




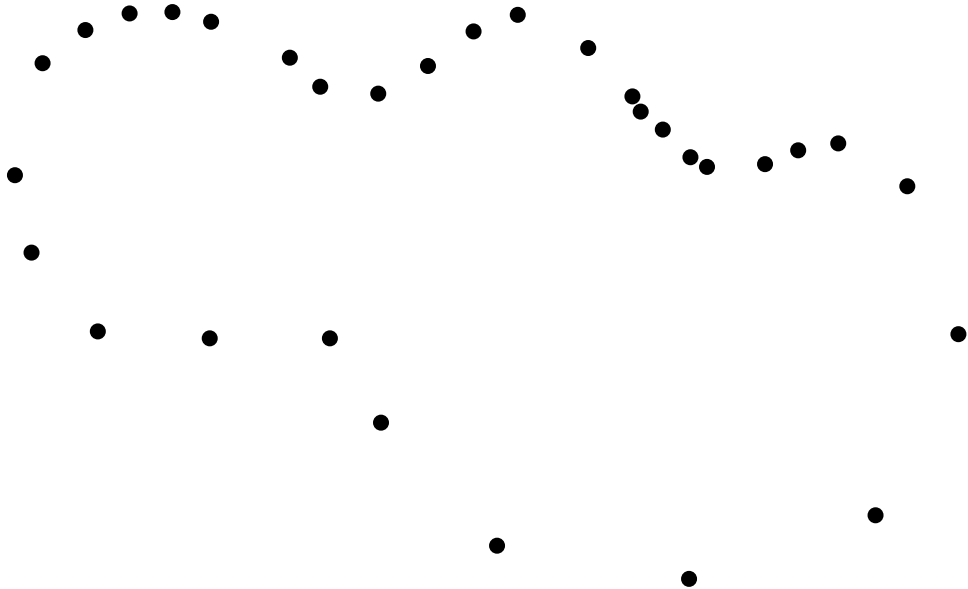
De la géométrie algorithmique
au calcul géométrique
l'exemple
de la triangulation de Delaunay

Application à la reconstruction

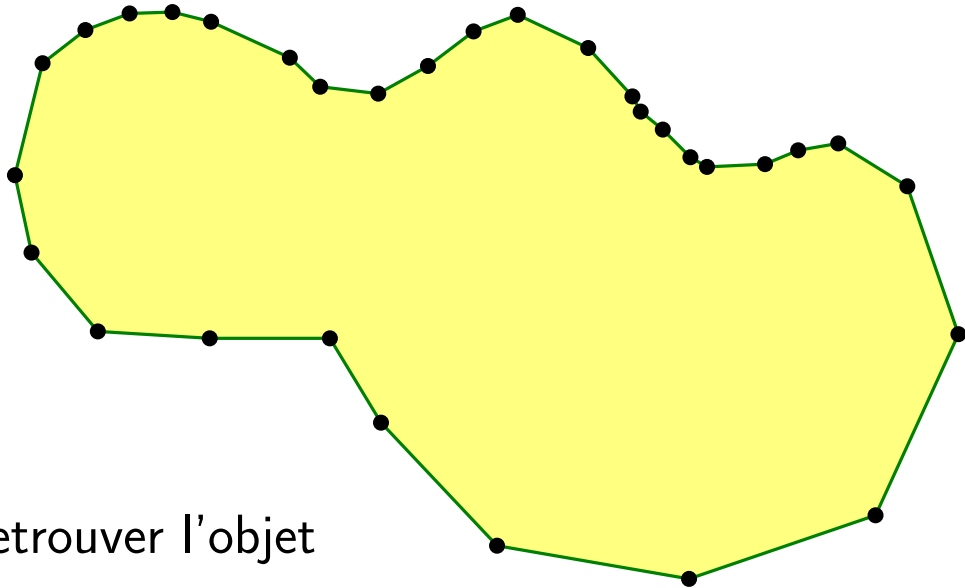


Le problème de
reconstruction

Des points sur le bord d'un objet

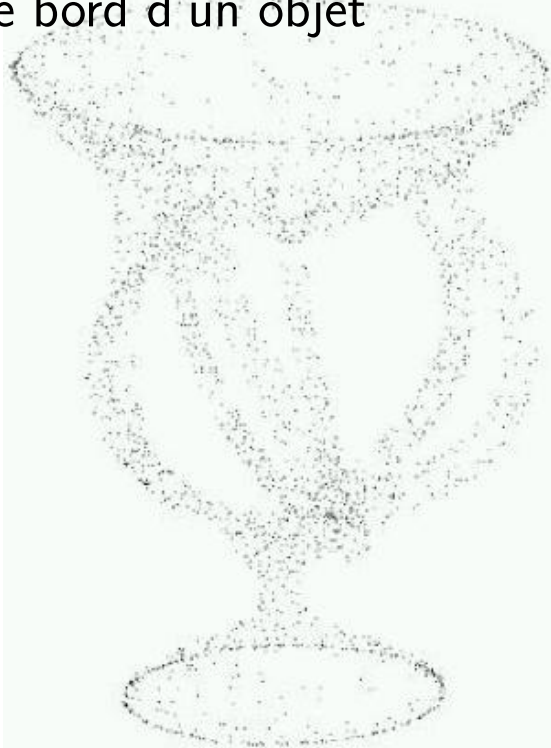


Des points sur le bord d'un objet

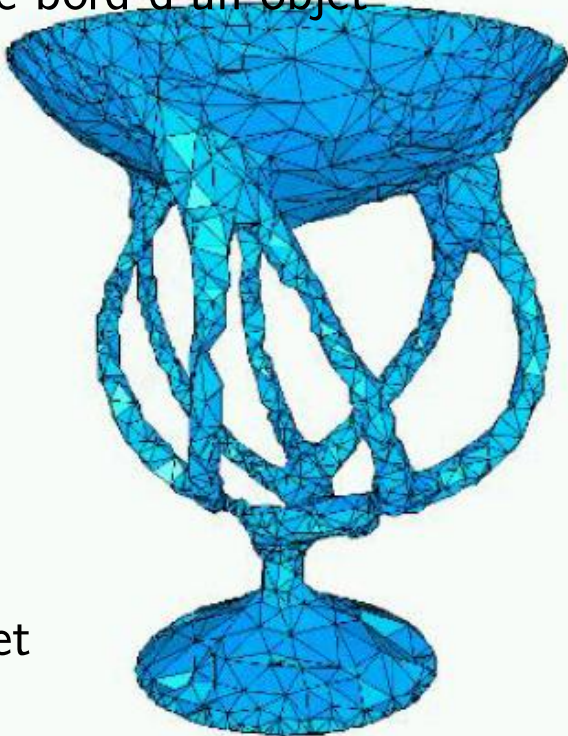


Retrouver l'objet

Des points sur le bord d'un objet



Des points sur le bord d'un objet



Retrouver l'objet

CAO

Capteurs

laser
lumière structurée
mécanique

Applications

Ingénierie inverse (maquette, espionnage industriel)
prototypage
contrôle qualité

Médical

Capteurs

scanner
échographie

...

Applications

Diagnostic

Simulation d'endoscopie

Planification de chirurgie

Géographie, Géologie

Capteurs

Photo aérienne, satellite
carotage
sismique

Applications

Cartographie, Modèles de terrains
Modélisation radio (téléphone)
"Image" sous-sol (tunnels, pétrole...)

Discrétisation d'objets "mathématiques"

Capteurs

Algorithme
en dimension $d - 1$

Applications

Visualisation d'équations algébriques
Planification de robot (x, y, θ)



Lien avec
la triangulation de Delaunay

Si l'objet est bien échantillonné
les arêtes de la reconstruction
sont des arêtes de Delaunay

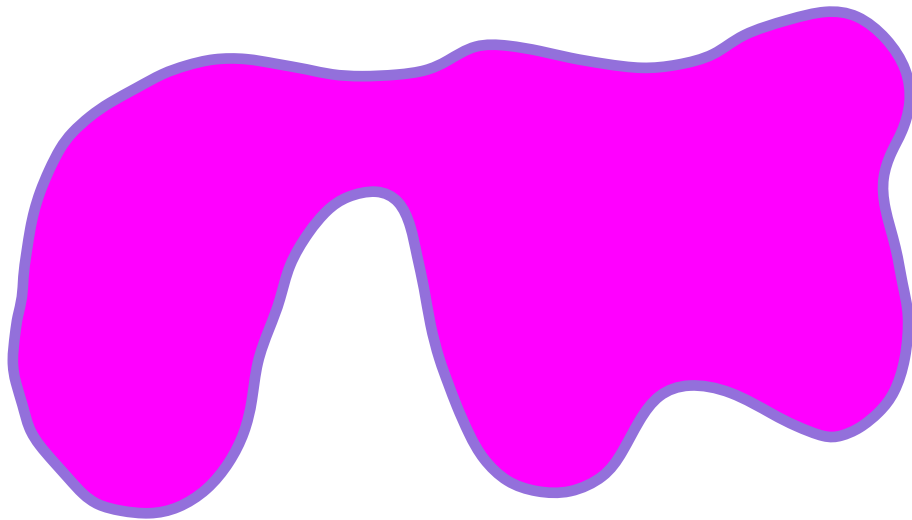
Si l'objet est bien échantillonné
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sont des arêtes de Delaunay

