

गुठी GUDHI

Geometric Understanding in Higher Dimensions



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Geometric Understanding in Higher Dimensions

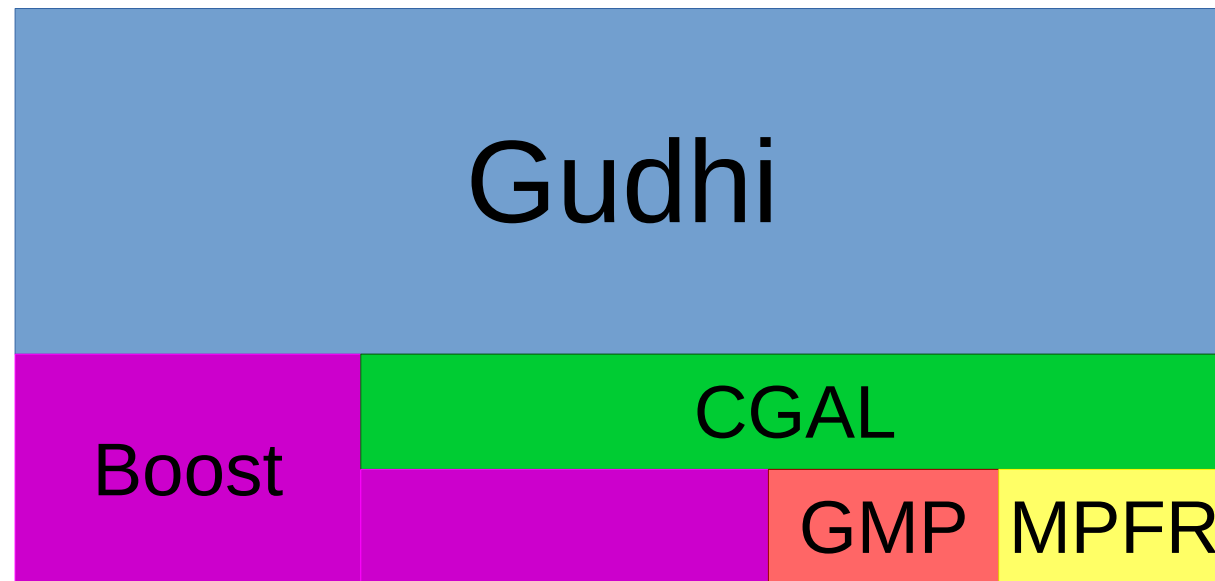
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GUDHI is a five years project supported by a Grant of the European Research Council and hosted by INRIA



- develop and understand geometrical data structures
- develop associated statistical, geometric and topological functions

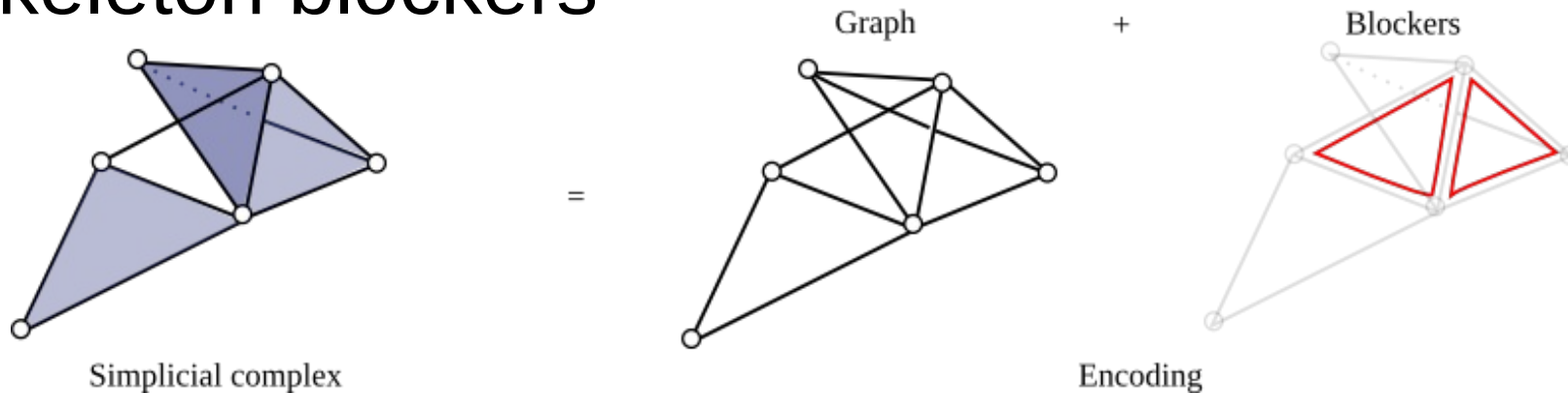
C++11 “header only” open source library:



