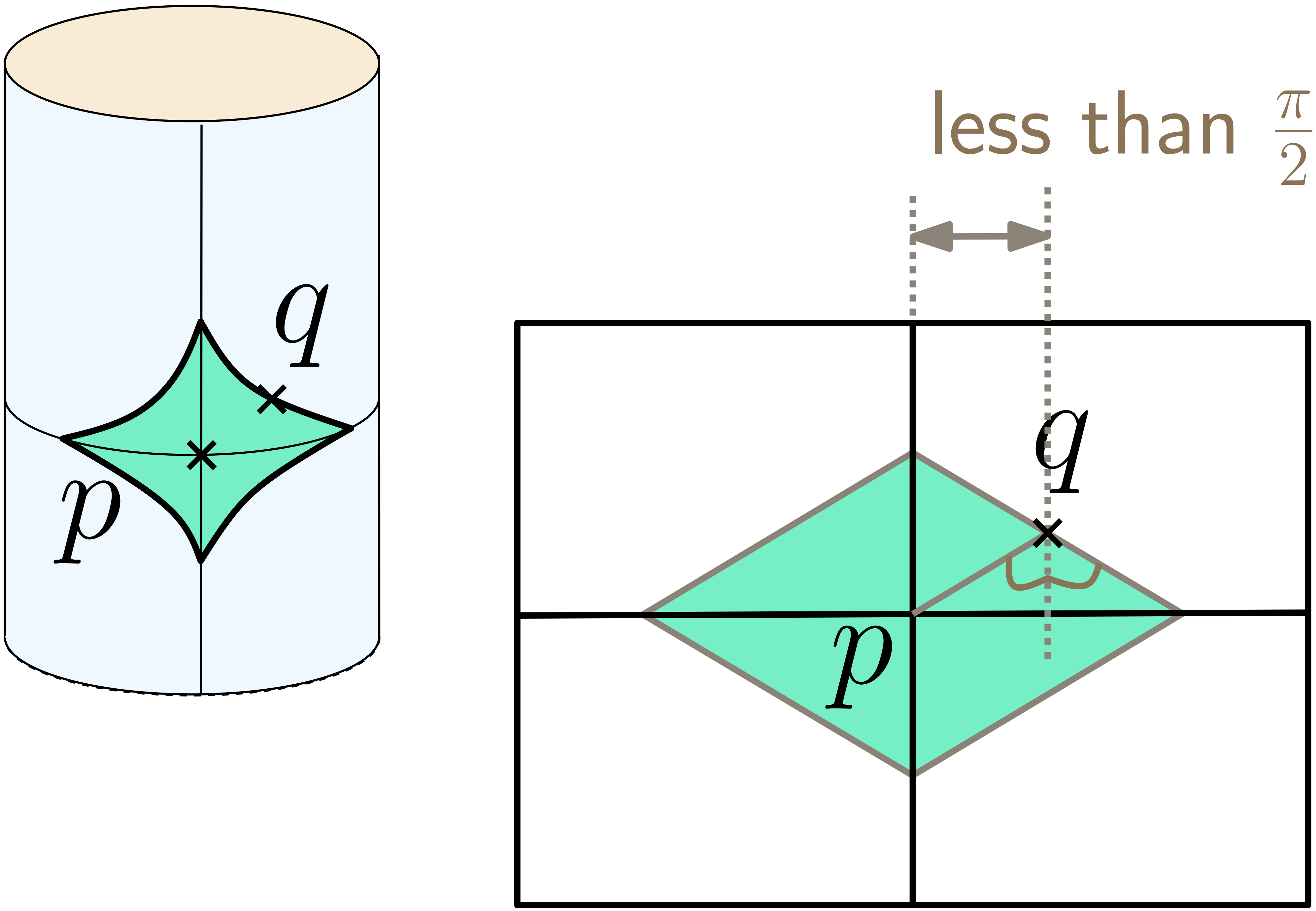


Rhombus graph

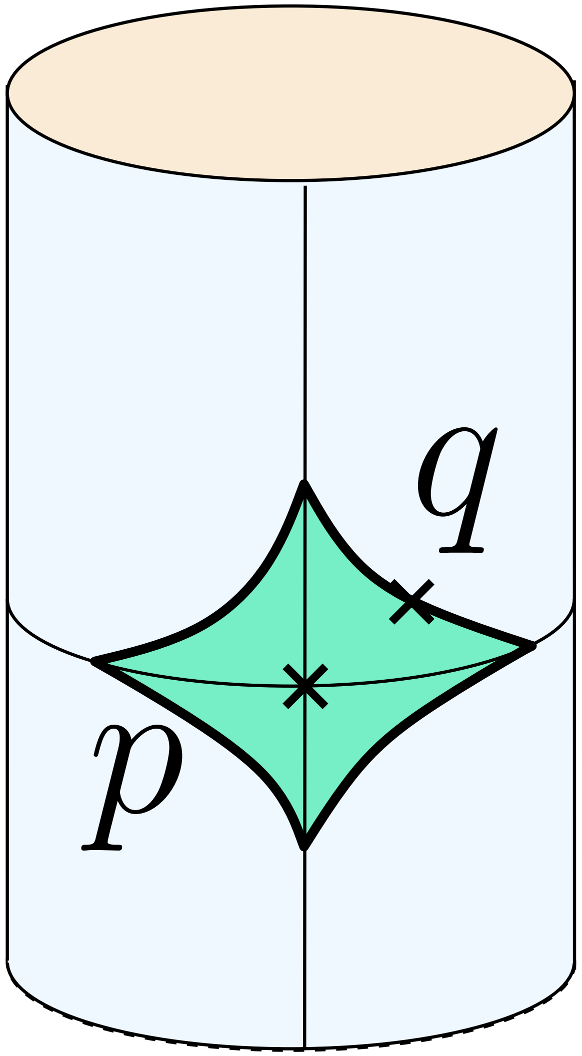


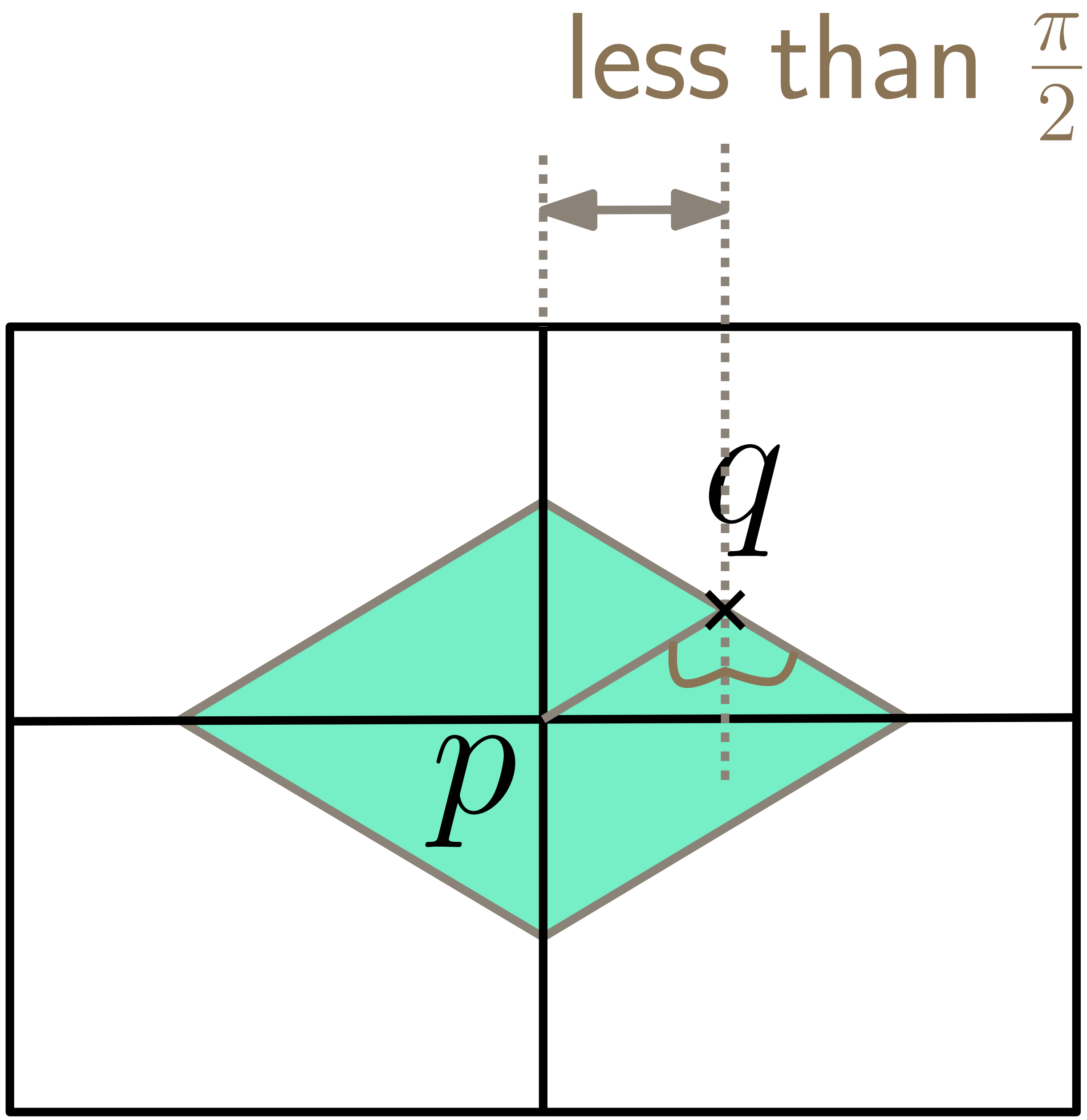


Rhombus graph



Rhombus graph





pq edge of RG

if rhombus empty