Définition squelette

Définition ε échantillon

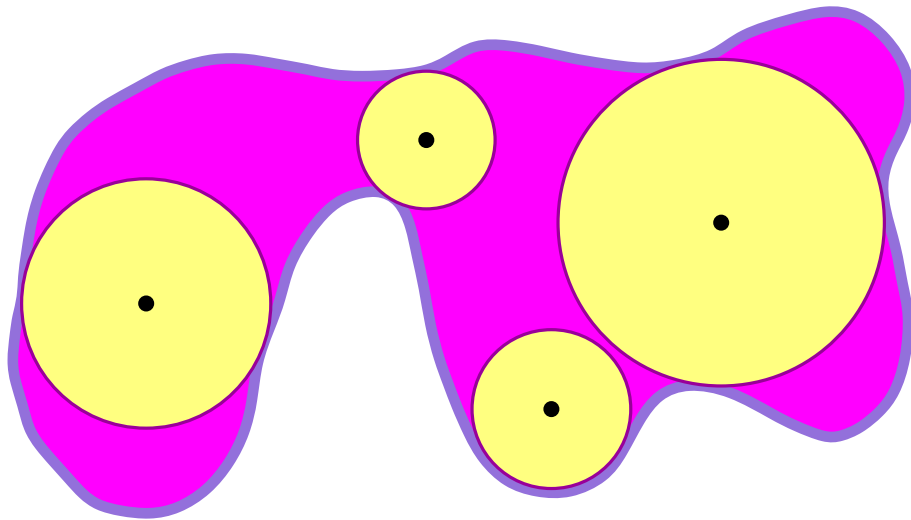
Reconstruction \subset Delaunay

Définition squelette



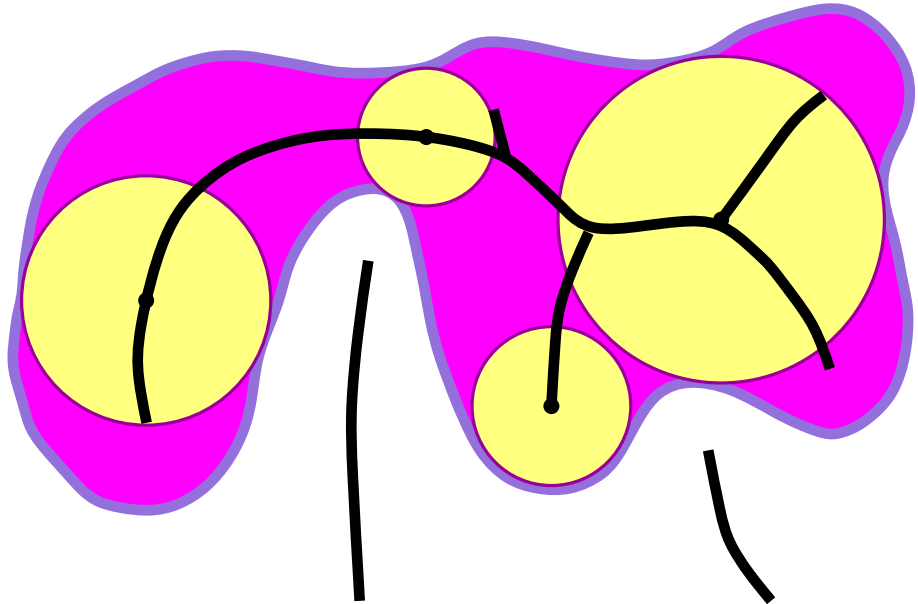
Définition squelette

Centres des cercles maximaux

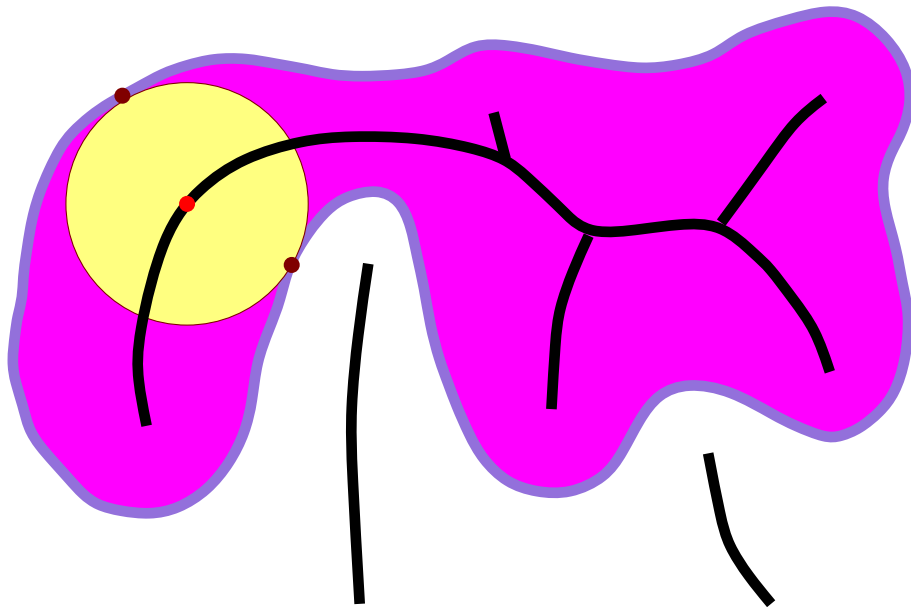


Définition squelette

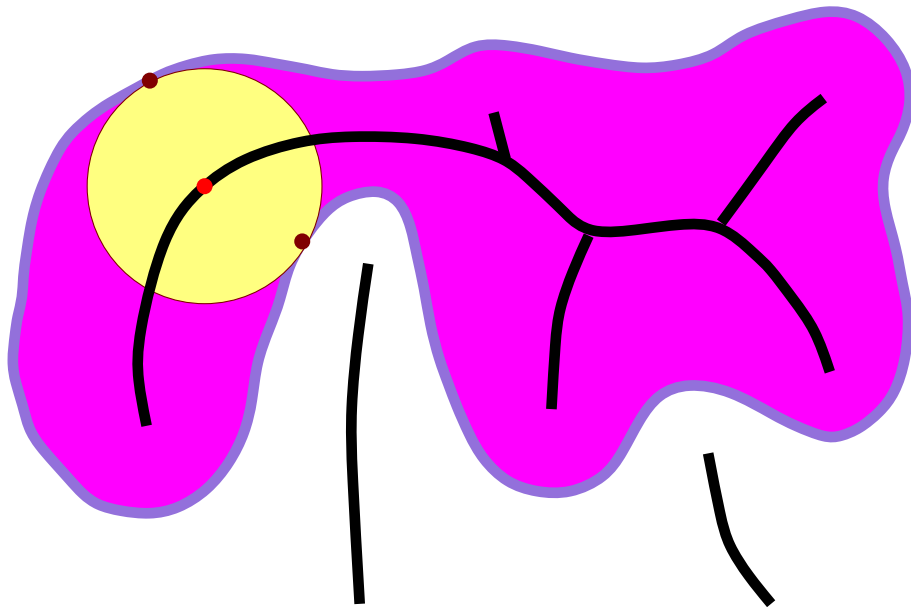
Centres des cercles maximaux



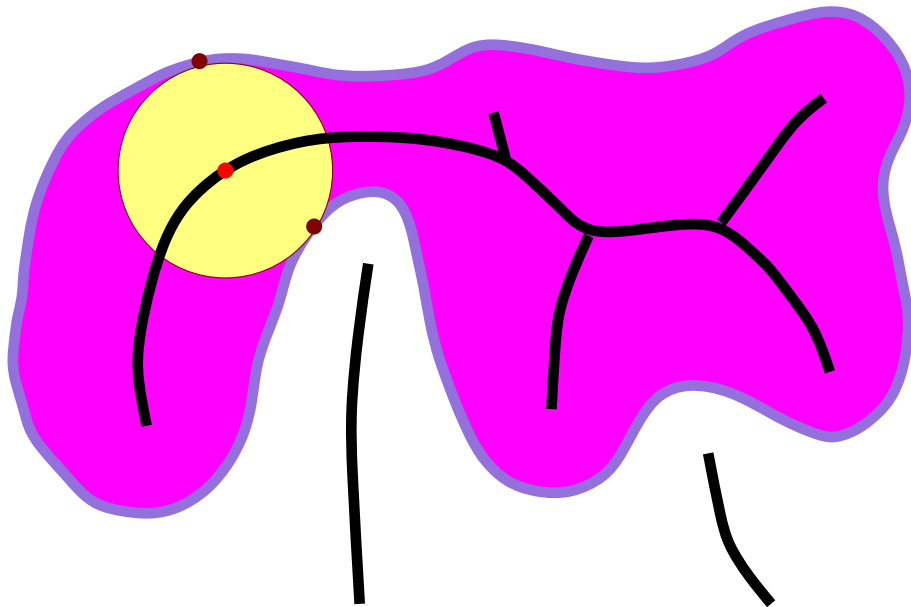
Définition squelette



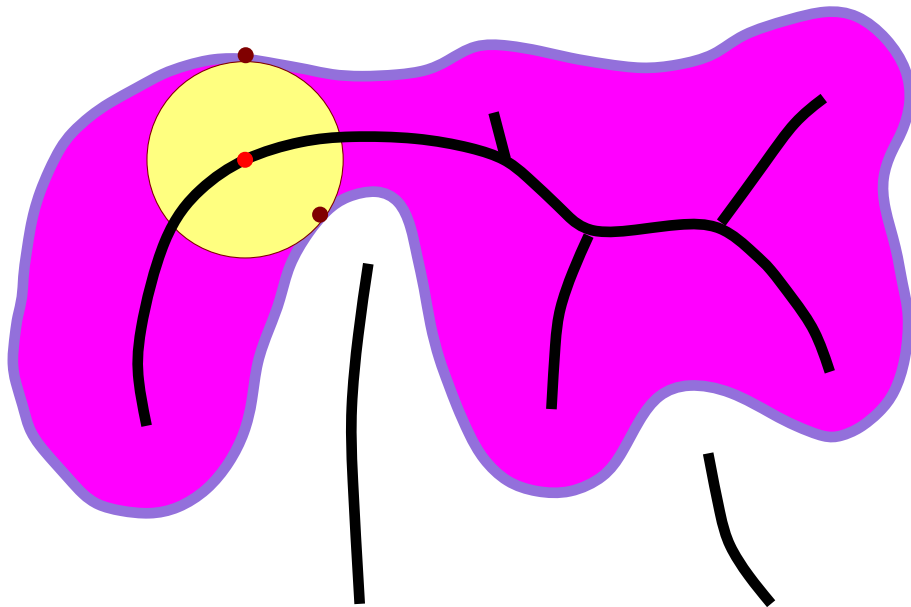
Définition squelette



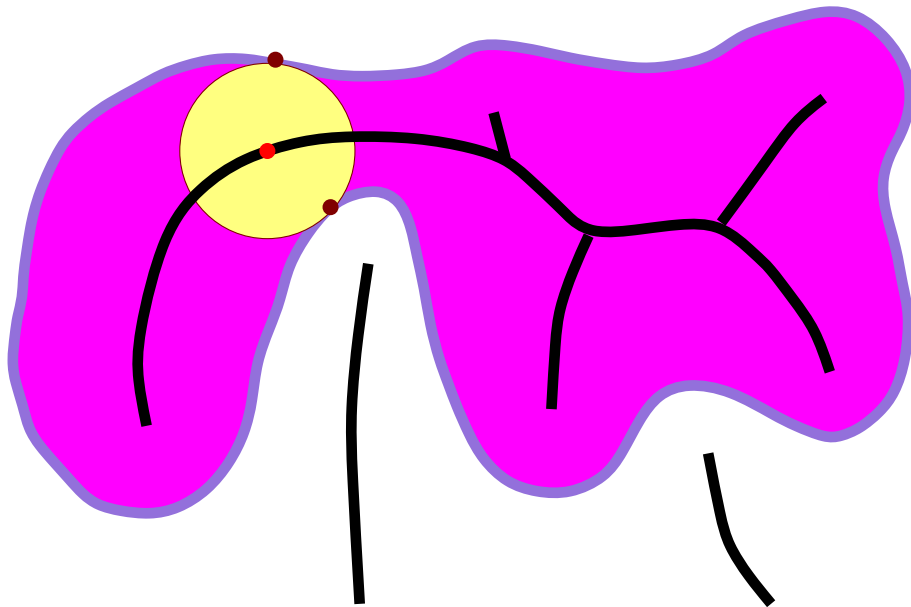
Définition squelette



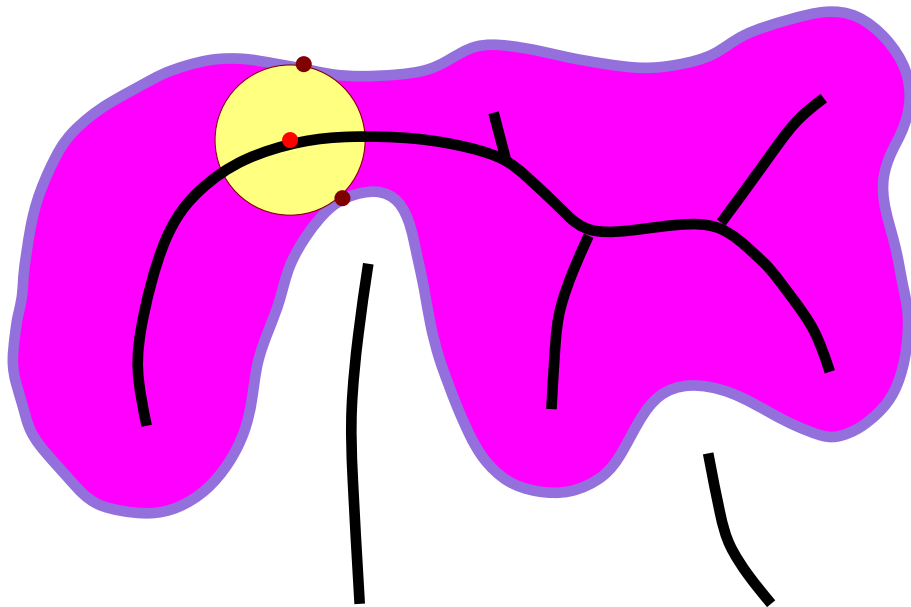
Définition squelette



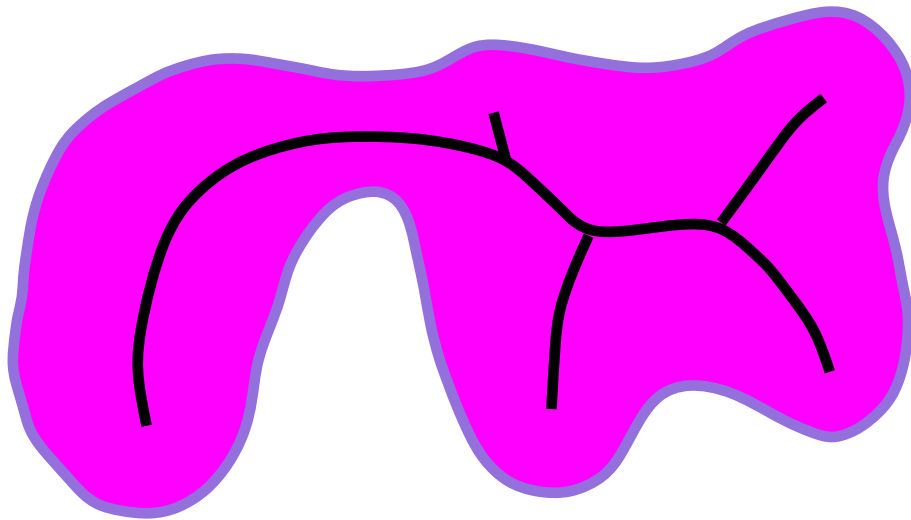
Définition squelette



Définition squelette

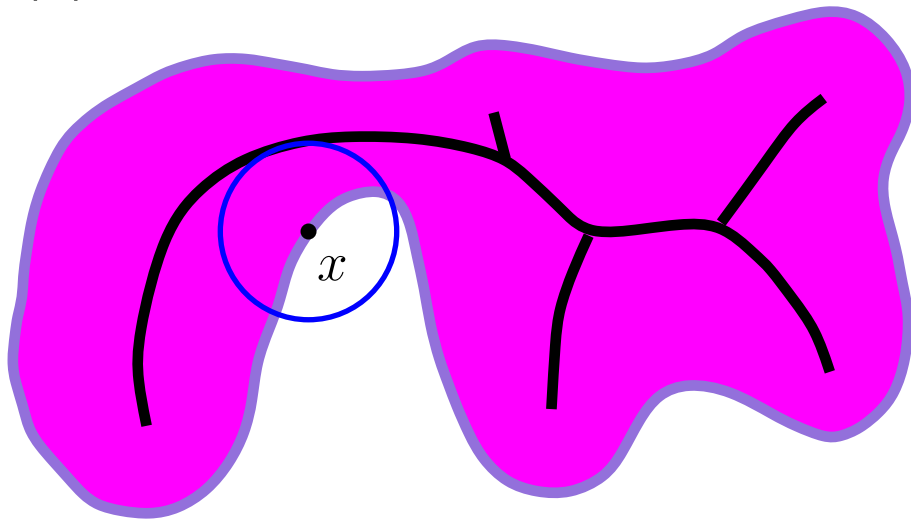


Définition ε échantillon



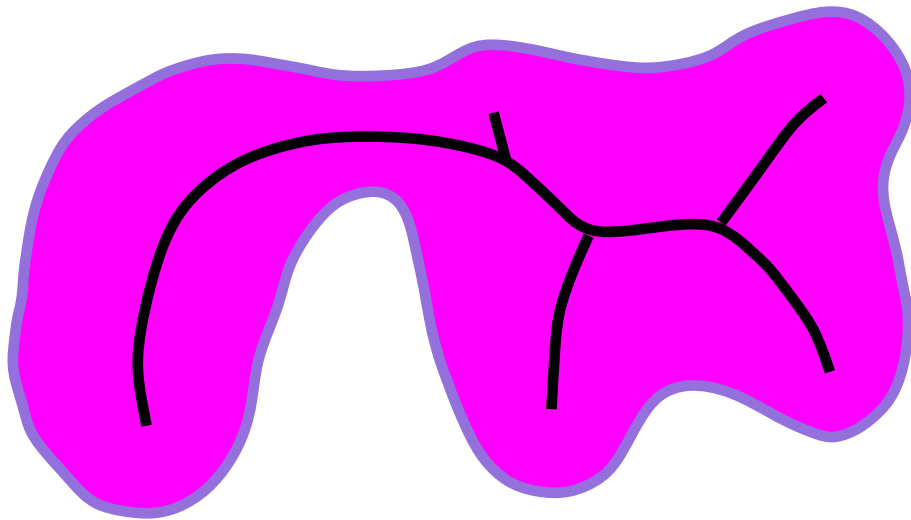
Définition ε échantillon

$\text{ifs}(x)$ (local feature size)



