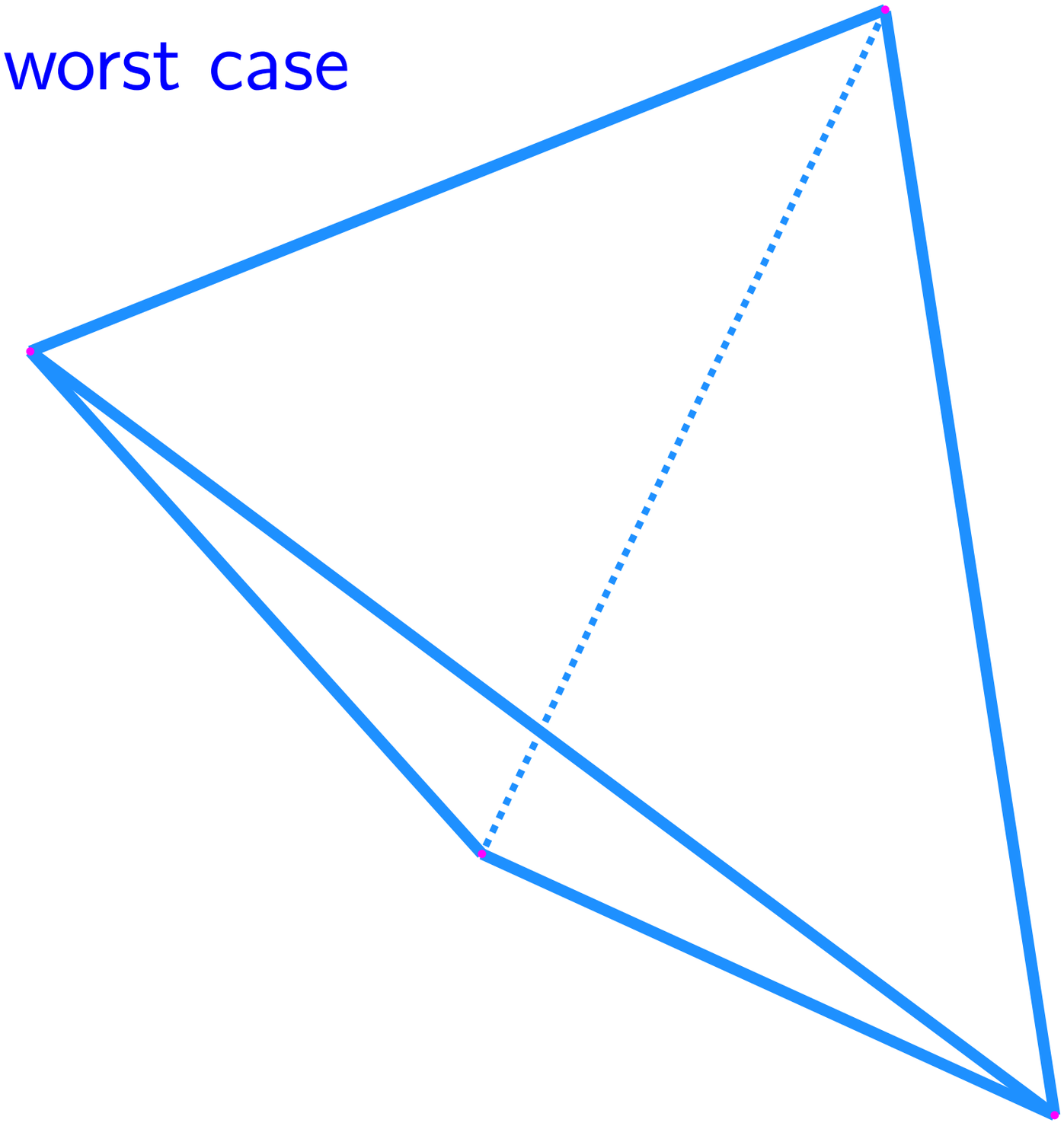
Random sampling of a cylinder yields a not so nasty Delaunay triangulation

Olivier Devillers

Xavier Goaoc

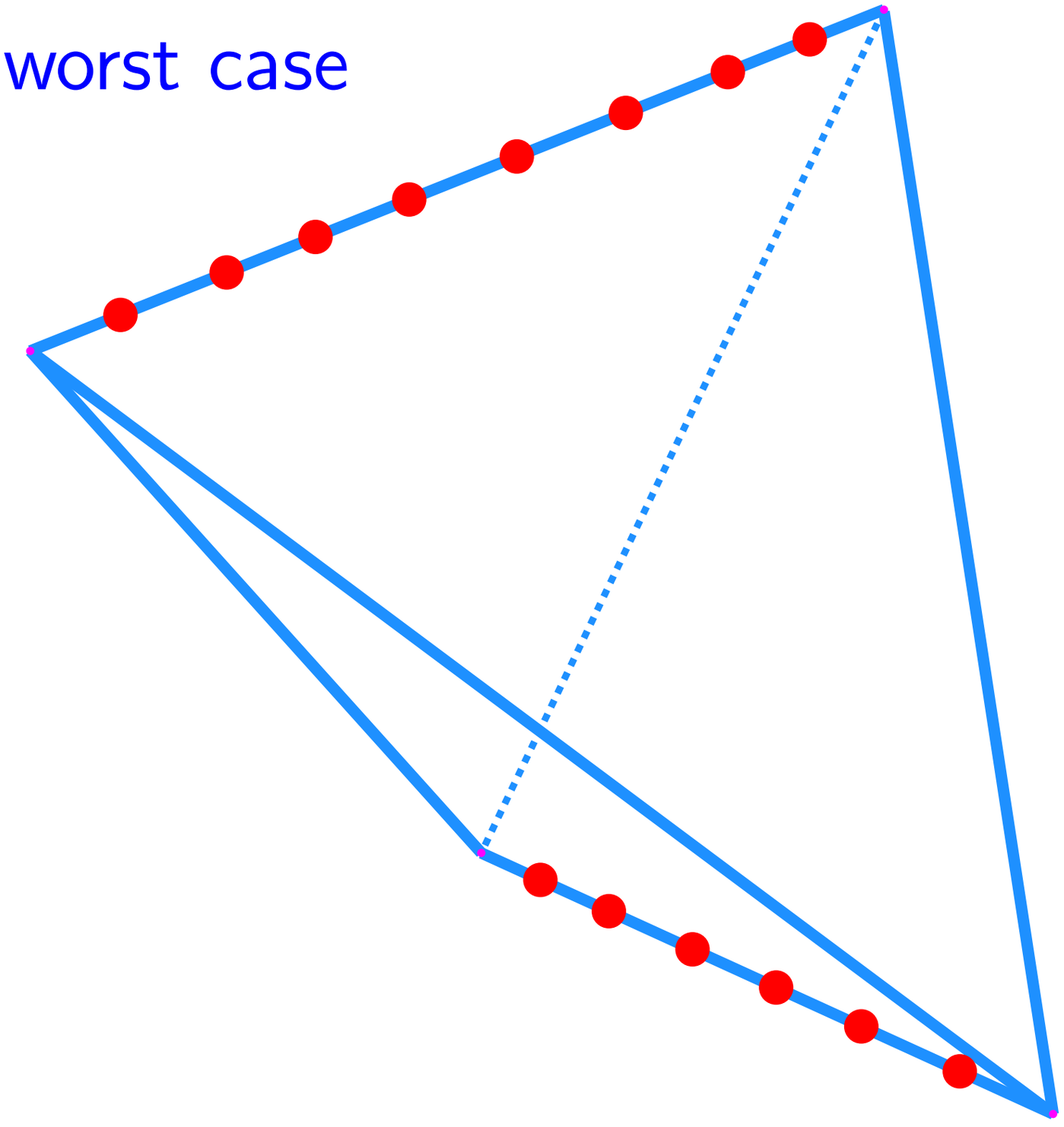
3D Delaunay triangulation

Quadratic in the worst case



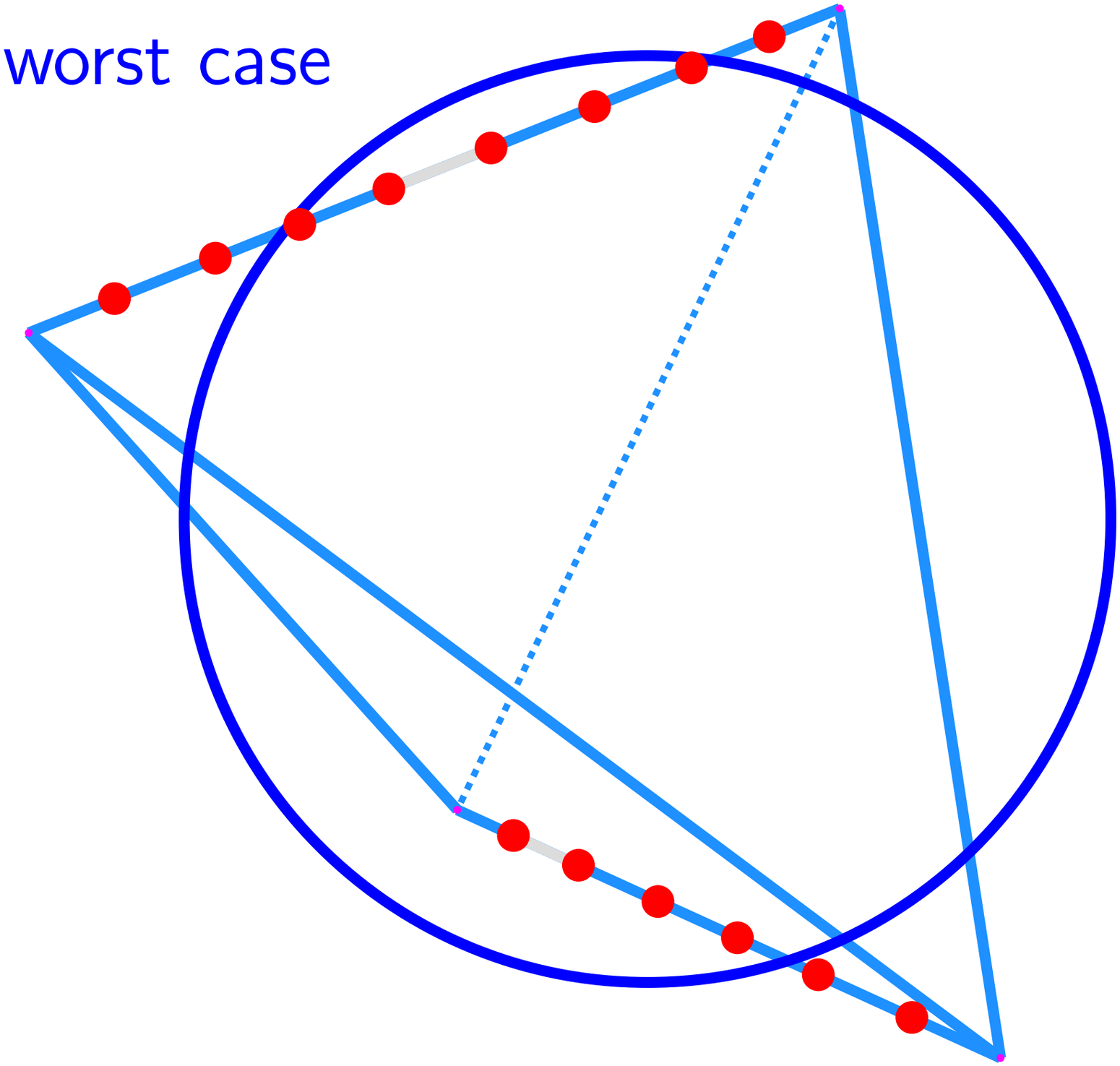
3D Delaunay triangulation

Quadratic in the worst case



3D Delaunay triangulation

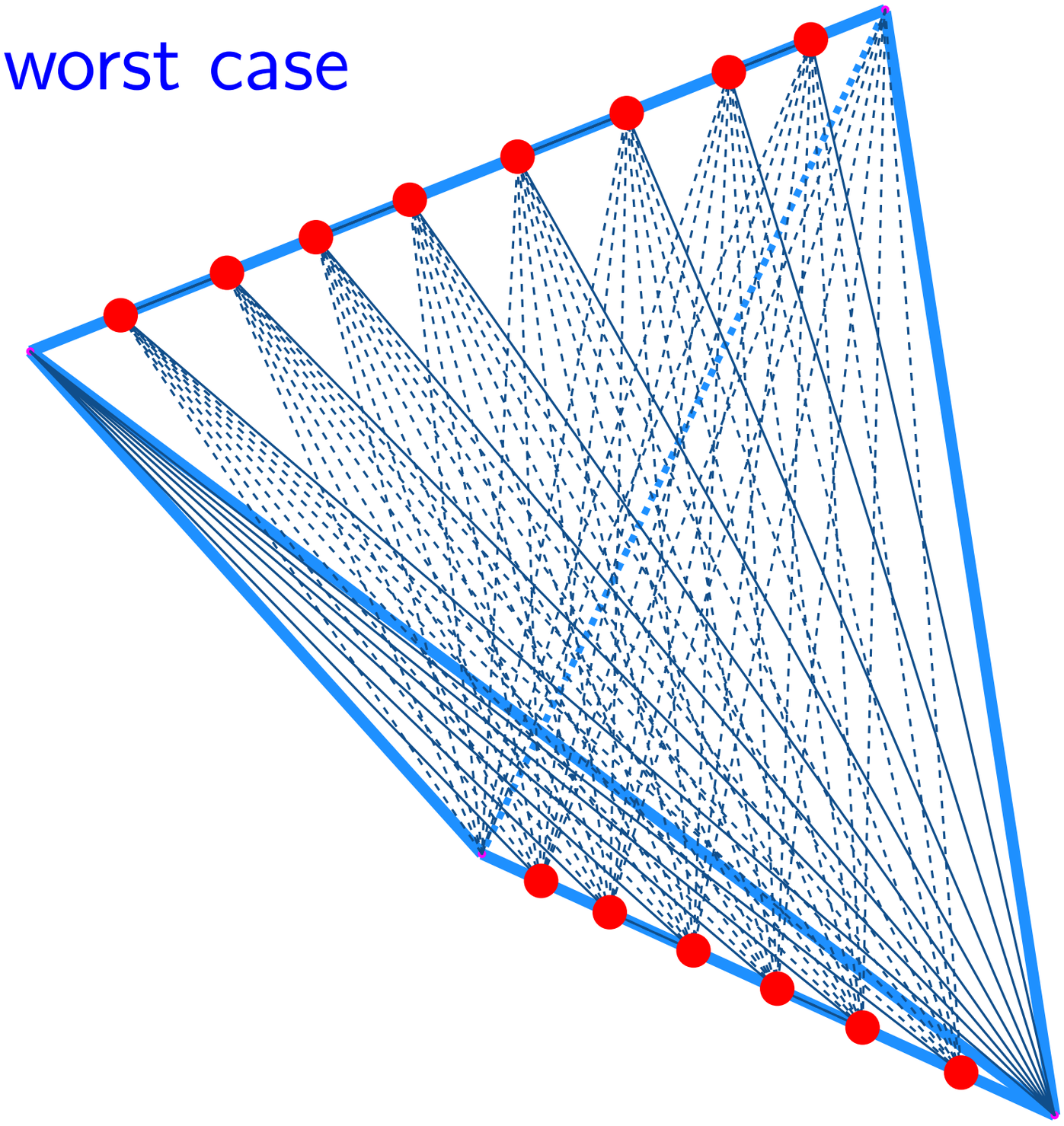
Quadratic in the worst case



3D Delaunay triangulation

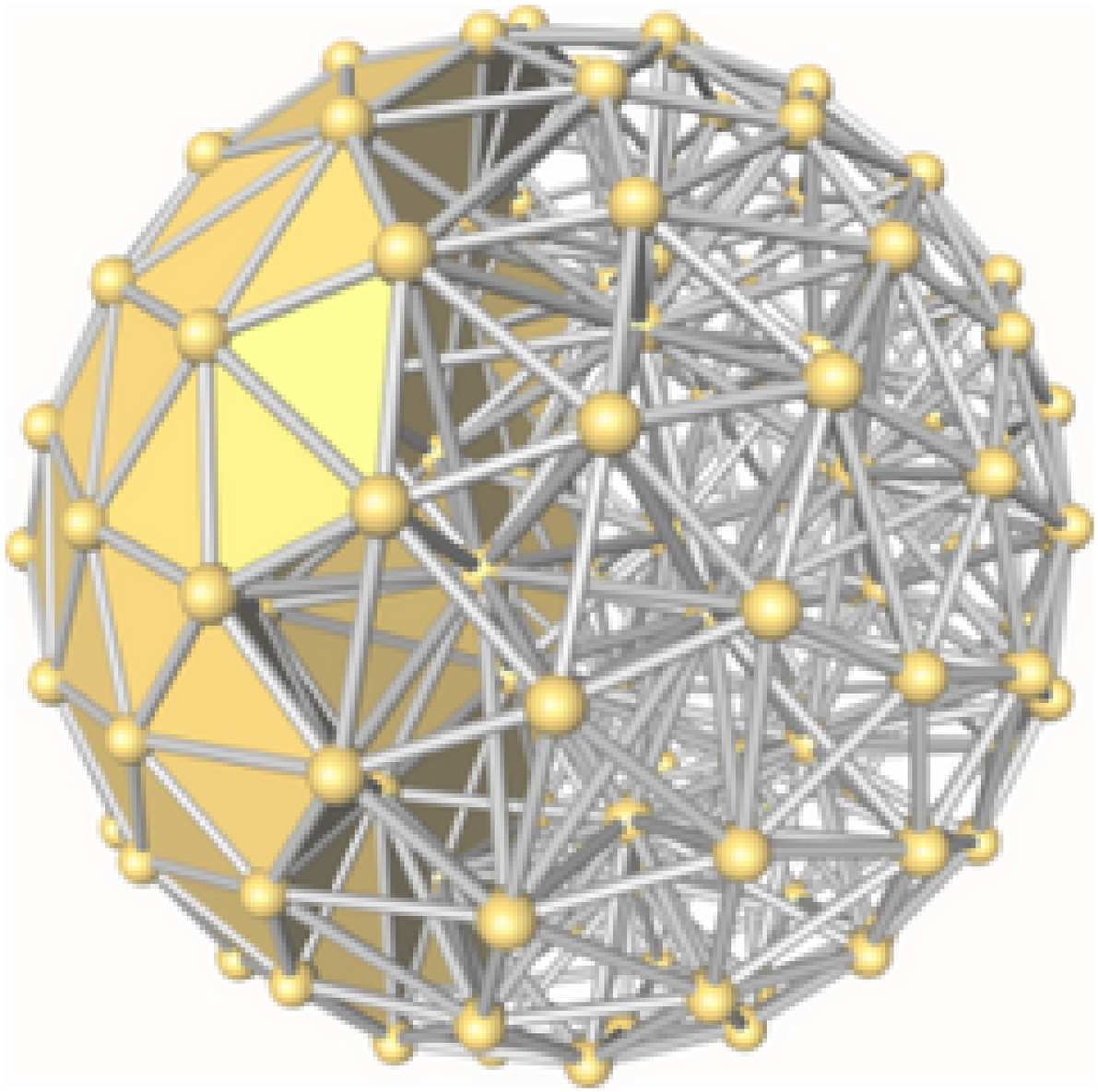
Quadratic in the worst case

$$\Omega(n^2)$$



Linear in random case

$$O(n)$$



[Dwyer]

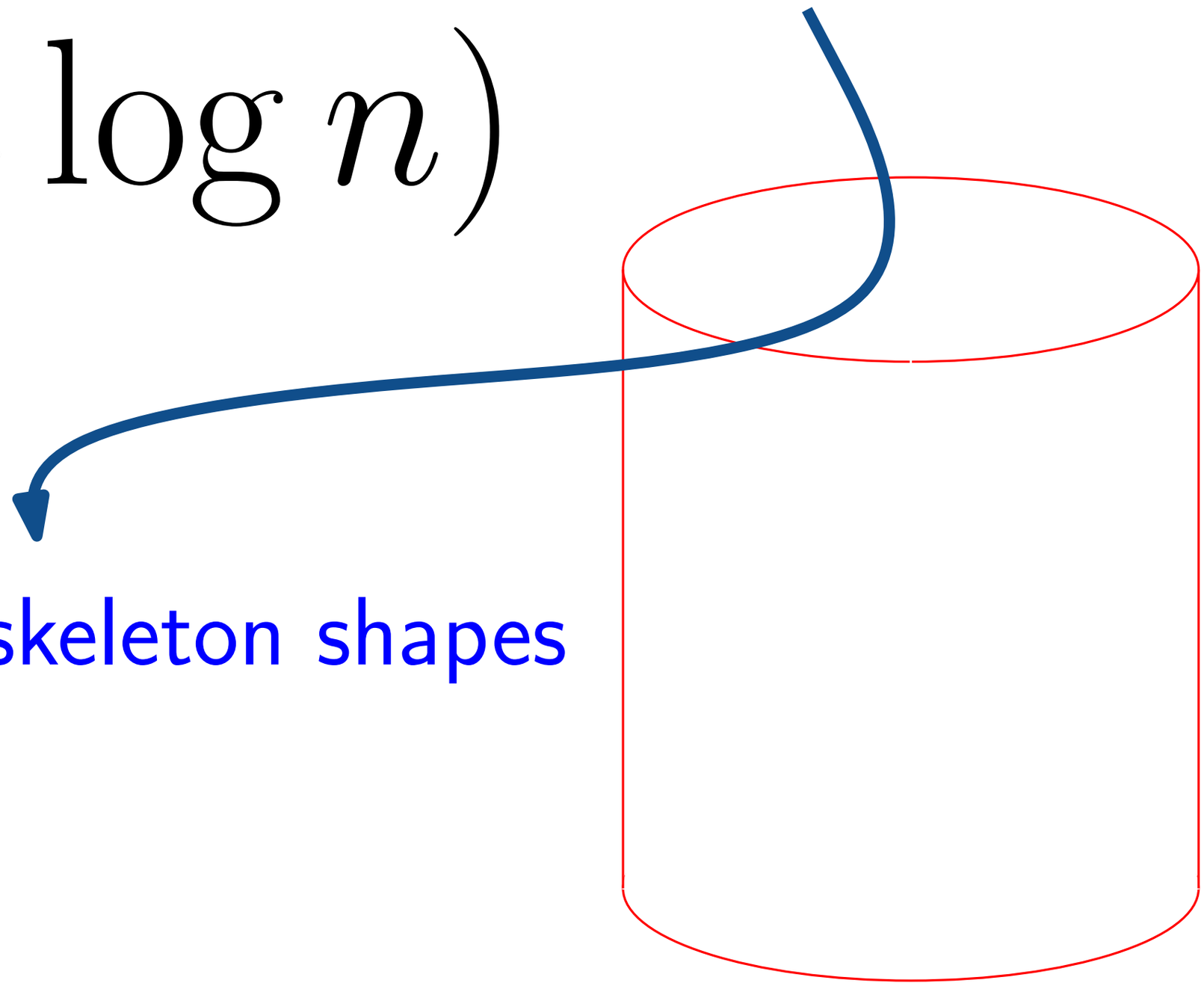
Almost linear for "well" sampled generic surface

$$O(n \log n)$$

[Attali Boissonnat Lieutier]

Almost linear for "well" sampled generic surface

$$O(n \log n)$$



no 1D skeleton shapes

[Attali Boissonnat Lieutier]

Almost linear for "well" sampled generic surface

$$O(n \log n)$$

evenly distributed points

$$O(n \log^3 n)$$

[Attali Boissonnat Lieutier]

Spread dependant

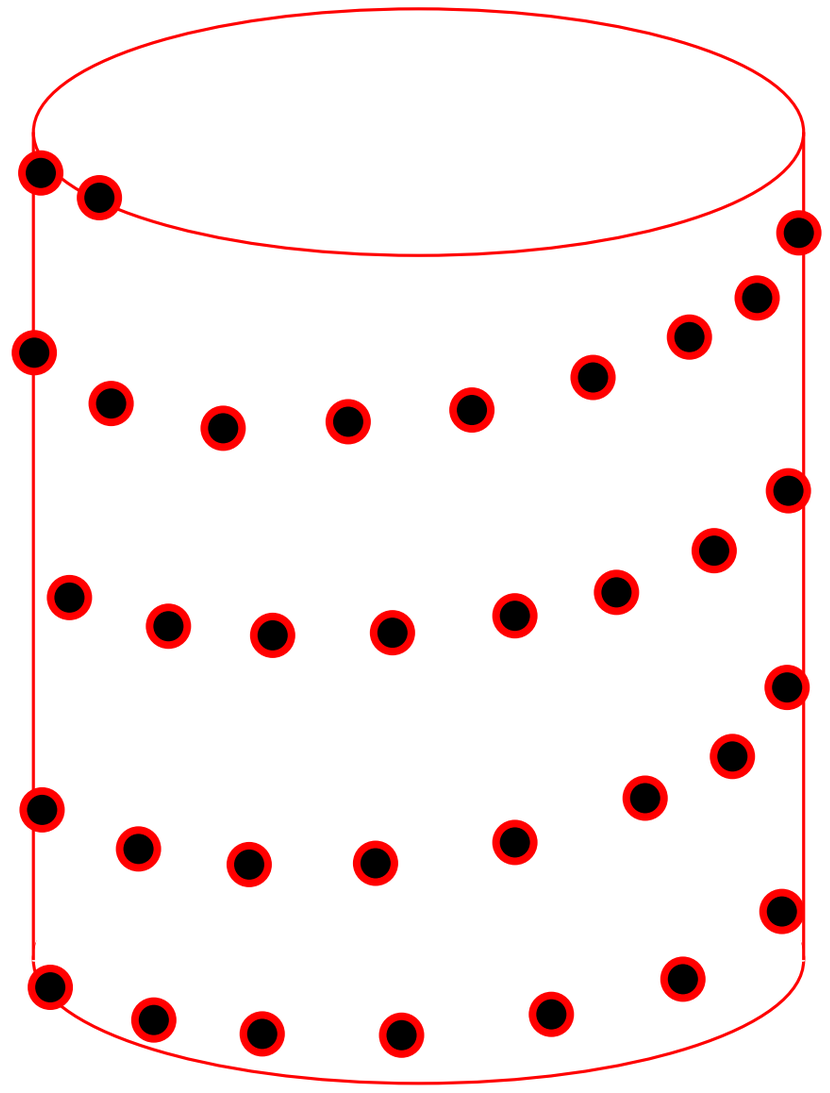
$$O(n\Delta)$$

[Erickson]

Spread dependant

$$O(n\Delta)$$

$$O(n\sqrt{n})$$



[Erickson]

Spread dependant

$$O(n\Delta)$$

evenly distributed points

$$O(n\sqrt{n\log n})$$

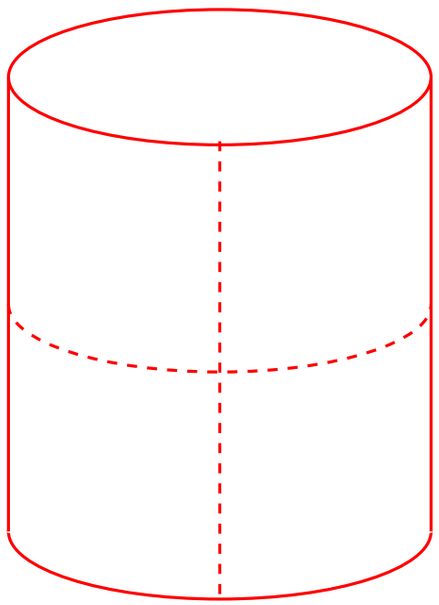
[Erickson]