RG $\subset$ Delaunay

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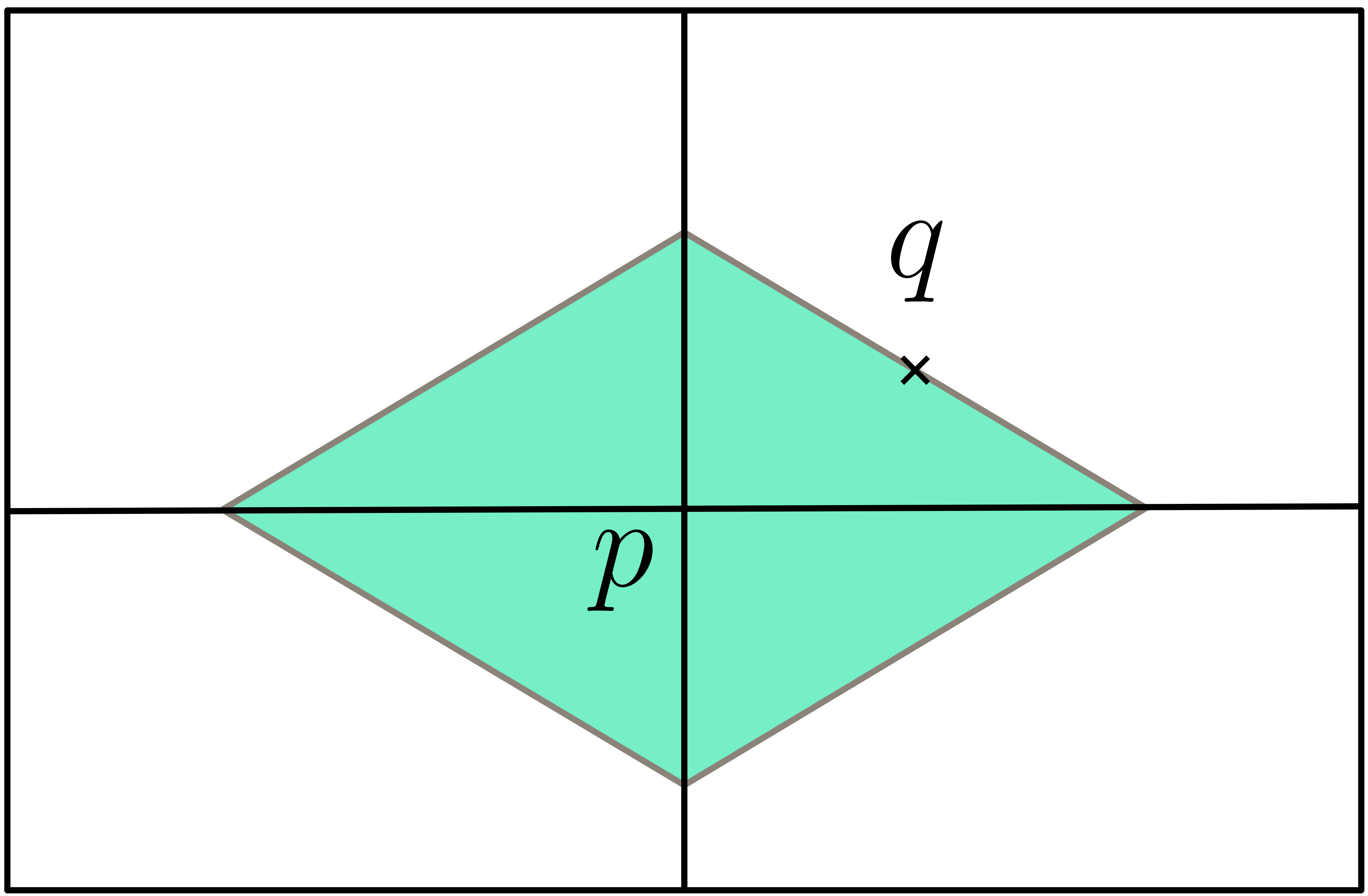
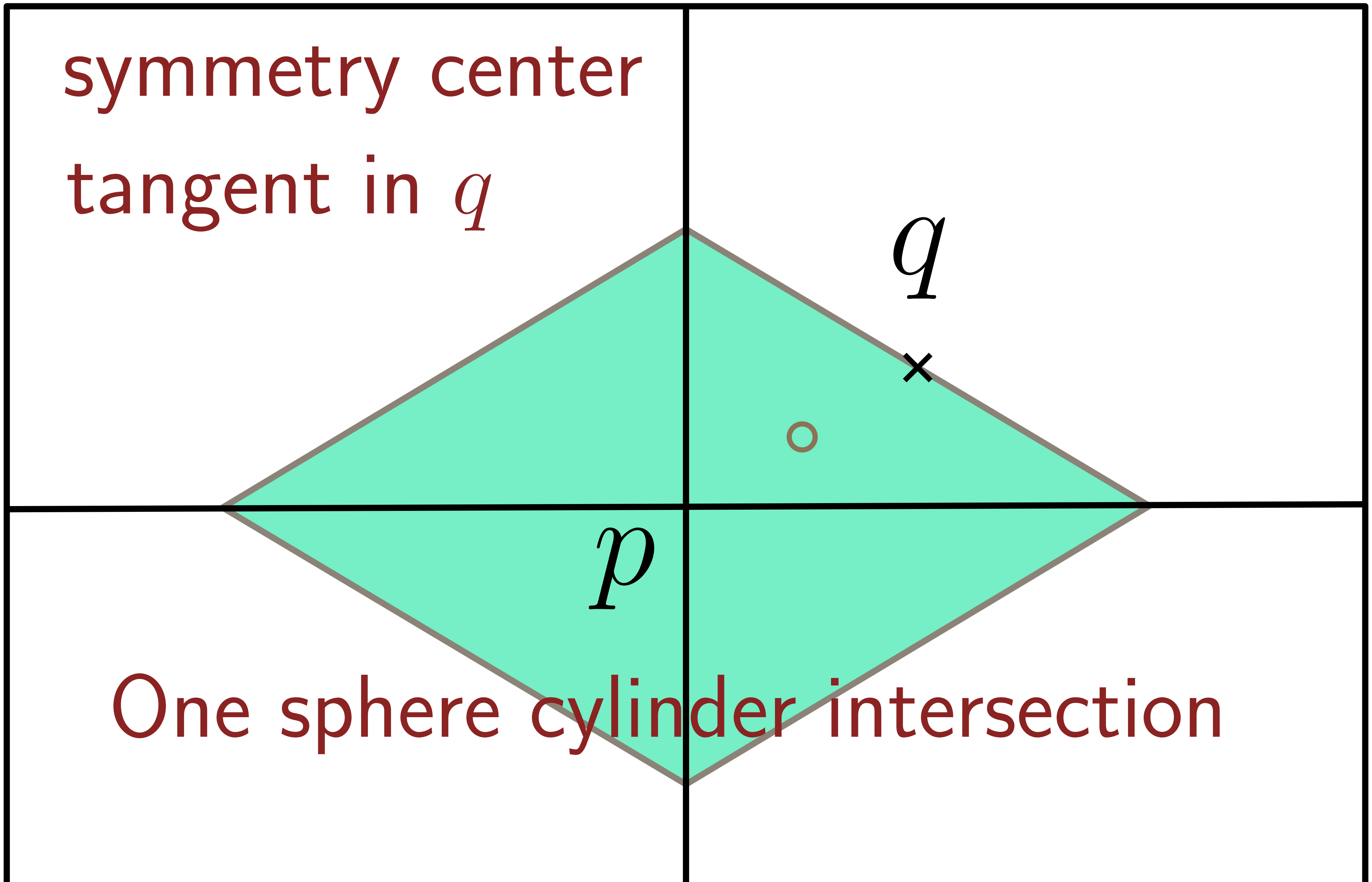
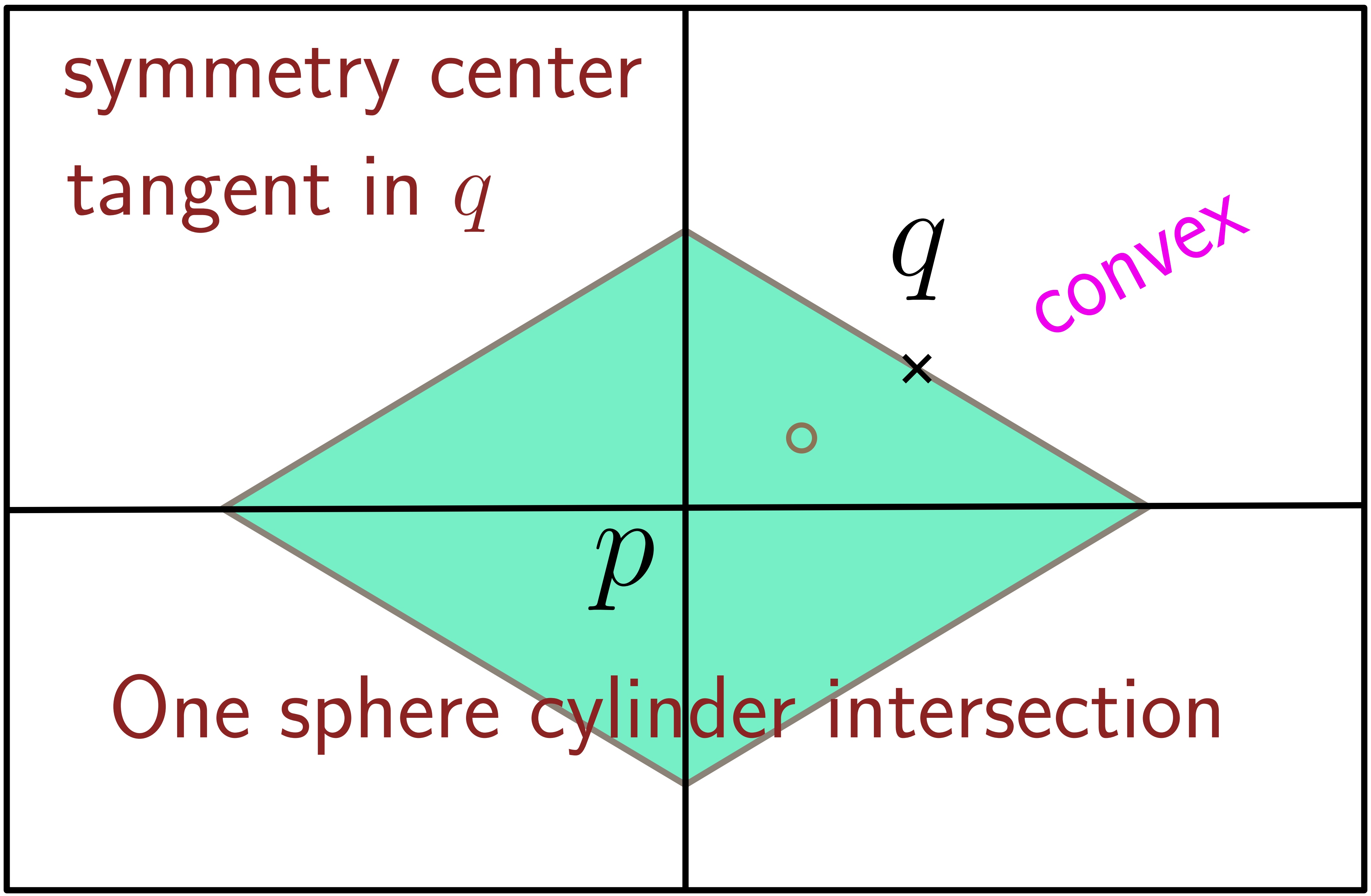