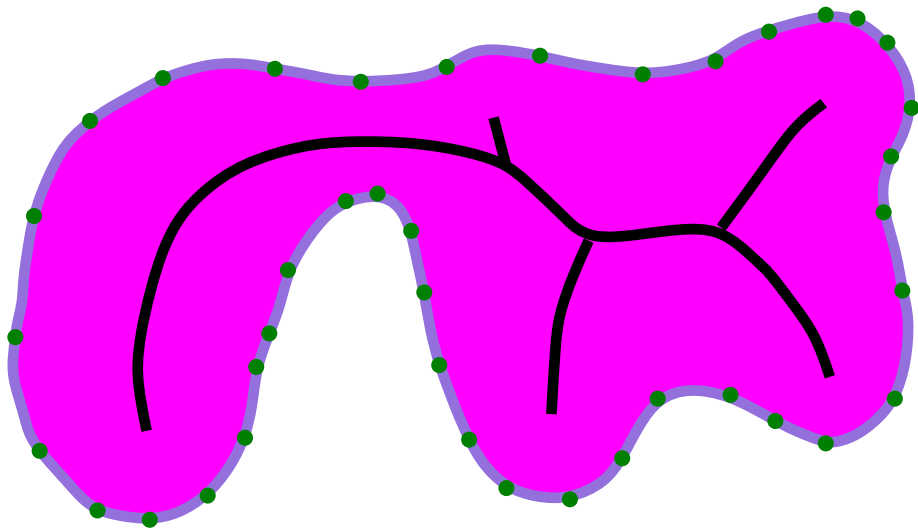
Définition ε échantillon

Échantillon



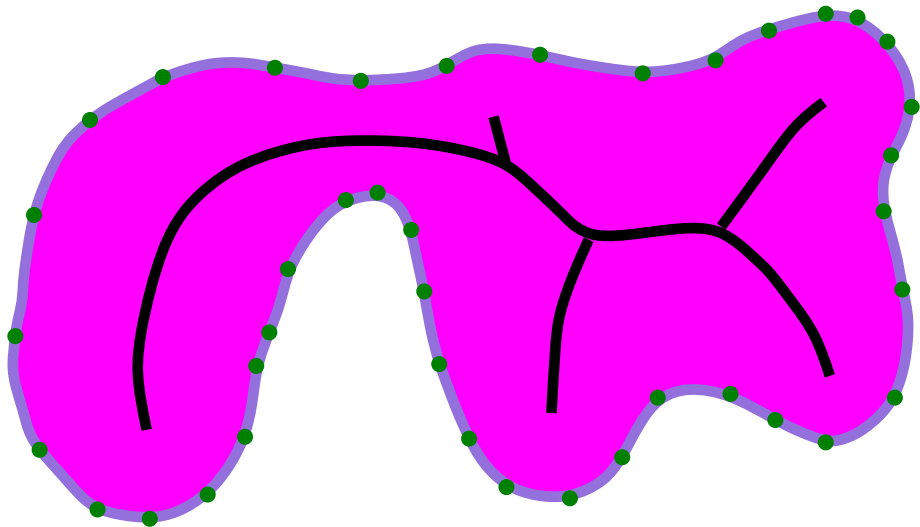
Définition ε échantillon

Échantillon



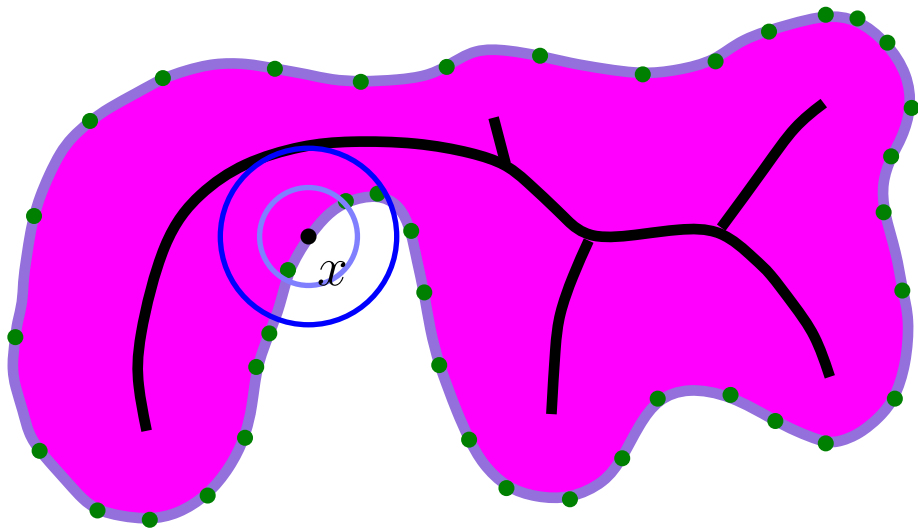
Définition ε échantillon

ε échantillon



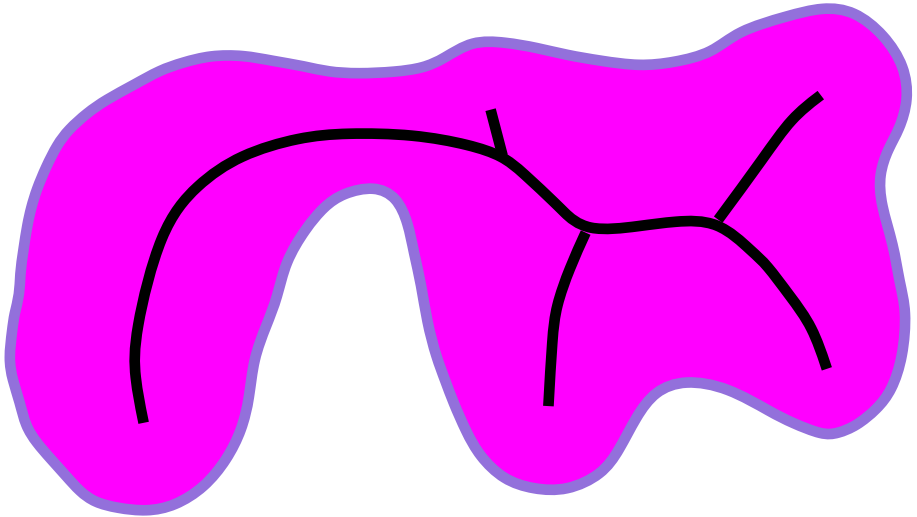
Définition ε échantillon

ε échantillon



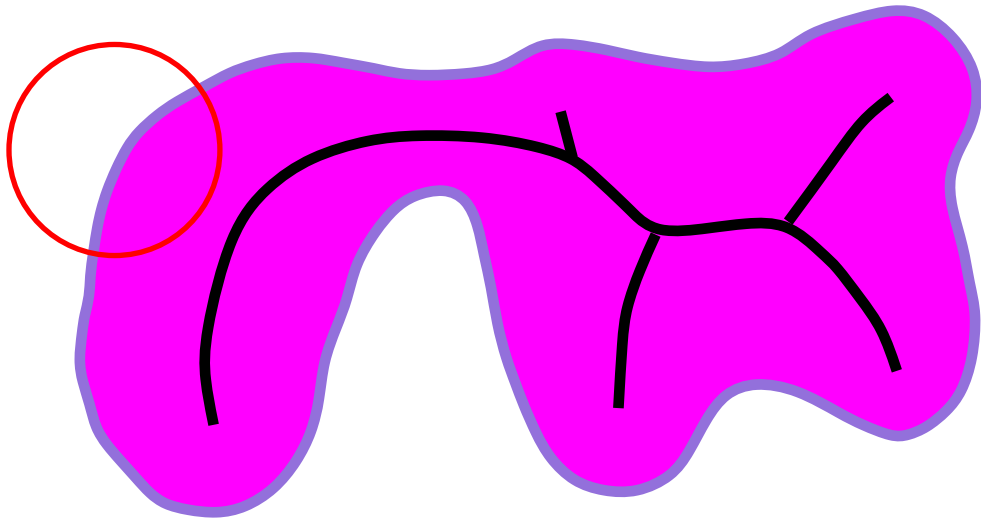
$\forall x$, plus proche \bullet de x sur le bord $< \varepsilon$ lfs(x)

Lemme



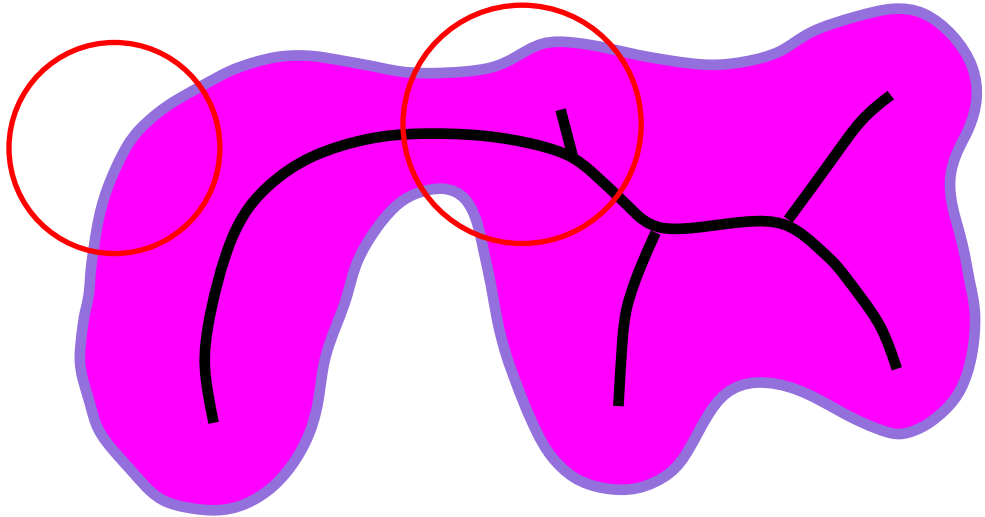
Lemme

Cercle \cap courbe a une seule composante connexe



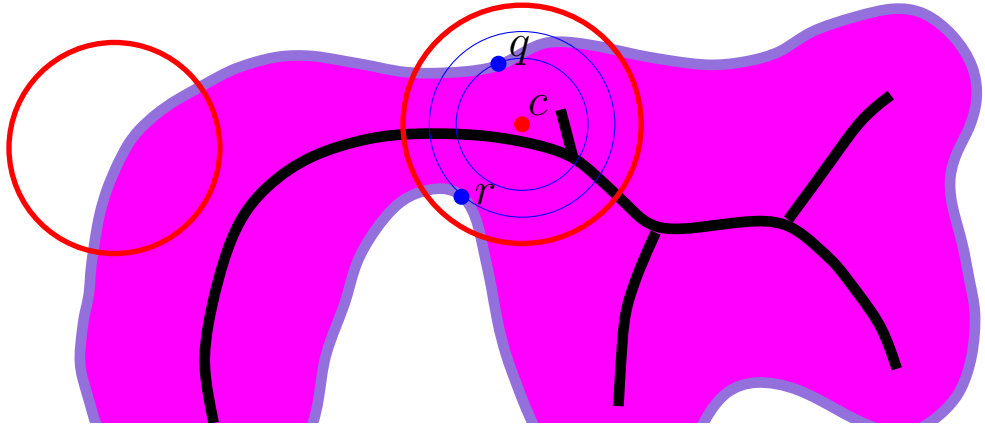
Lemme

Cercle \cap courbe a une seule composante connexe
ou alors Cercle \cap squelette $\neq \emptyset$



Lemme

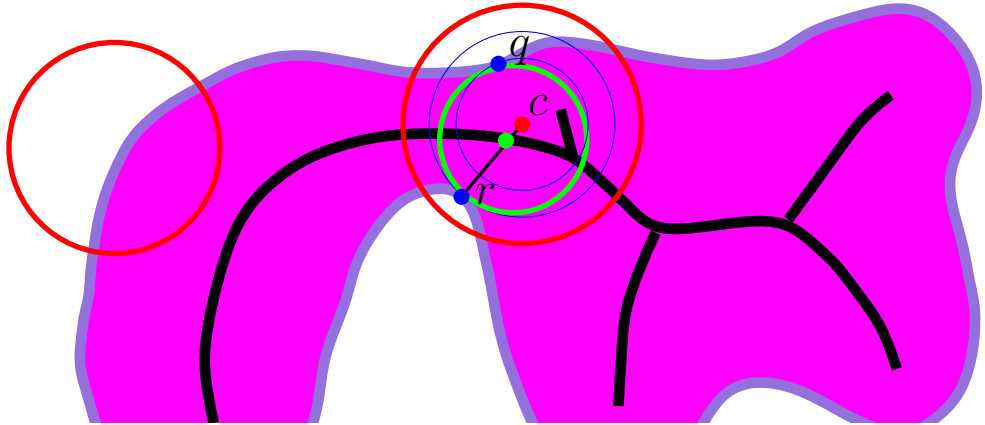
Cercle \cap courbe a une seule composante connexe
ou alors Cercle \cap squelette $\neq \emptyset$



q, r plus proche c dans 2 composantes

Lemme

Cercle \cap courbe a une seule composante connexe
ou alors Cercle \cap squelette $\neq \emptyset$