This paper

evenly distributed points on a cylinder

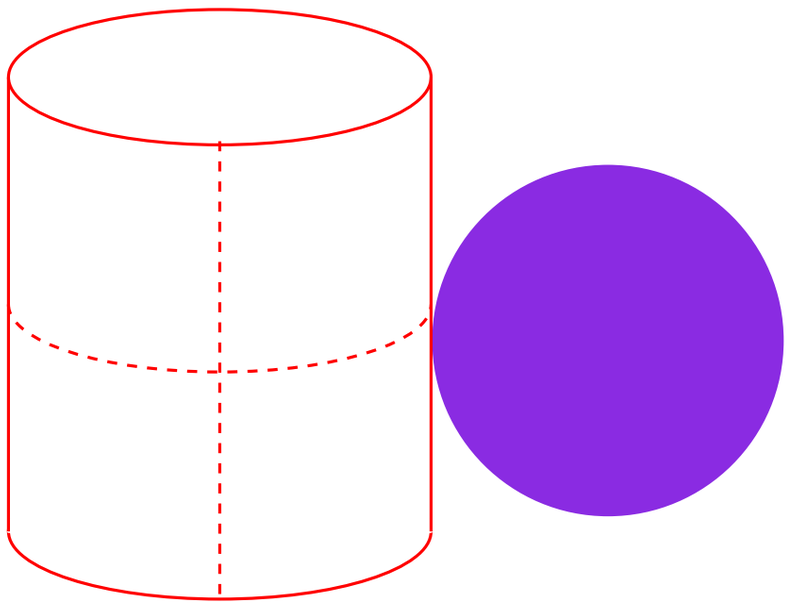
$$\Theta(n \log n)$$

Intersection sphere/cylinder

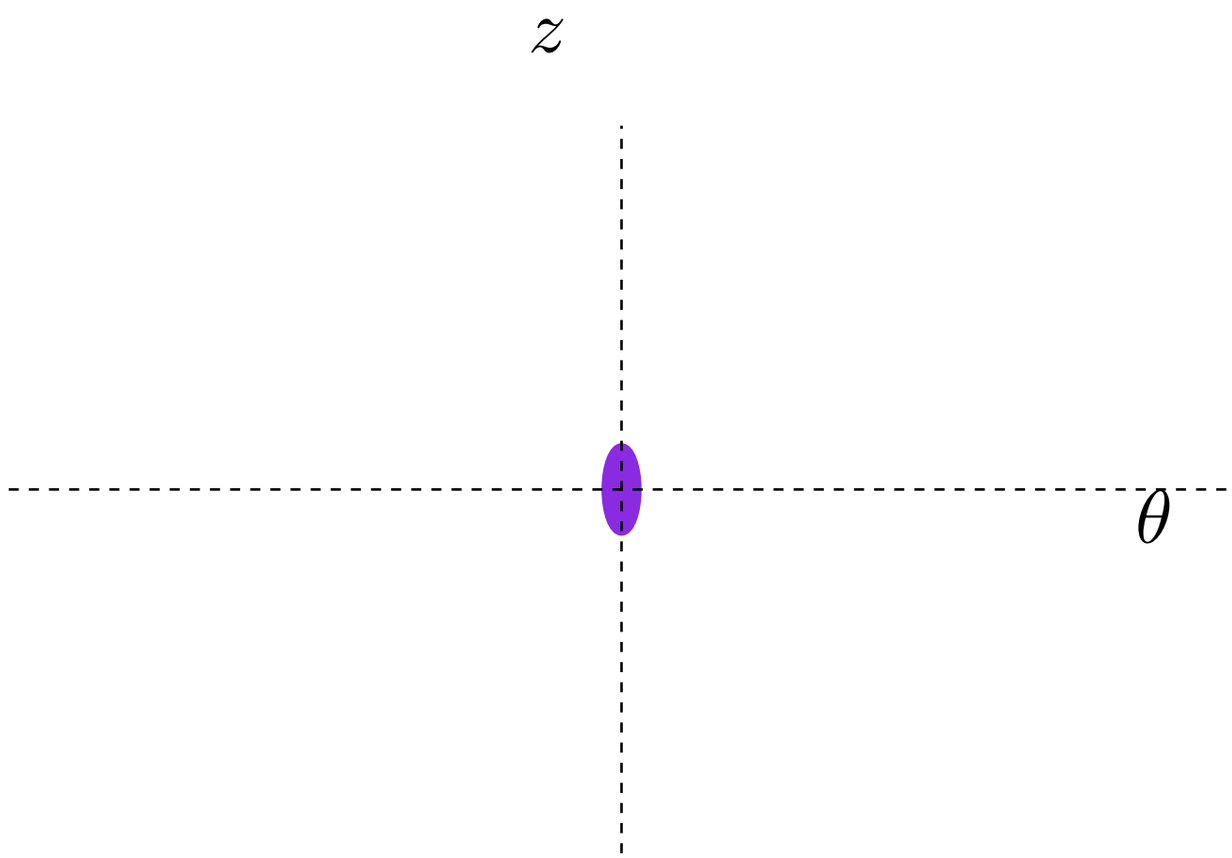


$r = 0.9$

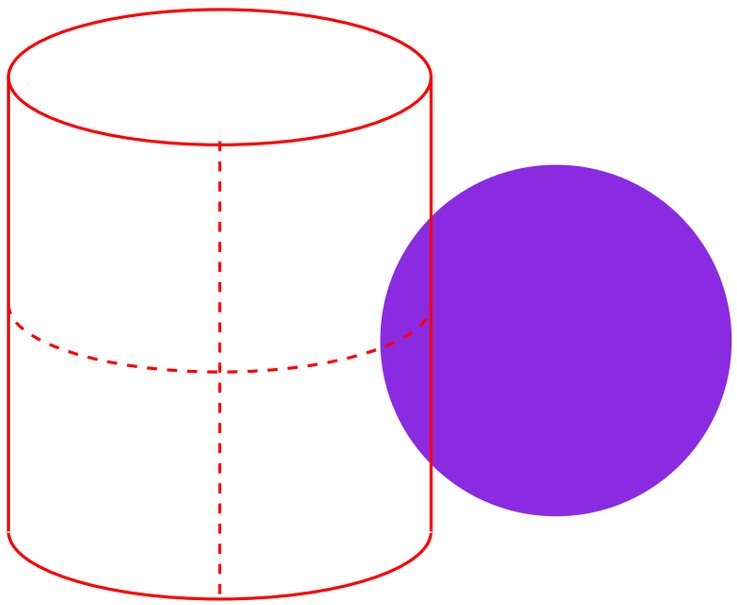
Intersection sphere/cylinder



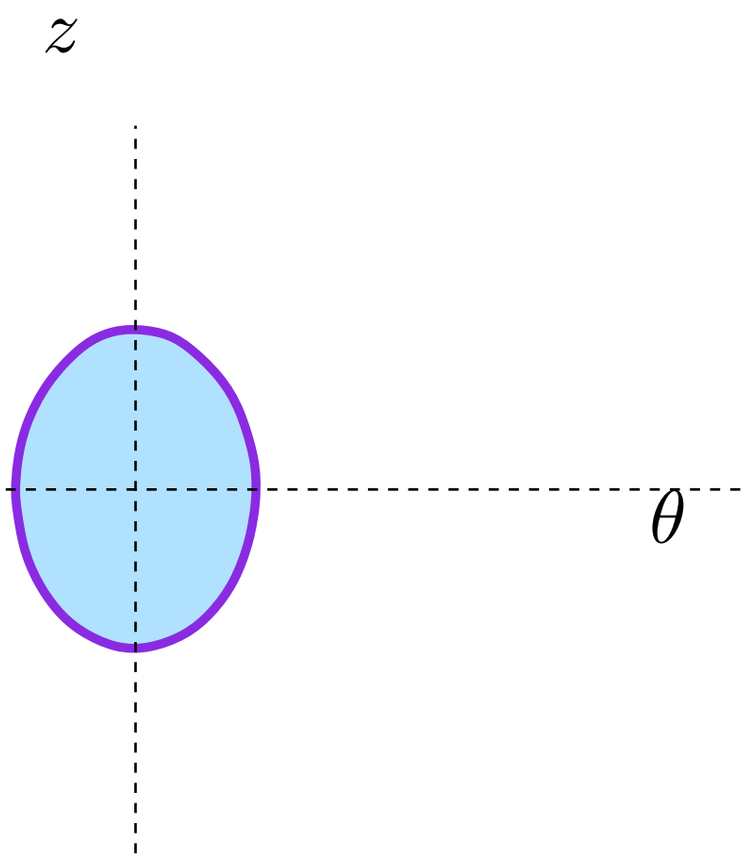
$r = 0.9$



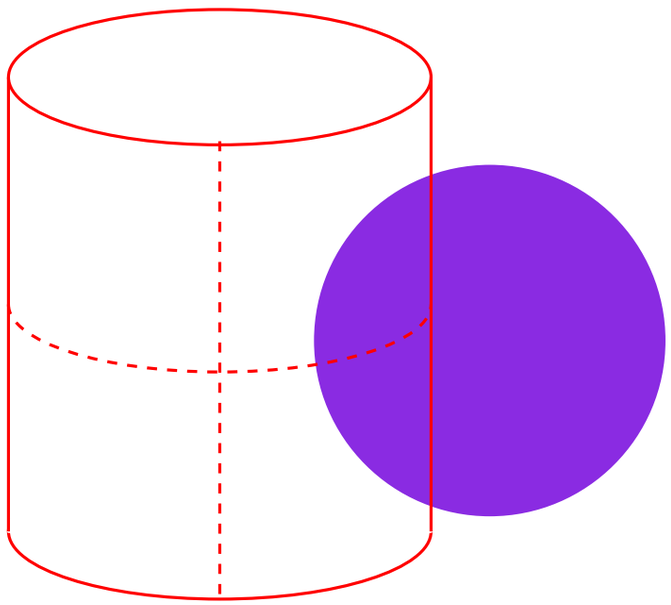
Intersection sphere/cylinder



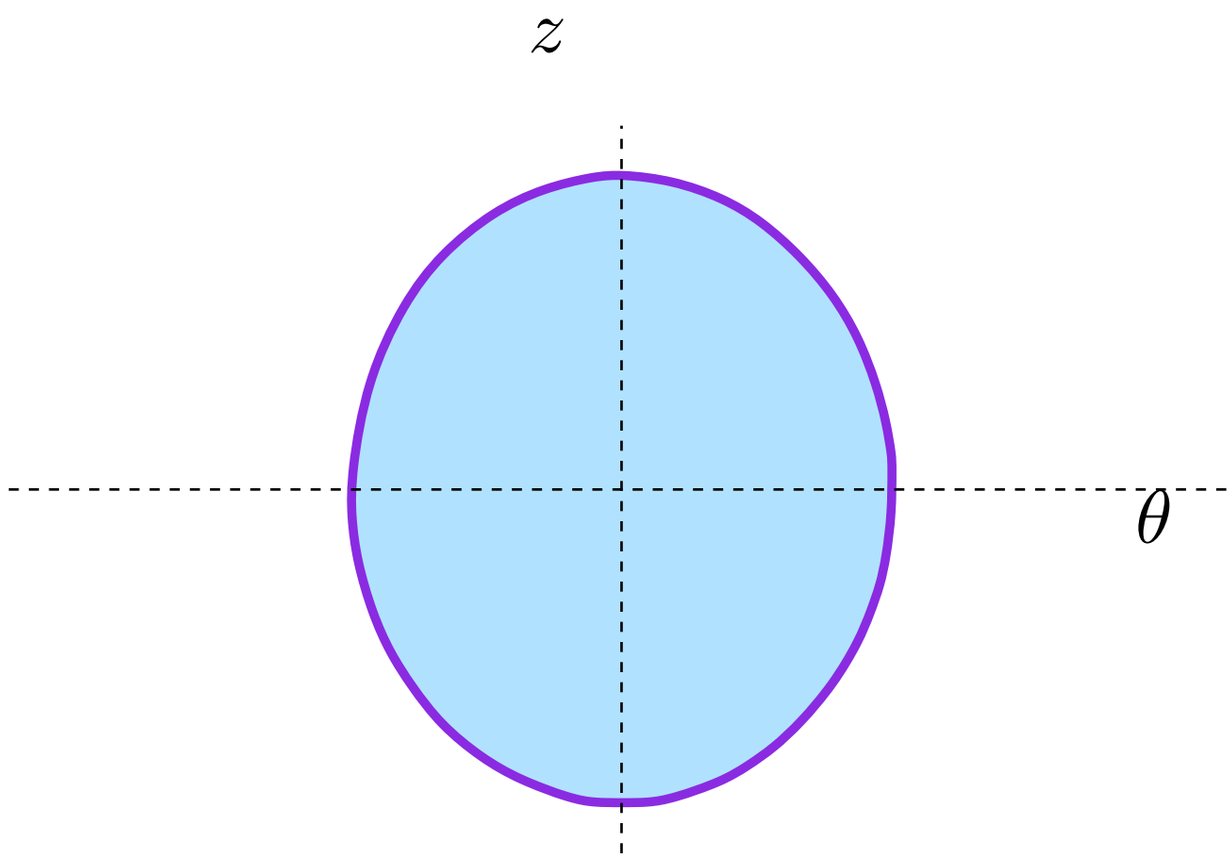
$r = 0.9$



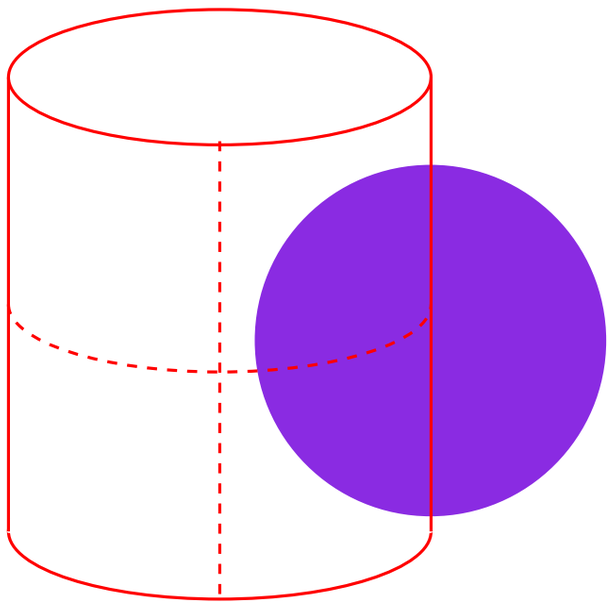
Intersection sphere/cylinder



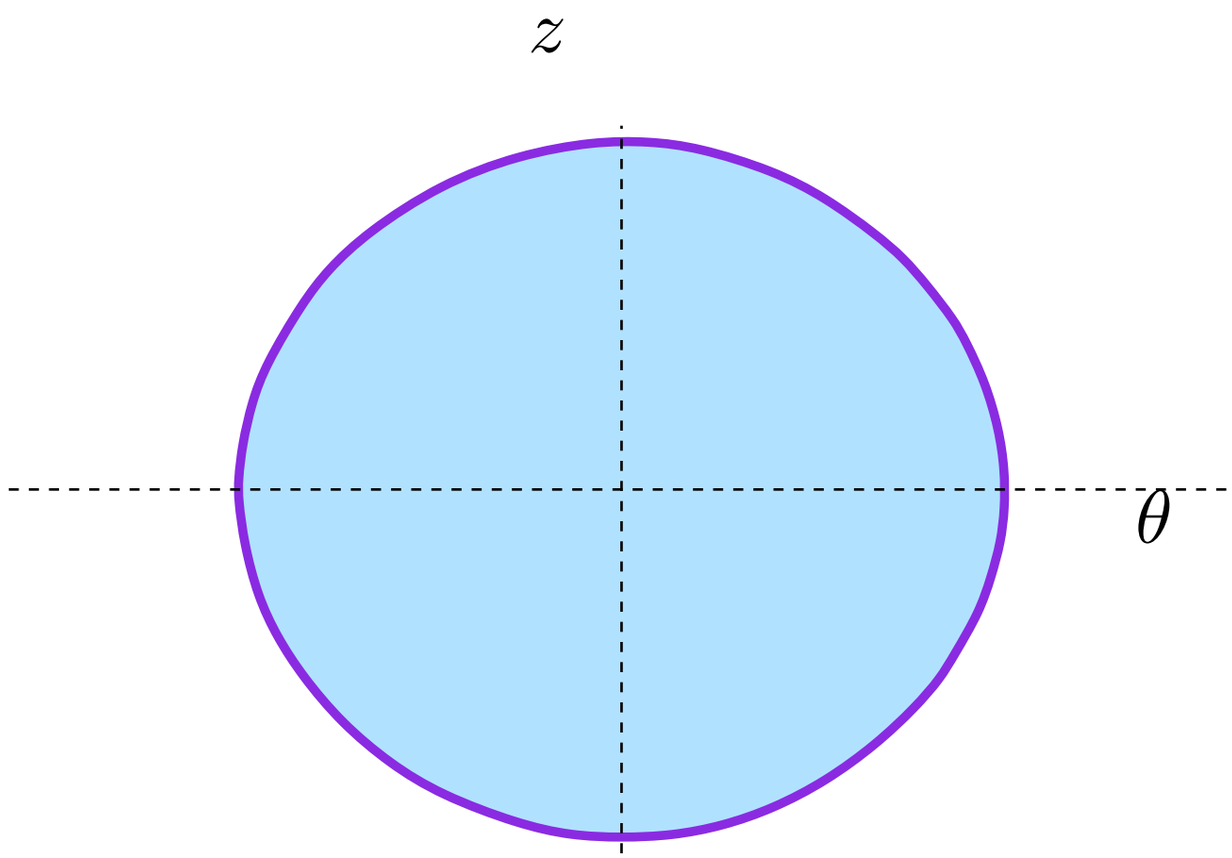
$r = 0.9$



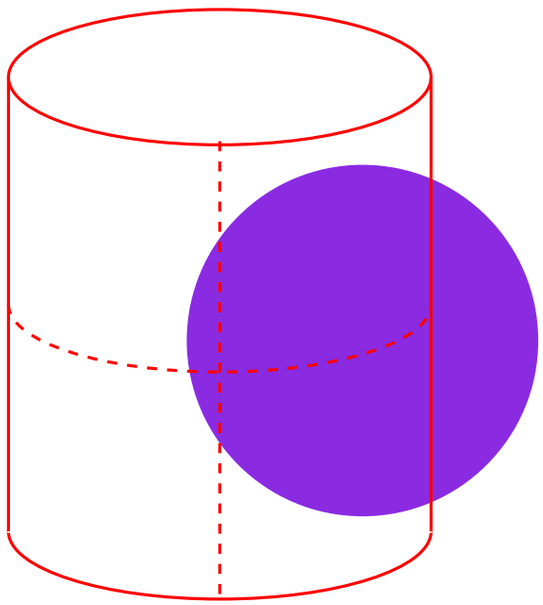
Intersection sphere/cylinder



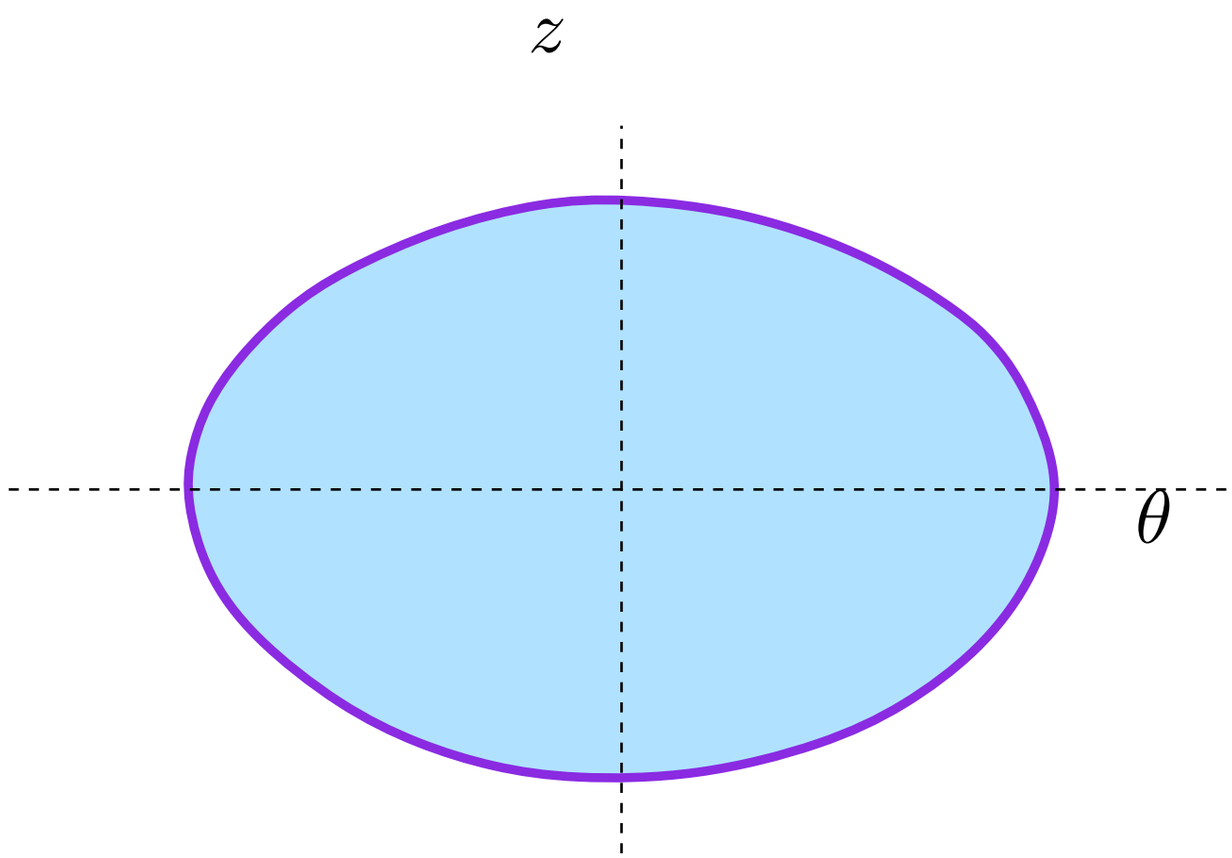
$r = 0.9$



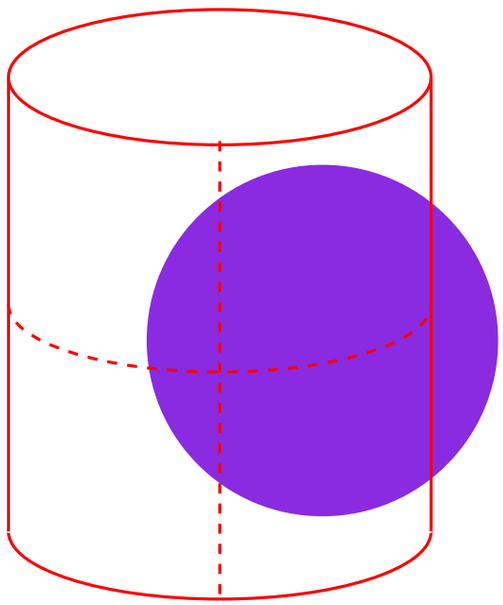
Intersection sphere/cylinder



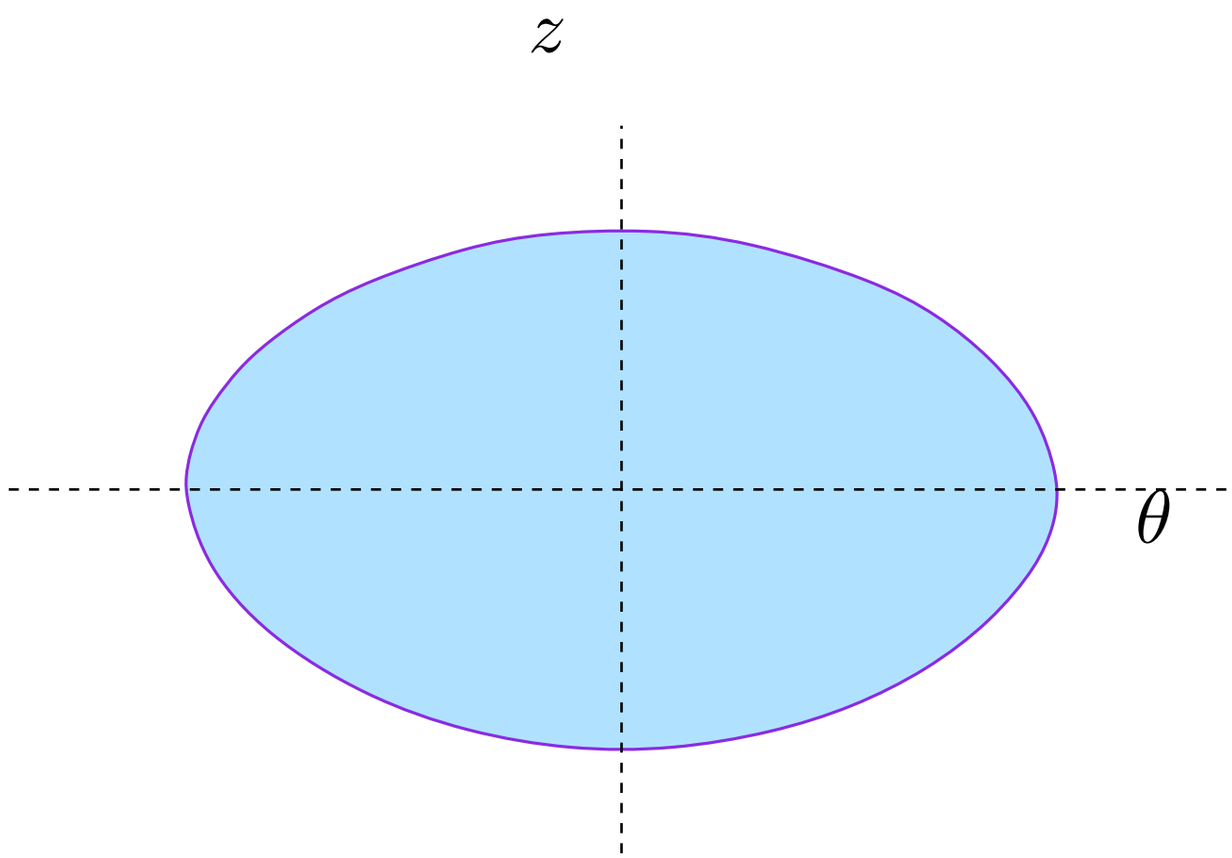
$r = 0.9$



Intersection sphere/cylinder

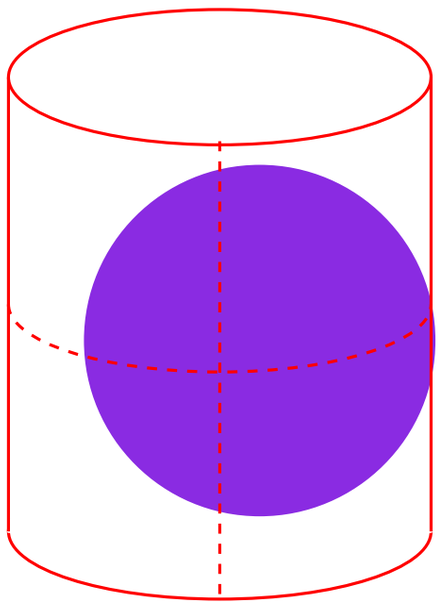


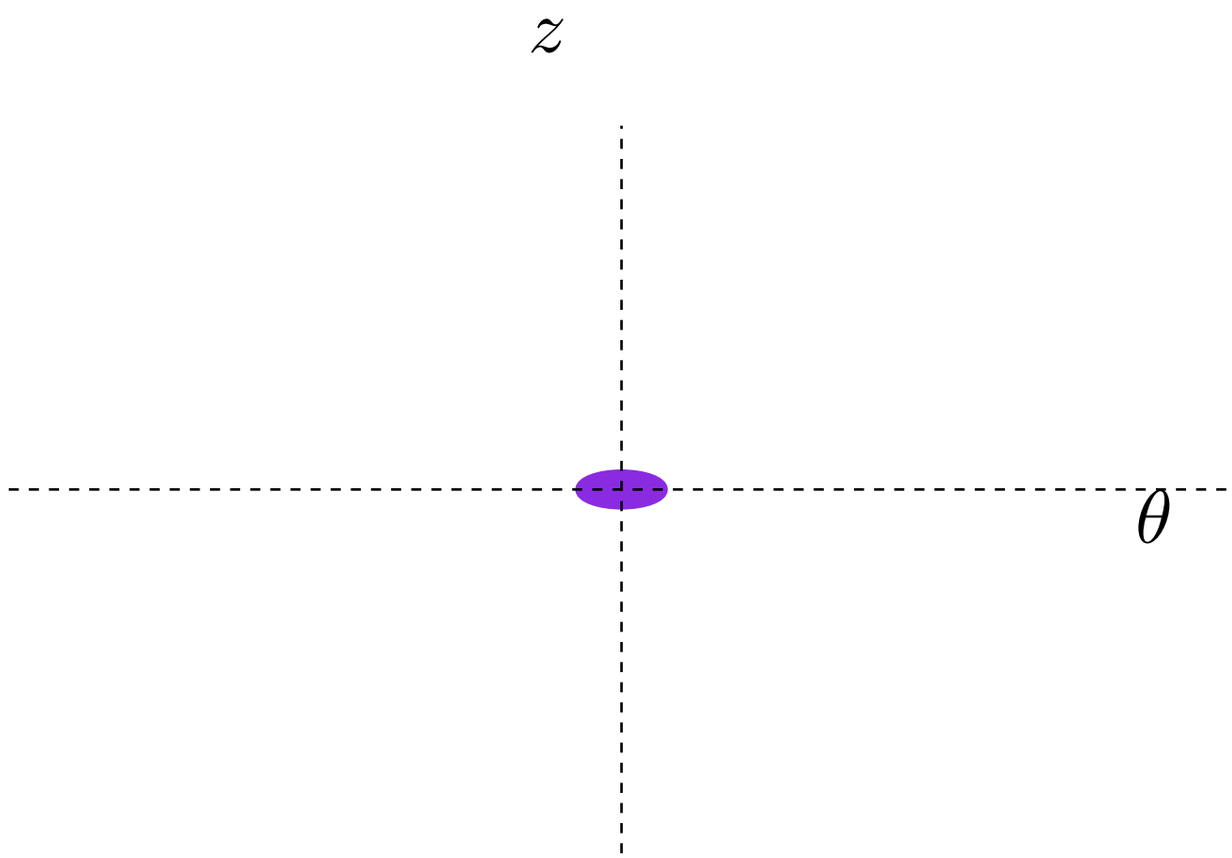
$r = 0.9$



Intersection sphere/cylinder

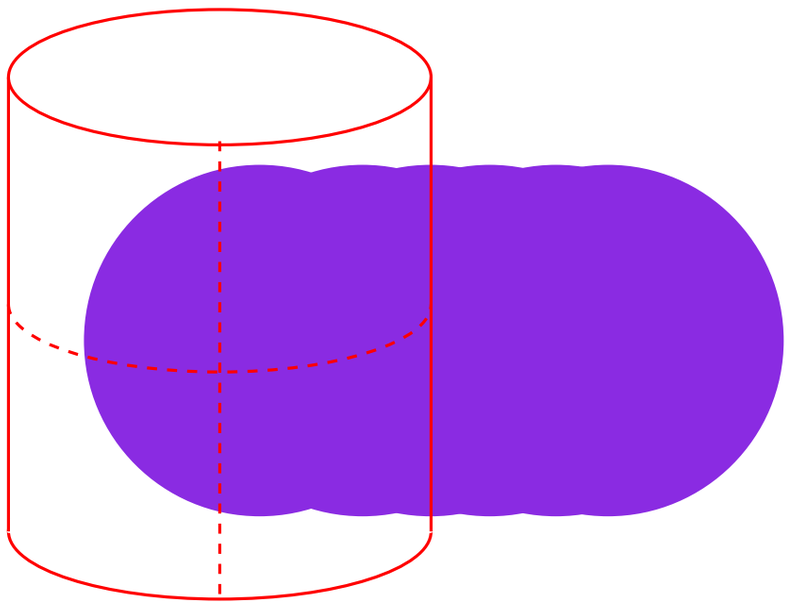
small radius $r = 0.9$

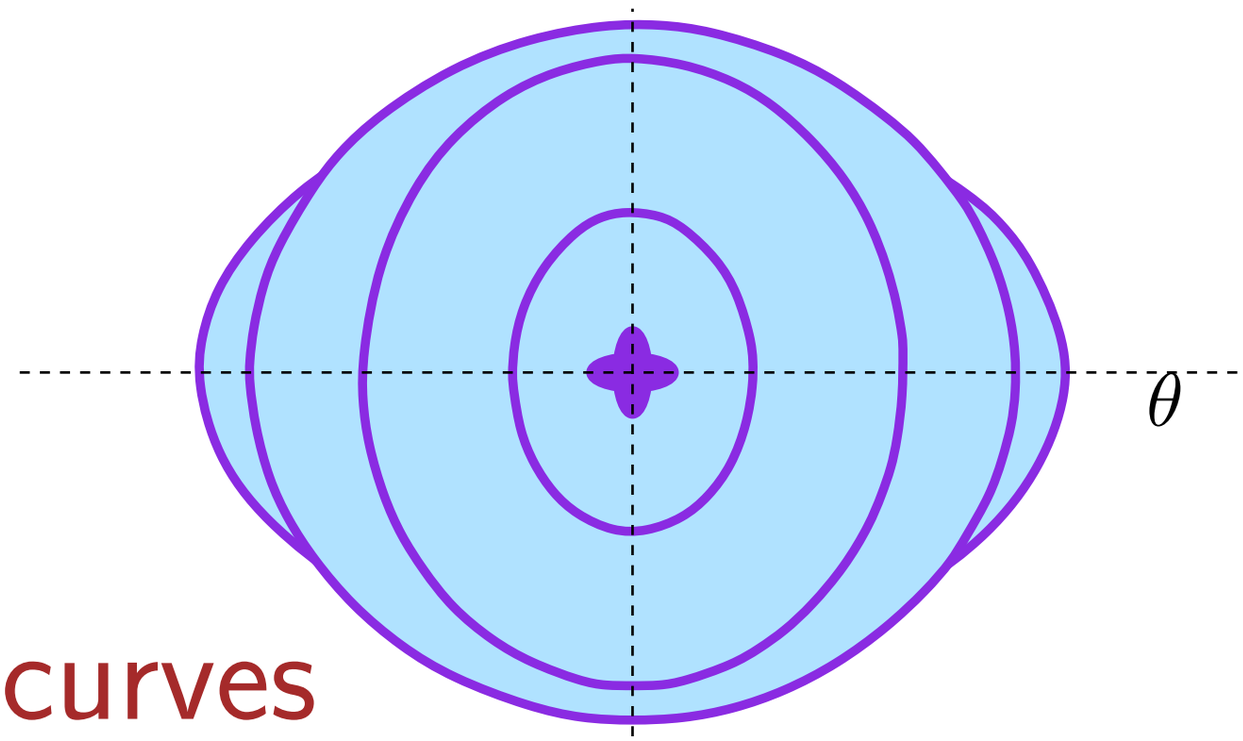




Intersection sphere/cylinder

small radius $r = 0.9$

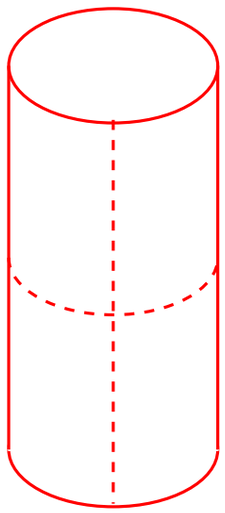