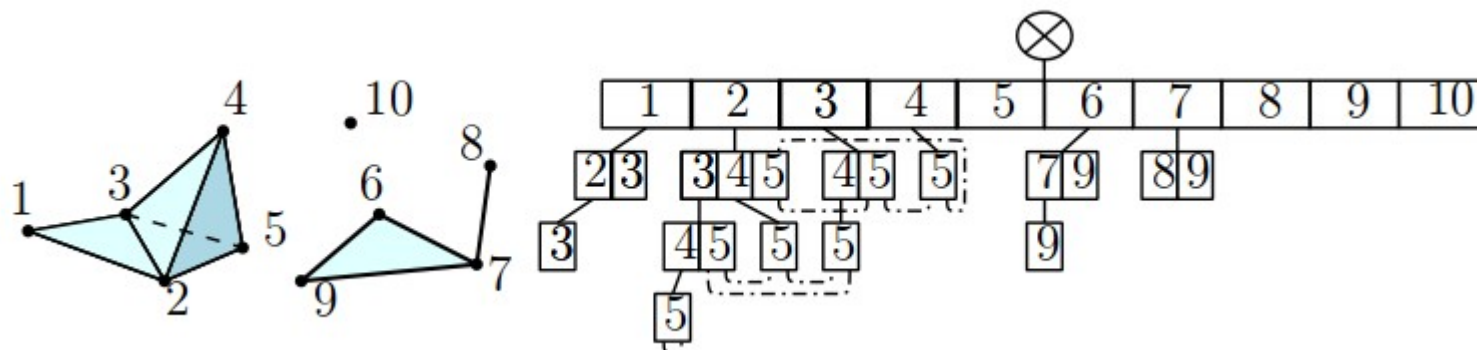
- data structures
- associated functions
- toolbox

Simplicial complexes data structures

- Skeleton blockers



- Simplex tree



Other simplicial complexes data structures

- Simplex array list
- Cubical complex
- Tangential complex

Particular simplicial complexes

- Witness complex
- Rips complex
- Alpha complex
- Weighted alpha complex

Simplicial complexes functions

- iteration
- size
- insert, remove
- faces, cofaces
- link, star
- collapse
- edge contraction
- find simplex
- is_pseudo_manifold
- info (optional)