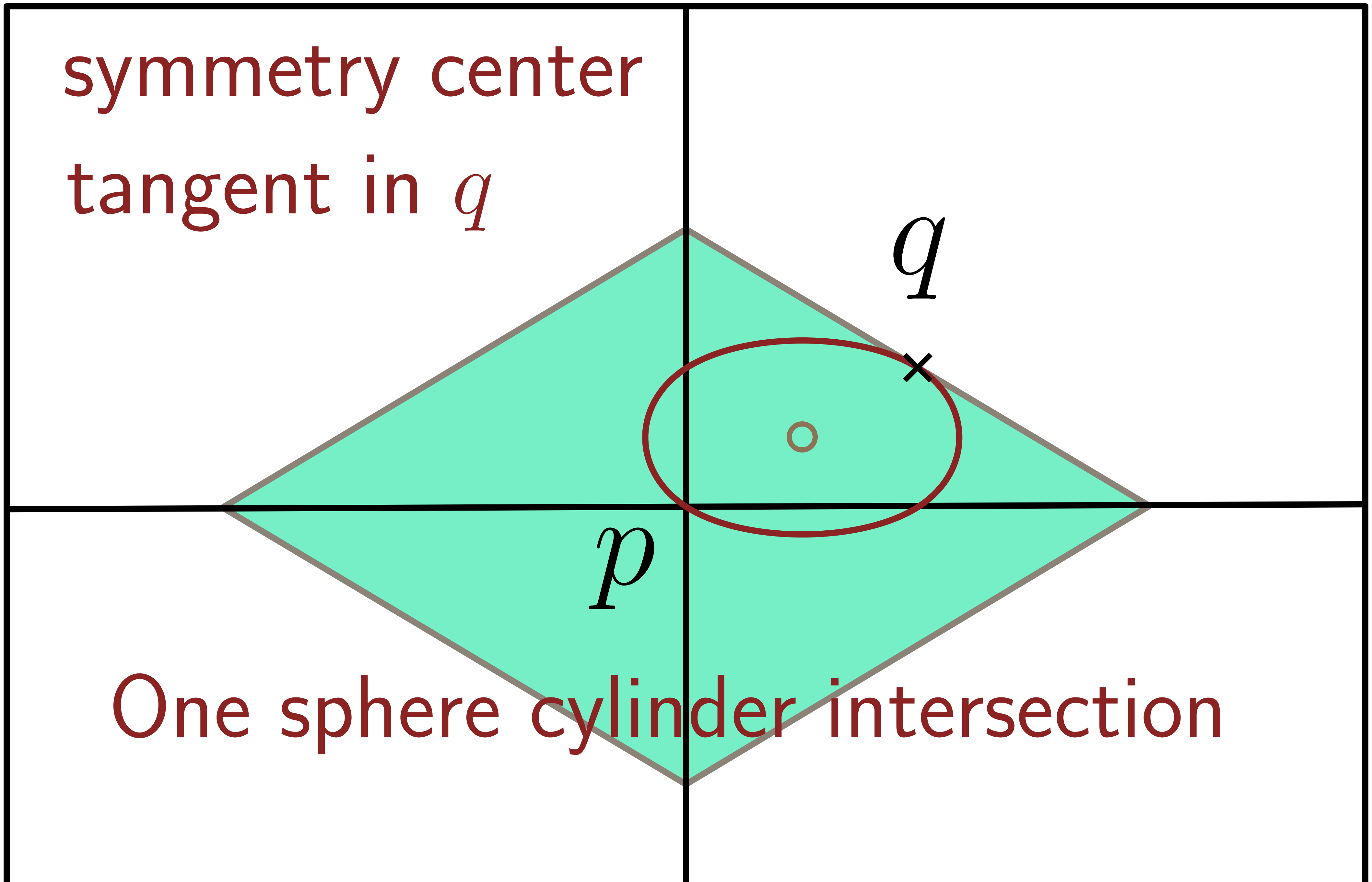
RG $\subset$ Delaunay



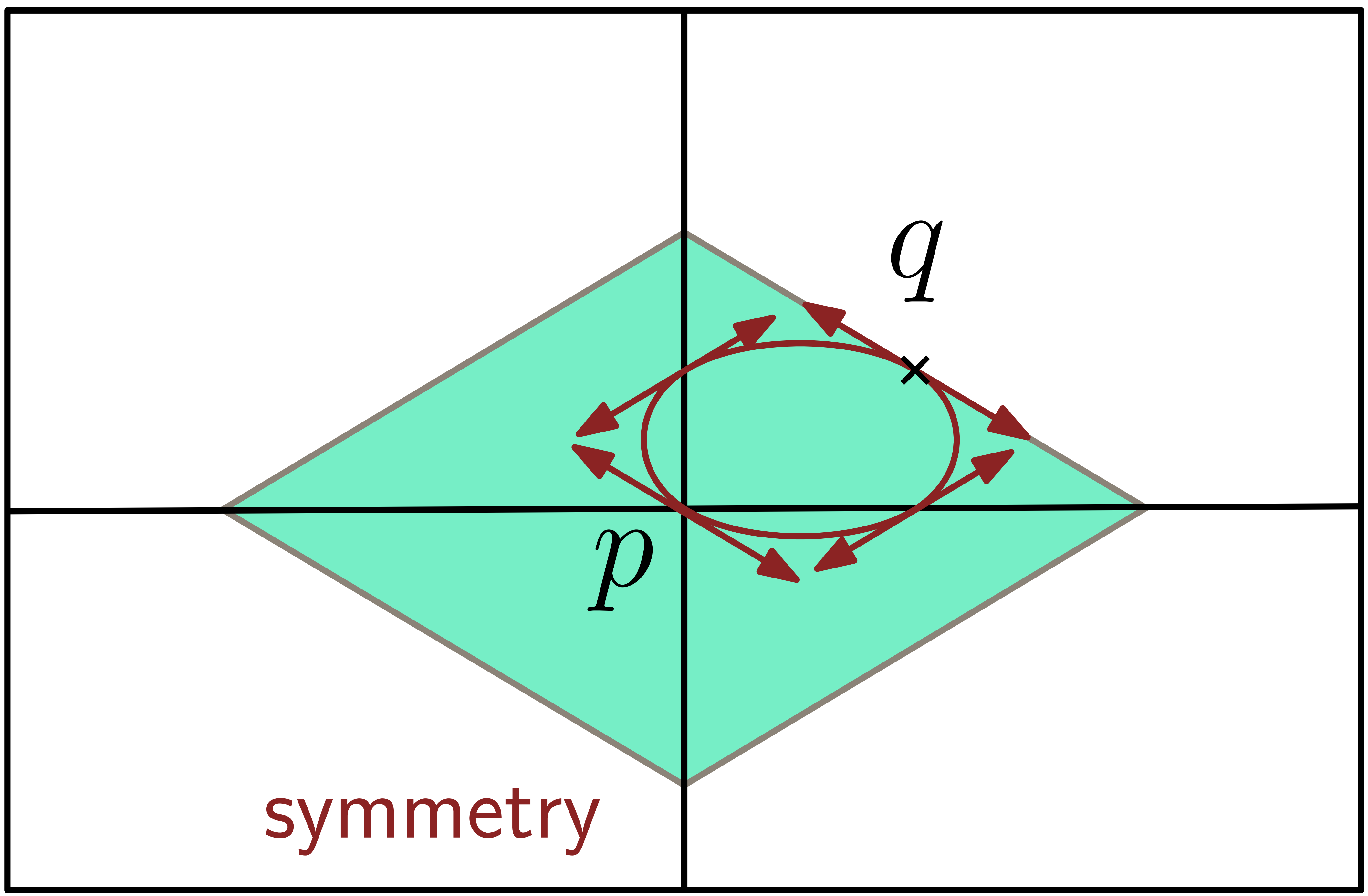
RG $\subset$ Delaunay



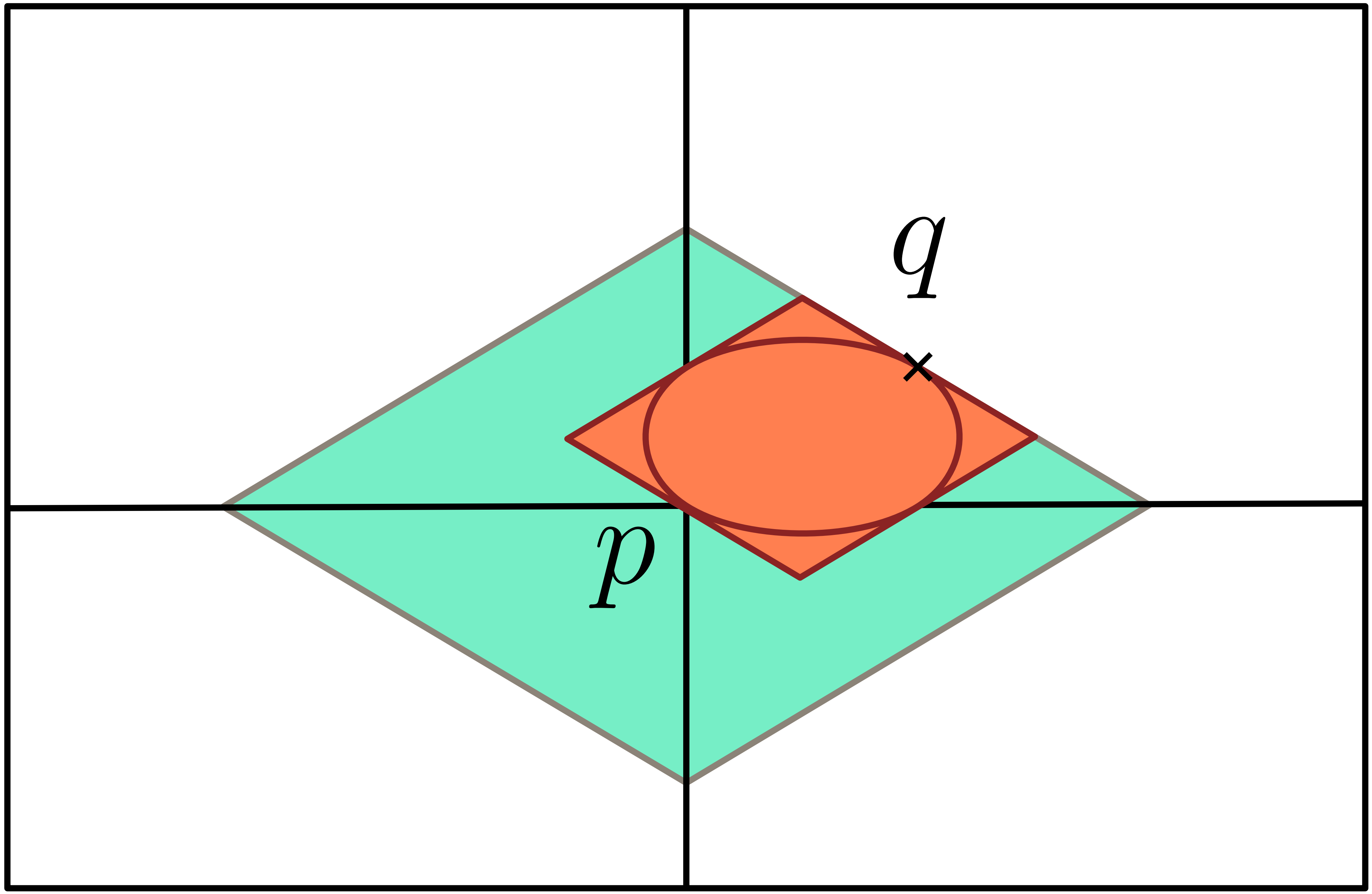
RG ⊂ Delaunay

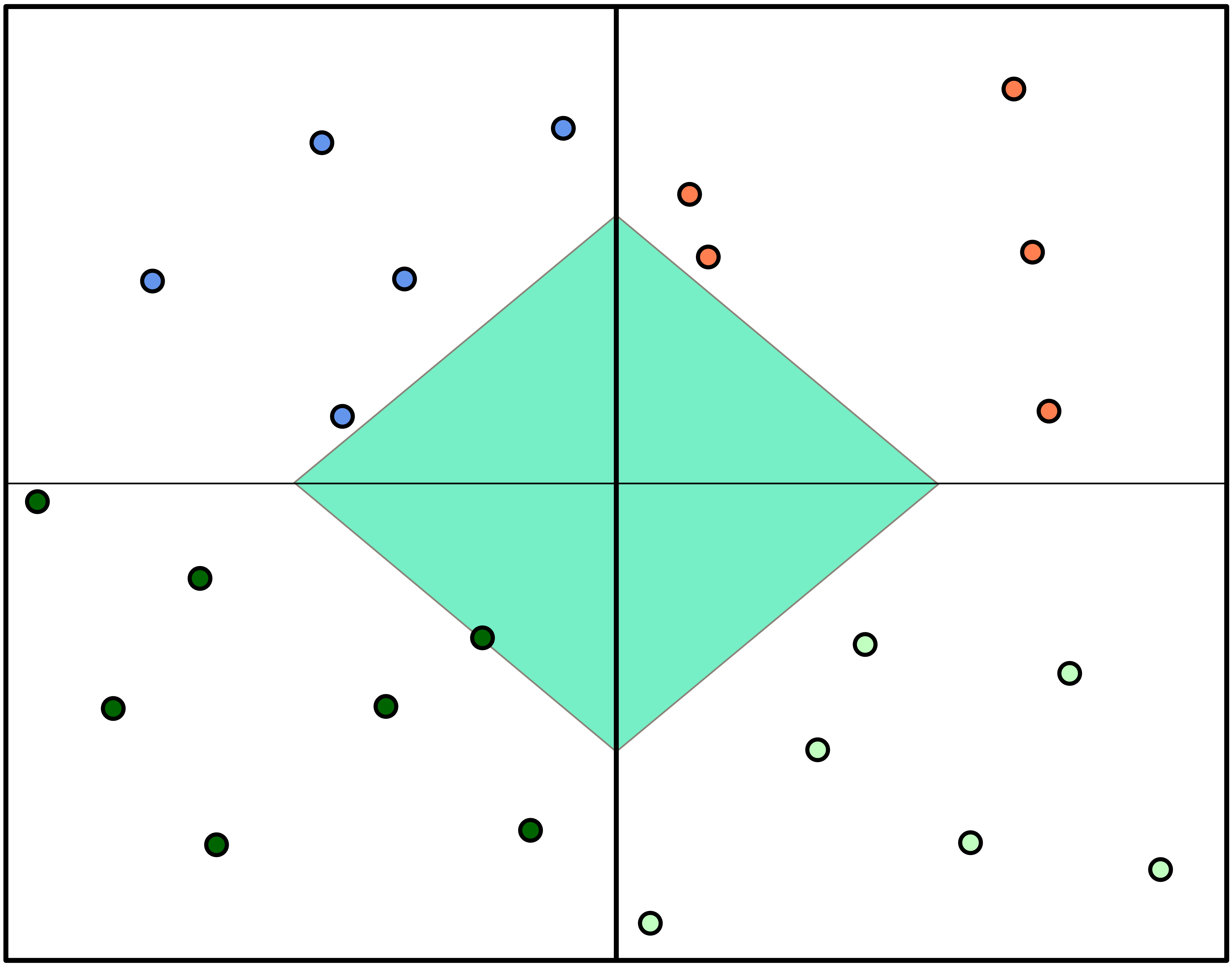

symmetry center