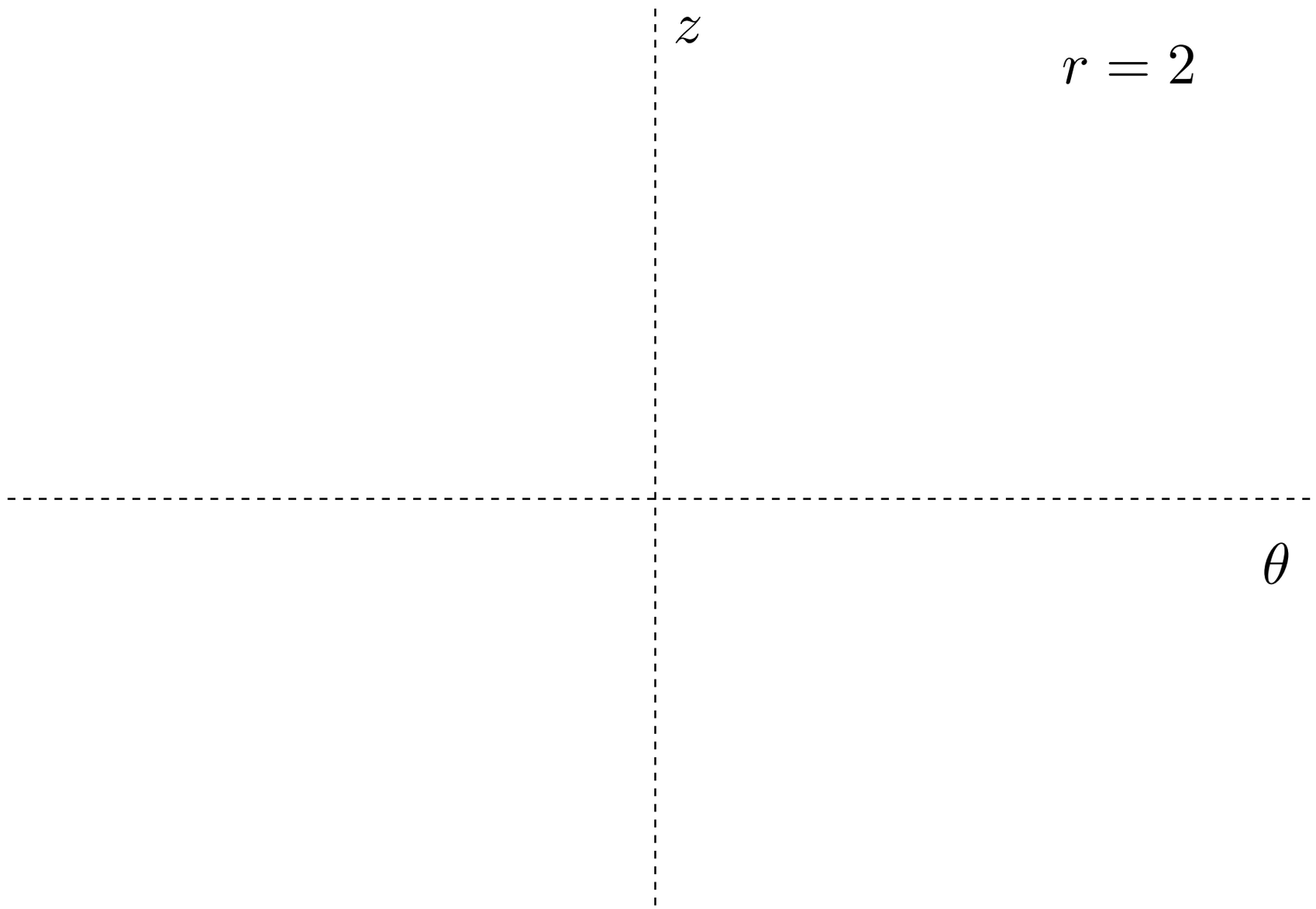


convex closed curves

Intersection sphere/cylinder

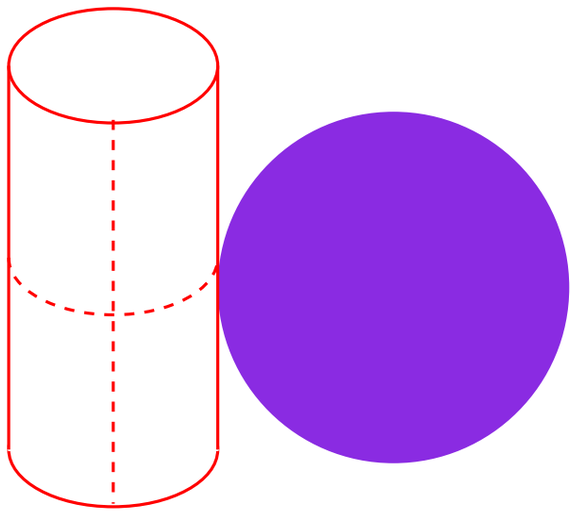
big radius

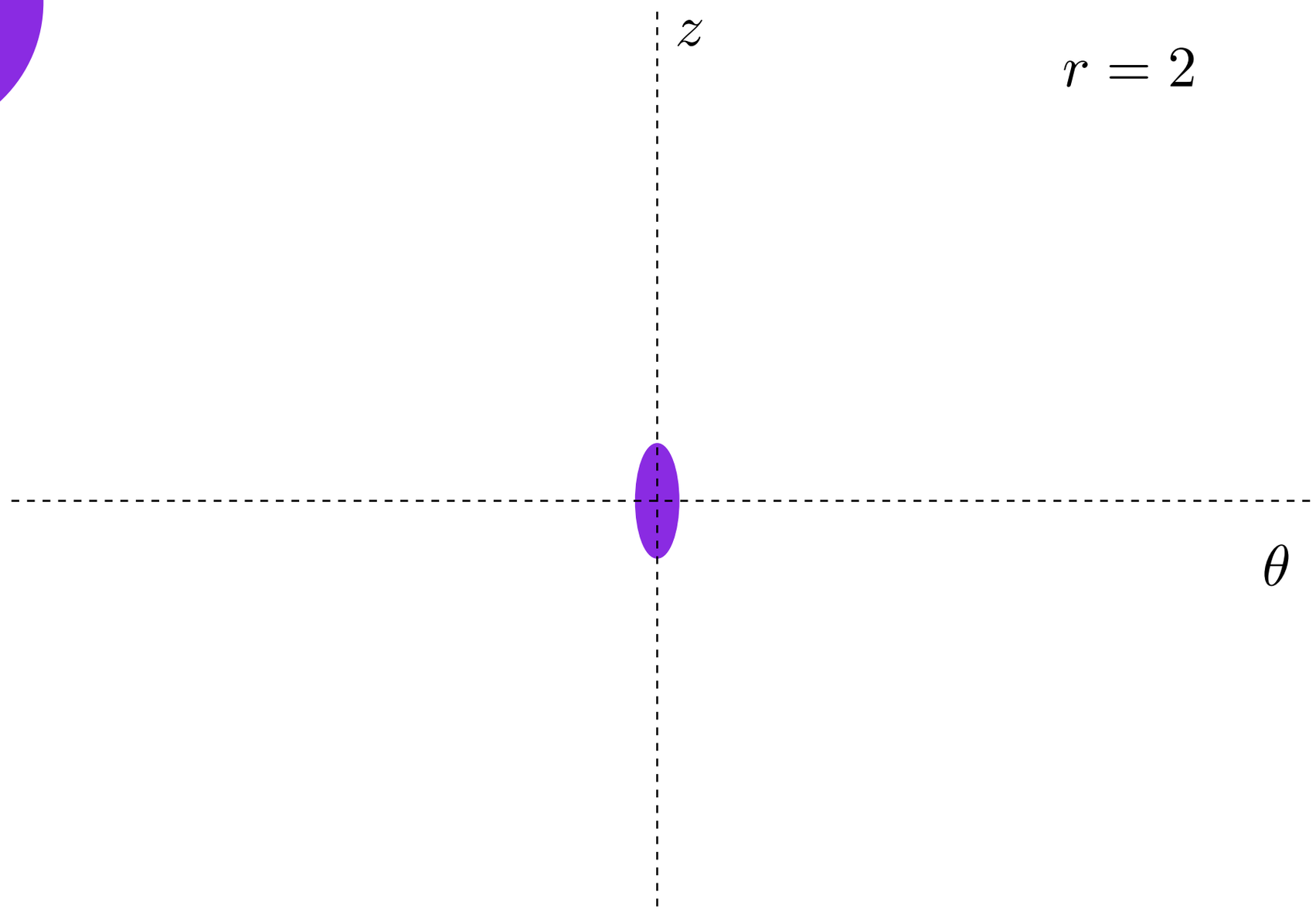




Intersection sphere/cylinder

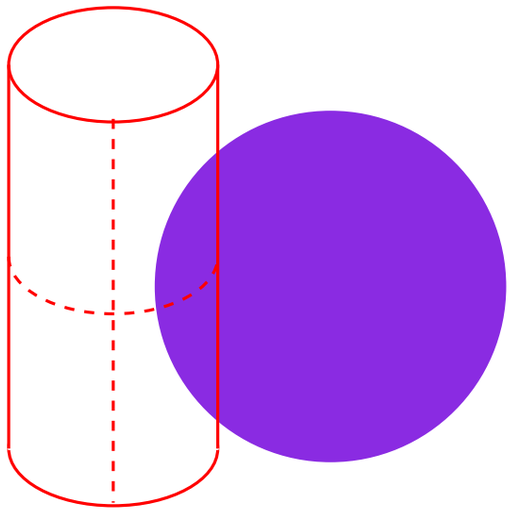
big radius

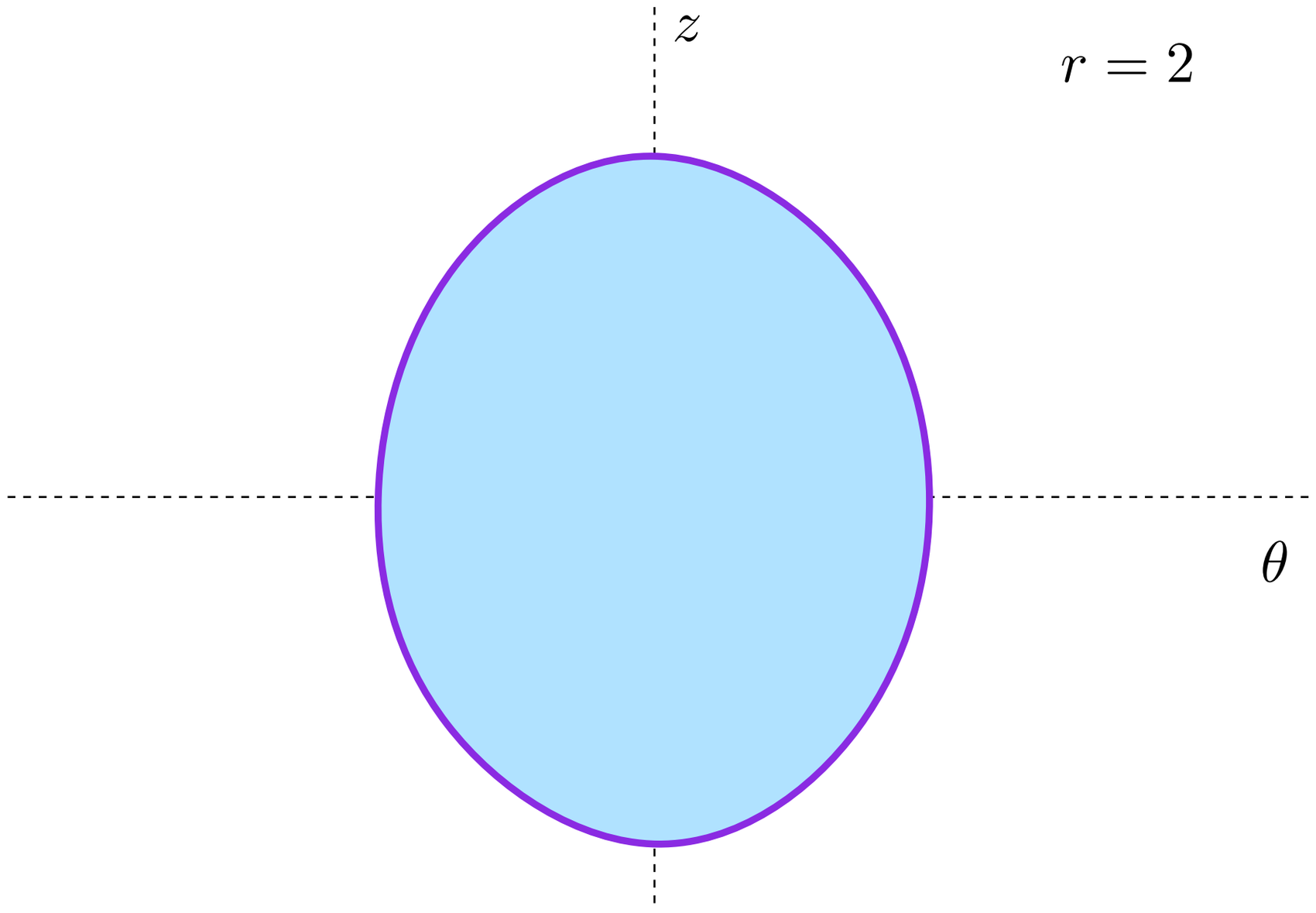




Intersection sphere/cylinder

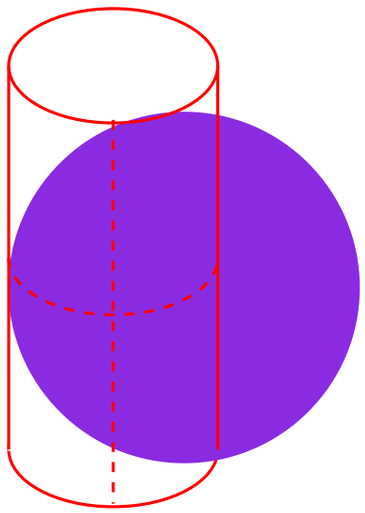
big radius

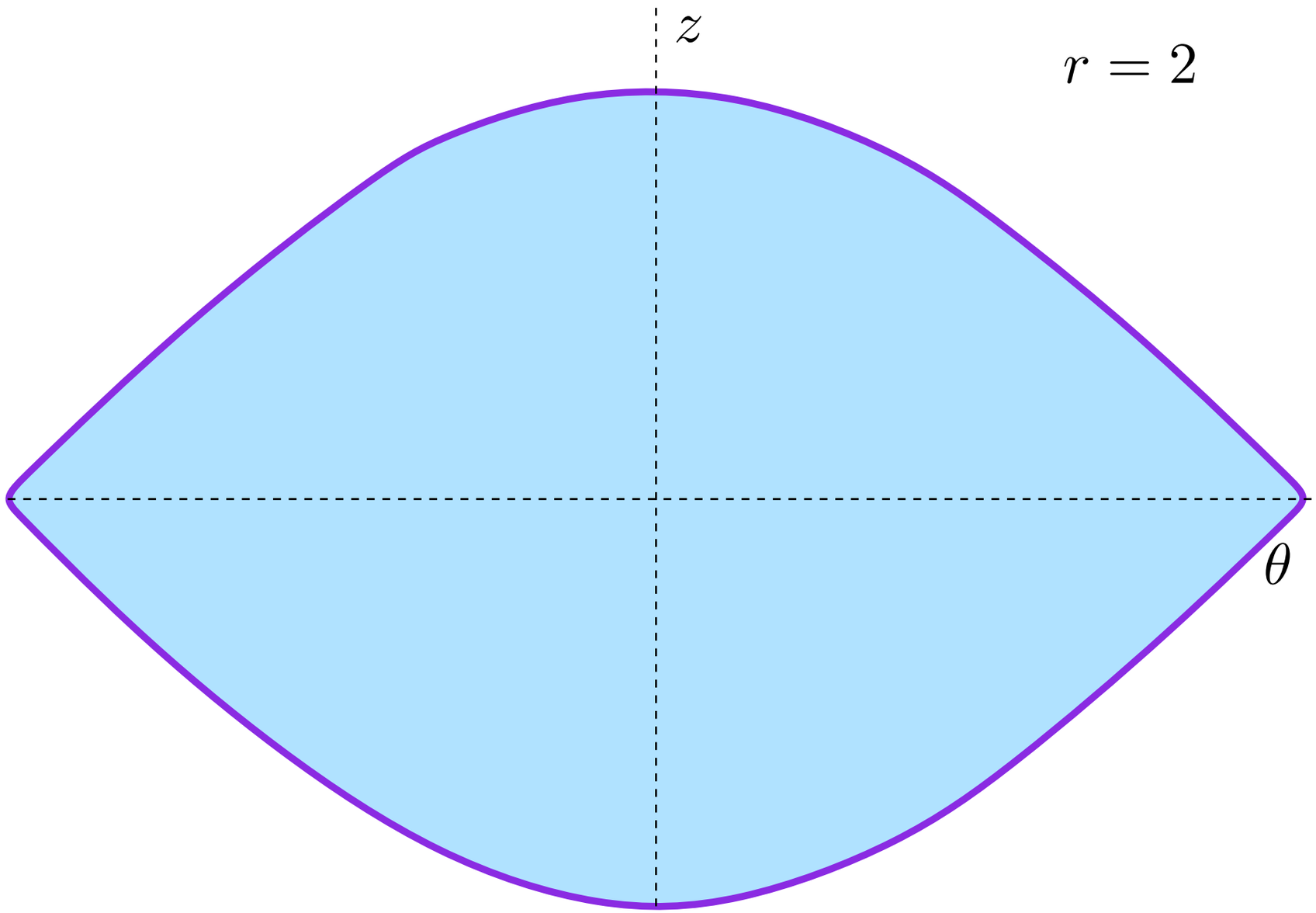




Intersection sphere/cylinder

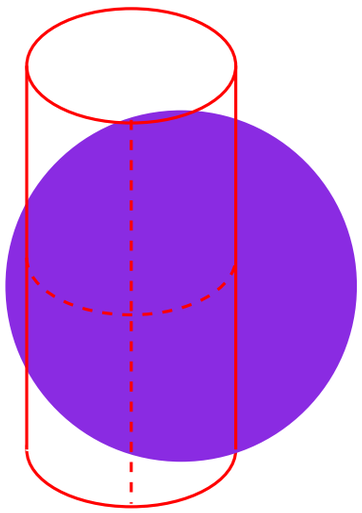
big radius

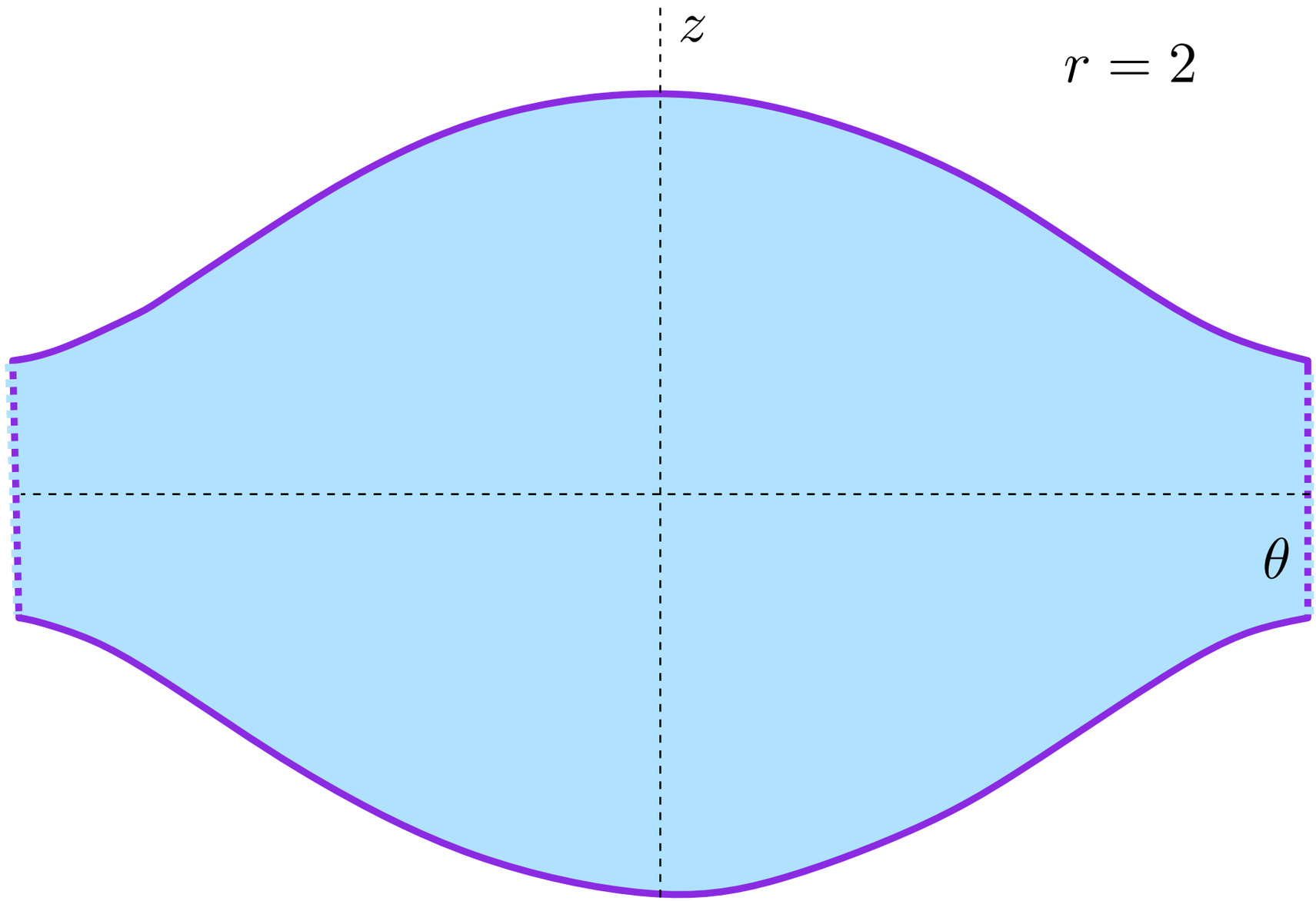




Intersection sphere/cylinder

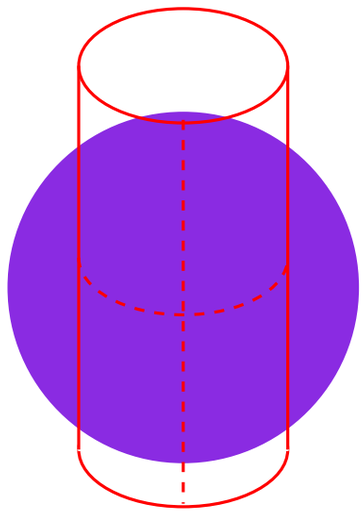
big radius

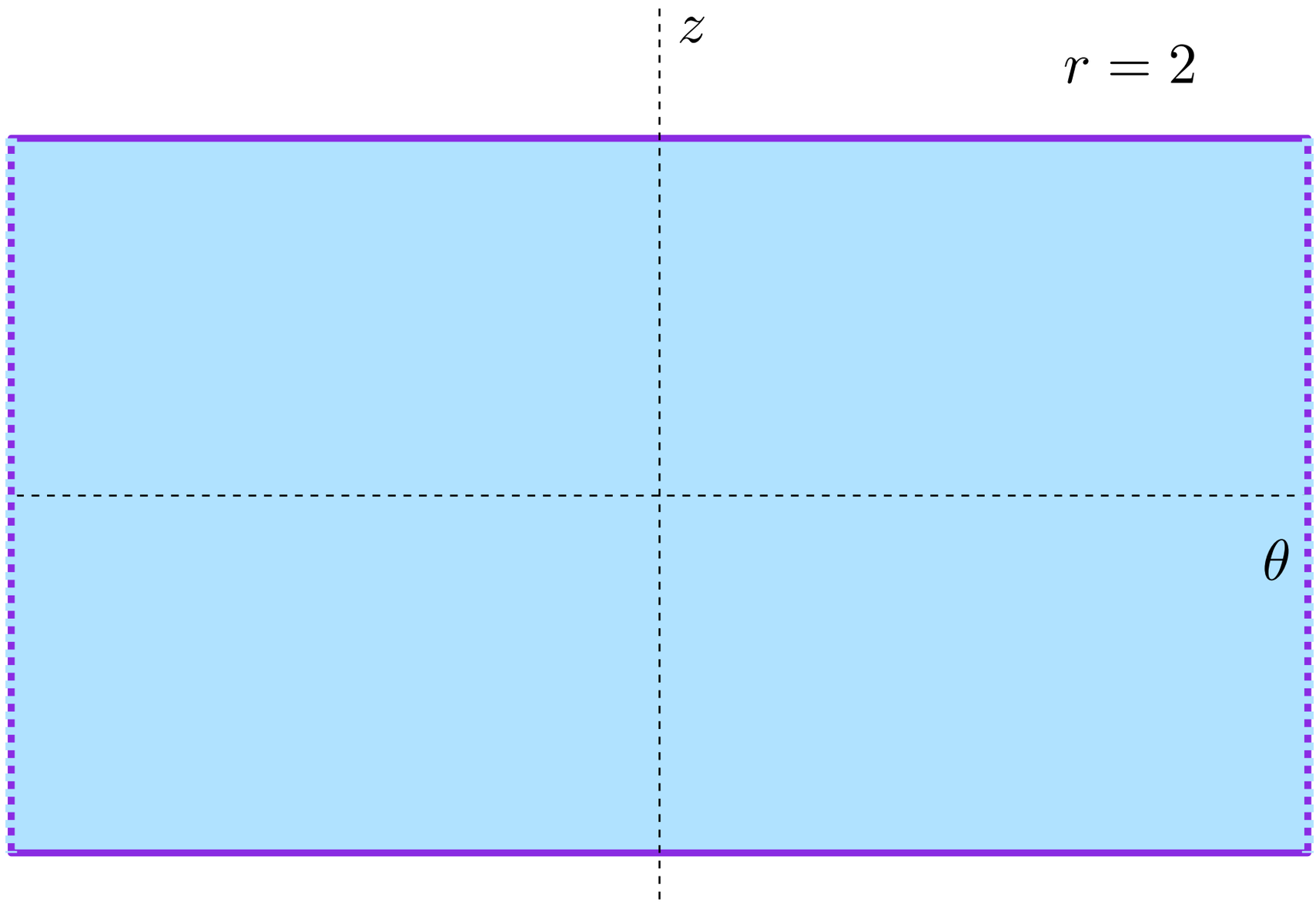




Intersection sphere/cylinder

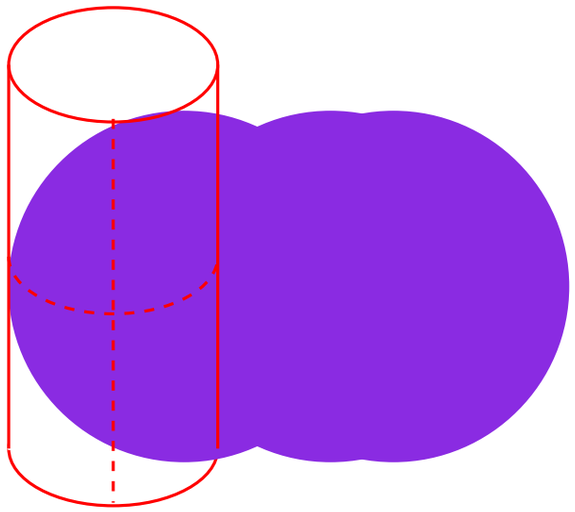
big radius

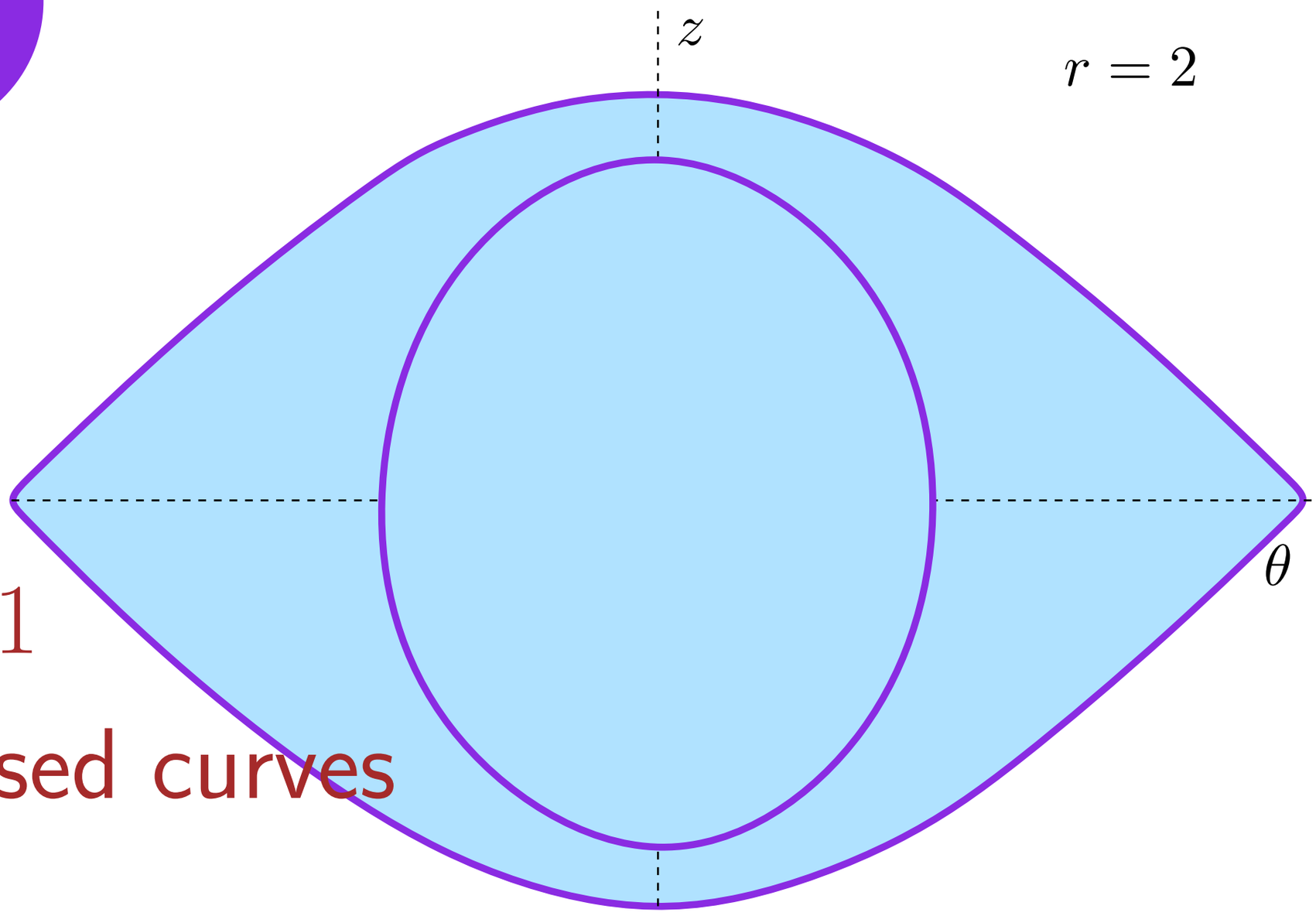




Intersection sphere/cylinder

big radius



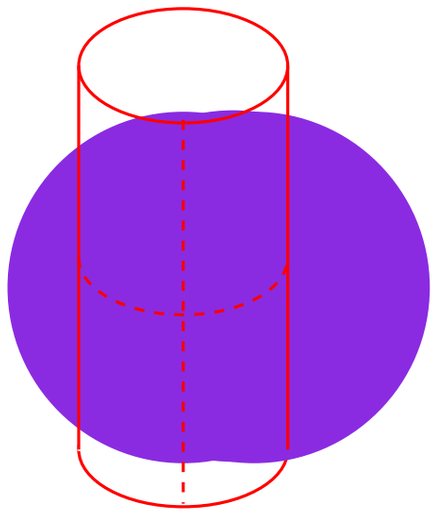


$$r \leq R + 1$$

convex closed curves

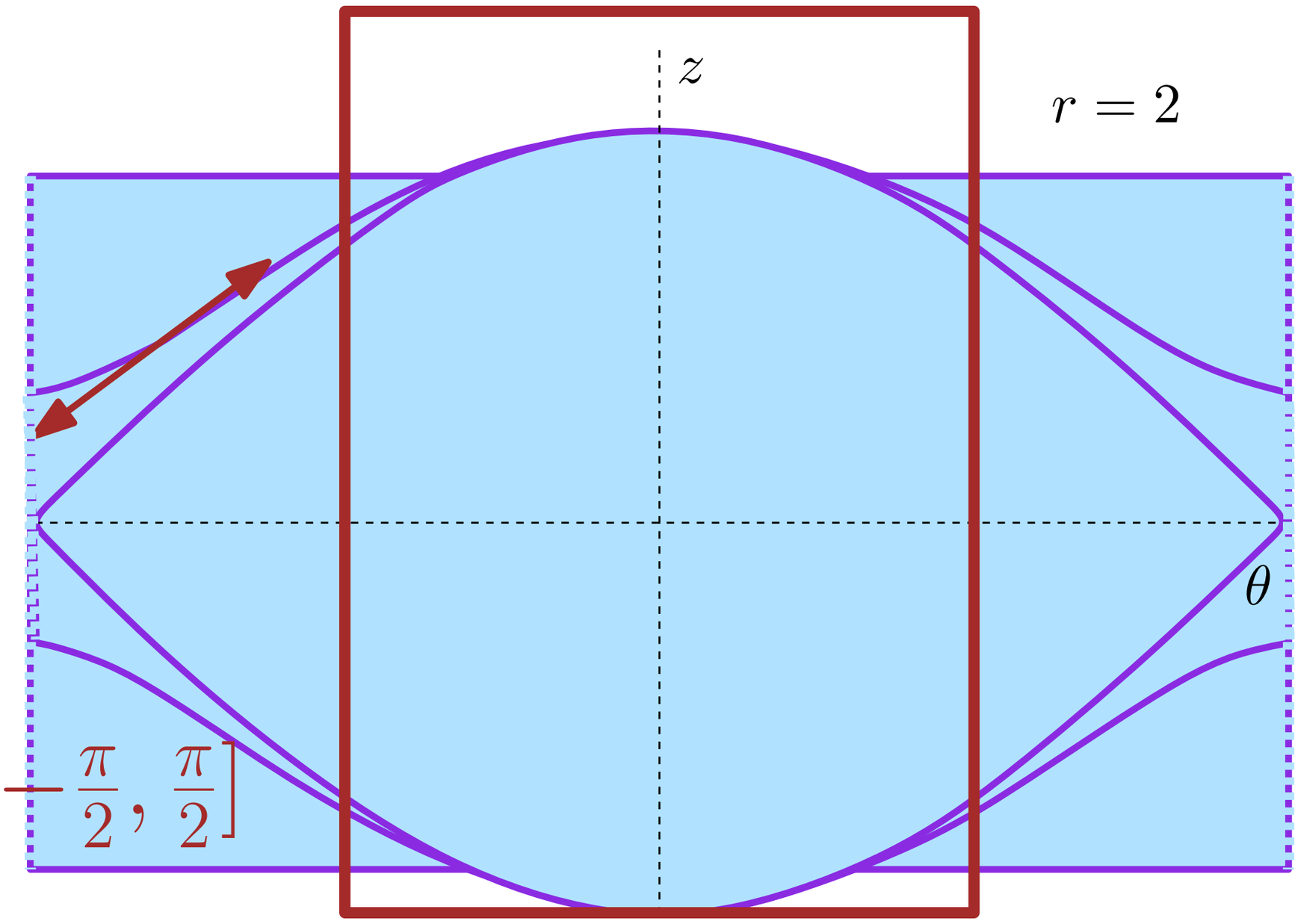
Intersection sphere/cylinder

big radius

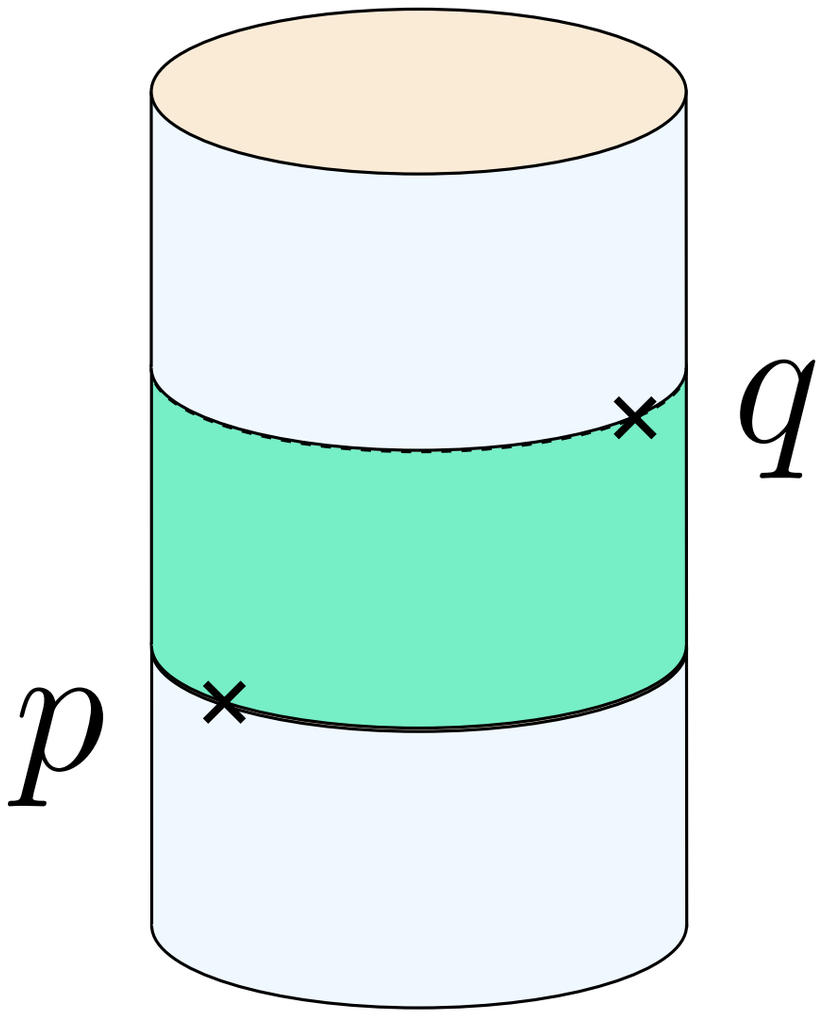


$r \geq R + 1$

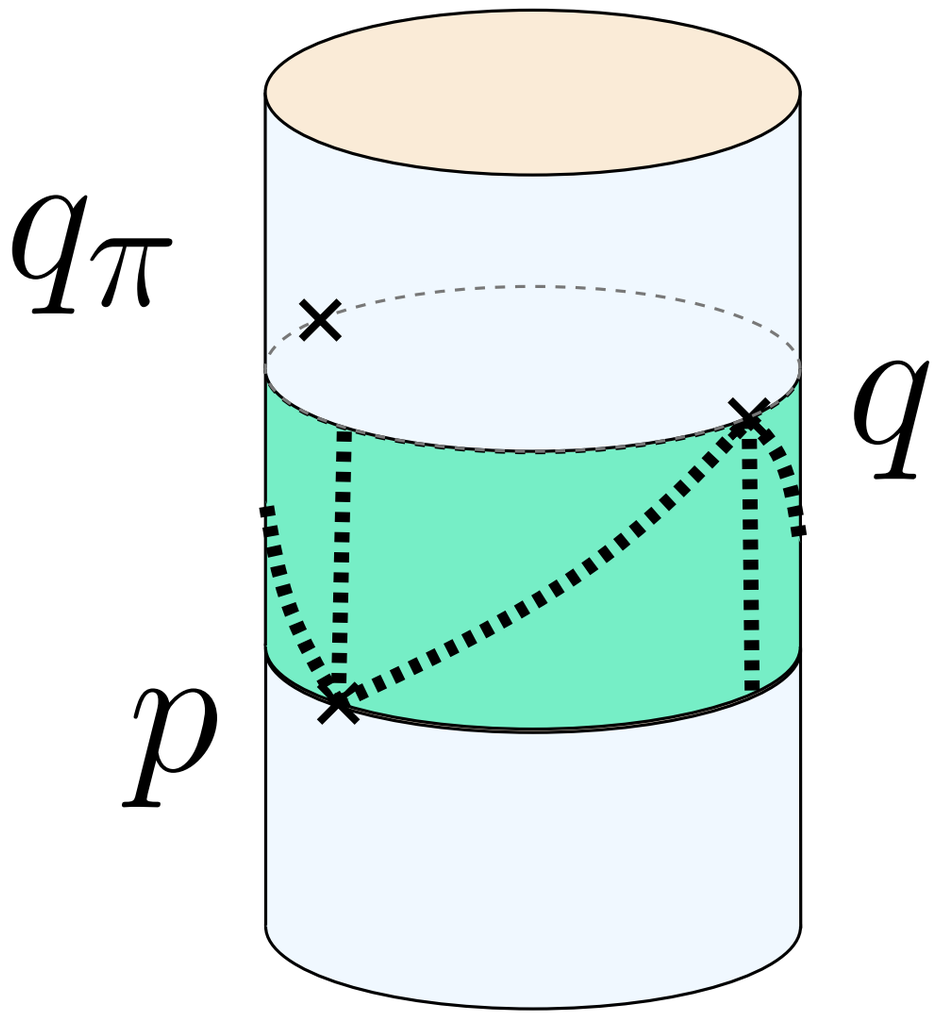
convex on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