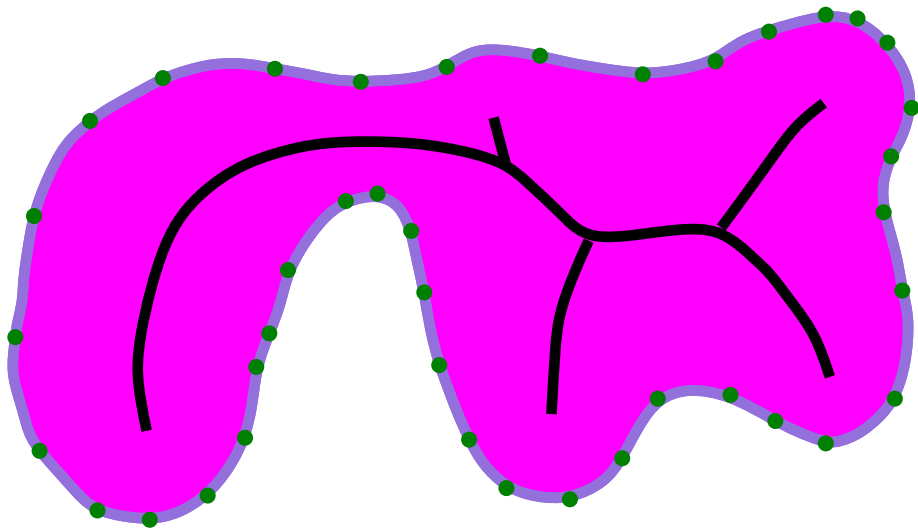


q, r plus proche c dans 2 composantes

cr coupe le squelette

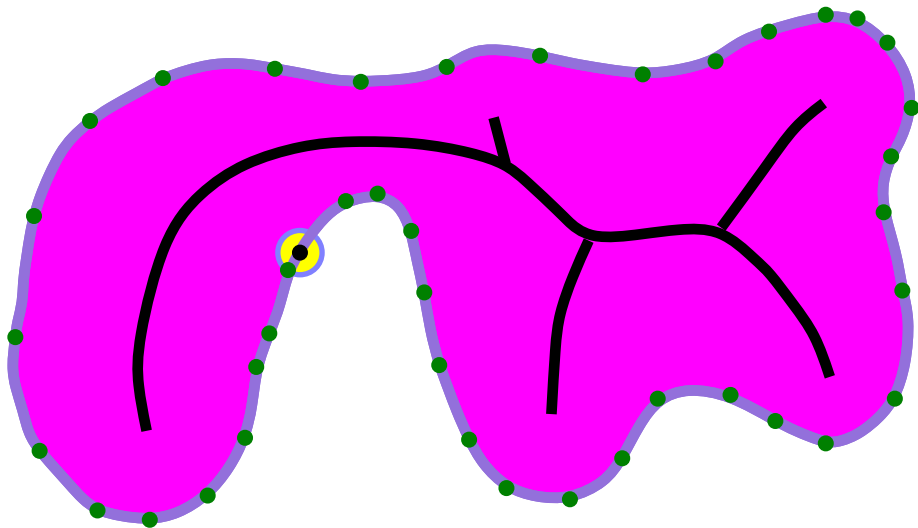
Reconstruction \subset Delaunay

1 échantillon



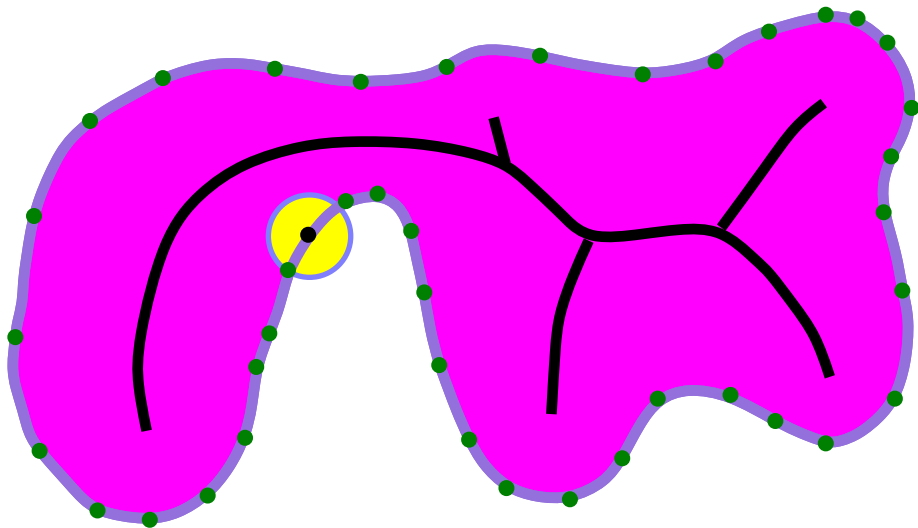
Reconstruction \subset Delaunay

1 échantillon



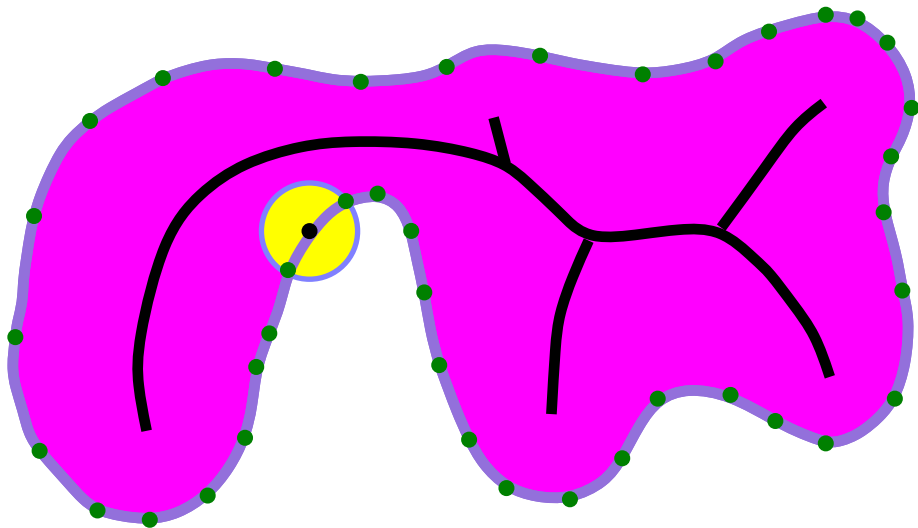
Reconstruction \subset Delaunay

1 échantillon



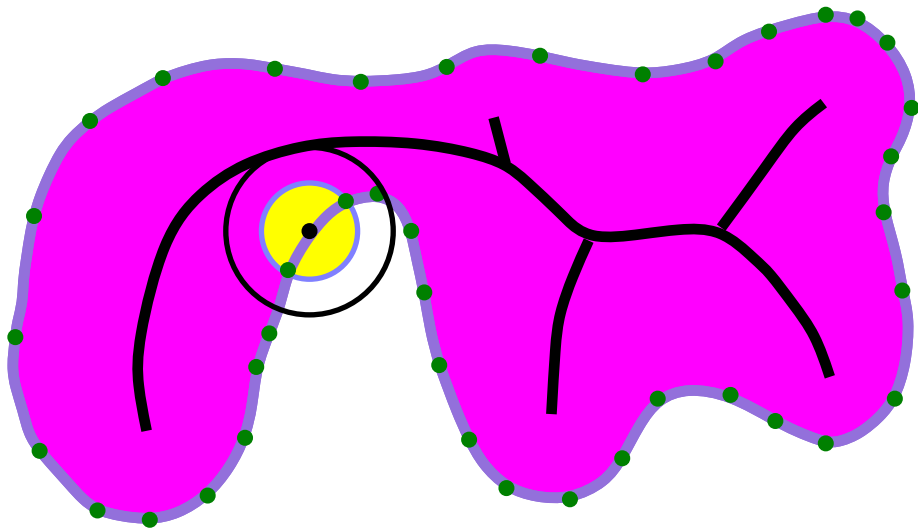
Reconstruction \subset Delaunay

1 échantillon



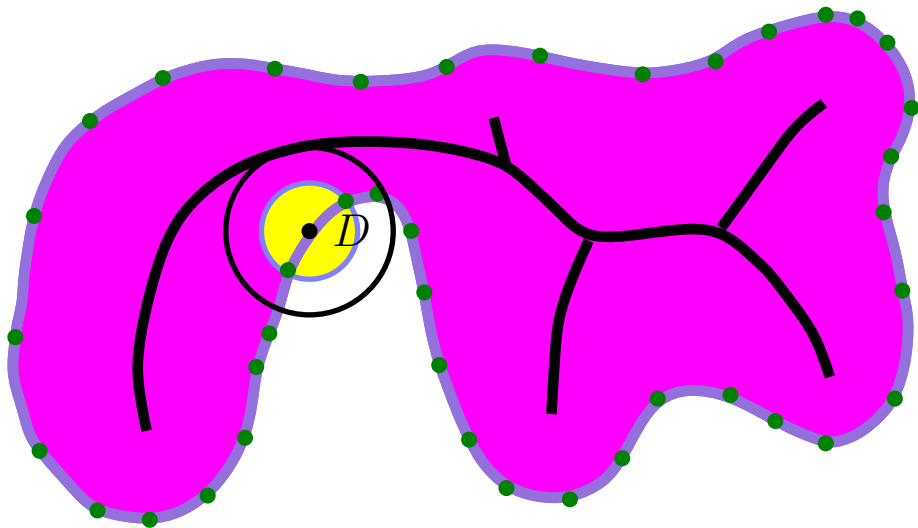
Reconstruction \subset Delaunay

1 échantillon



Reconstruction \subset Delaunay

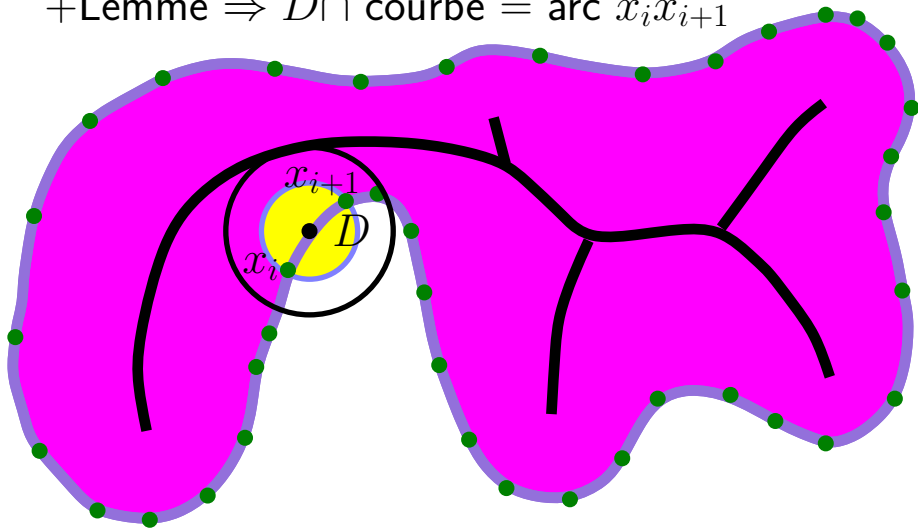
1 échantillon $\Rightarrow D \cap \text{squelette} = \emptyset$



Reconstruction \subset Delaunay

1 échantillon $\Rightarrow D \cap \text{squelette} = \emptyset$

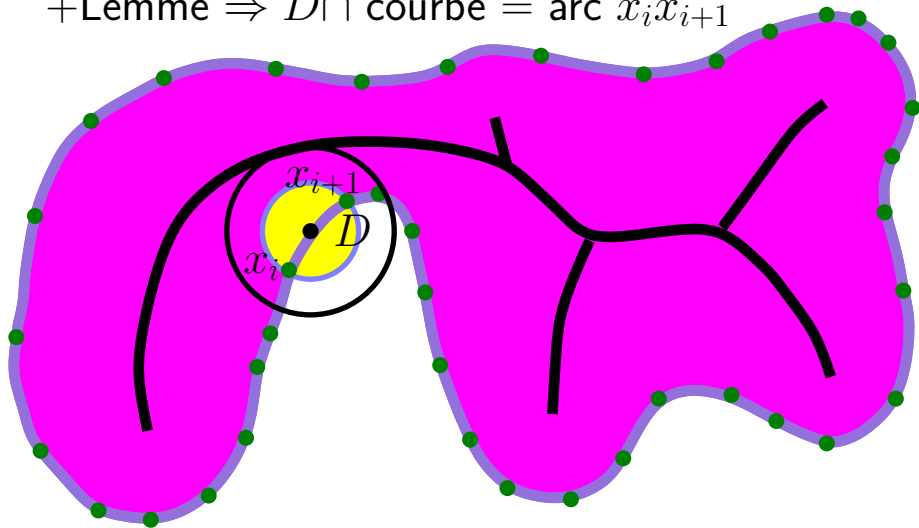
+Lemme $\Rightarrow D \cap \text{courbe} = \text{arc } x_i x_{i+1}$