tangent in q
q
p
One sphere cylinder intersection

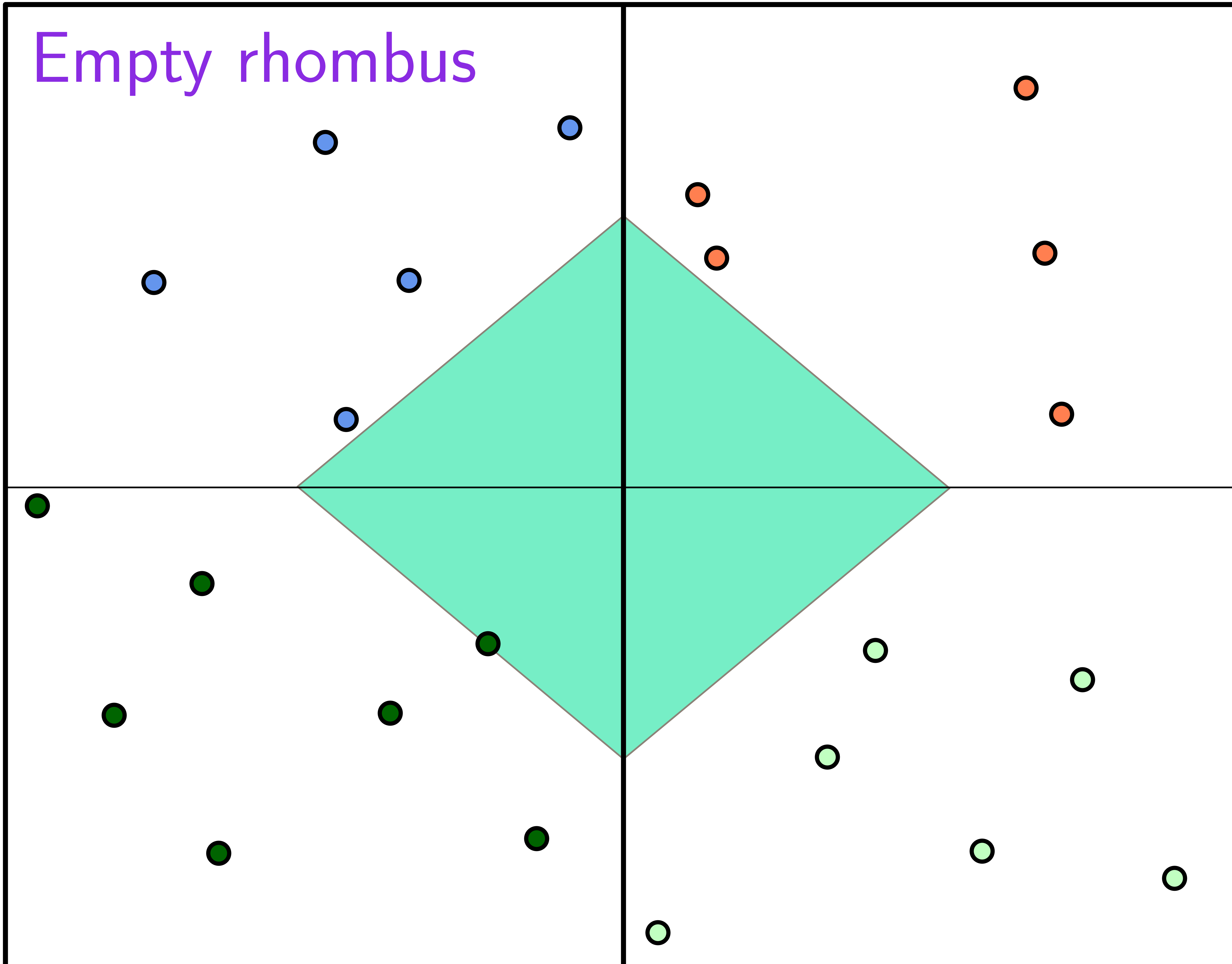
RG ⊂ Delaunay

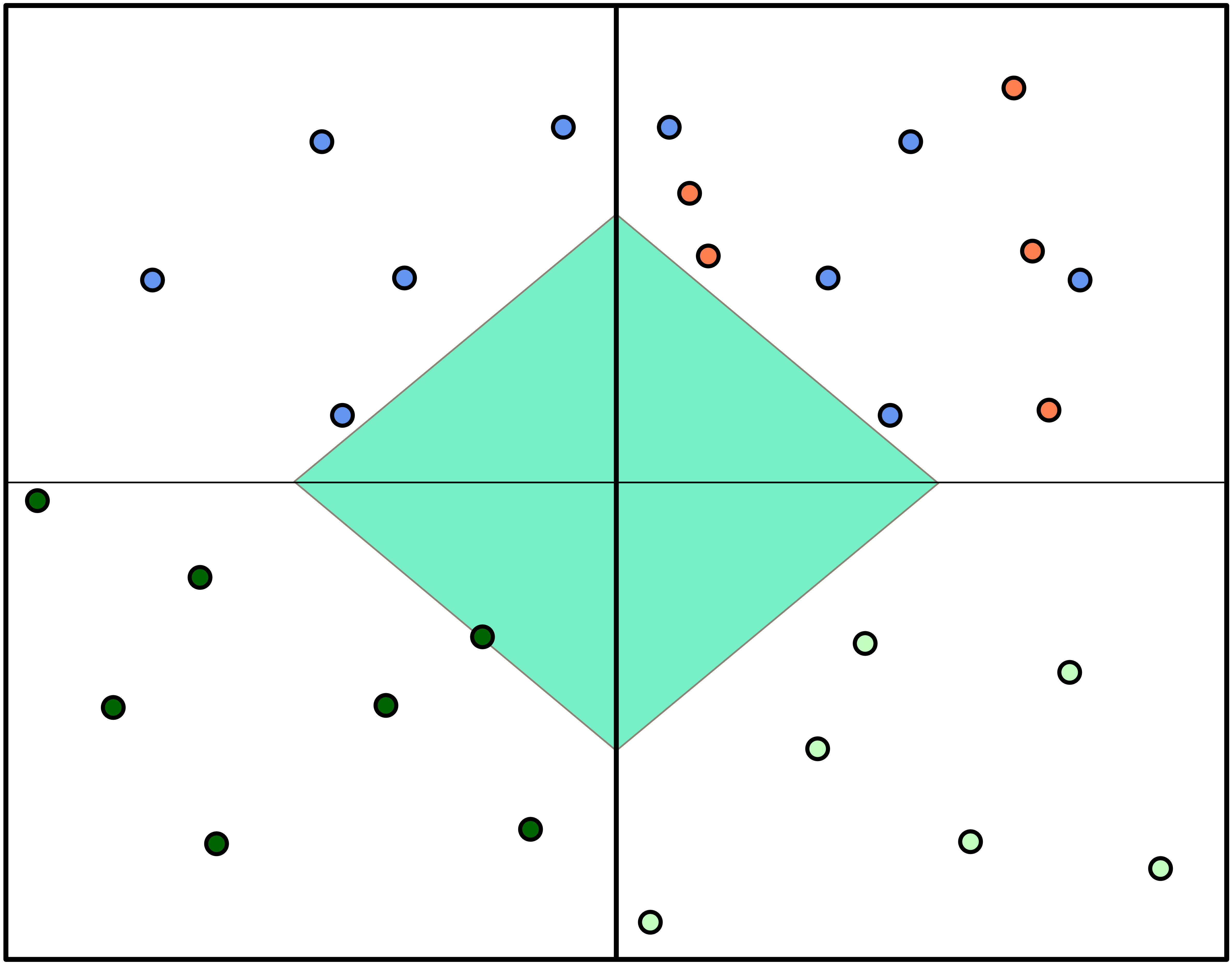


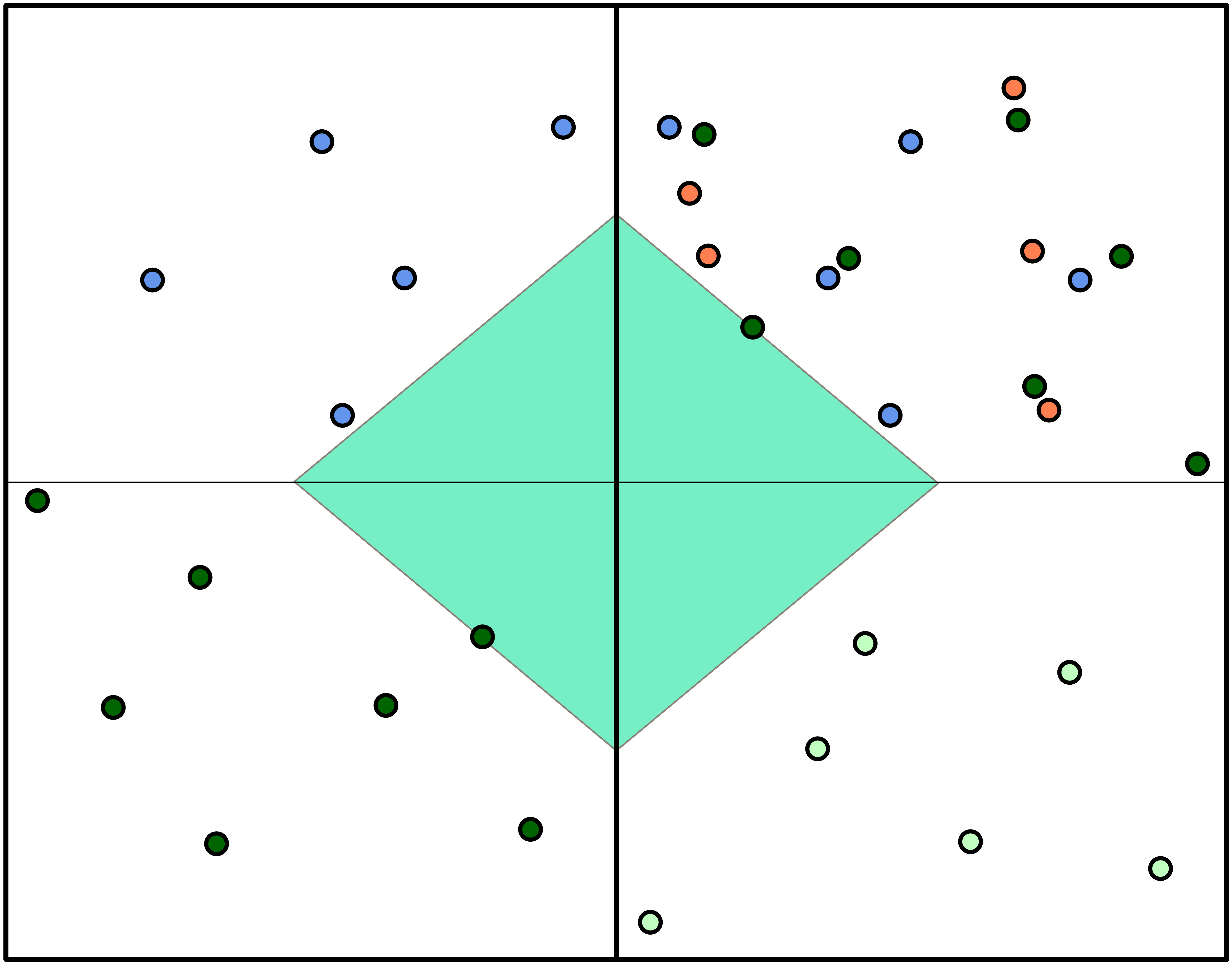
RG ⊂ Delaunay

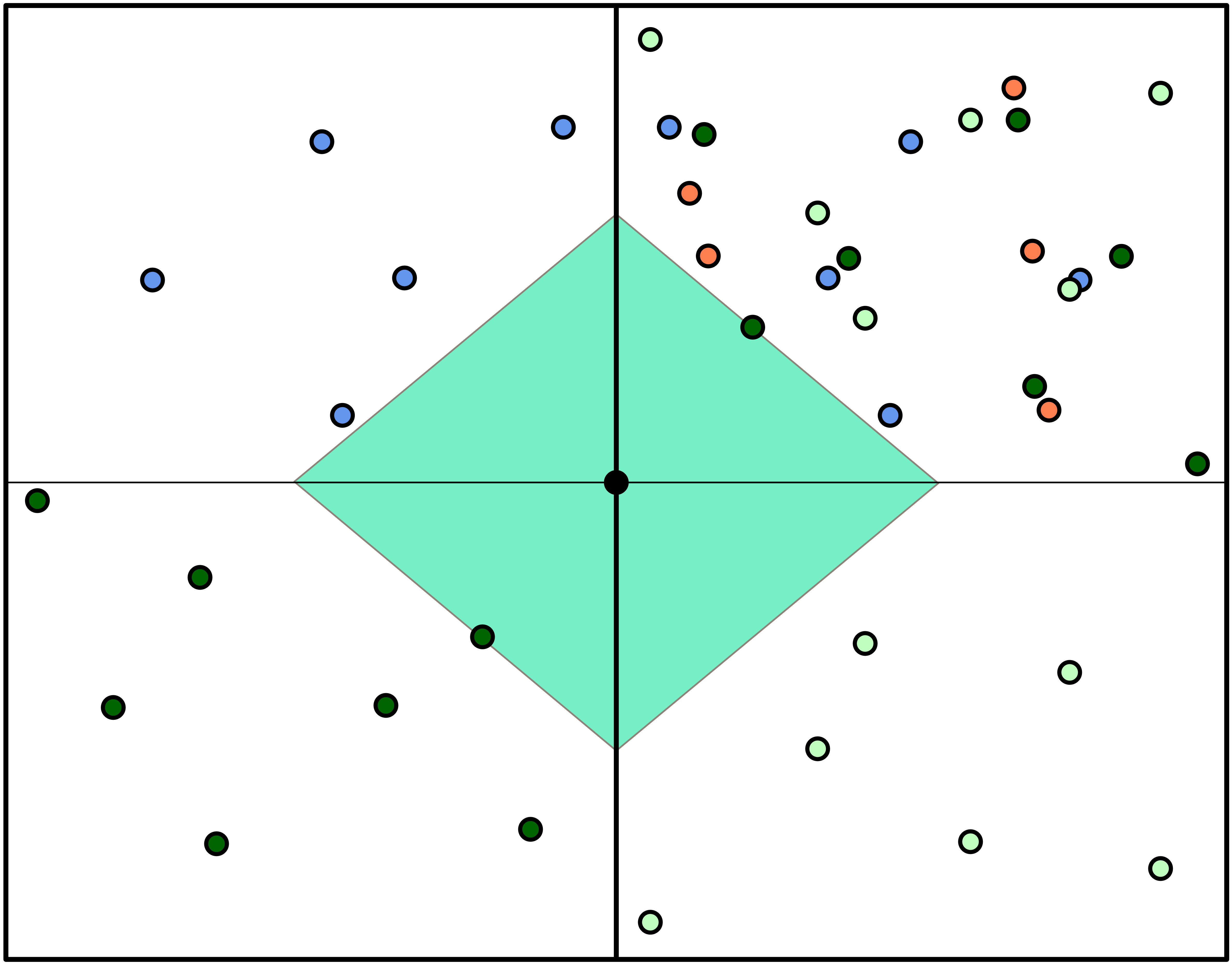