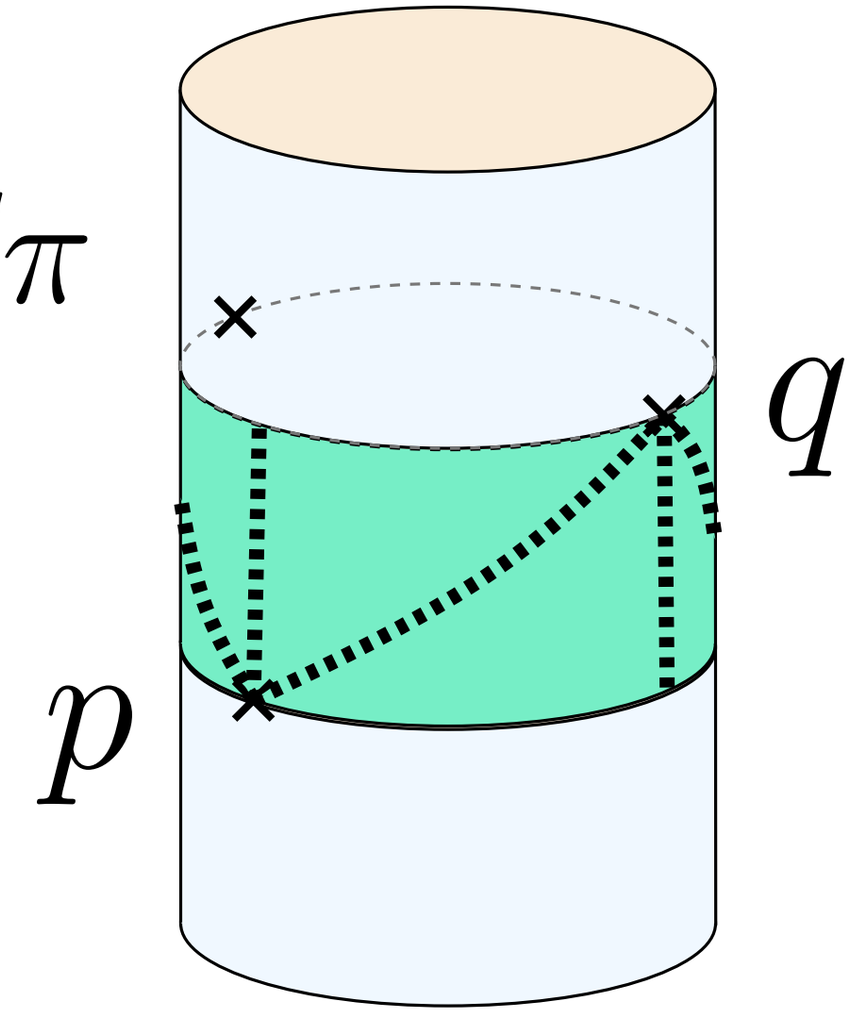
Triangulated slab graph

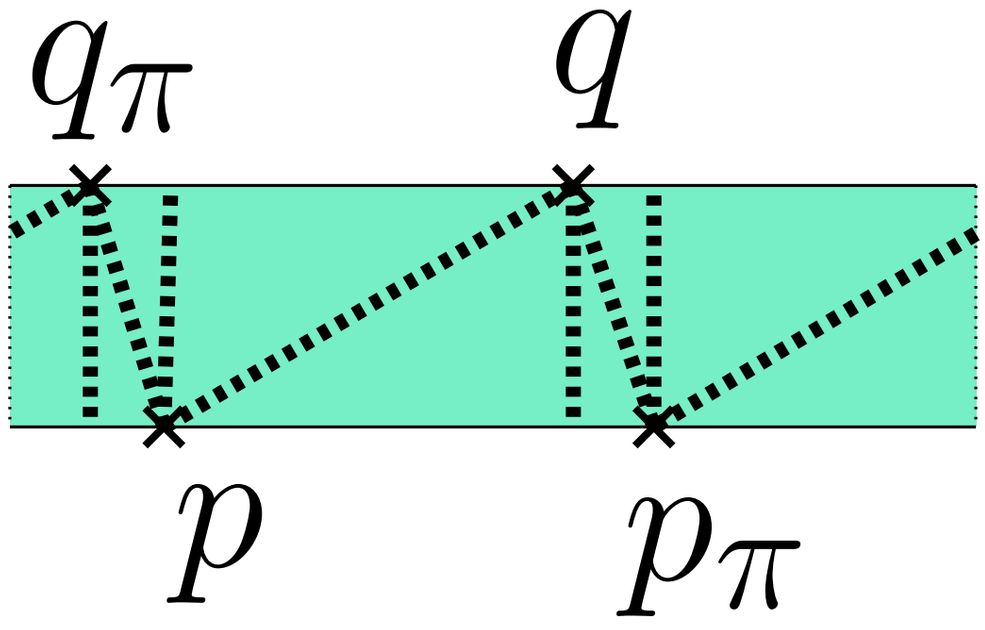


Triangulated slab graph

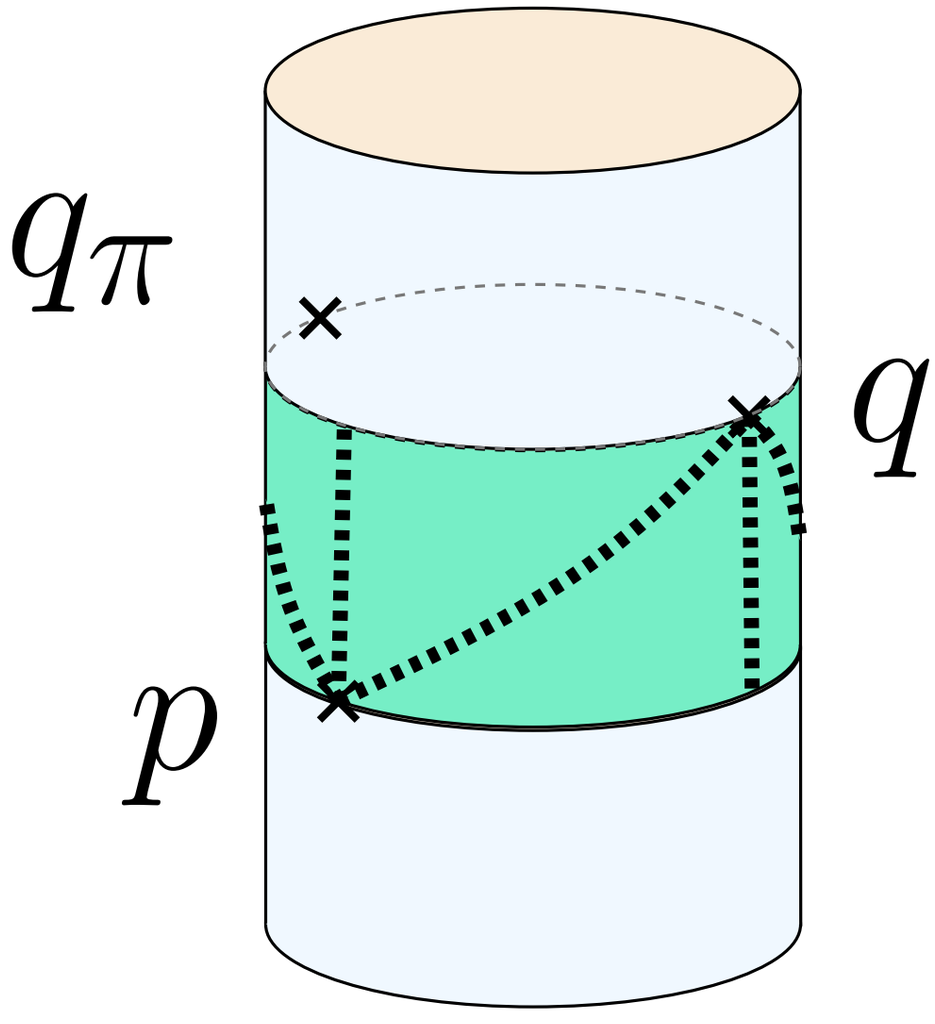


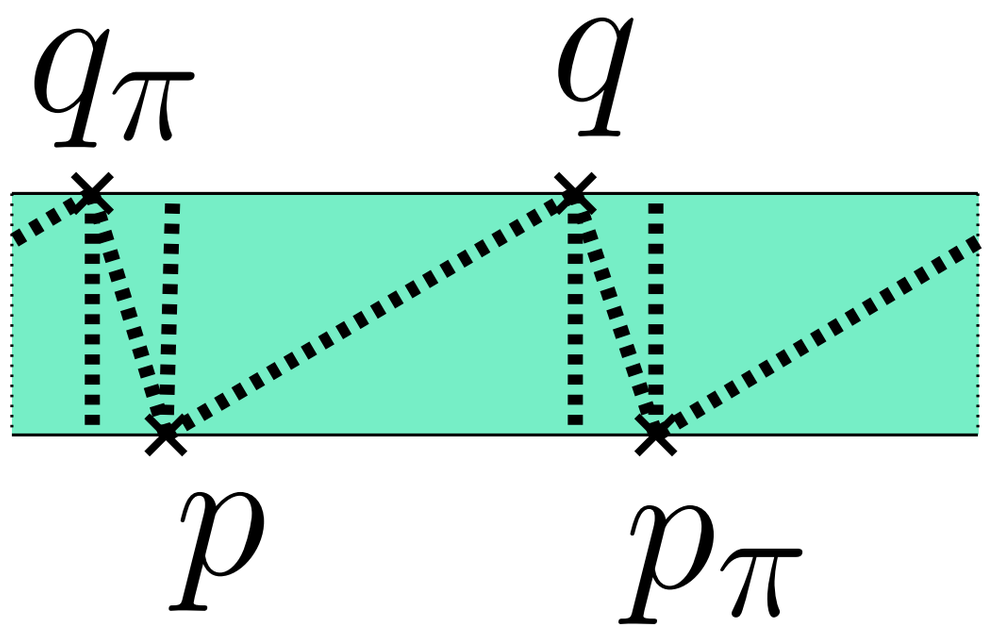
Triangulated slab graph





Triangulated slab graph





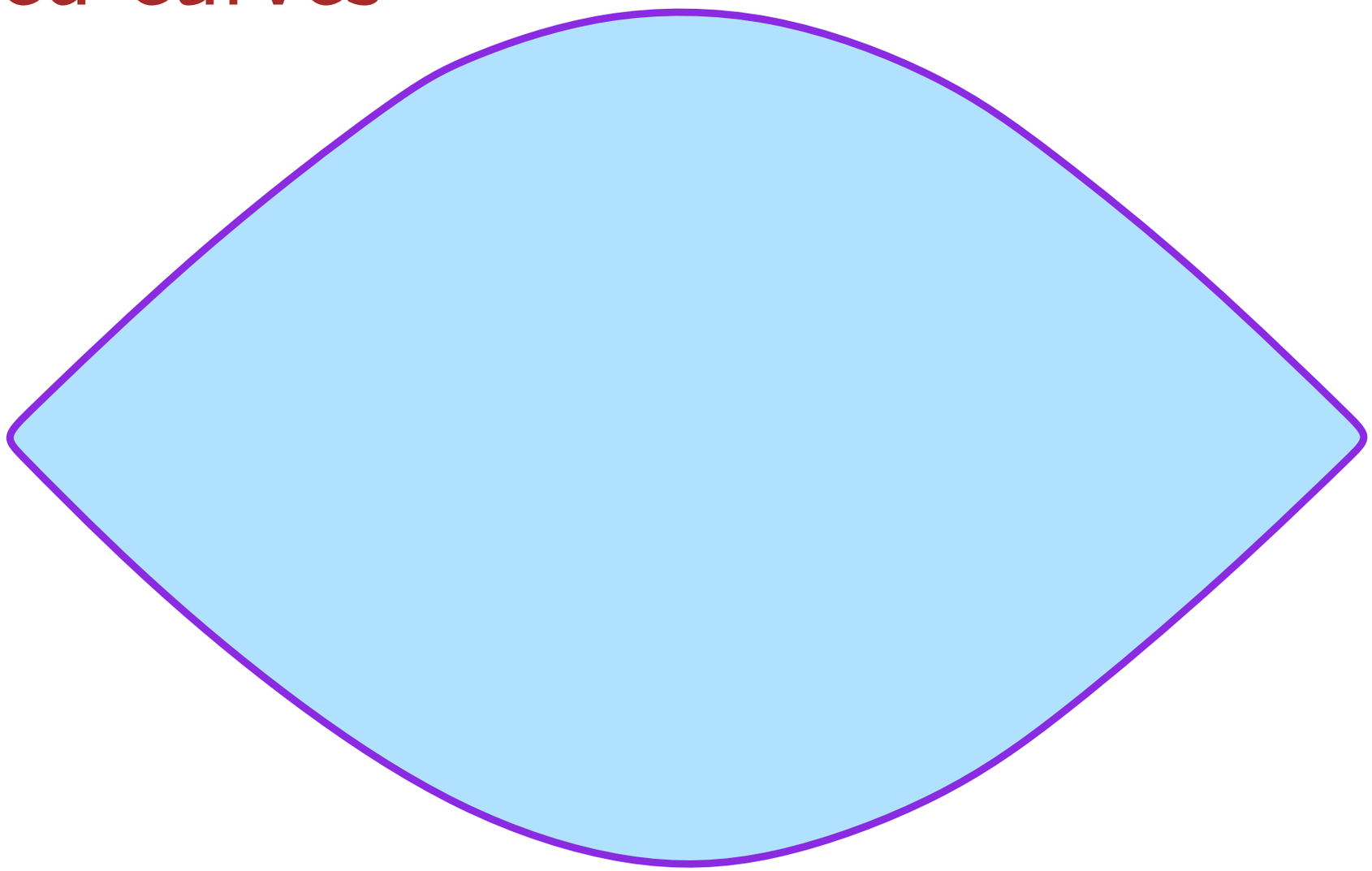
pq edge of TSG

if one of the 8 triangles empty

Delaunay $\subset$ TSG

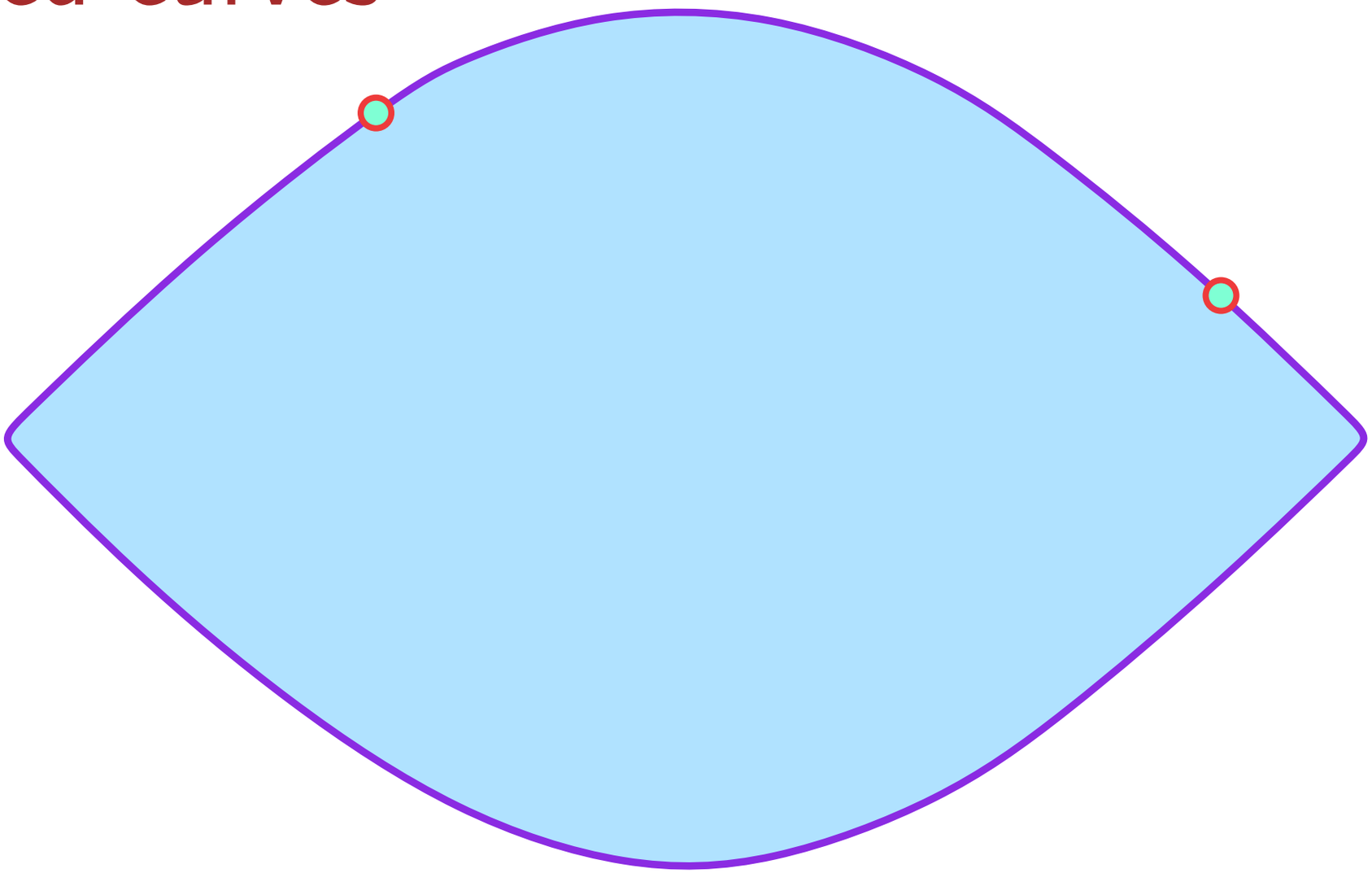
Delaunay $\subset$ TSG

convex closed curves



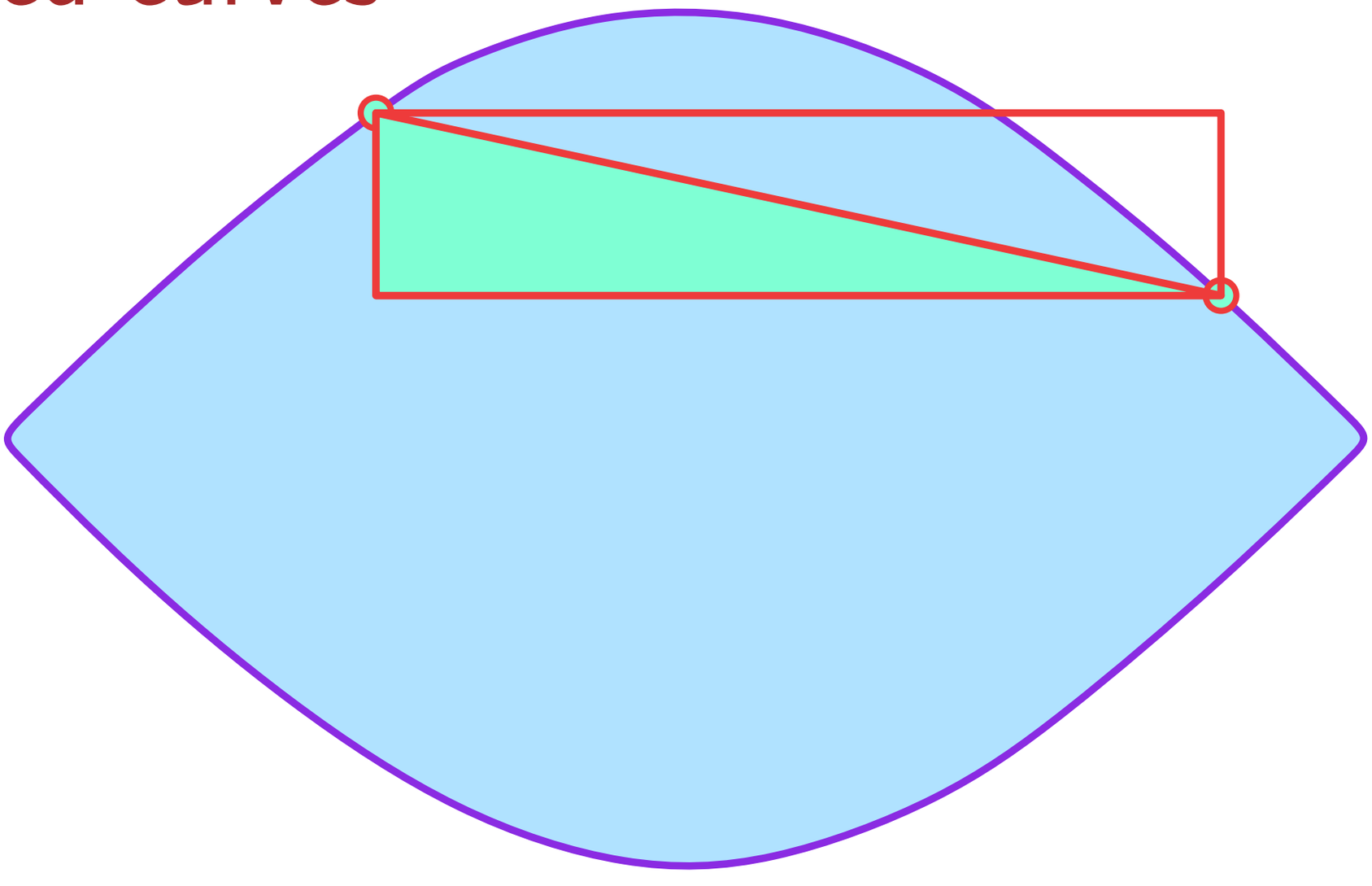
Delaunay $\subset$ TSG

convex closed curves



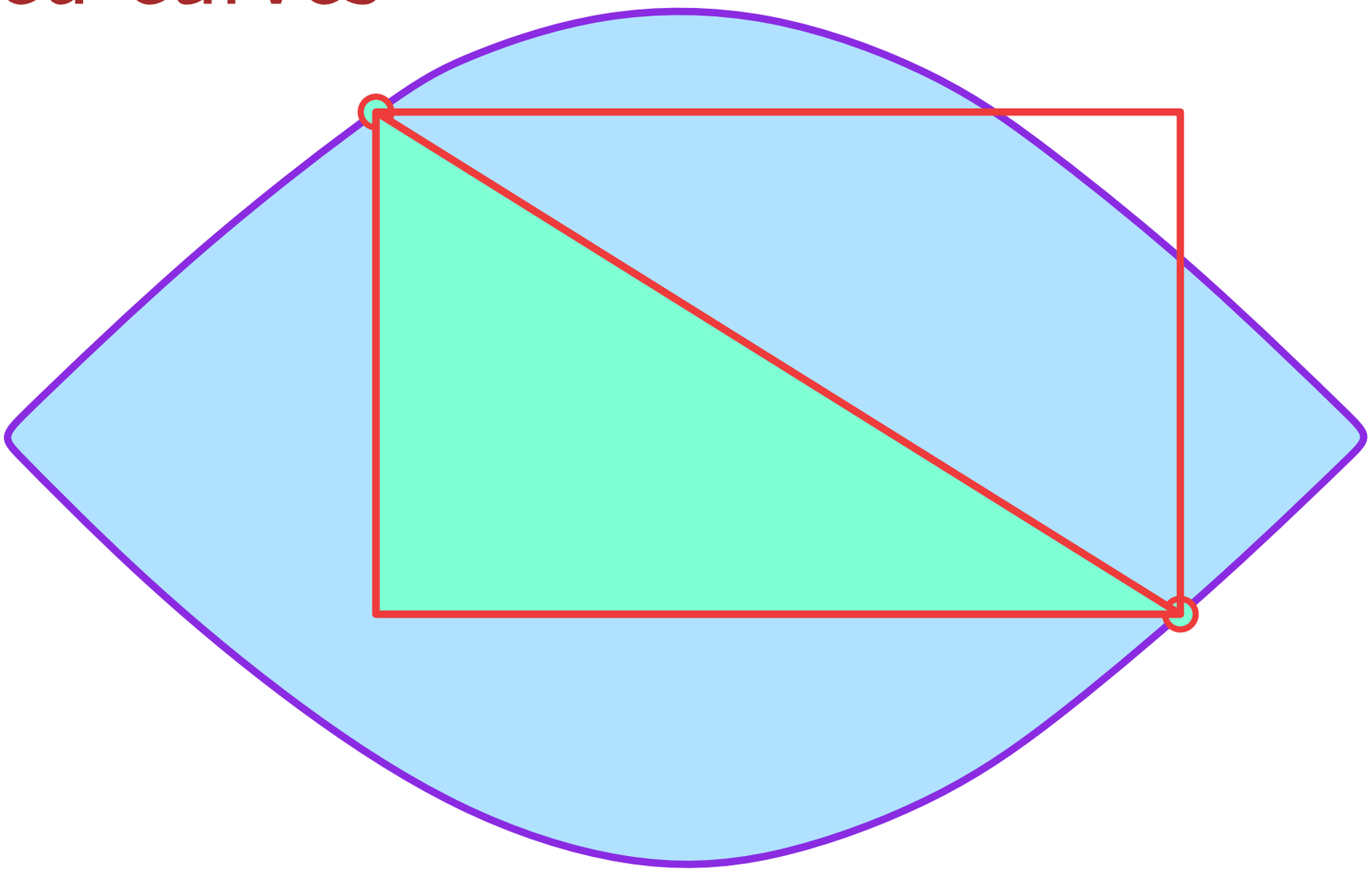
Delaunay $\subset$ TSG

convex closed curves

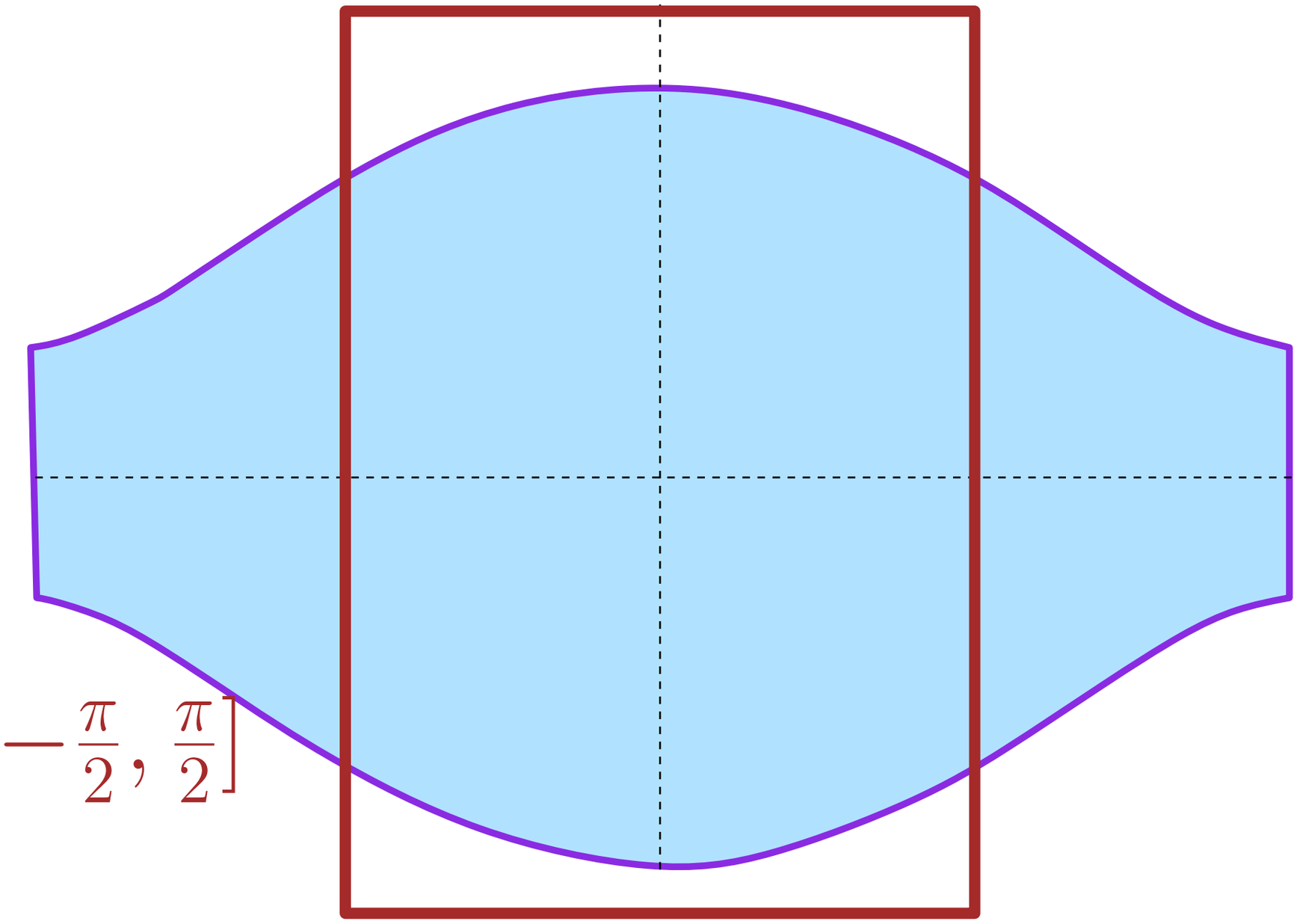


Delaunay $\subset$ TSG

convex closed curves



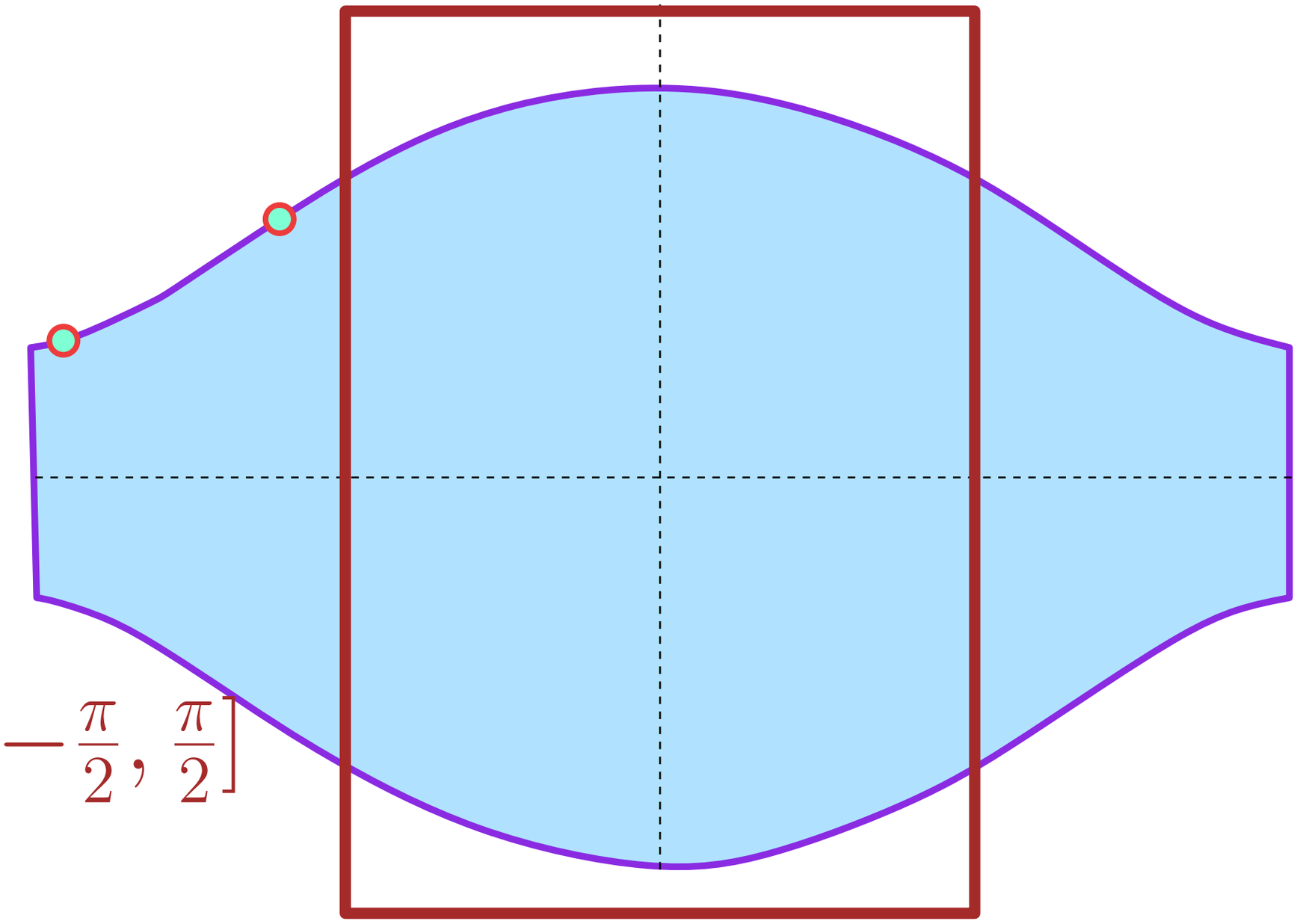
Delaunay $\subset$ TSG



$r \geq R + 1$

convex on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

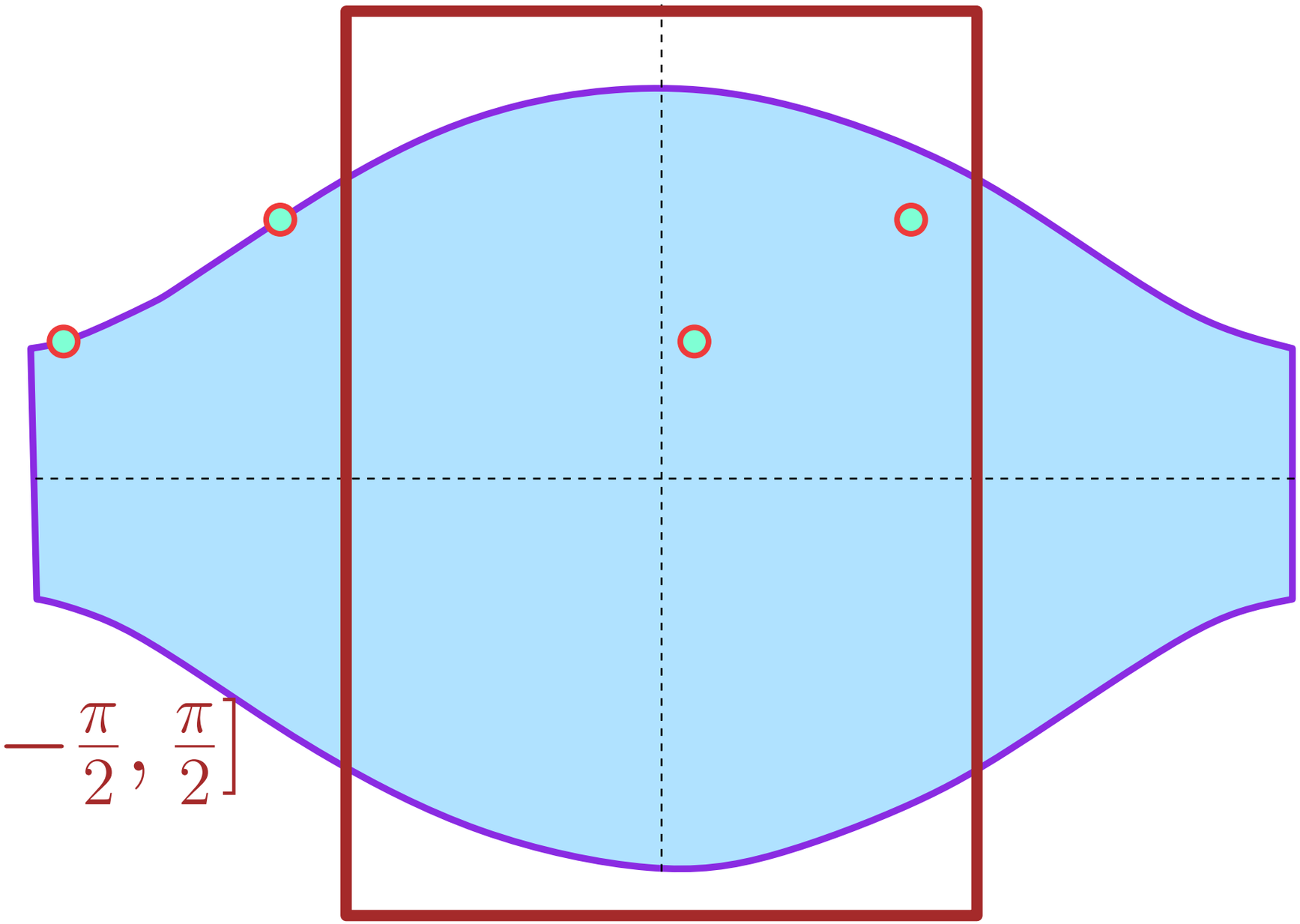
Delaunay $\subset$ TSG



$r \geq R + 1$

convex on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

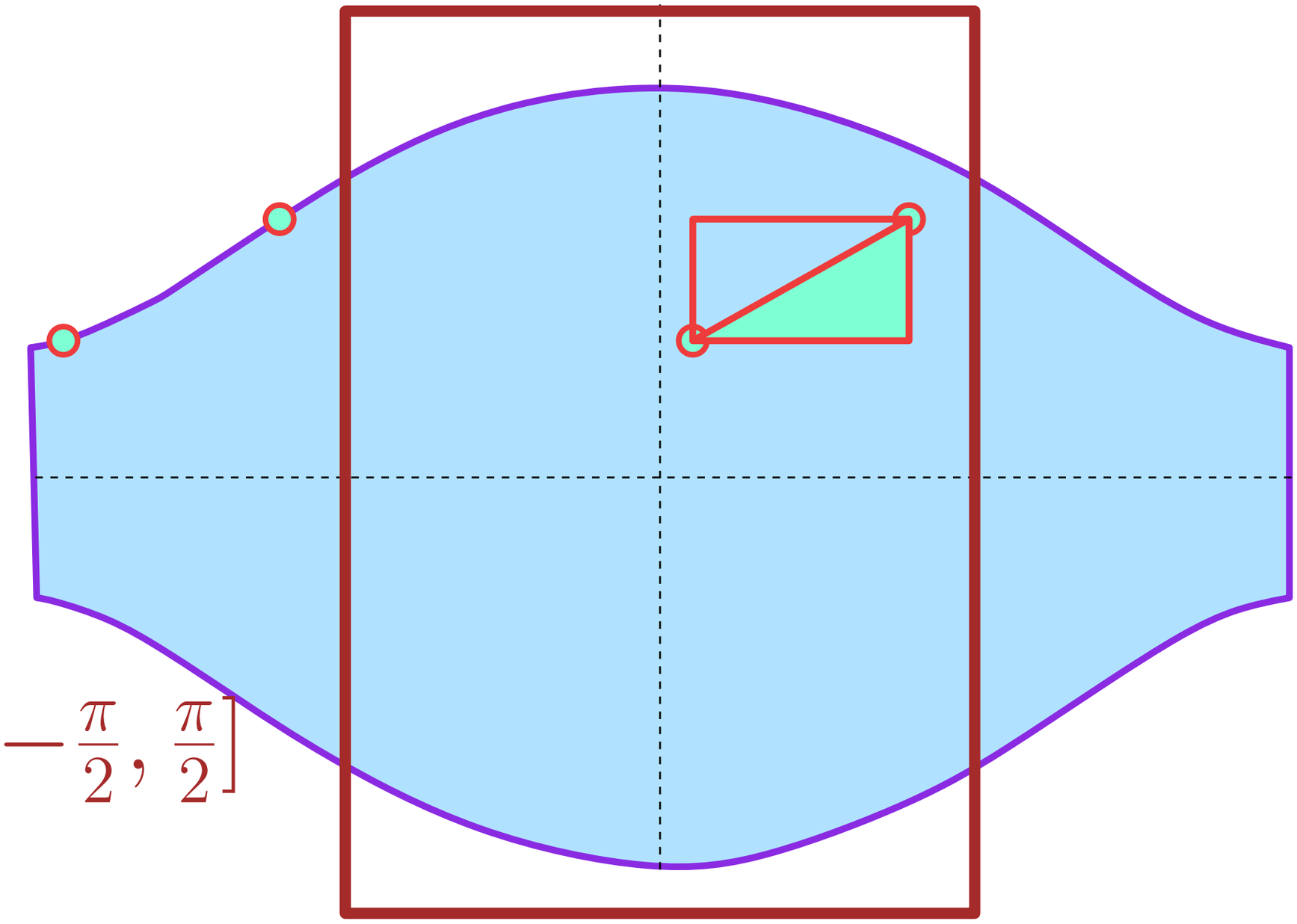
Delaunay $\subset$ TSG



$r \geq R + 1$

convex on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Delaunay $\subset$ TSG



$r \geq R + 1$

convex on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$$|\mathsf{TSG}| = O(n \log n)$$

$$Pr(pq \in \mathsf{TSG})$$

$$|\mathsf{TSG}| = O(n \log n)$$

$$Pr(pq \in \mathsf{TSG})$$
$$= \int_{z_q} \int_{\theta_q} Pr(pq \in \mathsf{TSG}) d\theta dz$$

$$|\mathsf{TSG}| = O(n \log n)$$

$$\begin{aligned} & Pr(pq \in \mathsf{TSG}) \\ &= \int_{z_q} \int_{\theta_q} Pr(pq \in \mathsf{TSG}) d\theta dz \\ &\leq O\left(\int \int Pr(\text{first of the 8 triangle is empty}) d\theta dz \right) \end{aligned}$$

$$|\mathsf{TSG}| = O(n \log n)$$

$$\begin{aligned}
&Pr(pq \in \mathsf{TSG}) \\
&= \int_{z_q} \int_{\theta_q} Pr(pq \in \mathsf{TSG}) d\theta dz \\
&\leq O\left(\int \int Pr(\text{first of the 8 triangle is empty}) d\theta dz \right) \\
&\leq O\left(\int \int (1 - z\theta)^{n-2} d\theta dz \right)
\end{aligned}$$

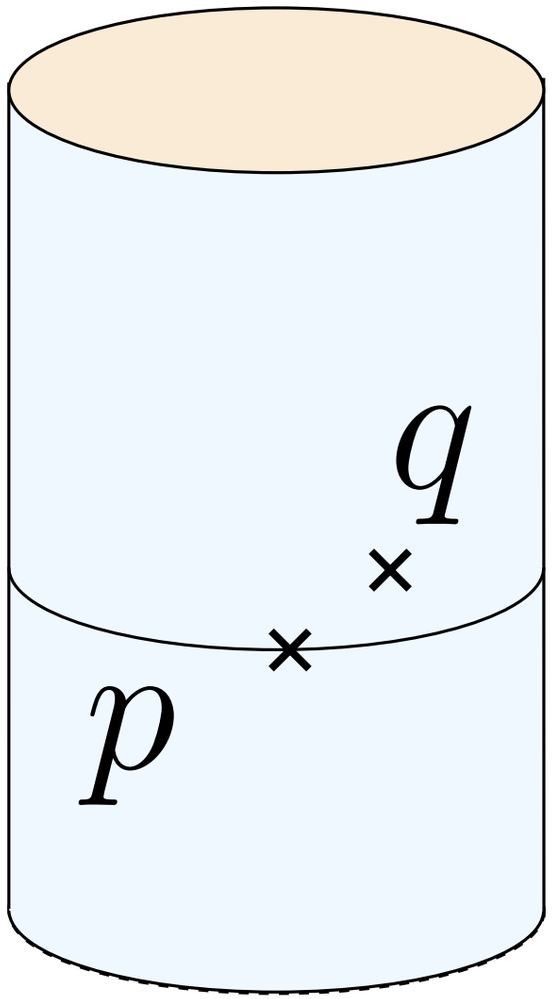
$$|\mathsf{TSG}| = O(n \log n)$$

$$\begin{aligned}
&Pr(pq \in \mathsf{TSG}) \\
&= \int_{z_q} \int_{\theta_q} Pr(pq \in \mathsf{TSG}) d\theta dz \\
&\leq O\left(\int \int Pr(\text{first of the 8 triangle is empty}) d\theta dz \right) \\
&\leq O\left(\int \int (1 - z\theta)^{n-2} d\theta dz \right) \\
&\leq O\left(\int \int e^{nz\theta} d\theta dz \right)
\end{aligned}$$