Reconstruction \subset Delaunay

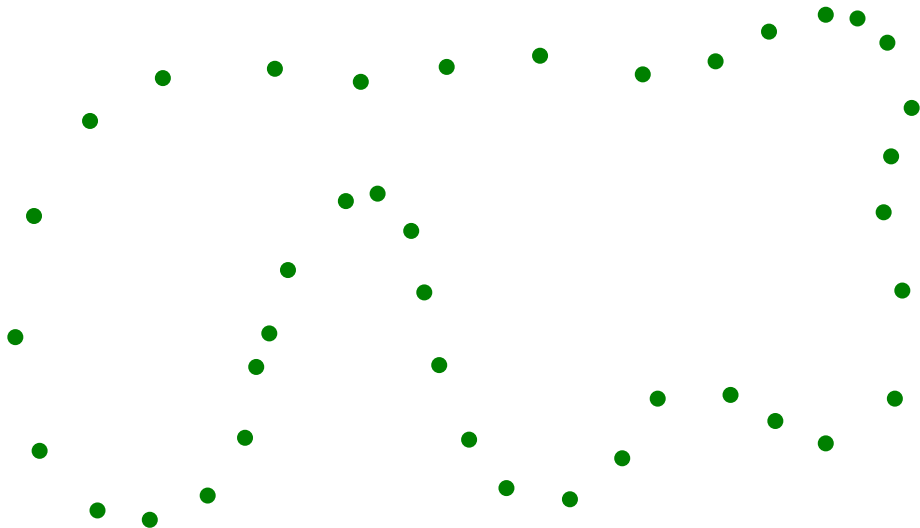
1 échantillon $\Rightarrow D \cap \text{squelette} = \emptyset$

+Lemme $\Rightarrow D \cap \text{courbe} = \text{arc } x_i x_{i+1}$

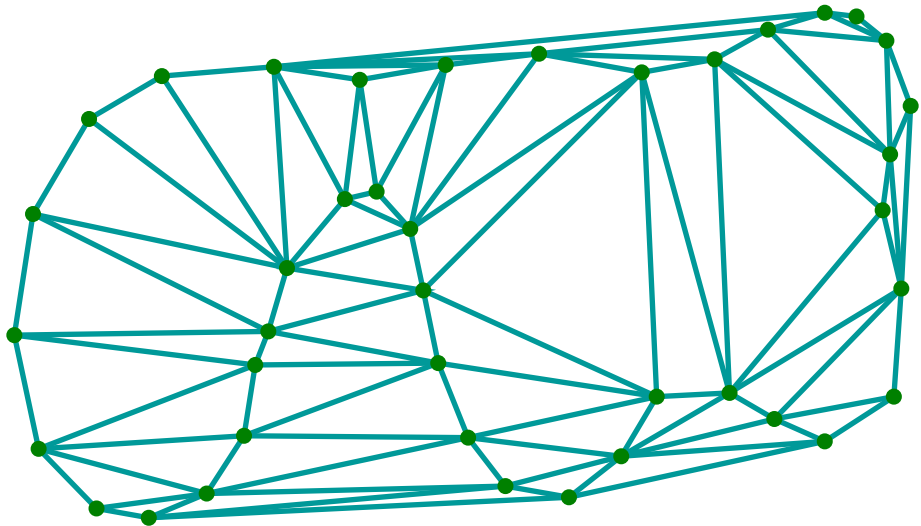


$\Rightarrow D \cap \text{échantillon} = \emptyset$ ($x_i x_{i+1}$ de Delaunay)