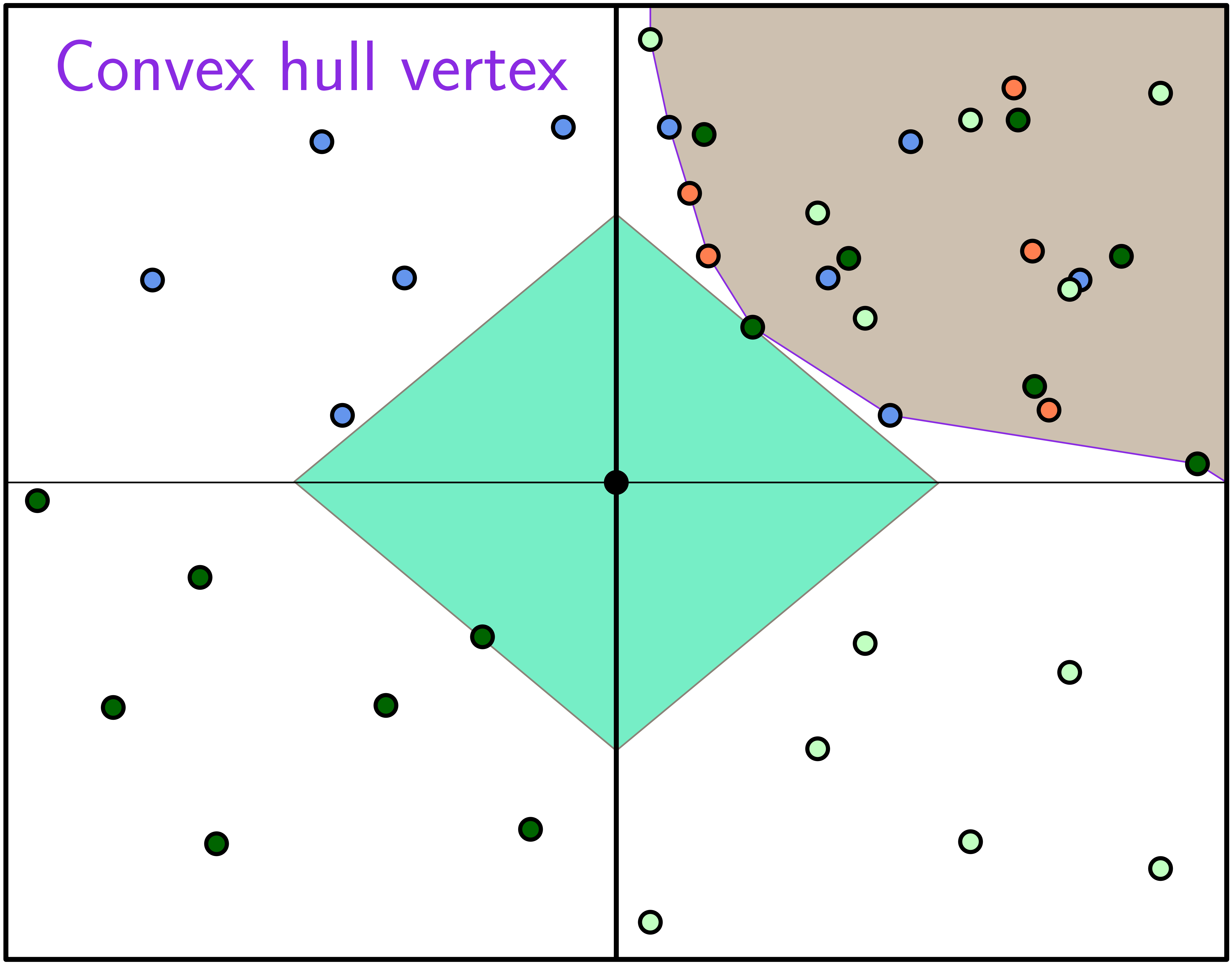


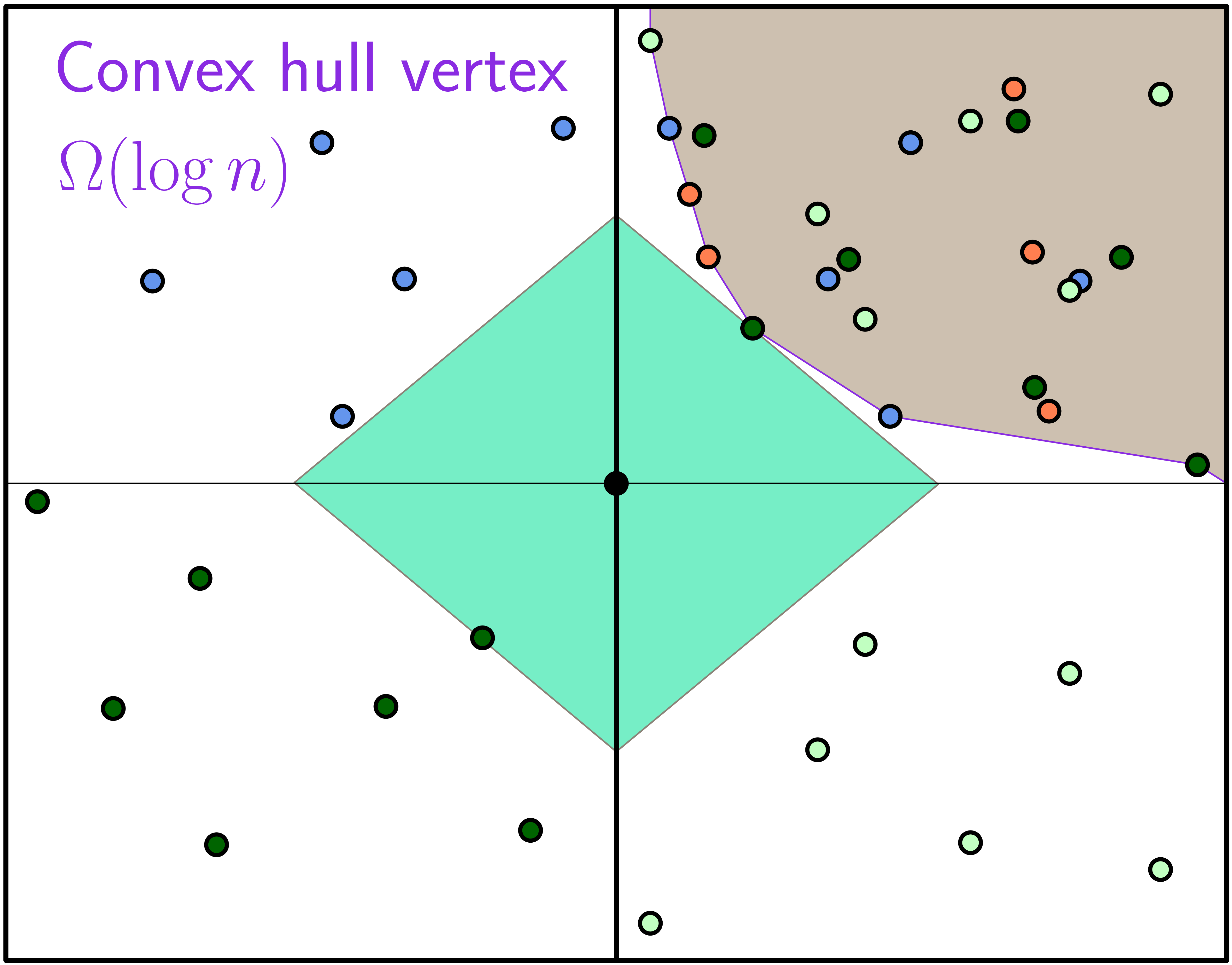

Empty rhombus

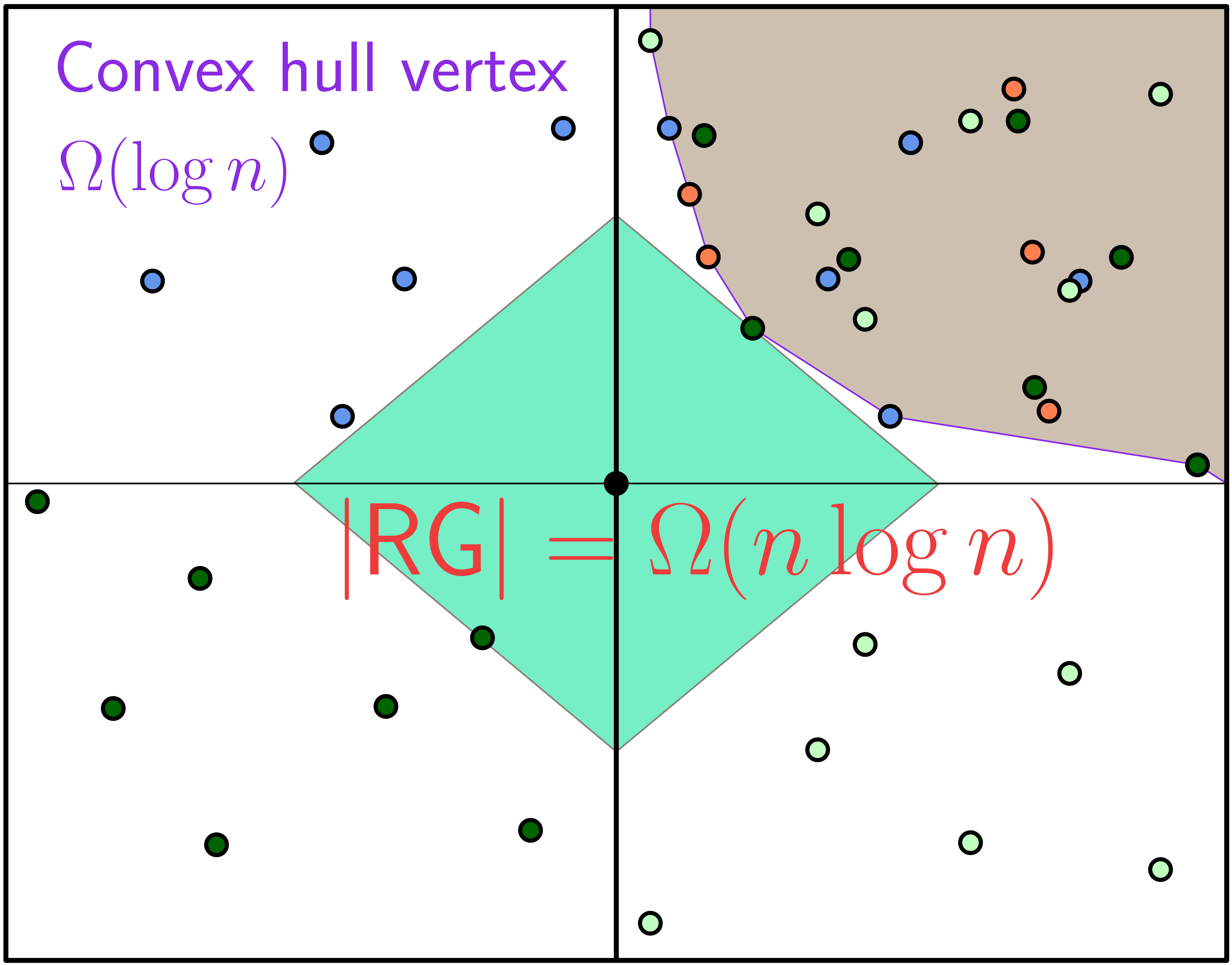




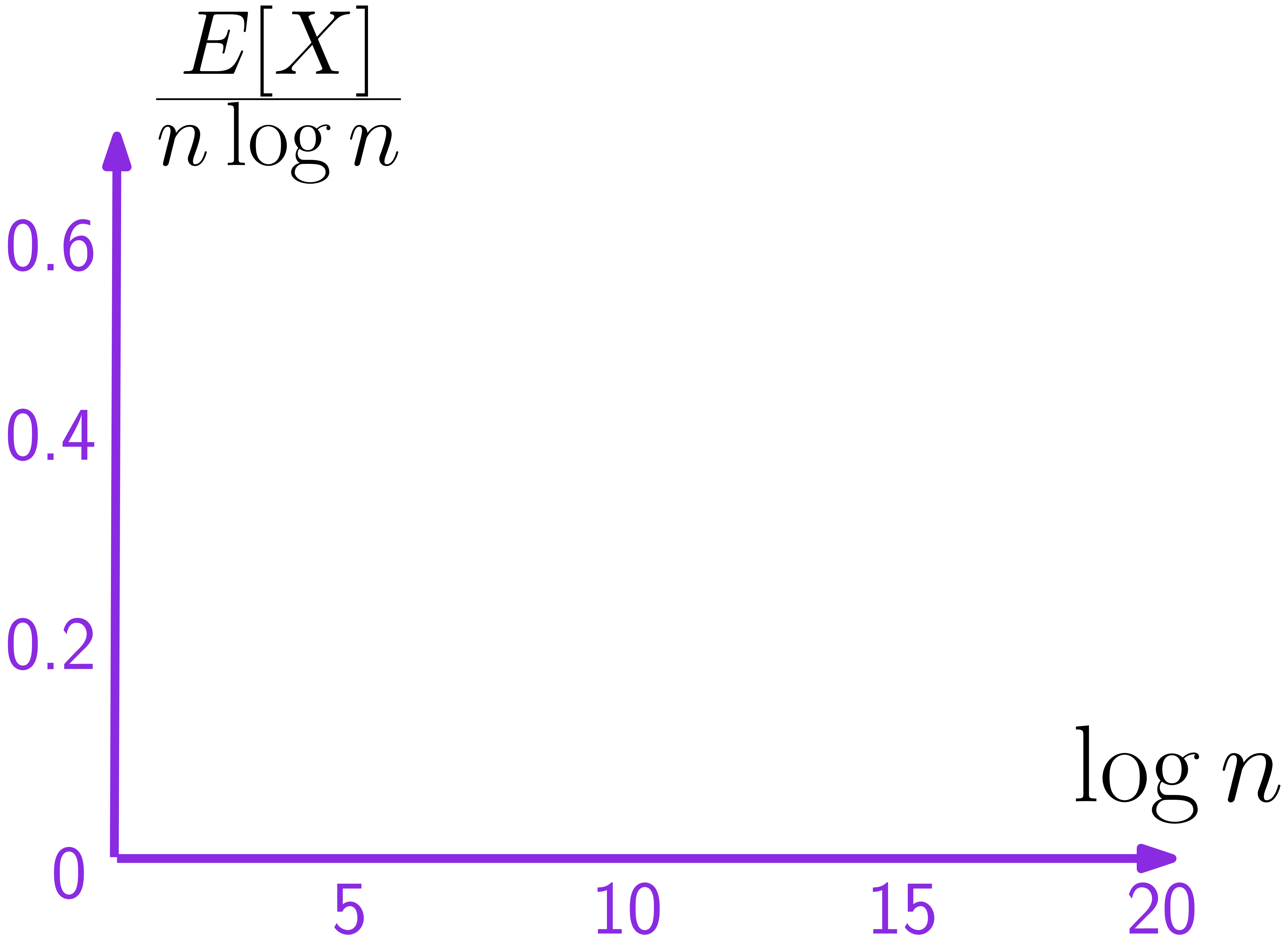



Convex hull vertex

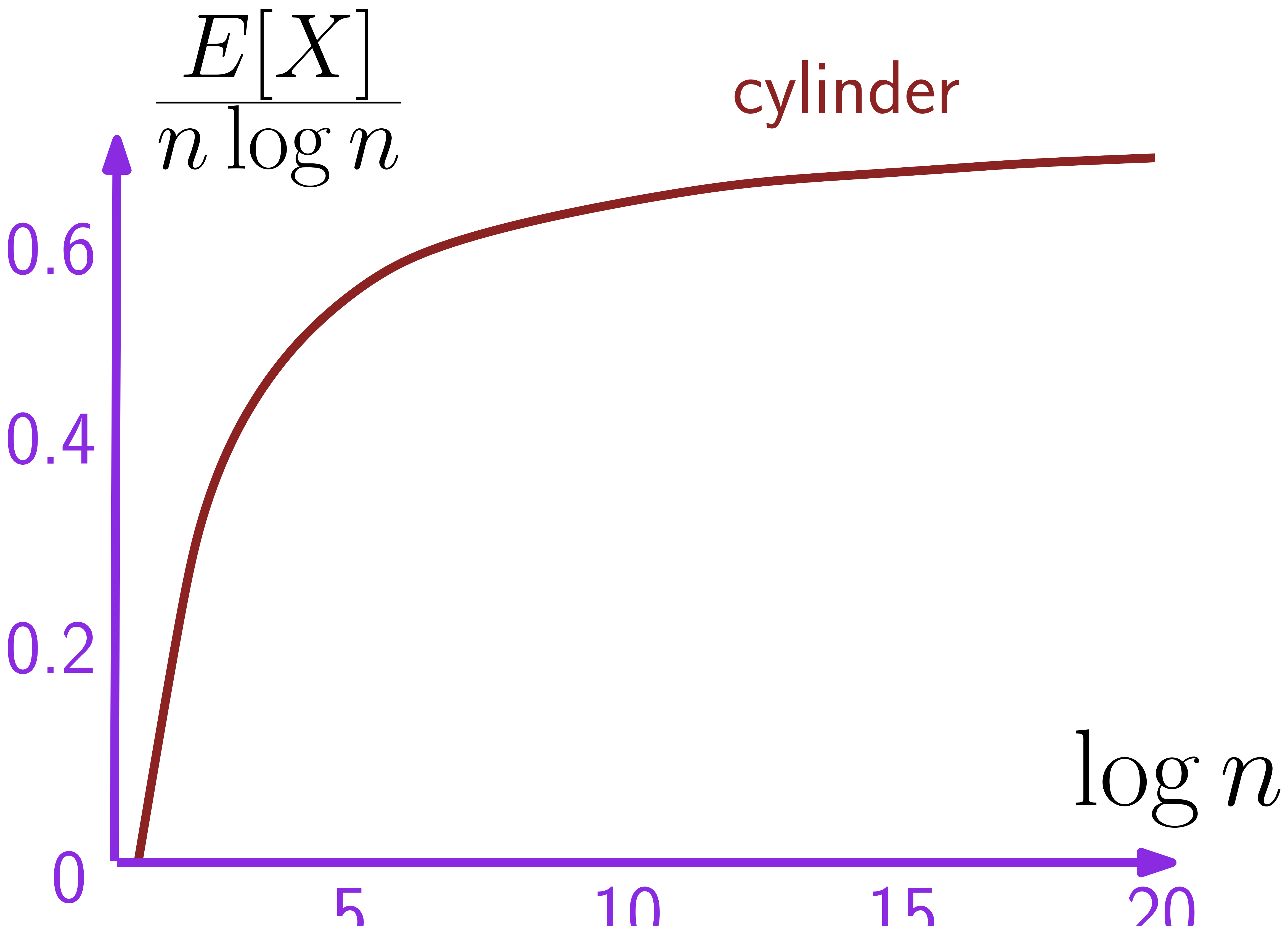

