Discover differences between networks with functional map.

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INRIA-Geometrica

Ongoing work with Frédéric Chazal and Maks Ovsjanikov

Oct 20, 2015

Problem

- Def:
  $G = (V, E)$ is a graph, V is its node set and E is the edge set.
  In general, we consider the weighted graph, i. e., map
  $w : E \to R^+$ indicates the similarity between the pair of nodes
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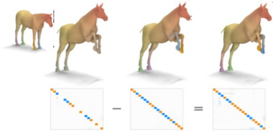
- Problem:
  Given two graphs G_1 and G_2 and a bijection between their vertices set. Can we explore the differences between them?

Motivation

Functional Maps: A Flexible Representation of Maps Between Shapes

Maks Ovsjanikov† Mirela Ben-Chen‡ Justin Solomon‡ Adrian Butscher‡ Leonidas Guibas‡

† LIX, École Polytechnique ‡ Stanford University



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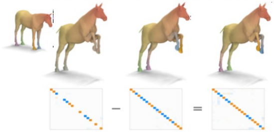

Analysis and Visualization of Maps Between Shapes

Maks Ovsjanikov[1] Mirela Ben-Chen[2] Frederic Chazal[3] Leonidas Guibas[4]

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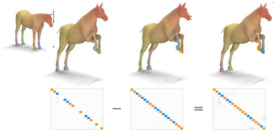

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Map-Based Exploration of Intrinsic Shape Differences and Variability

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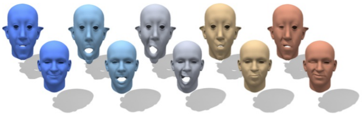

Motivation

SHAPE $\longrightarrow$ GRAPH

Laplacian-Beltrami $\longrightarrow$ Graph Laplacian

Functional Map

Given a bijection $T : G_1 \to G_2$, then one can naturally induce the bijection in the function space $T_F : \mathcal{F}(G_2) \to \mathcal{F}(G_1)$.

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For example, let $g : G_2 \to \mathcal{R}$, then $T_F(g) = g \circ T \in \mathcal{F}(G_1)$.

Property: T_F is a linear map.

Functional Map

Assume the $\{\phi_i\}, \{\psi_j\}$ are respectively the basis of G_1 and G_2, since T_F is a linear map from $\mathcal{F}(G_1)$ to $\mathcal{F}(G_2)$, it's determined by the basis and the bijection.

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In fact, let $f = \sum_i a_i \phi_i$, one have $g = T_F(f)$ and $g = \sum_j b_j \psi_j$. Thus in the finite case, T_F can be completely encoded by a matrix C, which exists uniquely satisfying $Ca = b$.

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With the reduced basis, one can approximate C.

Laplacian Basis

It's well-known that the eigenfunctions of laplacian operator are ordered from low-frequency (low eigenvalue) to high-frequency (high eigenvalue).

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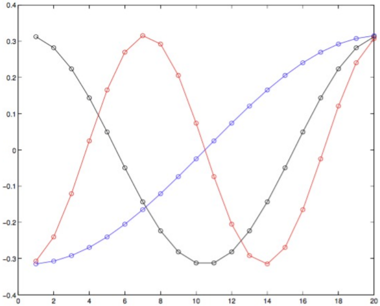

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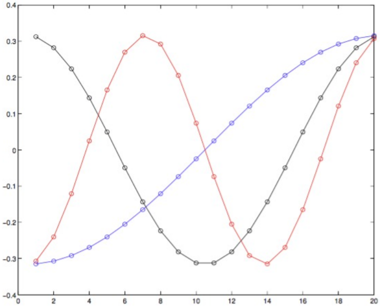

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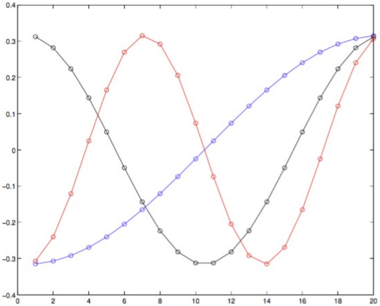

It provides a multi-scale description of the function space on the graphs. More importantly, we can approximate the T_F with limited basis!

Highlight Distortion

We adapt the formulation in "Analysis and Visualization of Maps Between Shapes". Given some measure μ_1, μ_2 on the nodes in G_1, G_2 and $T : G_2 \to G_1$, our goal is to find function f such that the following admits maximal:

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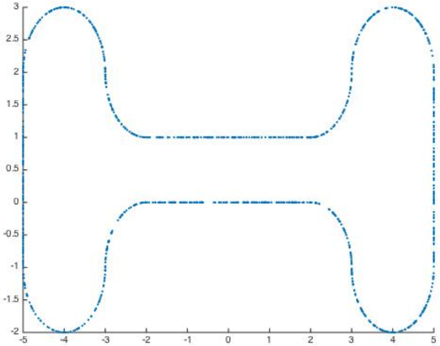
With some mild conditions, one will conclude that the coefficient of f is nothing more than the right singular vector corresponding to the largest singular value of C, the matrix representation of T_F.

Experiments: Bone



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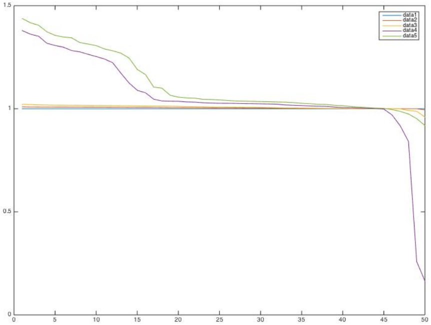

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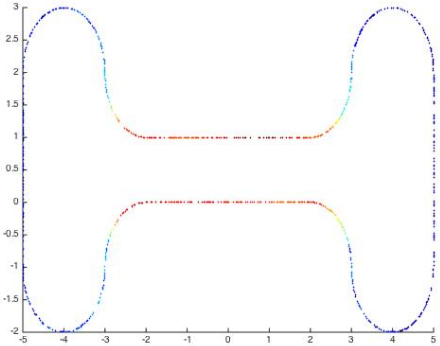

Experiments: Food Pairing

NA → EA	EA → NA
gruyere cheese	
milk fat	
parmesan cheese	
sour cherry	
raspberry	
nutmeg	
oatmeal	
pecan	
fennel	
sauerkraut	
orange peel	
grape	
lima bean	
...	

Experiments: Food Pairing

NA $\rightarrow$ EA	EA $\rightarrow$ NA
gruyere cheese	katsuobushi
milk fat	chinese cabbage
parmesan cheese	seaweed
sour cherry	sesame oil
raspberry	sake
nutmeg	mandarin peel
oatmeal	peanut oil
pecan	thai pepper
fennel	sesame seed
sauerkraut	litchi
orange peel	wasabi
grape	soybean
lima bean	soy sauce
...	...

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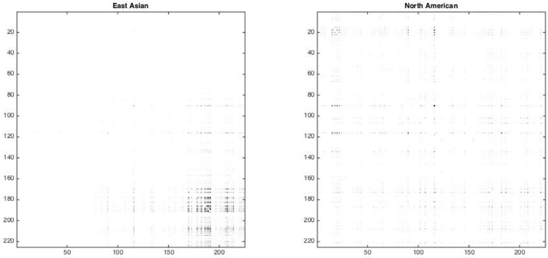

Discussion

- The practical network data are complicated, need to be studied case by case. And we usually don't have the GROUND-TRUTH.
- We hope this method can cooperate with other methods, say, Graph Drawing.
- More open problems, the formulation is not limited at all...

The End

Thanks for your attention.
Questions?