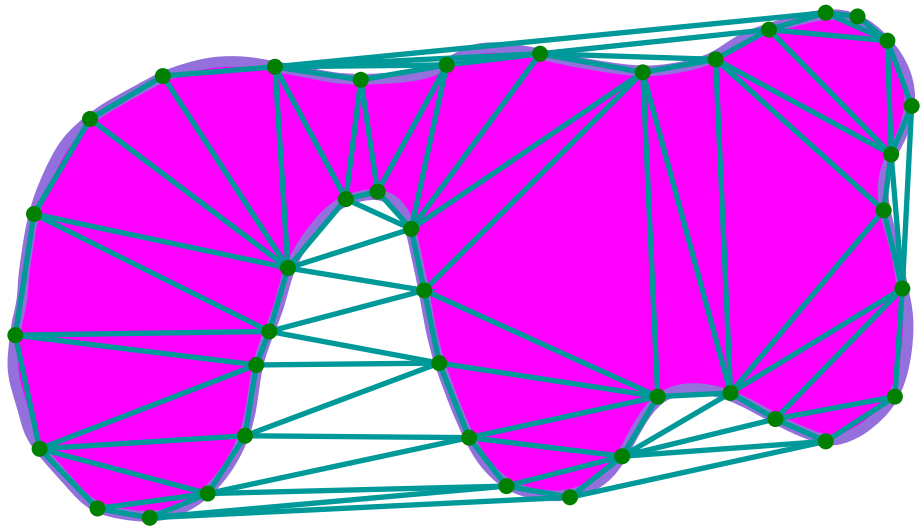
Chercher la courbe dans Delaunay

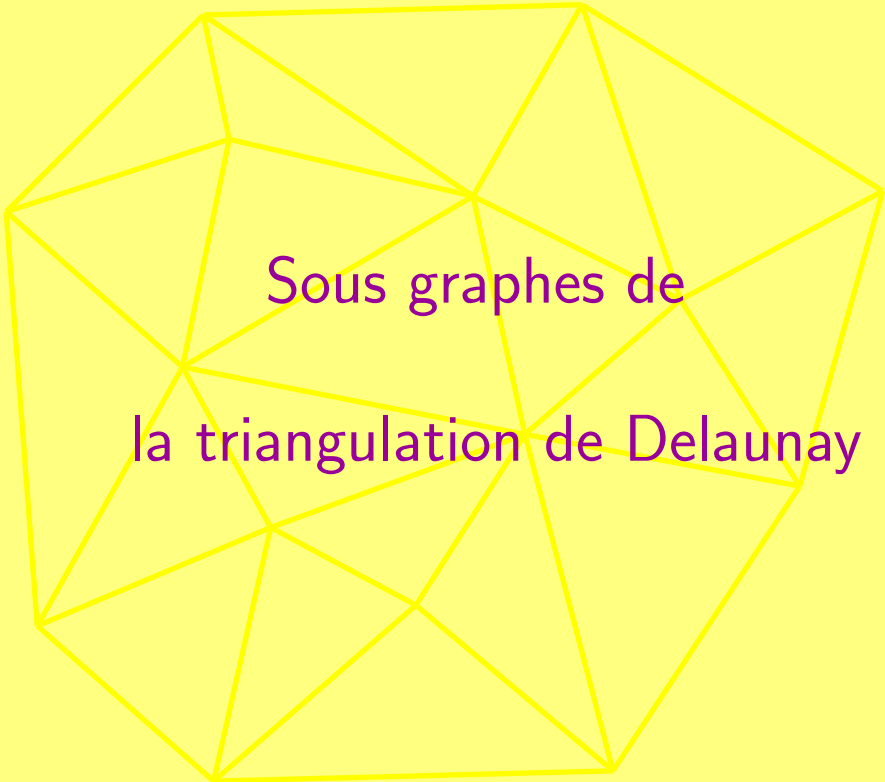


Chercher la courbe dans Delaunay



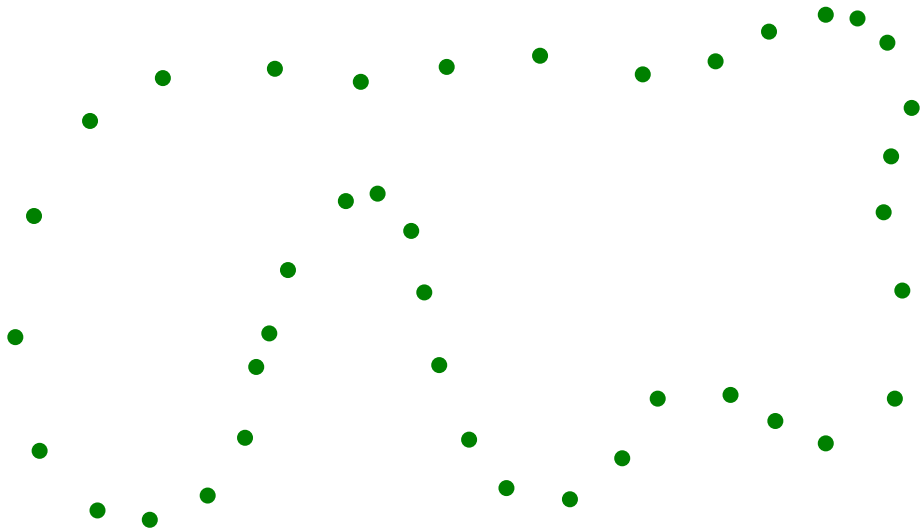
Chercher la courbe dans Delaunay



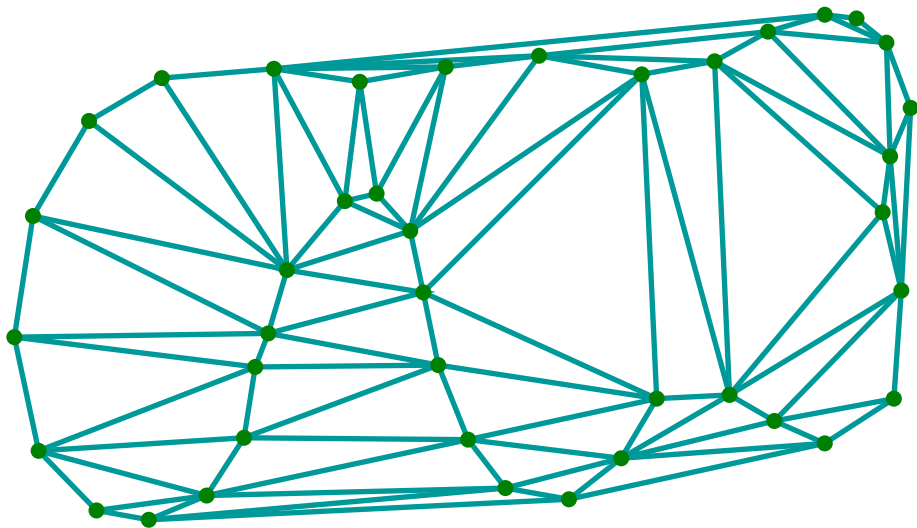


Sous graphes de
la triangulation de Delaunay

Graphe de Gabriel

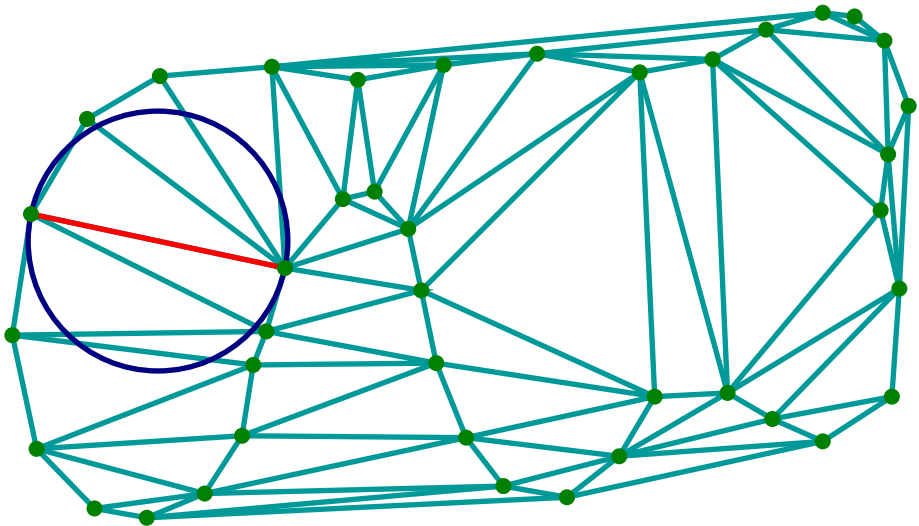


Graphe de Gabriel

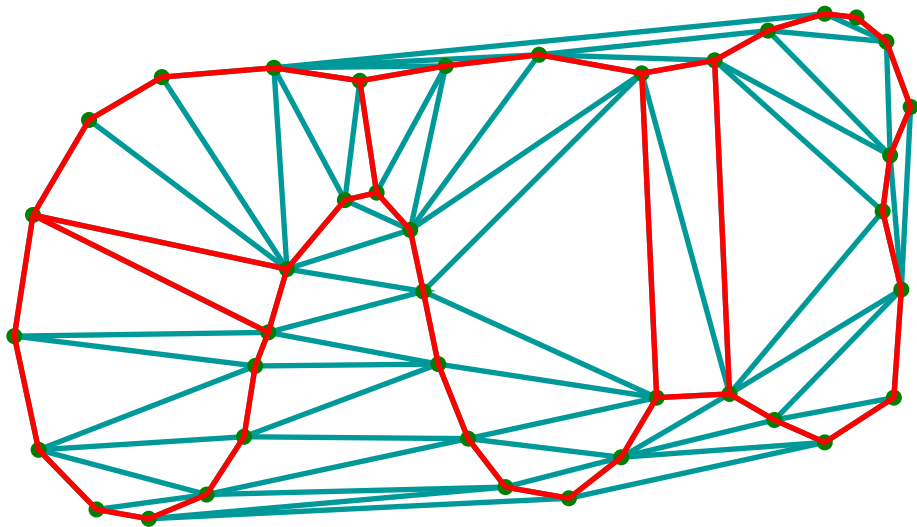


Graphe de Gabriel