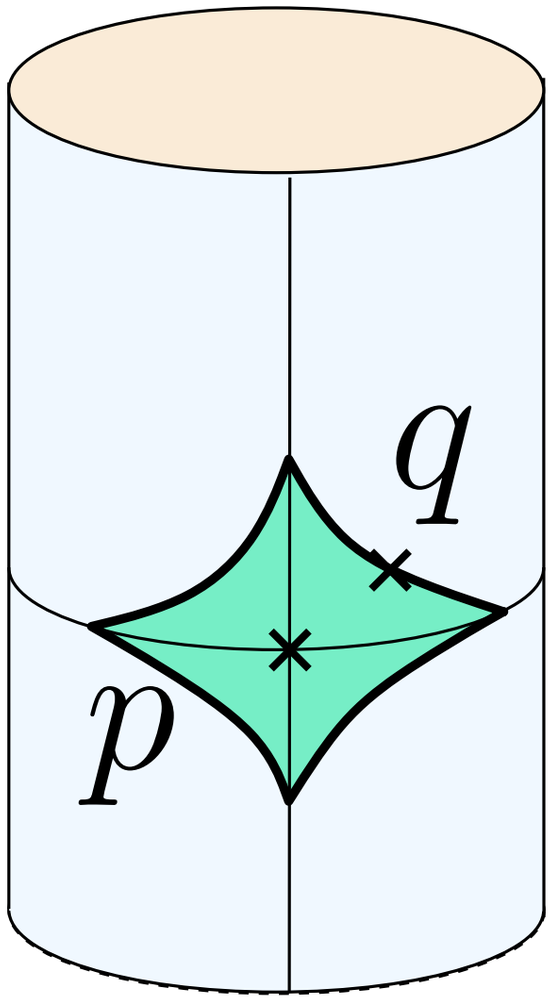
$$|\mathsf{TSG}| = O(n \log n)$$

$$\begin{aligned}
&Pr(pq \in \mathsf{TSG}) \\
&= \int_{z_q} \int_{\theta_q} Pr(pq \in \mathsf{TSG}) d\theta dz \\
&\leq O\left(\int \int Pr(\text{first of the 8 triangle is empty}) d\theta dz \right) \\
&\leq O\left(\int \int (1 - z\theta)^{n-2} d\theta dz \right) \\
&\leq O\left(\int \int e^{nz\theta} d\theta dz \right) \\
&\leq O\left(n \log n \right)
\end{aligned}$$