Convex hull vertex
$\Omega(\log n)$


Convex hull vertex
$\Omega(\log n)$
$|\text{RG}| = \Omega(n \log n)$

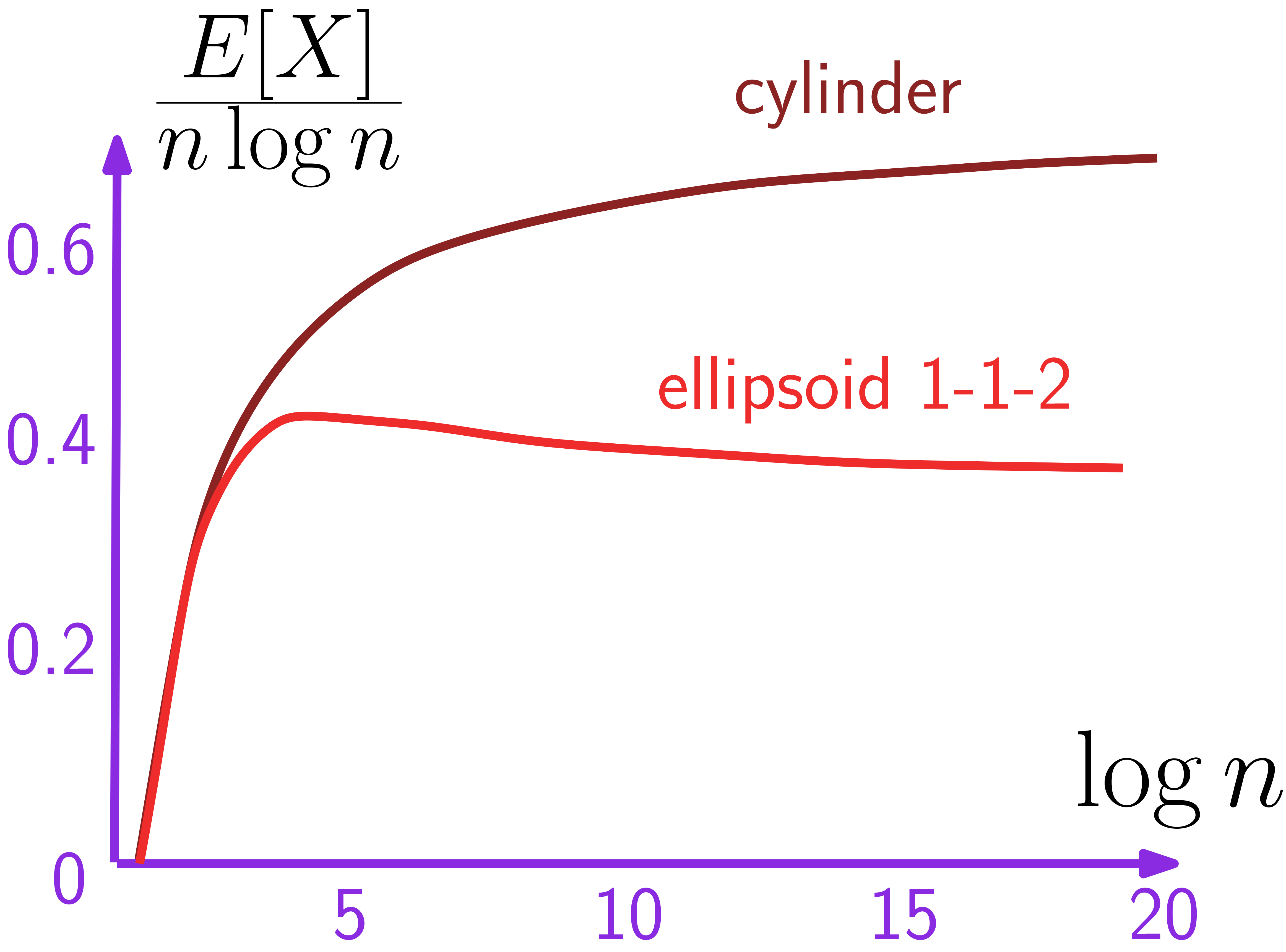
Experimental results



Experimental results



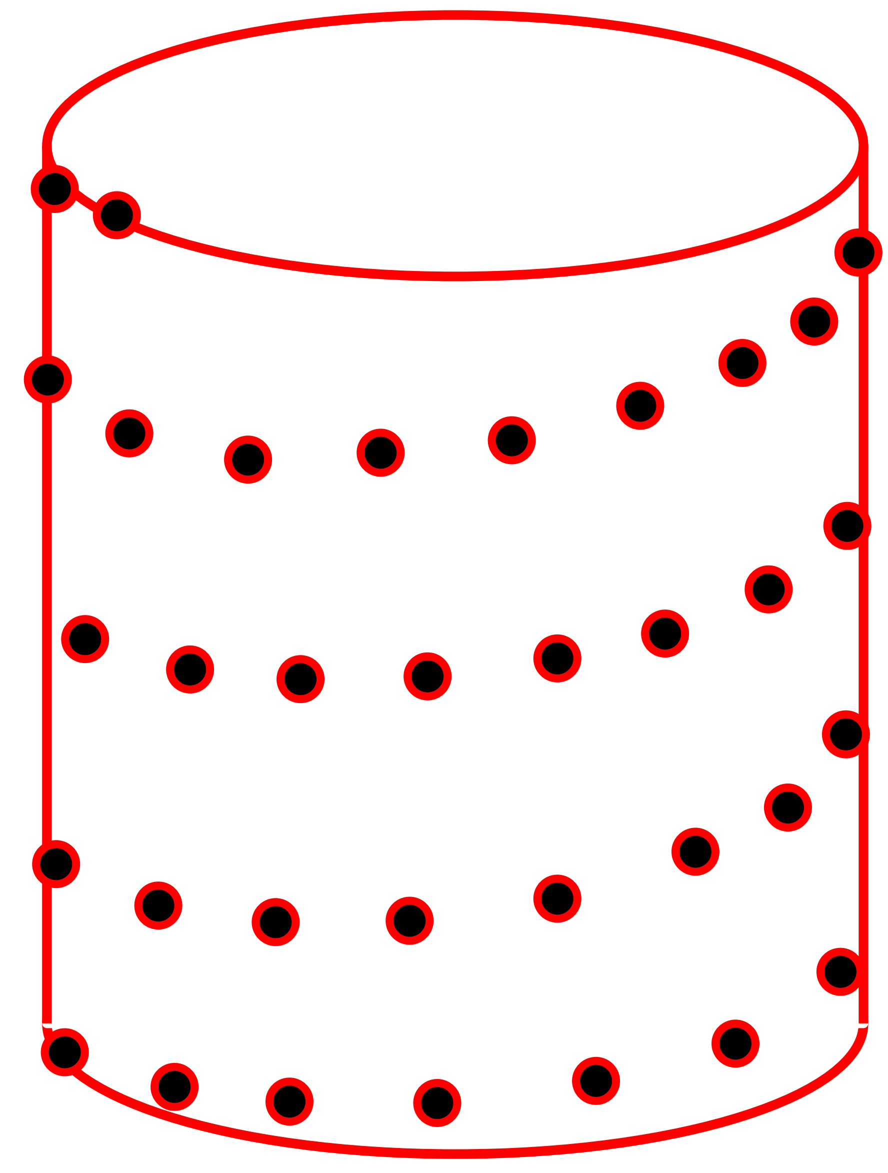