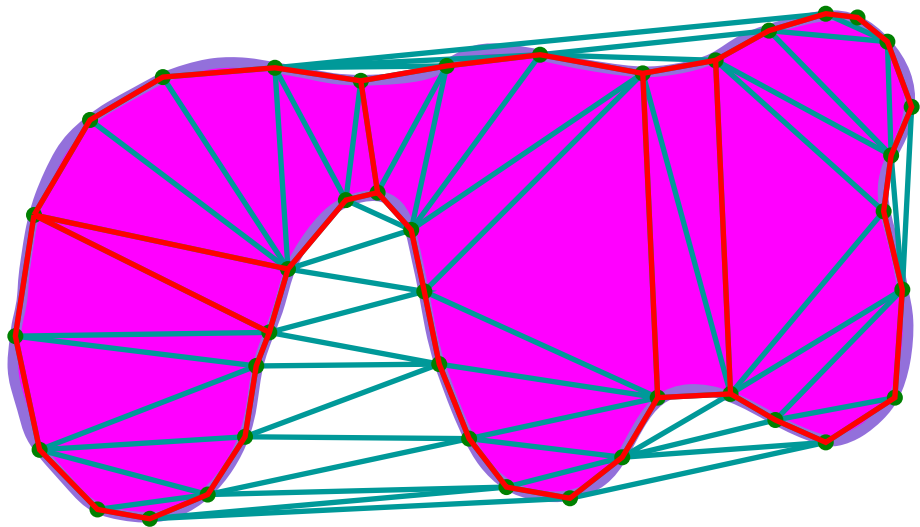
Cercle diamétral vide



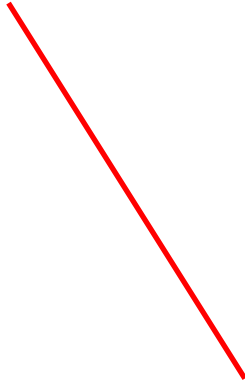
Graphe de Gabriel



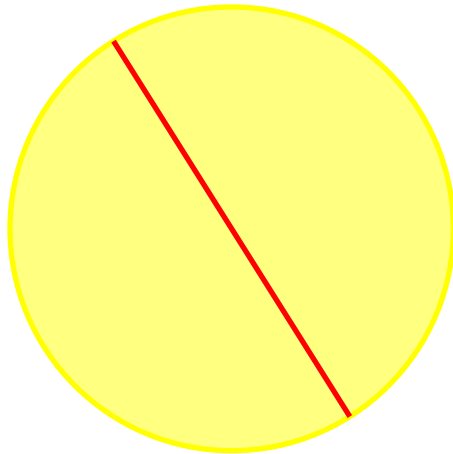
Graphe de Gabriel



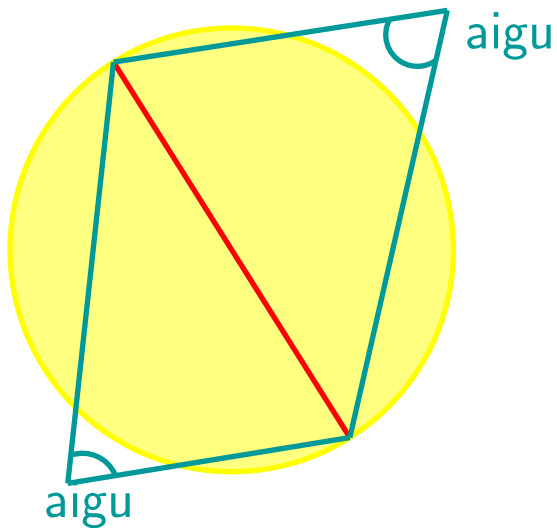
Graphe de Gabriel



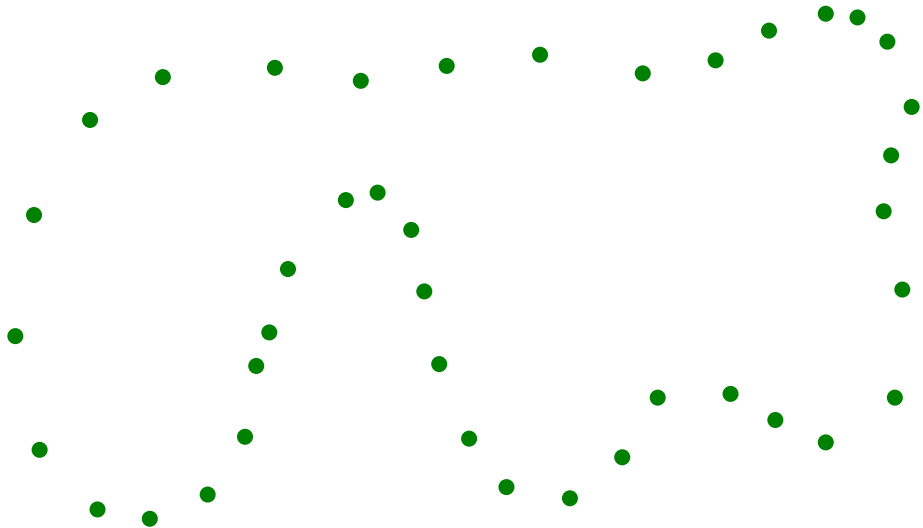
Graphe de Gabriel



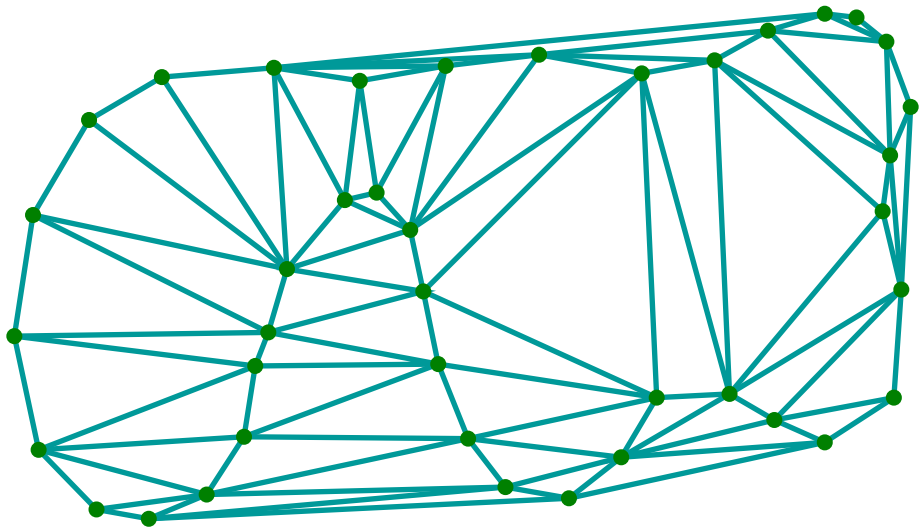
Graphe de Gabriel



α forme

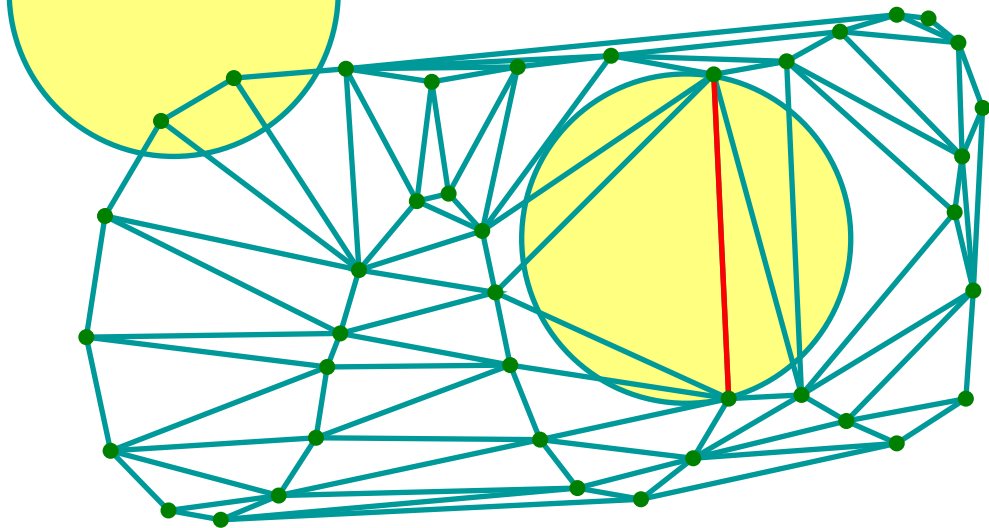


α forme

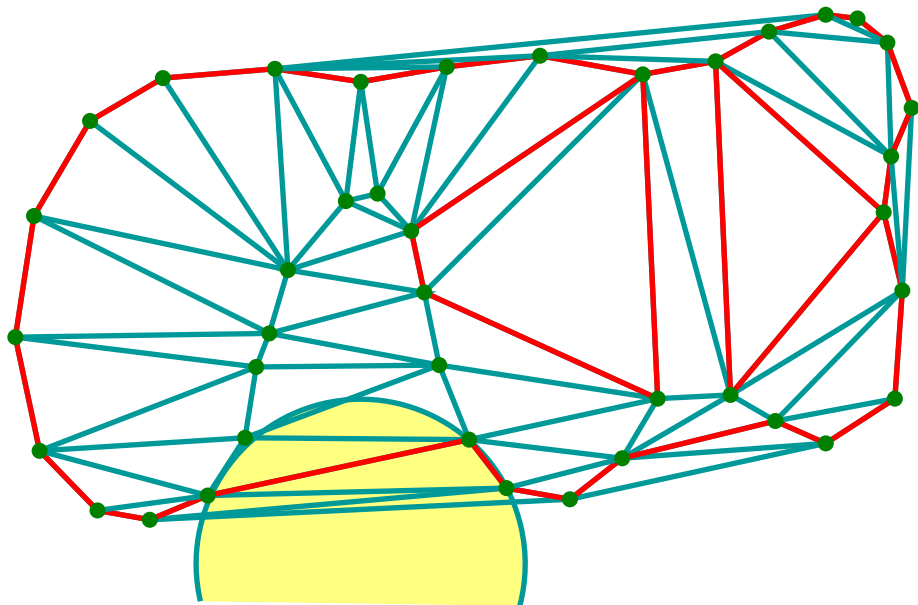