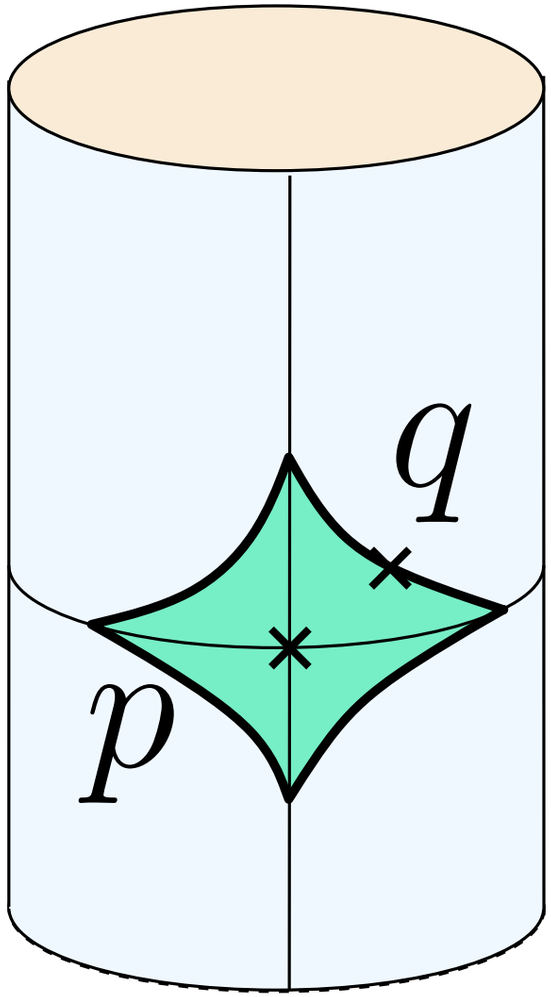
Rhombus graph

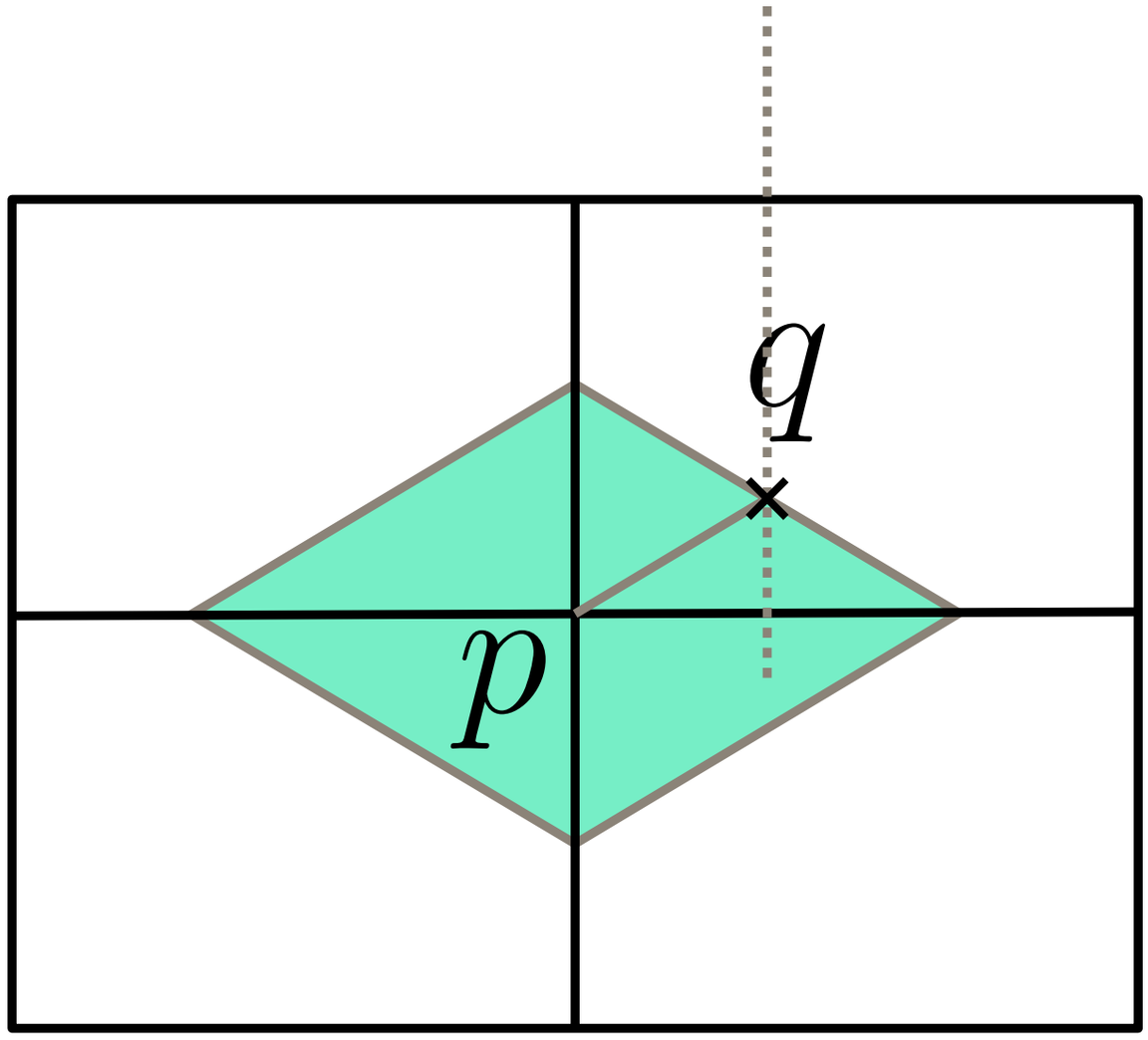


Rhombus graph

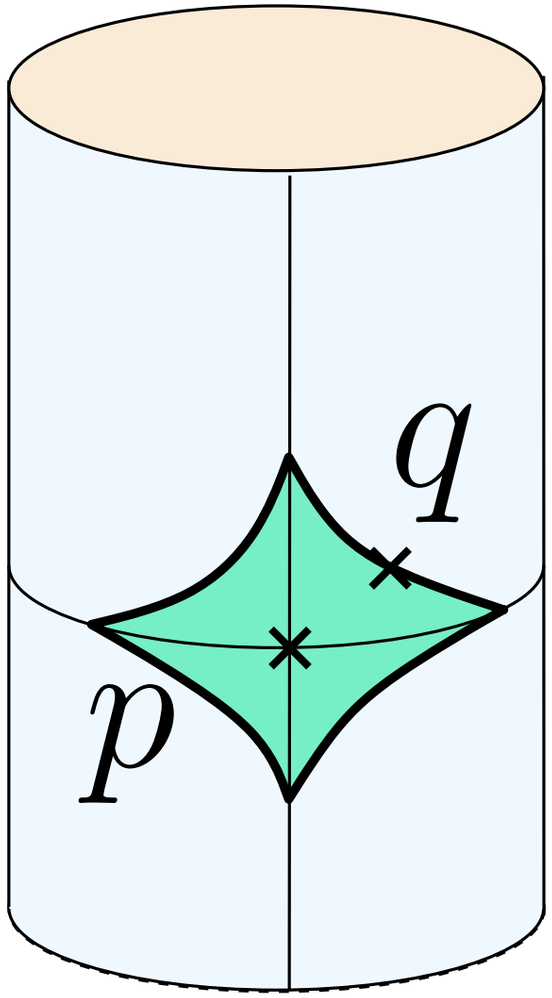


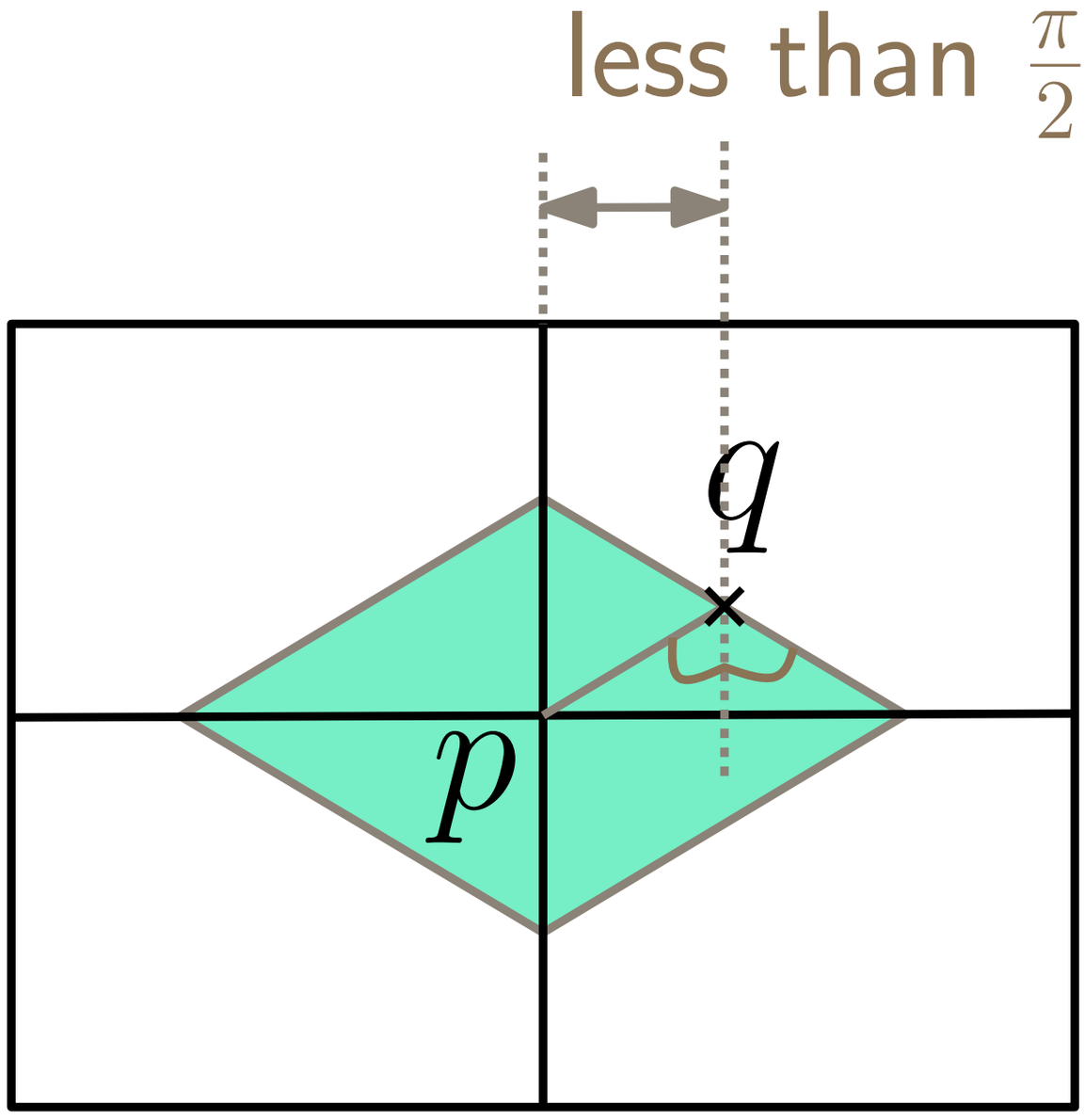
Rhombus graph



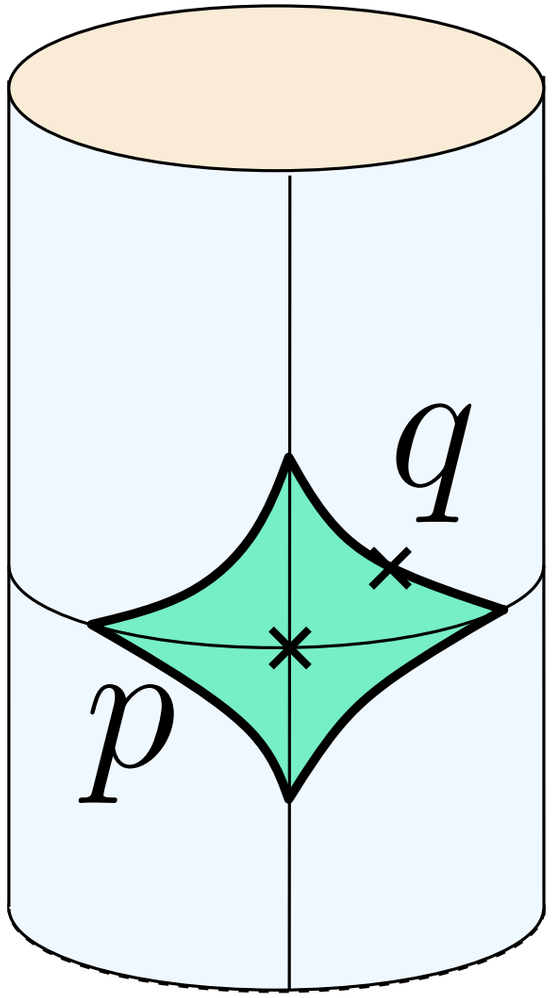


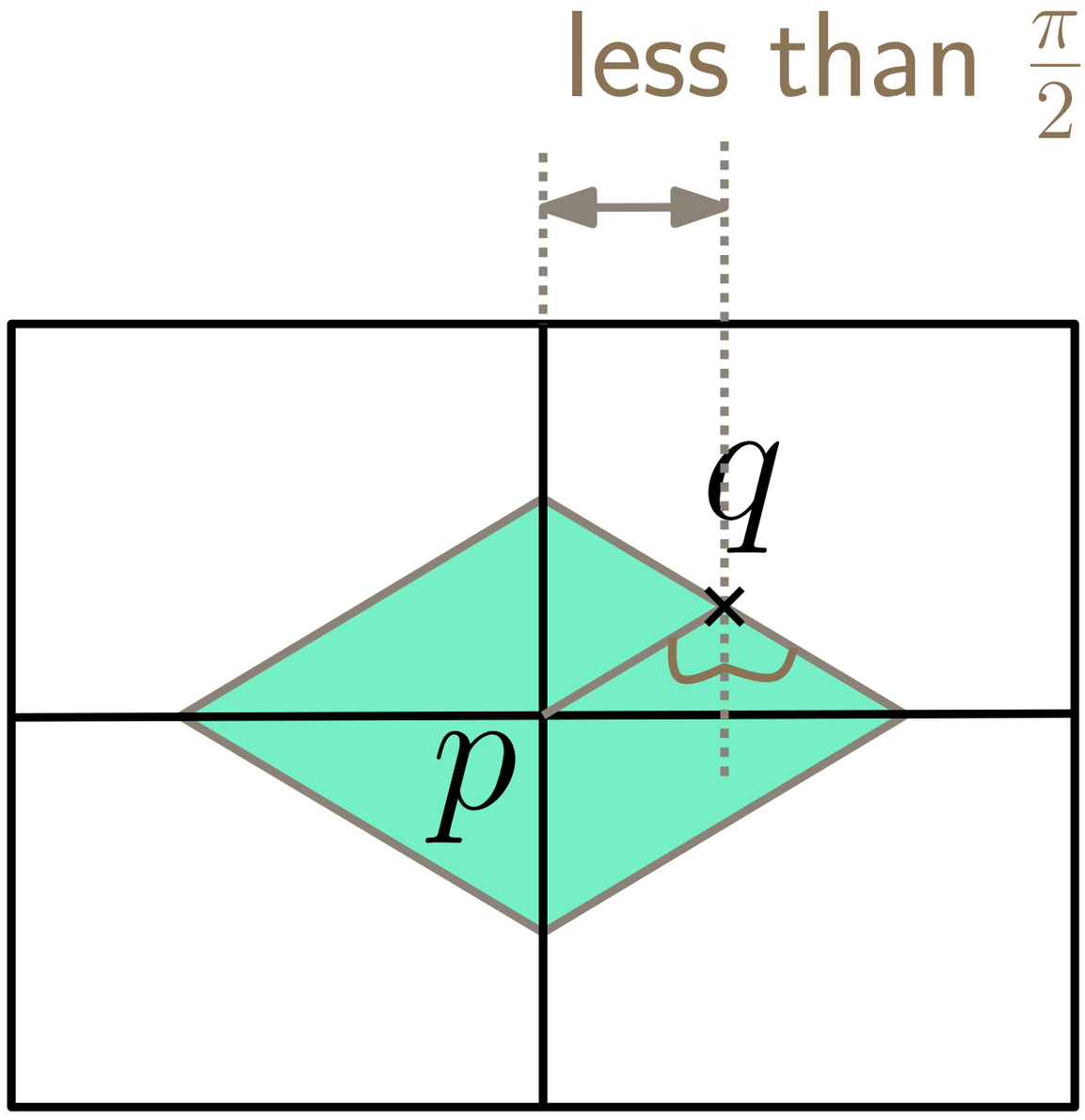
Rhombus graph





Rhombus graph



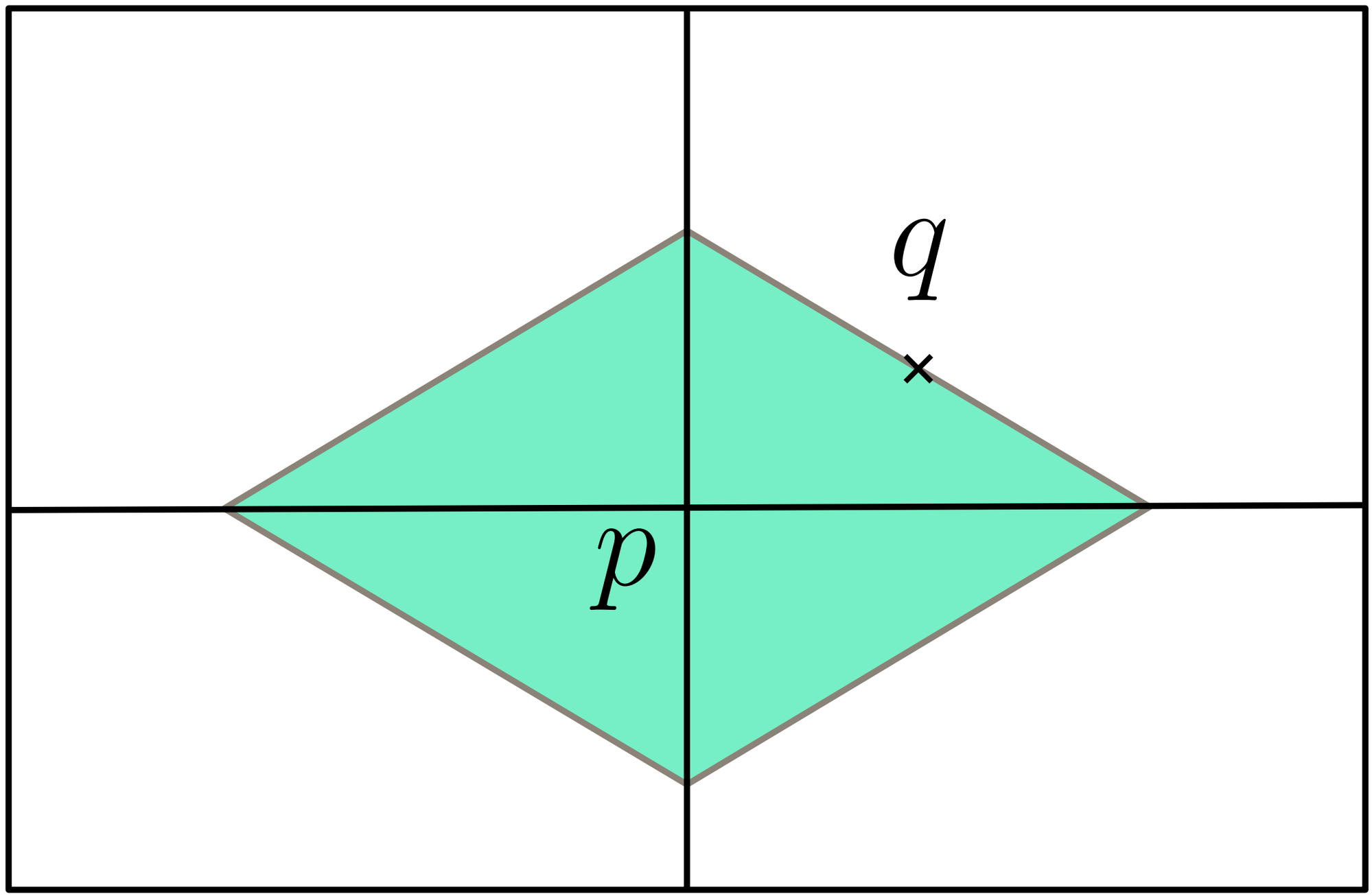


pq edge of RG

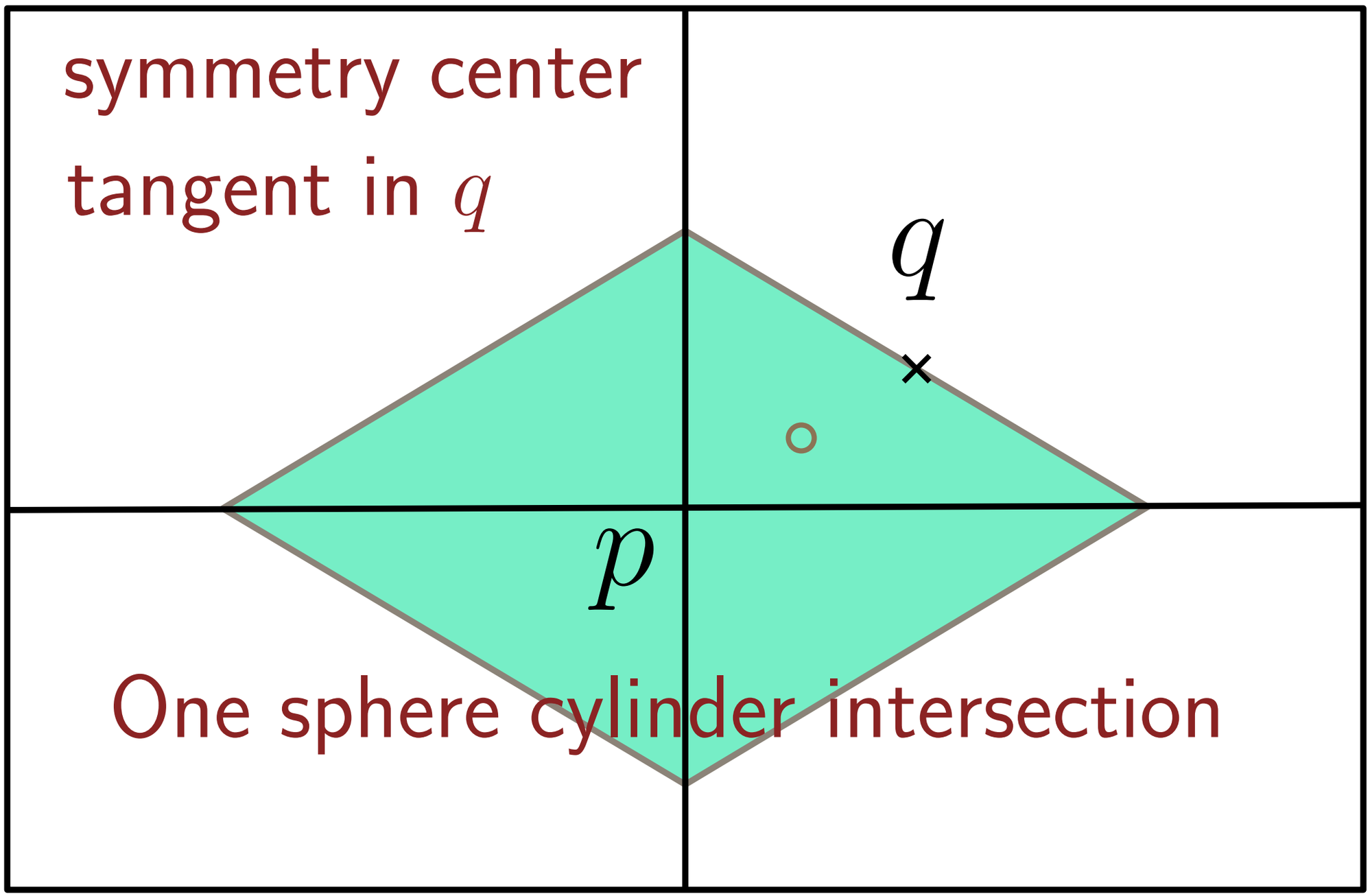
if rhombus empty

RG $\subset$ Delaunay

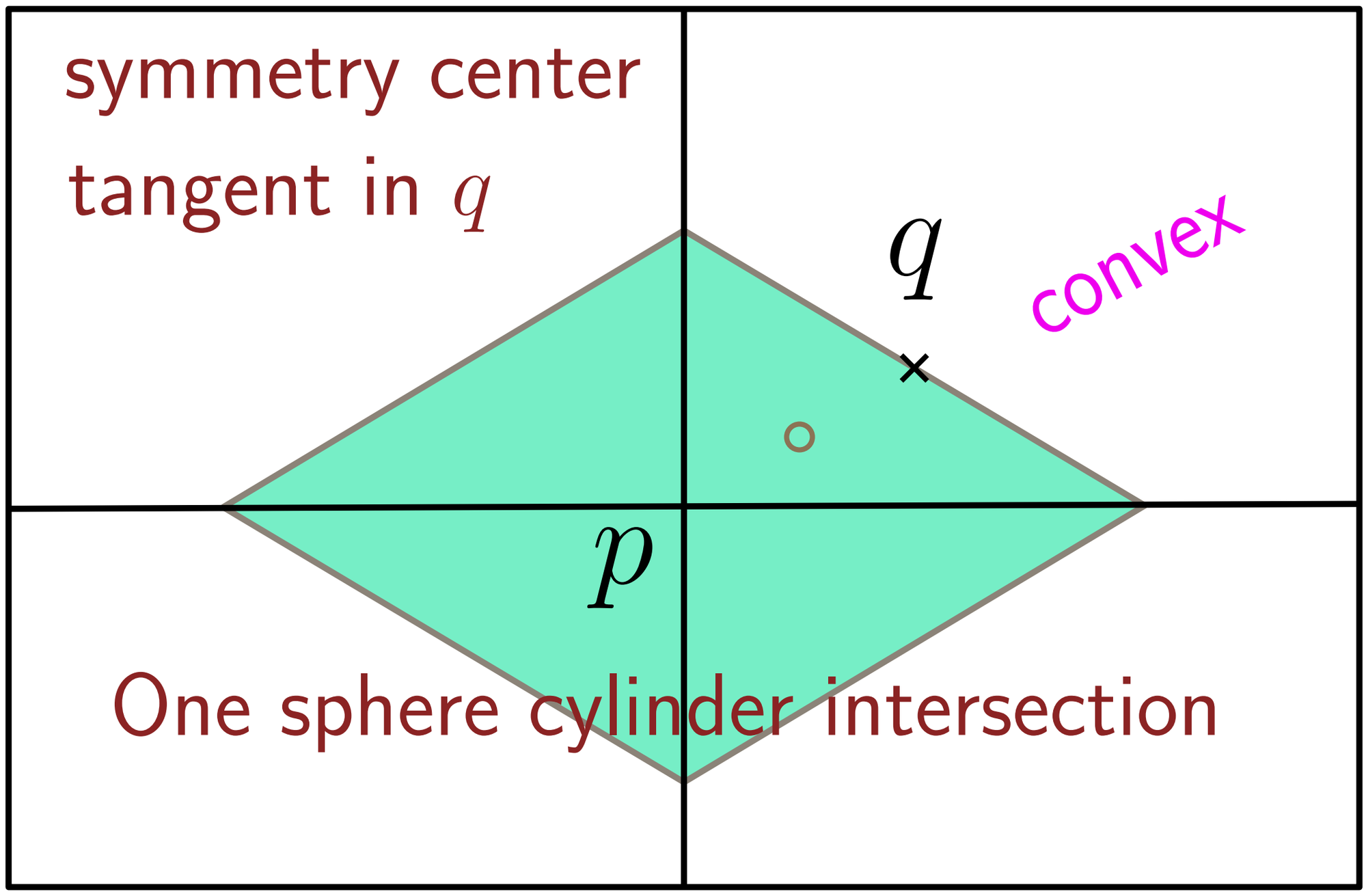
RG $\subset$ Delaunay



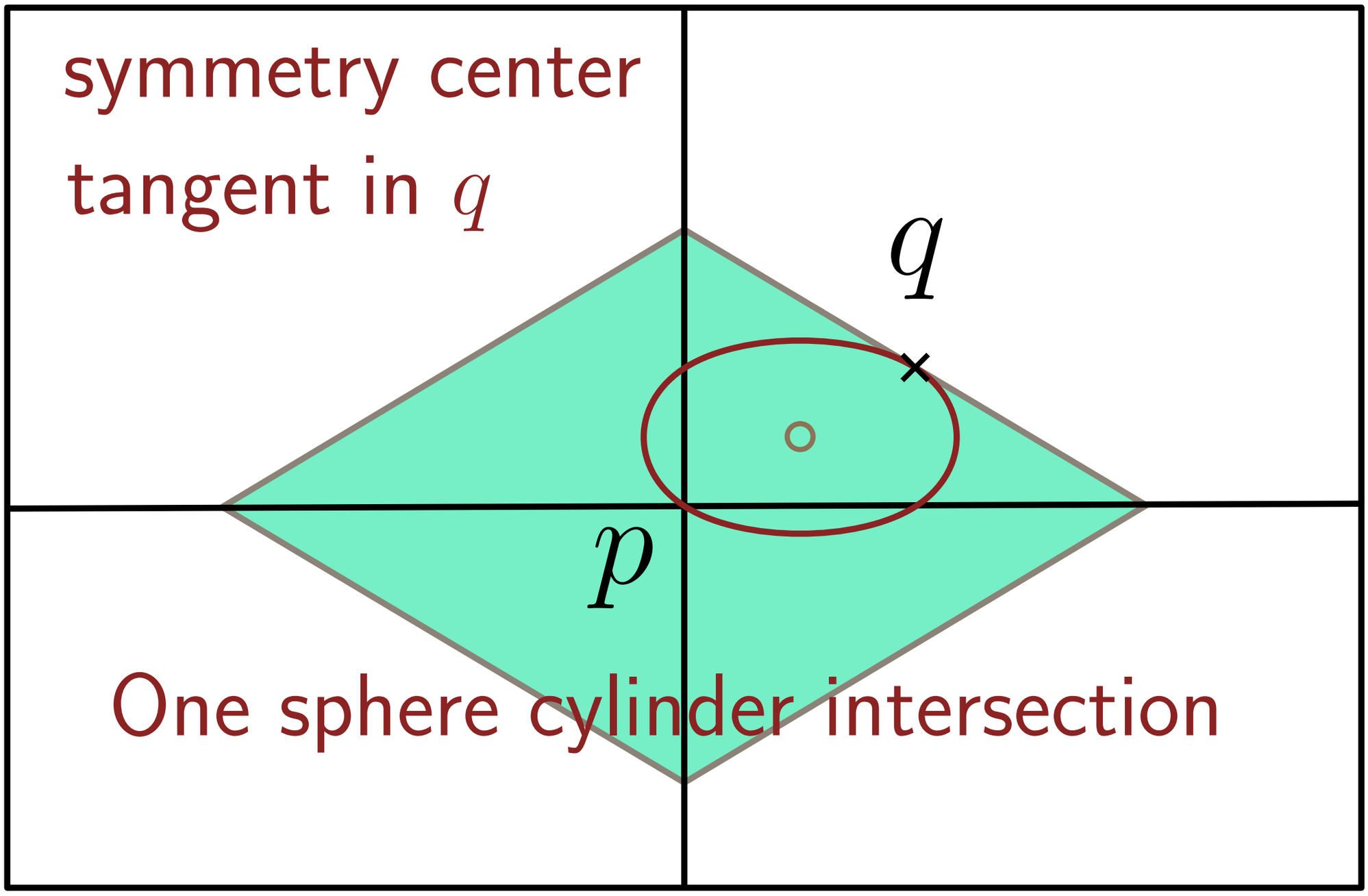
RG $\subset$ Delaunay



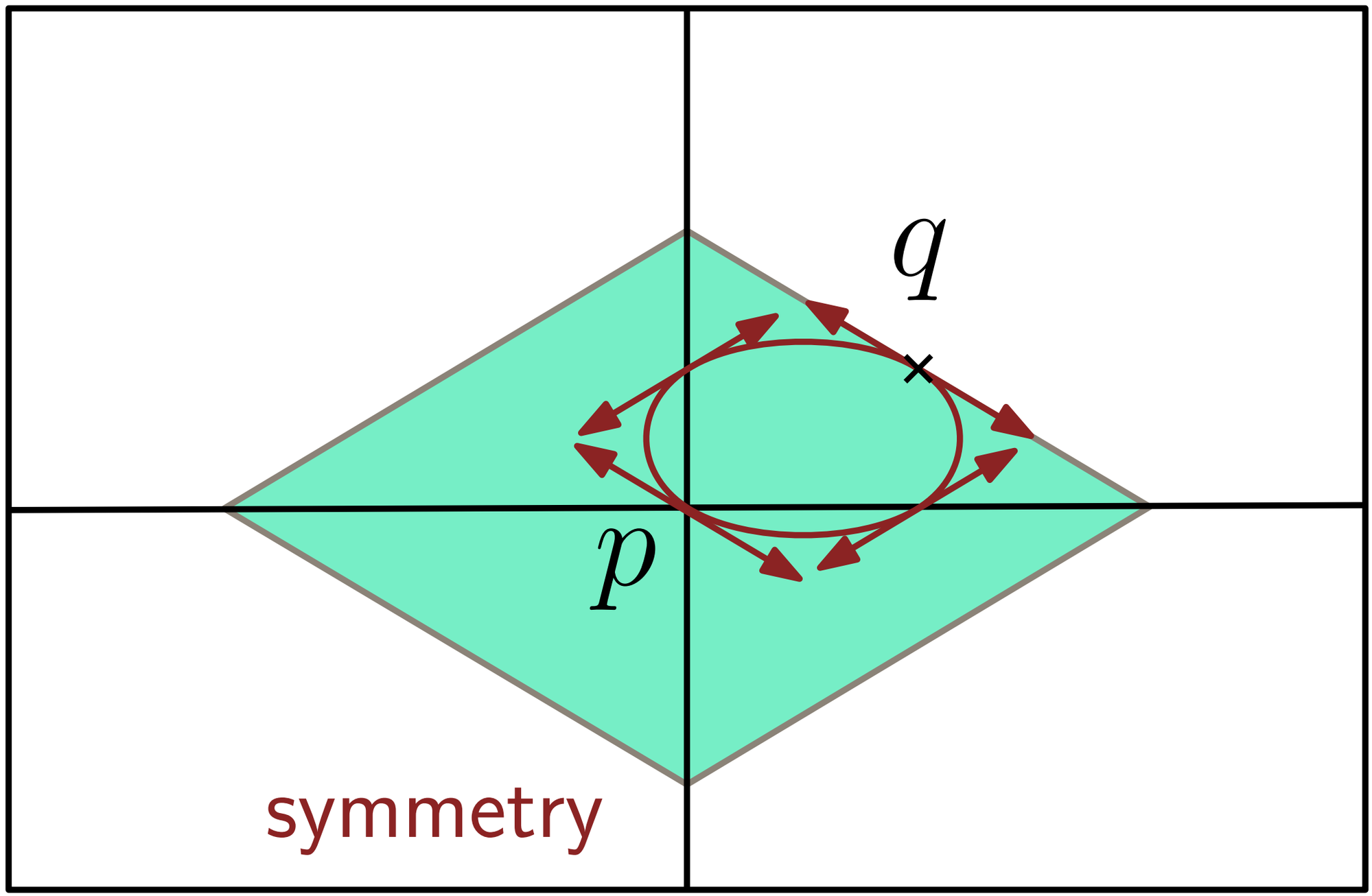