Experimental results



Generic skeleton hypotheses

$\Omega(n\sqrt{n})$

Erickson pathologic example



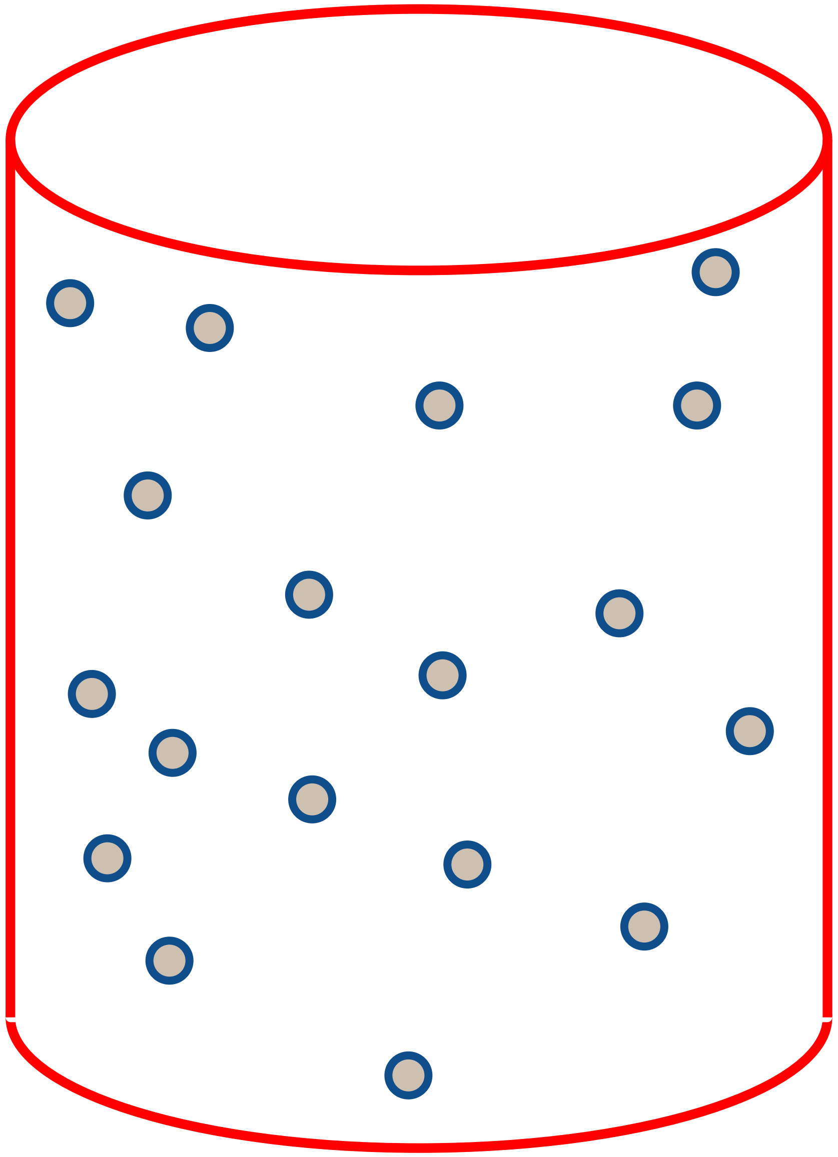
Generic skeleton hypotheses

are probably unnecessary

Erickson pathologic example

is really pathologic