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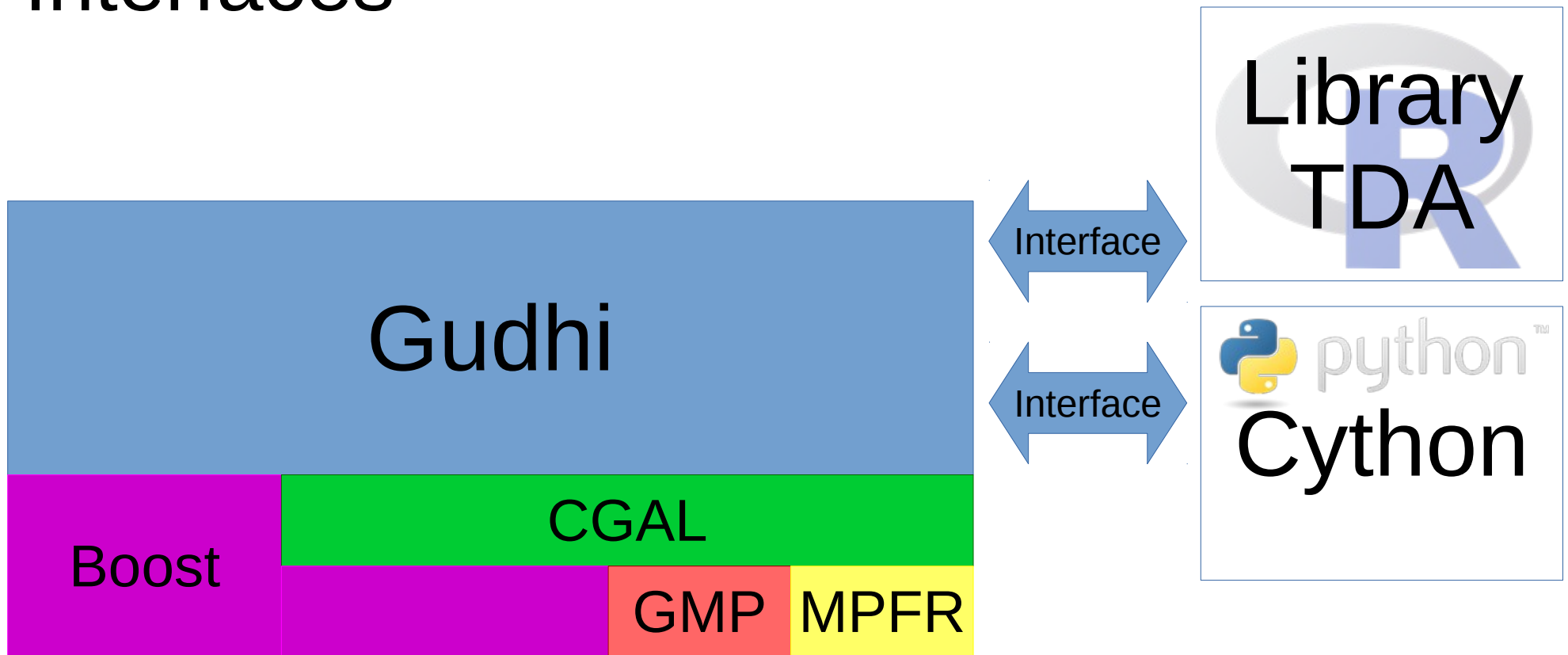


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Toolbox

- Persistence co-homology
- Contraction
- (in progress)
- Clustering (hard / soft)
- Bottleneck distance
- Distance to a measure

Interfaces



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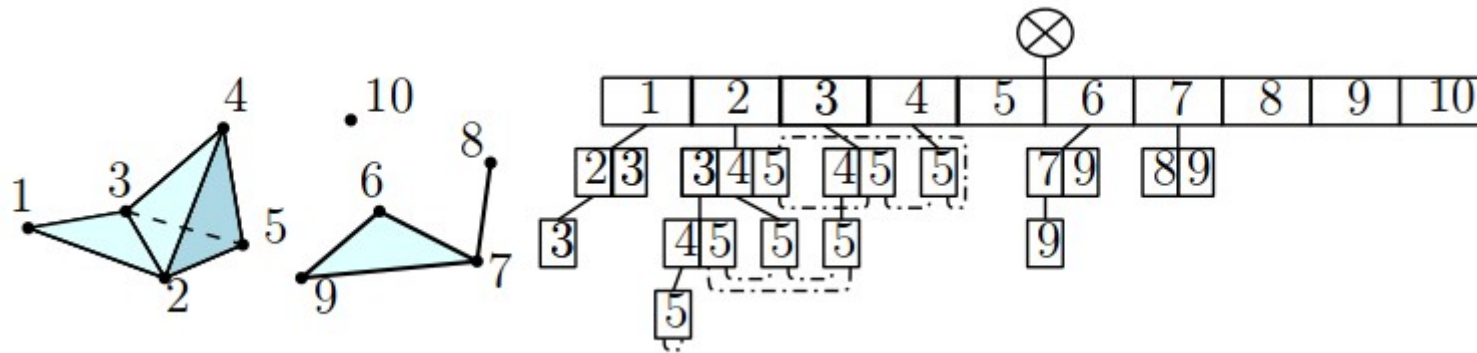
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Persistent homology from an alpha complex