RG $\subset$ Delaunay



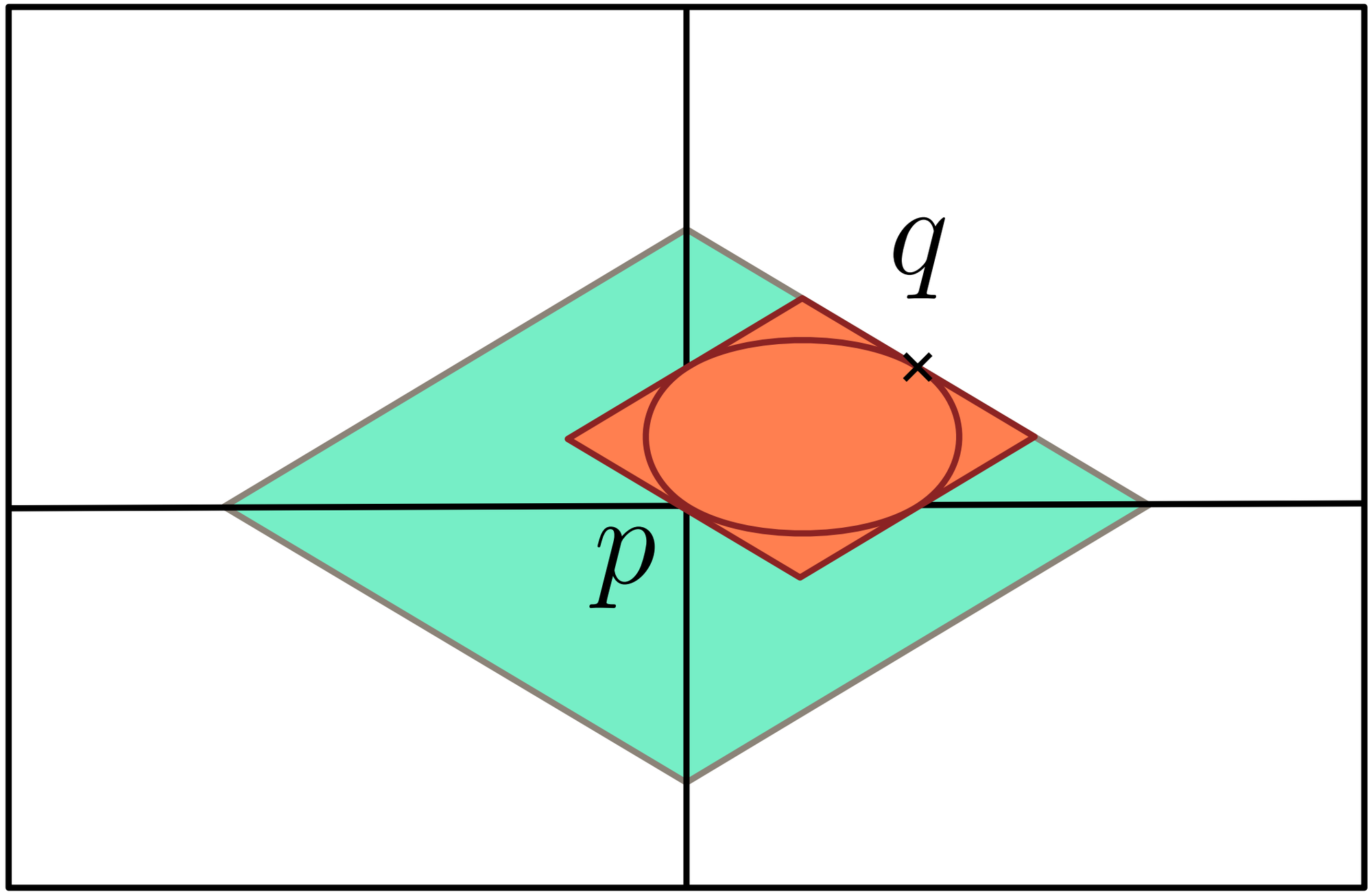
RG $\subset$ Delaunay

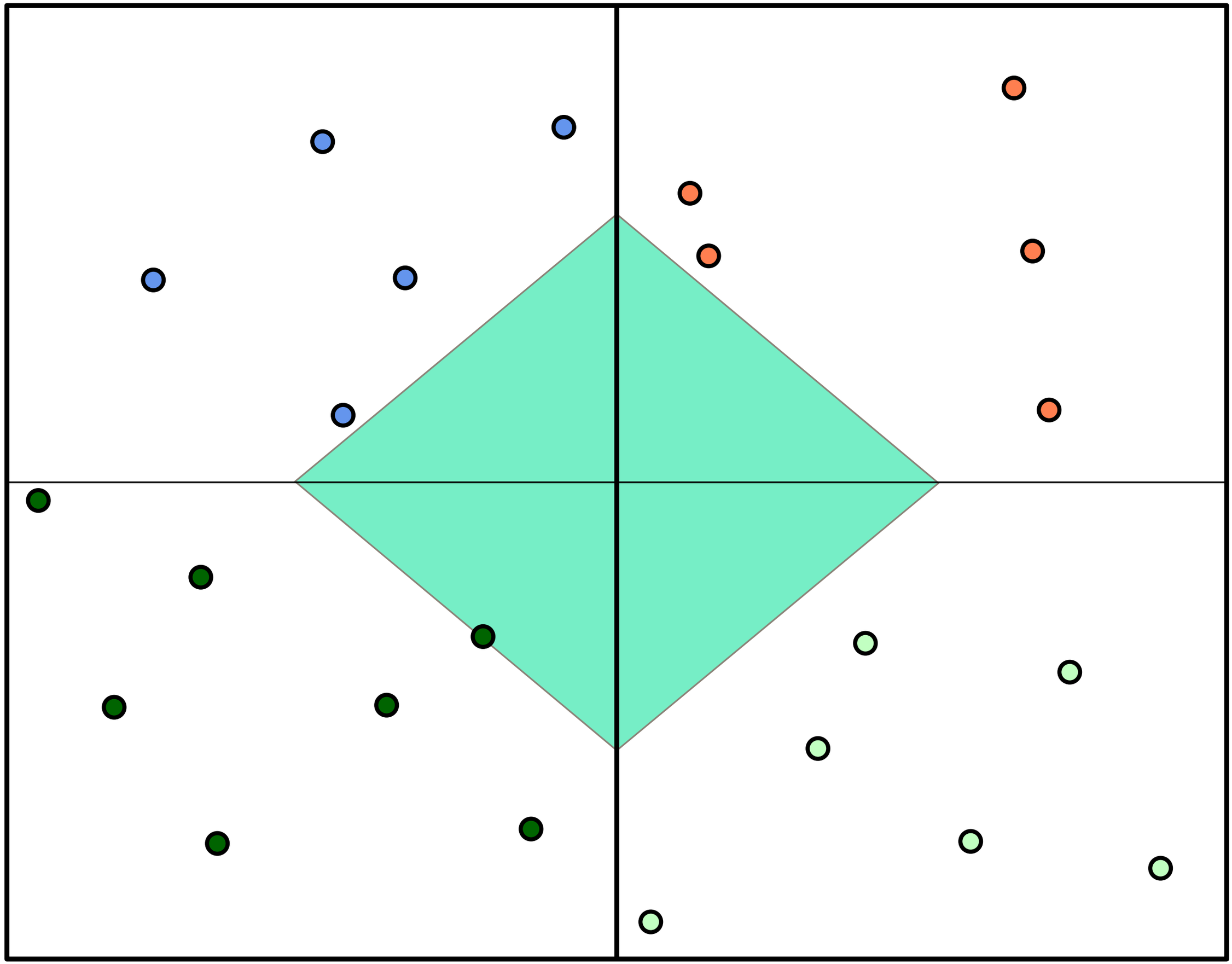


RG $\subset$ Delaunay

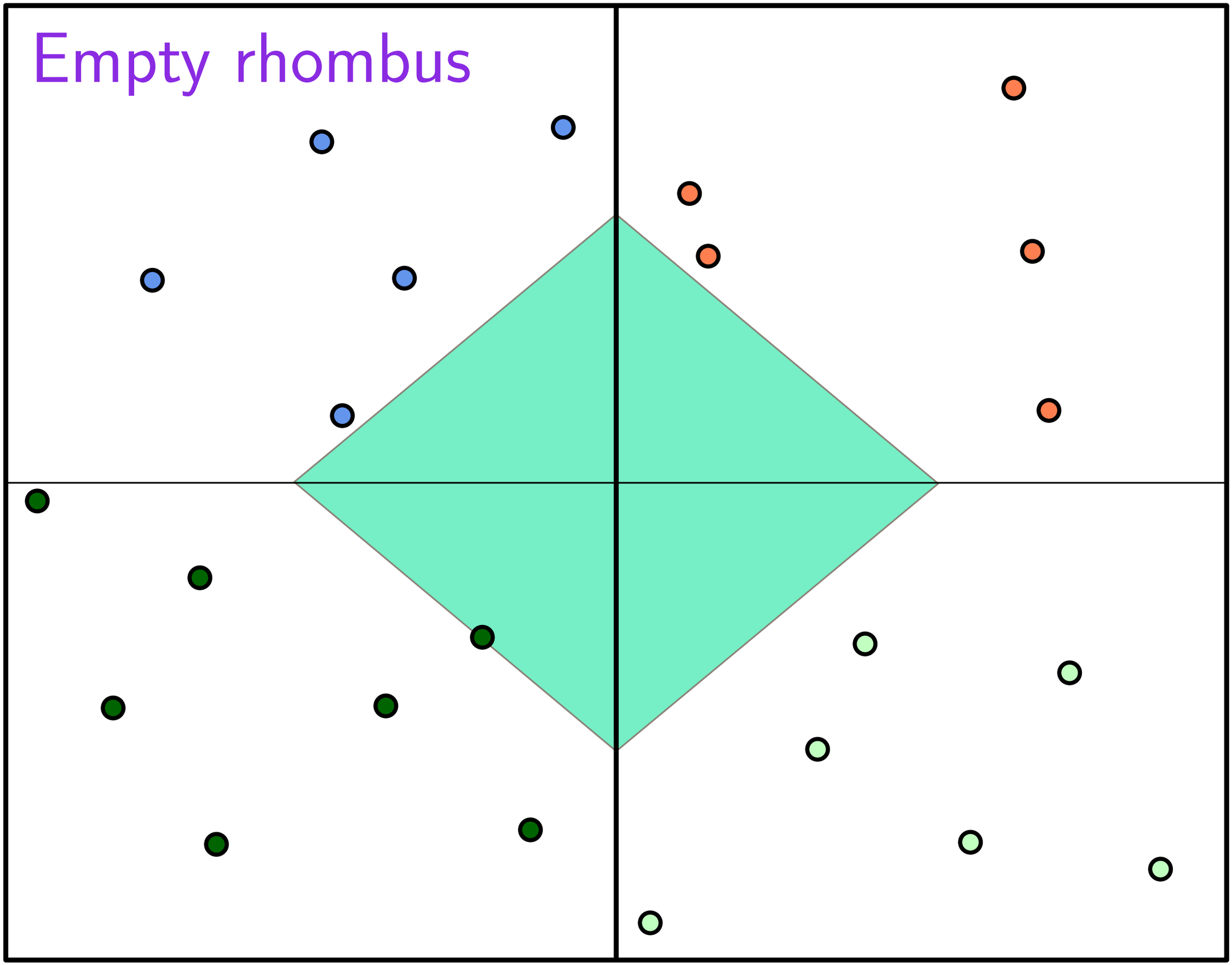


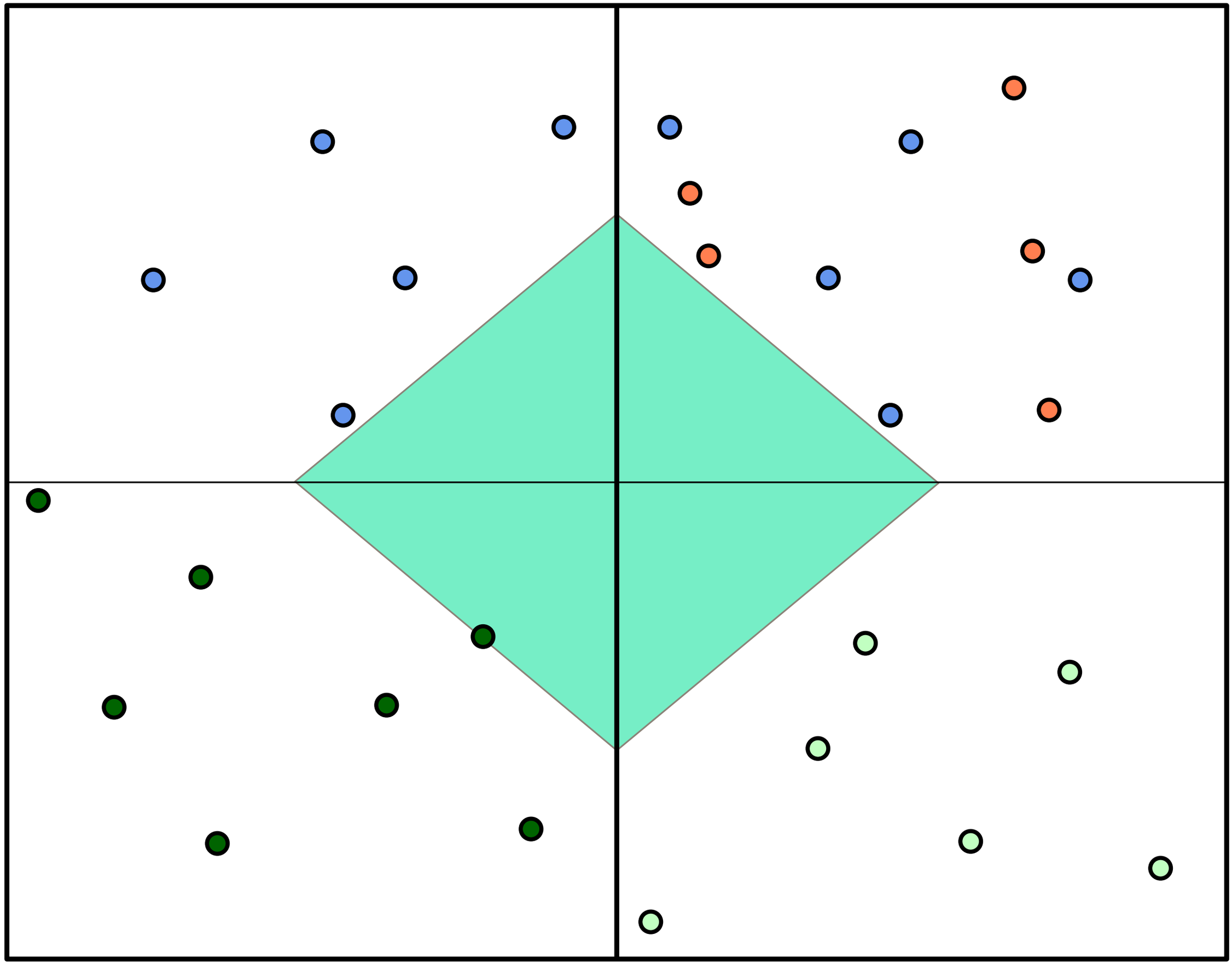
RG $\subset$ Delaunay

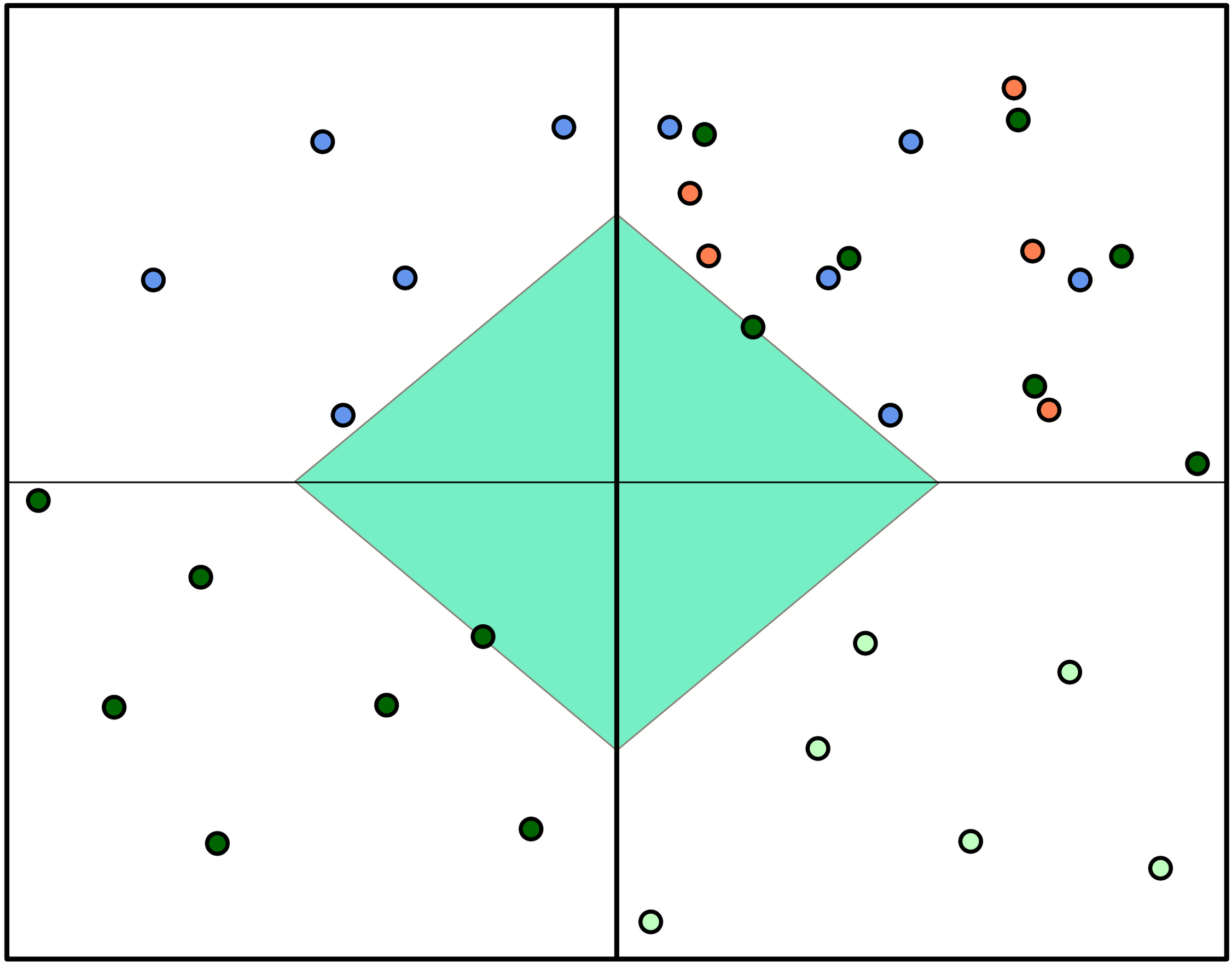


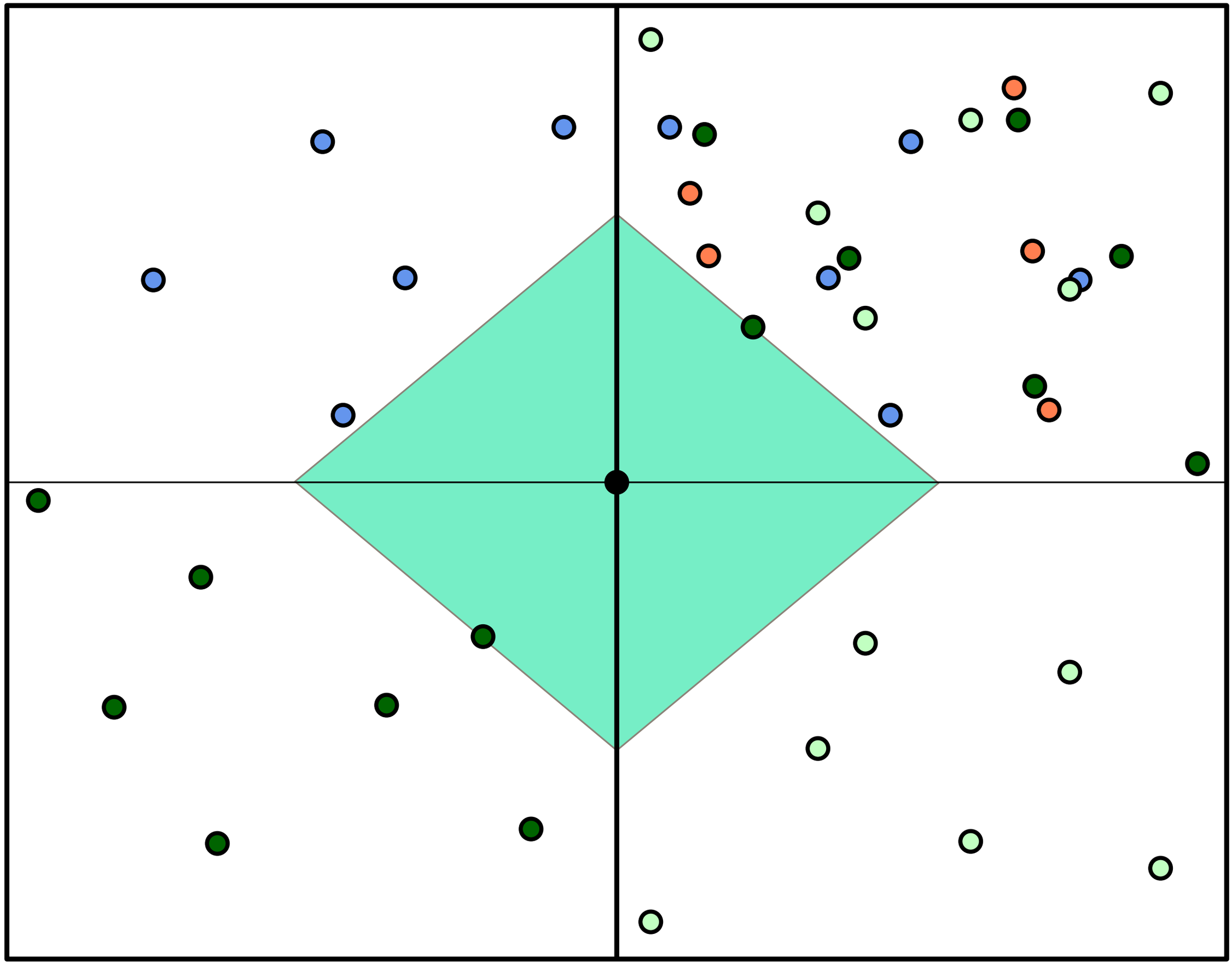


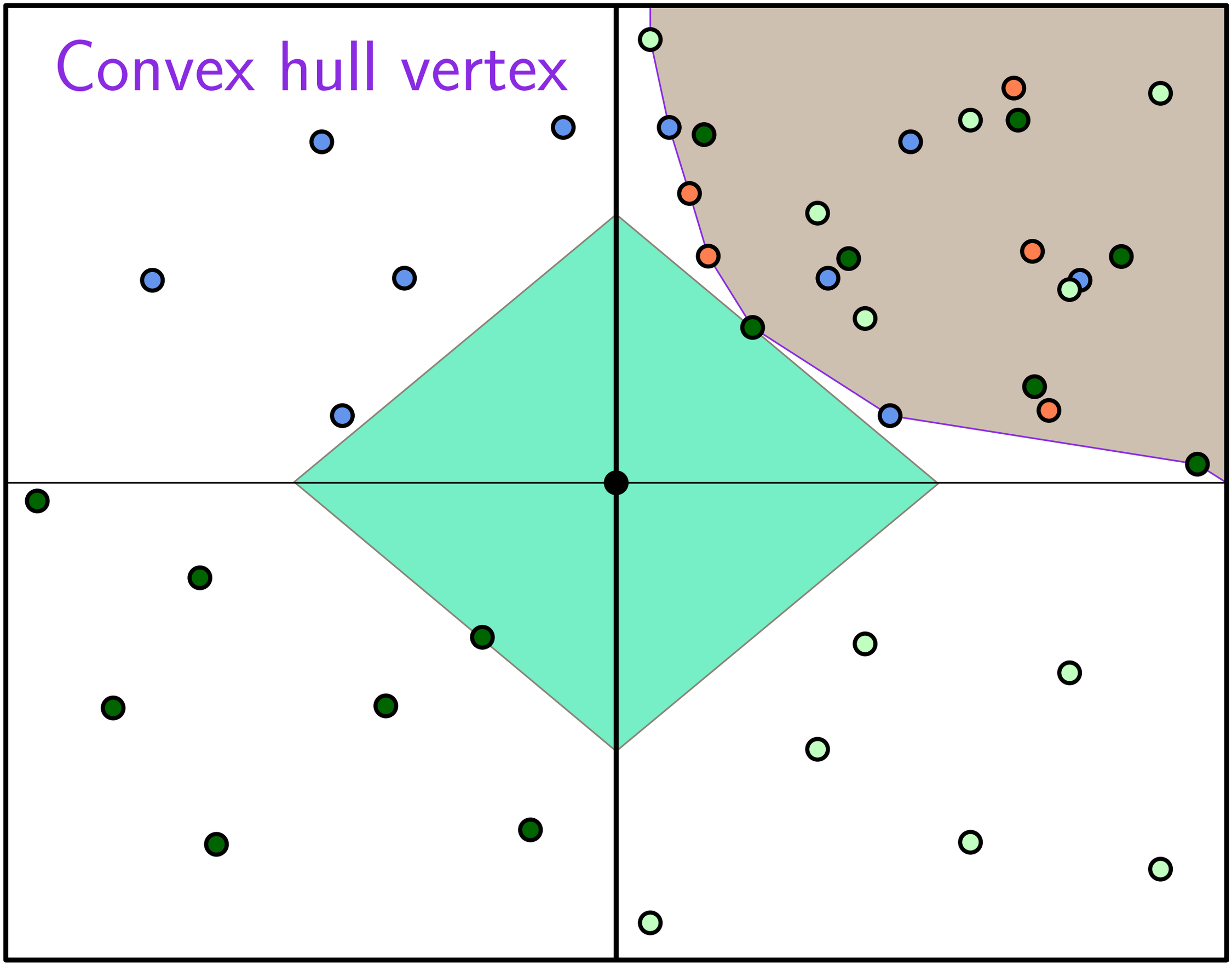
Empty rhombus

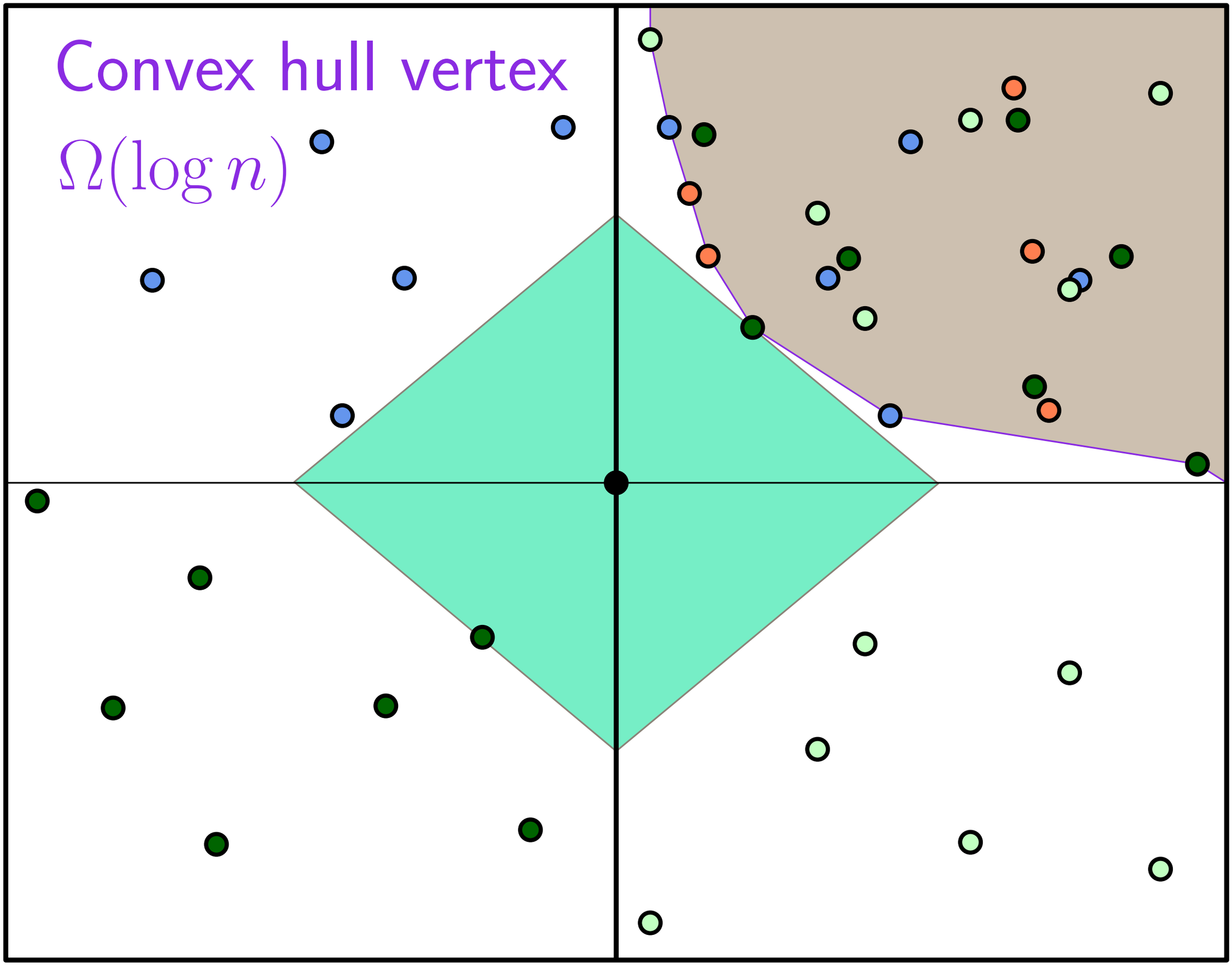









Convex hull vertex

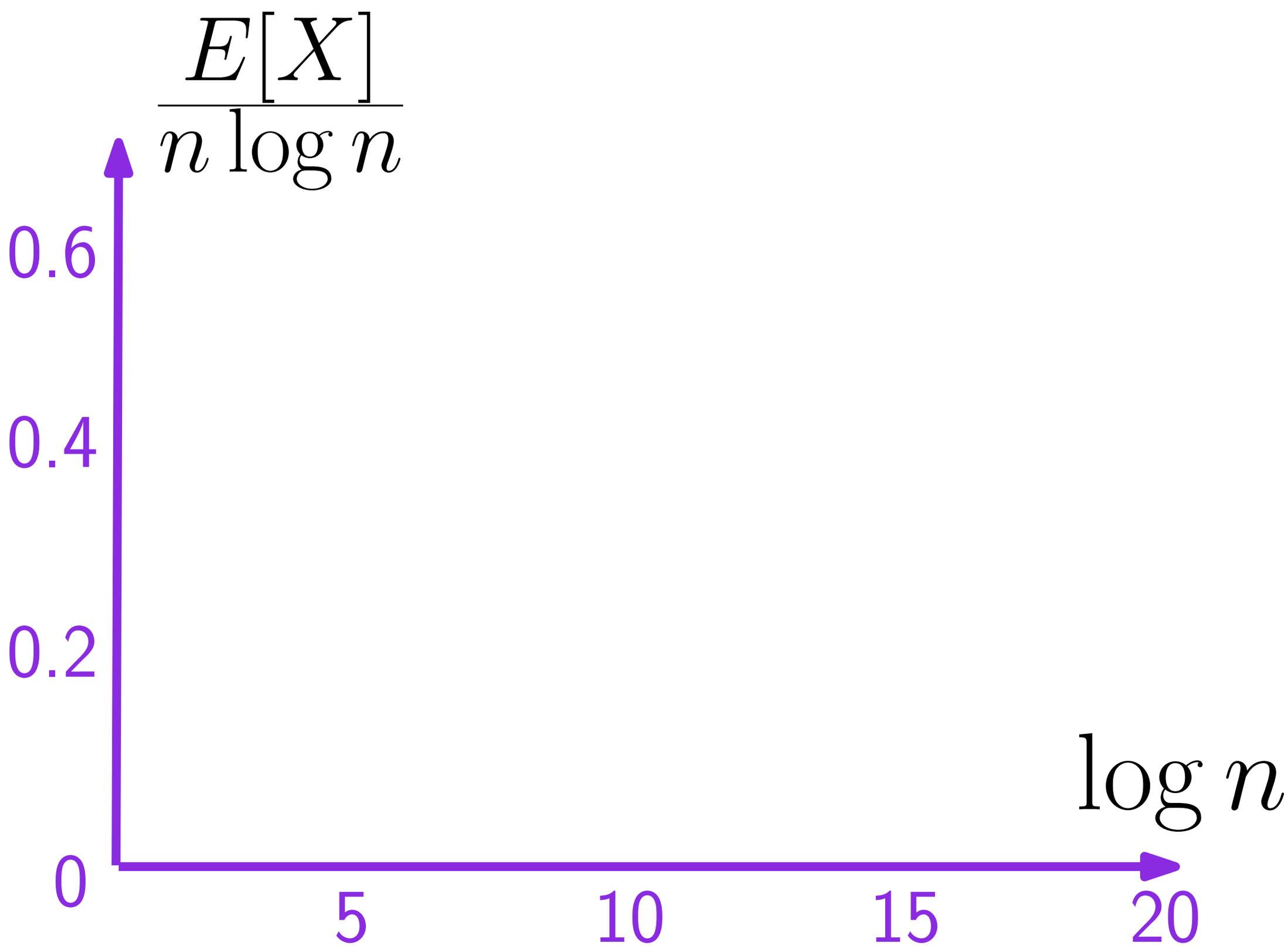

Convex hull vertex
$\Omega(\log n)$

Convex hull vertex

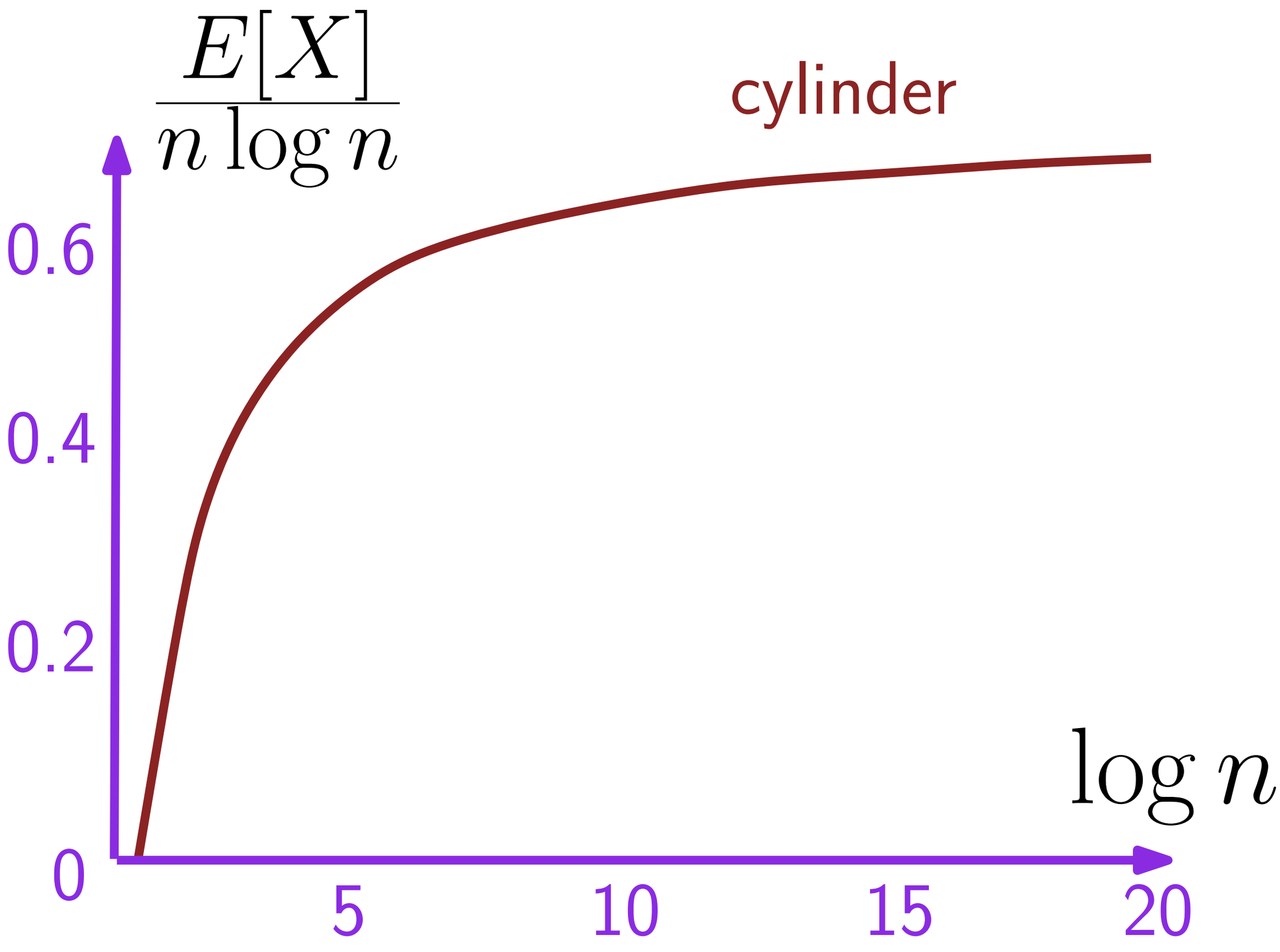
$\Omega(\log n)$