$\Theta(n \log n)$



Generic skeleton hypotheses

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Erickson pathologic example

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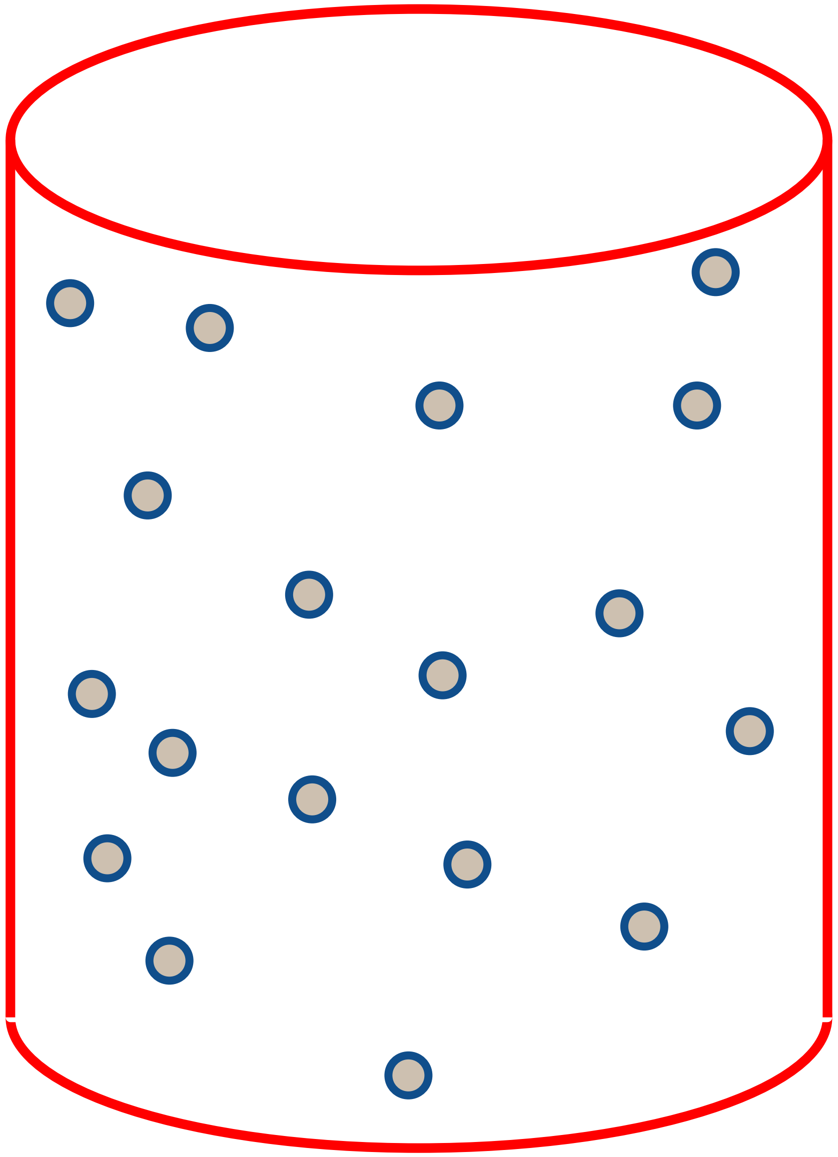

[Jeff simultaneous work]