Simplex tree

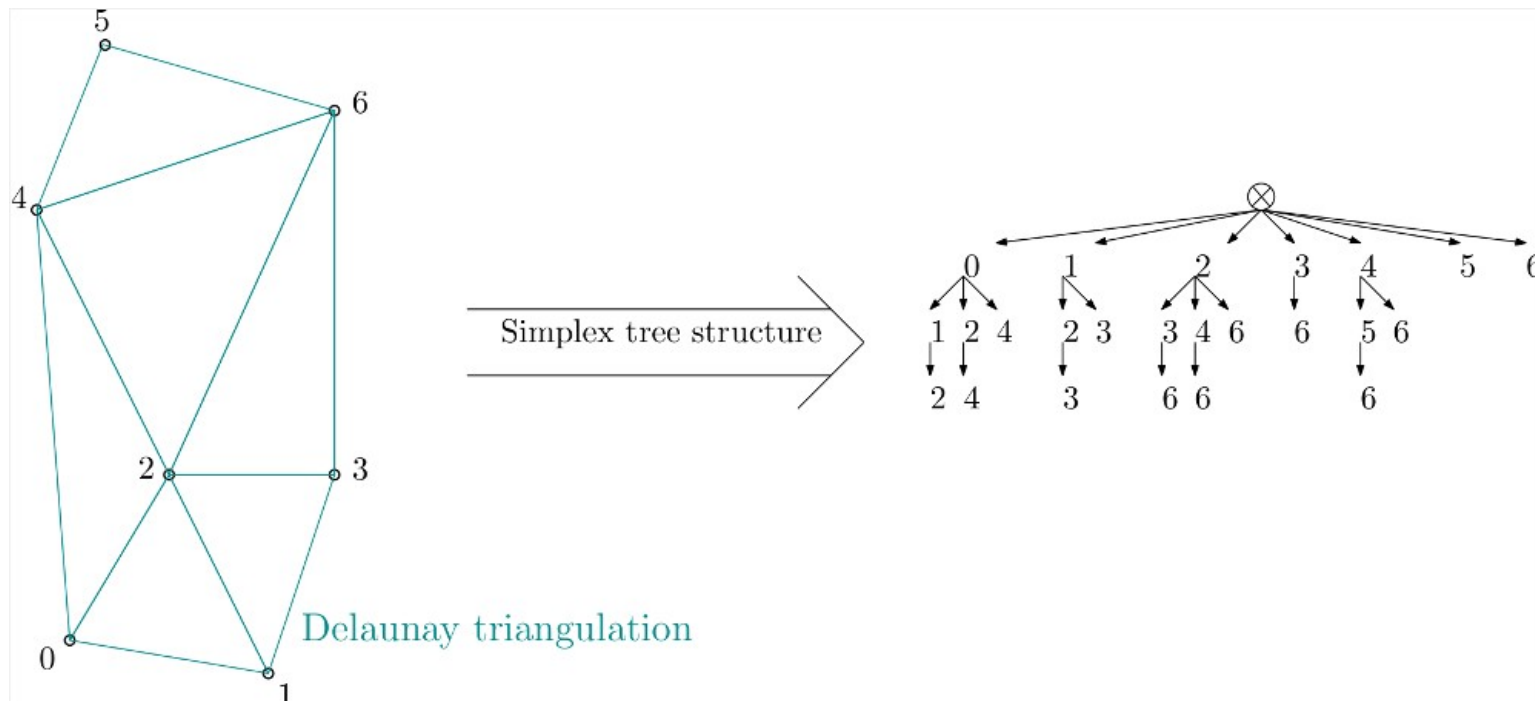
- Simplicial complexes can be represented by a trie data structure that represents all the simplices.



A simplicial complex on 10 vertices and its simplex tree.

Alpha complex

- fills a simplex tree data structure with a Delaunay Triangulation.



