GUDHI
Geometric Understanding in Higher Dimensions

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GUDHI is a five years project supported by a Grant of the European Research Council and hosted by INRIA

- develop and understand geometrical data structures
- develop associated statistical, geometric and topological functions
C++11 “header only” open source library:

- data structures
- associated functions
- toolbox
Simplicial complexes data structures
- Skeleton blockers

- Simplex tree
Other simplicial complexes data structures
- Simplex array list
- Cubical complex
- Tangential complex

Particular simplicial complexes
- Witness complex
- Rips complex
- Alpha complex
- Weighted alpha complex
Simplicial complexes functions

- iteration
- size
- insert, remove
- faces, cofaces
- link, star
- collapse
- edge contraction
- find simplex
- is_pseudo_manifold
- info (optional)
Toolbox

- Persistence co-homology
- Contraction
  (in progress)
- Clustering (hard / soft)
- Bottleneck distance
- Distance to a measure
Interfaces

Gudhi

Library
TDA

Cython

Boost  CGAL  GMP  MPFR
Persistent homology from an alpha complex
Simplex tree
- Simplicial complexes can be represented by a trie data structure that represents all the simplices.

A simplicial complex on 10 vertices and its simplex tree.
Alpha complex
- fills a simplex tree data structure with a Delaunay Triangulation.
Algorithm 1 Filtration value computation algorithm

for i : dimension $\rightarrow 1$ do
  for all $\sigma$ of dimension i do
    if filtration($\sigma$) is NaN then
      filtration($\sigma$) = $\alpha_i^2(\sigma)$
    end if
    for all $\tau$ face of $\sigma$ do
      if filtration($\tau$) is not NaN then
        filtration($\tau$) = min (filtration($\tau$), filtration($\sigma$))
      else
        if $\tau$ is not Gabriel for $\sigma$ then
          filtration($\tau$) = filtration($\sigma$)
        end if
      end if
    end for
  end for
end for

[0,4] is not Gabriel $\rightarrow \alpha_{40} = \alpha_{420}$

[2,0] is Gabriel $\rightarrow \alpha_{20}$ is not modified (NaN)

[2,4] is Gabriel $\rightarrow \alpha_{42}$ is not modified (NaN)

N.B.: Gabriel on a single point has no sense.
Persistent homology
- algebraic method for measuring the topological features (holes, cavities, ...)
- stable regard to noise
Examples
- random points on a 3D sphere:

<table>
<thead>
<tr>
<th>Ambient dimension</th>
<th>Birth</th>
<th>Death</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>inf</td>
</tr>
<tr>
<td>2</td>
<td>0.13947</td>
<td>0.999999</td>
</tr>
</tbody>
</table>
Thank you !