$|\mathsf{RG}| = \Omega(n \log n)$

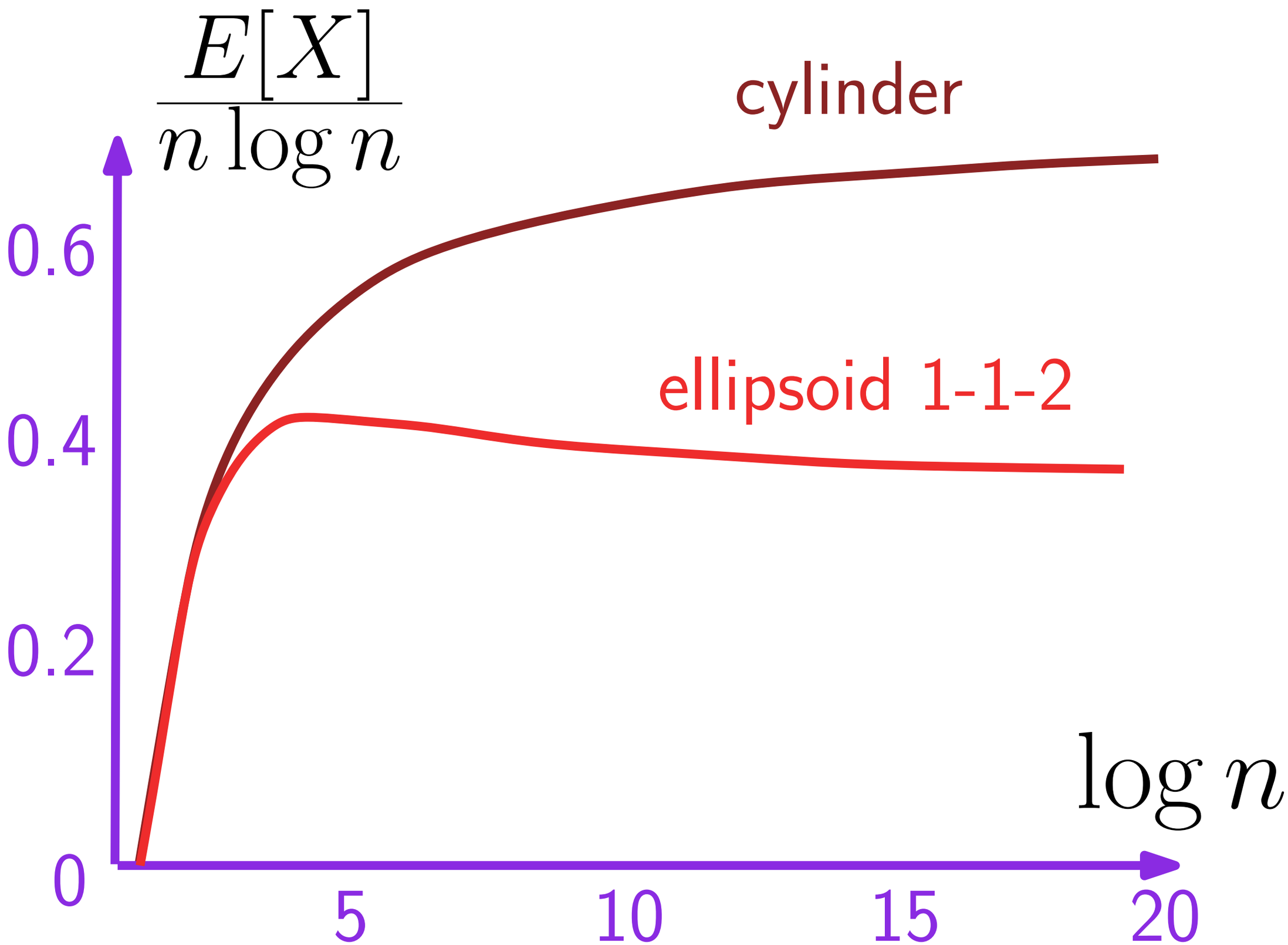
Experimental results



Experimental results



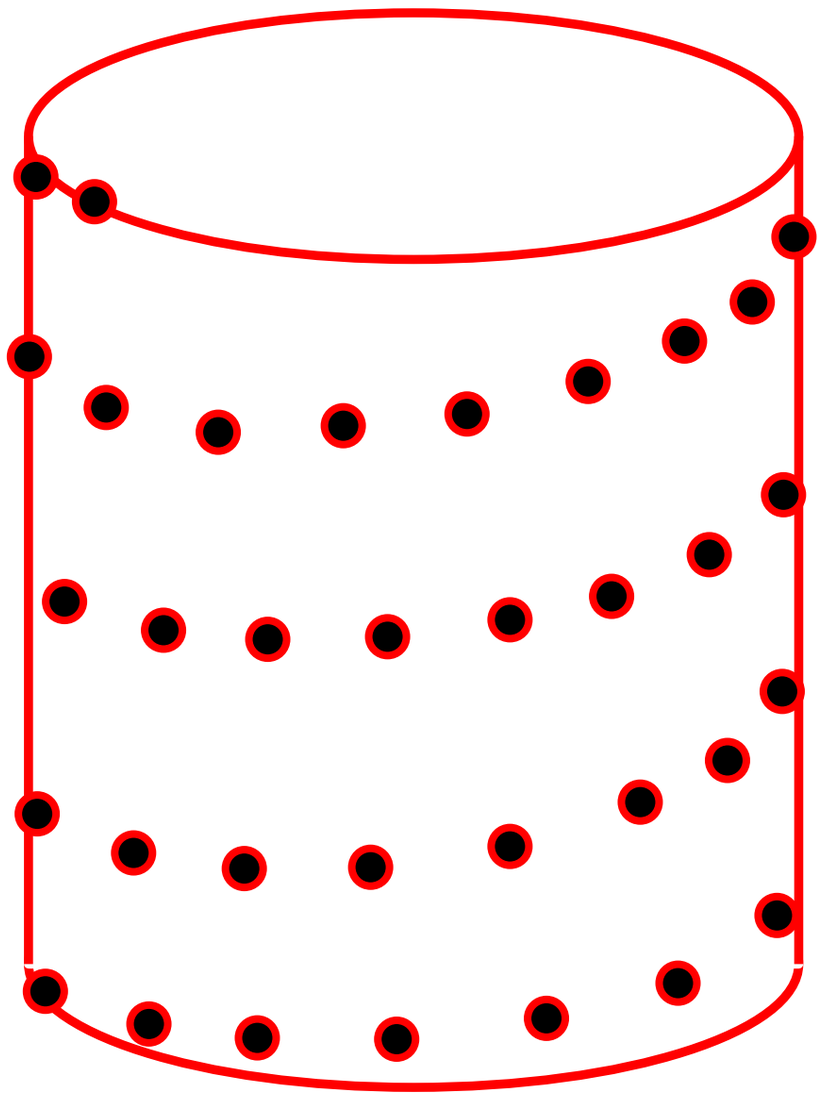
Experimental results



Generic skeleton hypotheses

$\Omega(n\sqrt{n})$

Erickson pathologic example



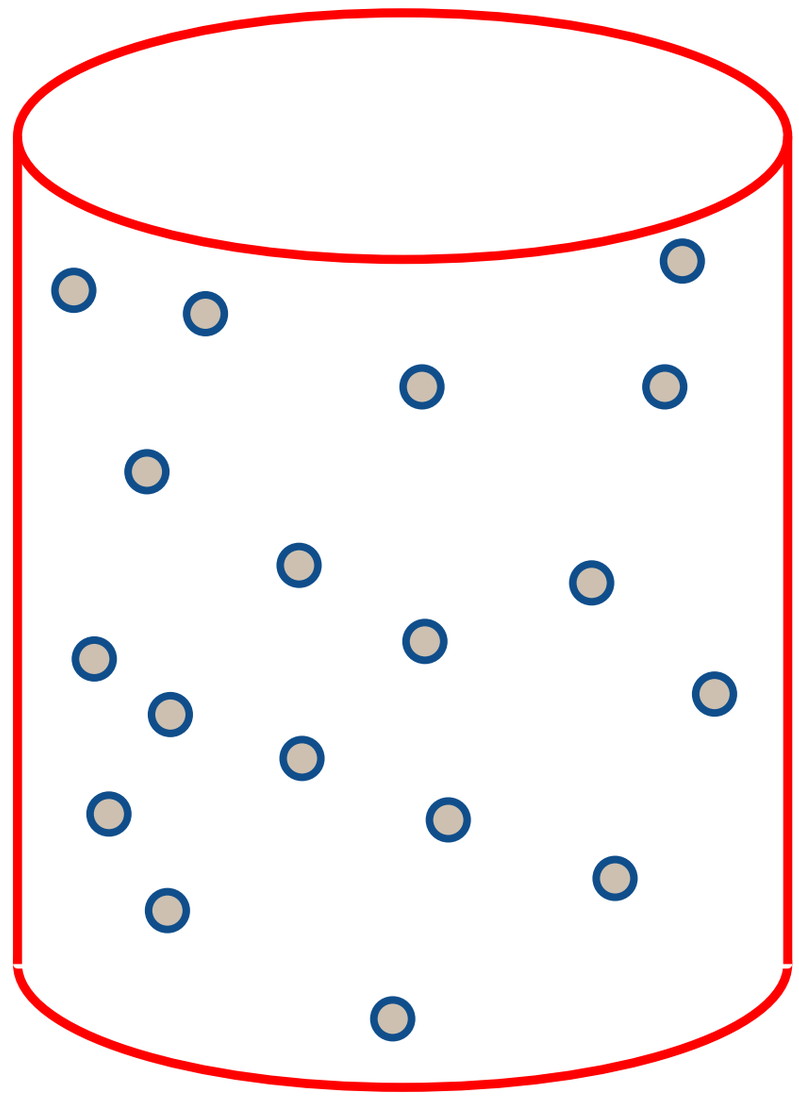
Generic skeleton hypotheses

are probably unnecessary

Erickson pathologic example

is really pathologic

$\Theta(n \log n)$



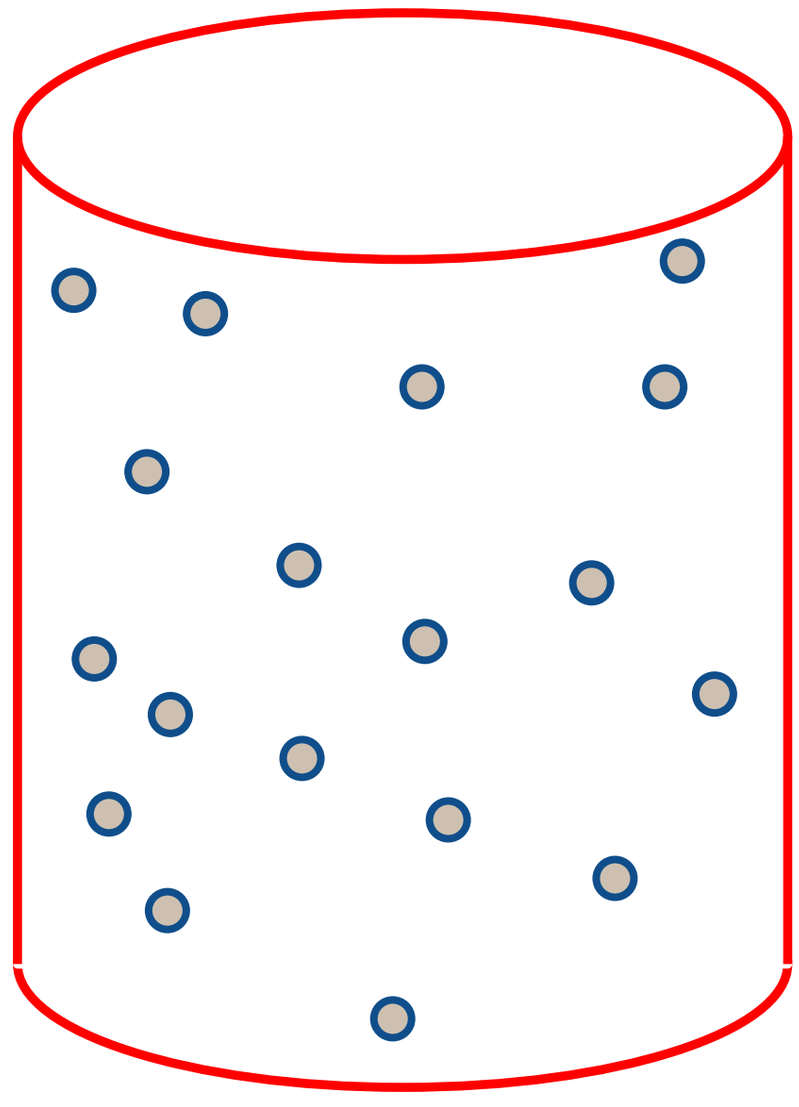
Generic skeleton hypotheses

are probably unnecessary

Erickson pathologic example

is really pathologic

$\Theta(n \log n)$



[Jeff simultaneous work]