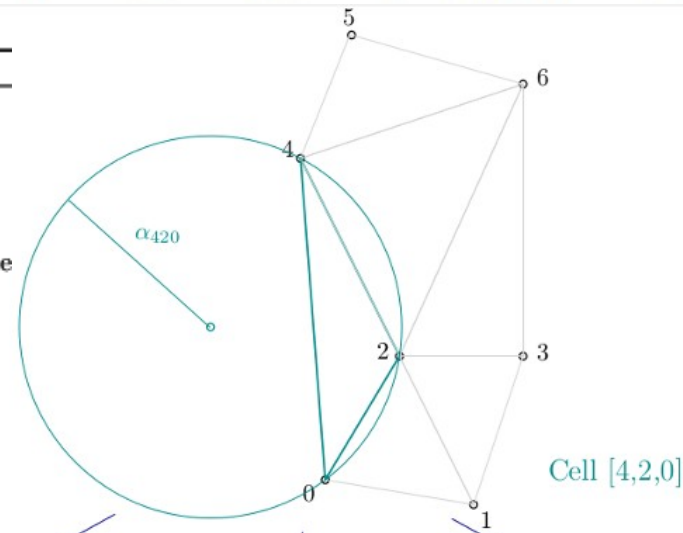
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Dimension = 2 - $\sigma = [4,2,0]$

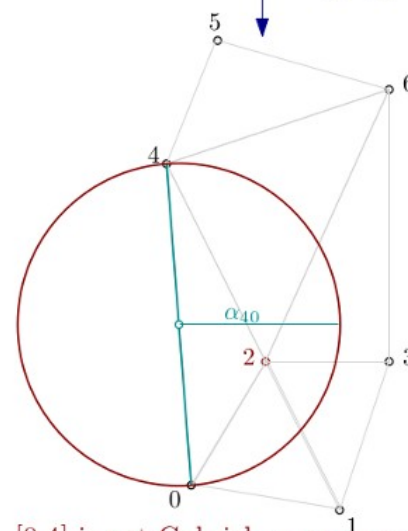
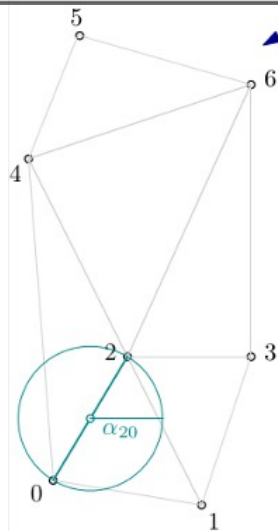
Algorithm 1 Filtration value computation algorithm

```

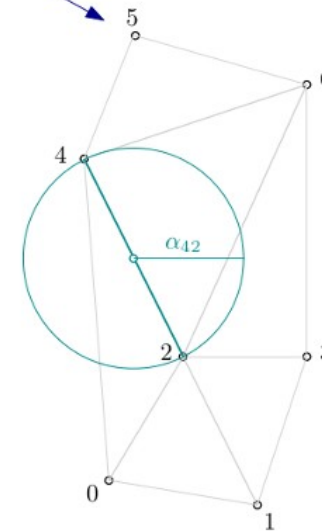
for i : dimension  $\rightarrow$  1 do
  for all  $\sigma$  of dimension i do
    if filtration( $\sigma$ ) is NaN then
      filtration( $\sigma$ ) =  $\alpha^2(\sigma)$ 
    end if
    for all  $\tau$  face of  $\sigma$  do ▷ propagate alpha filtration value
      if filtration( $\tau$ ) is not NaN then
        filtration( $\tau$ ) = min (filtration( $\tau$ ), filtration( $\sigma$ ))
      else
        if  $\tau$  is not Gabriel for  $\sigma$  then
          filtration( $\tau$ ) = filtration( $\sigma$ )
        end if
      end if
    end for
  end for
end for
  
```



For all faces of $[4,2,0]$



$[0,4]$ is not Gabriel $\rightarrow \alpha_{40} = \alpha_{420}$



$[2,4]$ is Gabriel $\rightarrow \alpha_{42}$ is not modified (NaN)

$[2,0]$ is Gabriel $\rightarrow \alpha_{20}$ is not modified (NaN)

N.B. : is Gabriel on a single point has no sense.

Persistent homology

- algebraic method for measuring the topological features (holes, cavities, ...)
- stable regard to noise

Examples

- random points on a 3D sphere:

Ambient dimension	Birth	Death
0	0	inf
2	0.13947	0.999999

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Thank you !