α forme

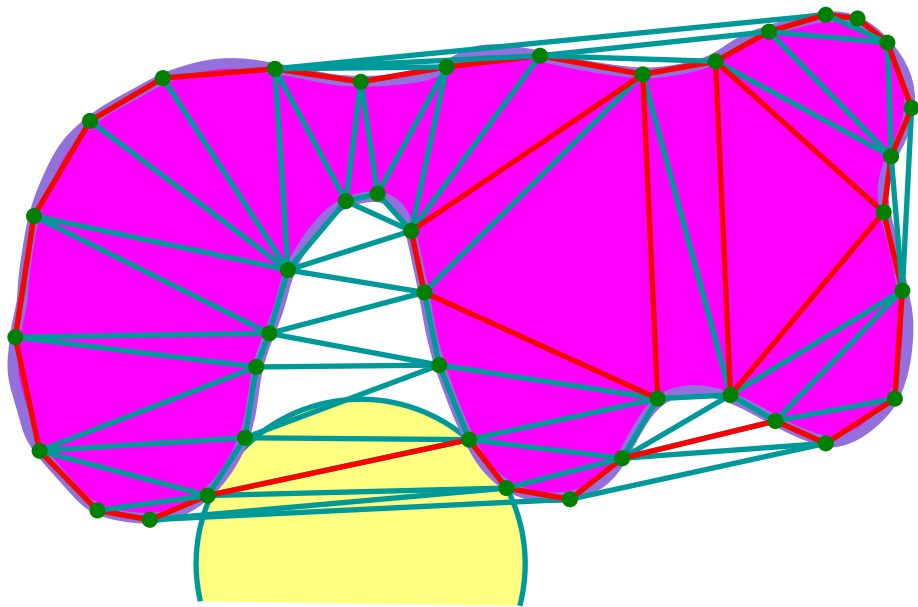
Cercle de rayon α vide



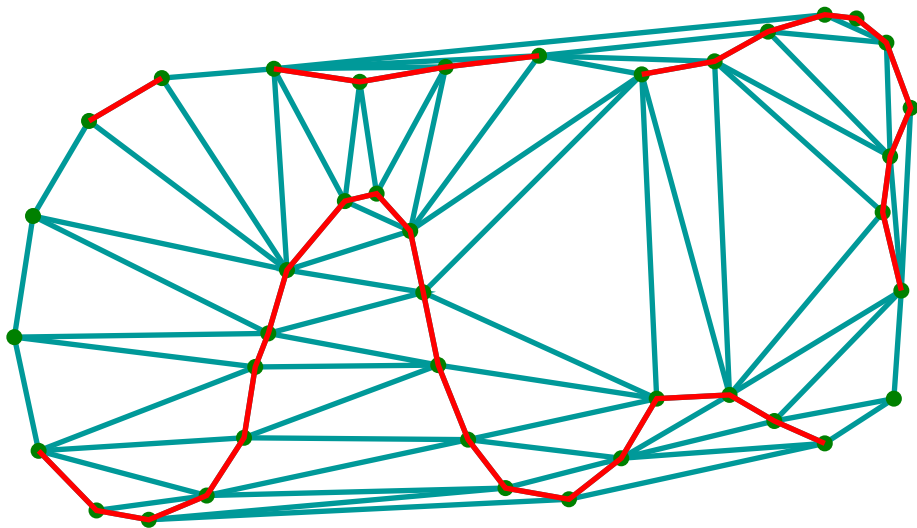
α forme



α forme

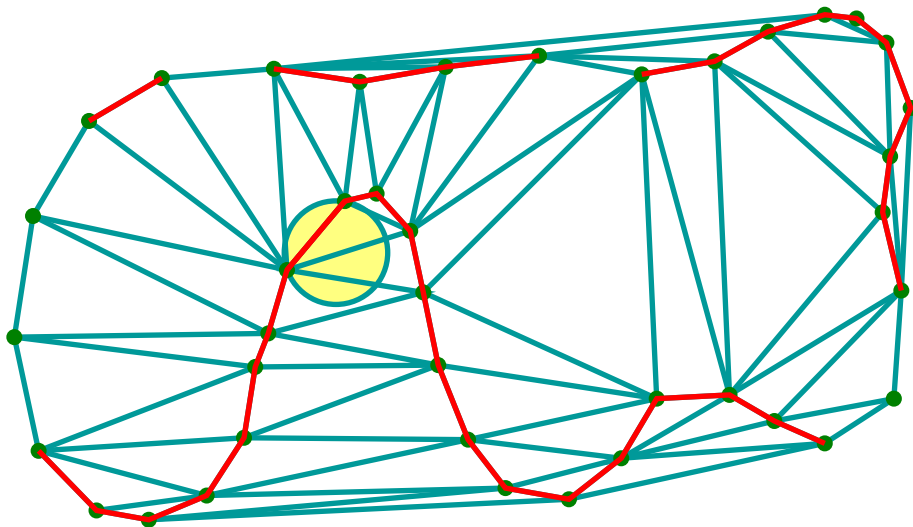


α forme

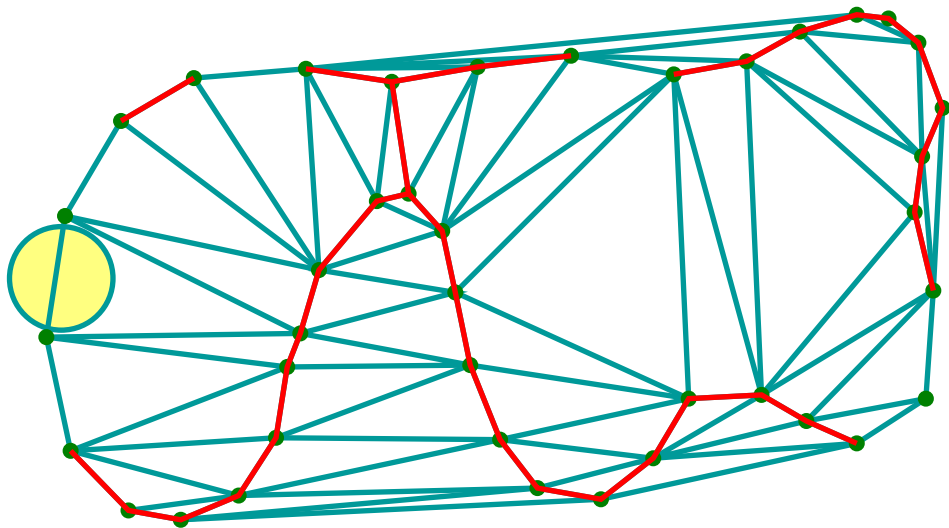
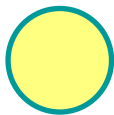


α forme

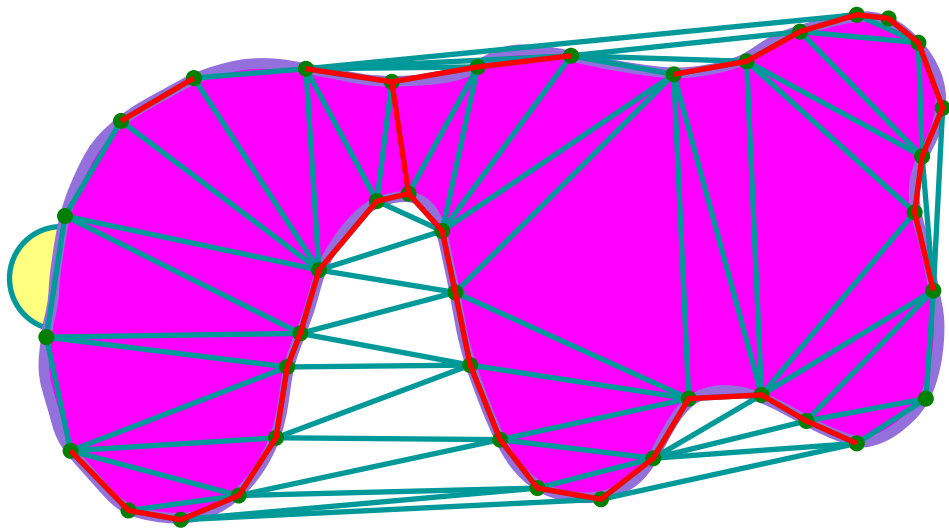
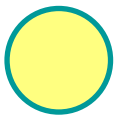
Cercle de rayon α vide



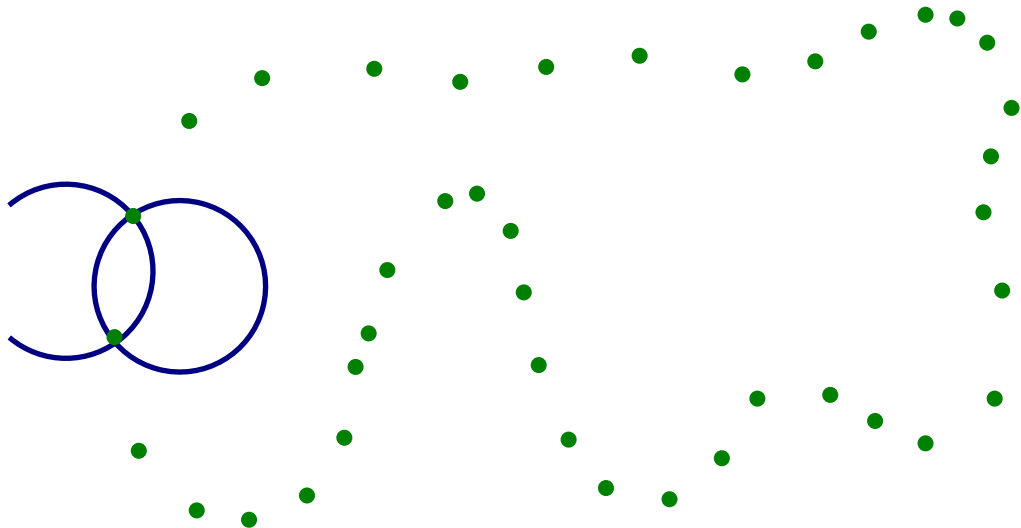
α forme



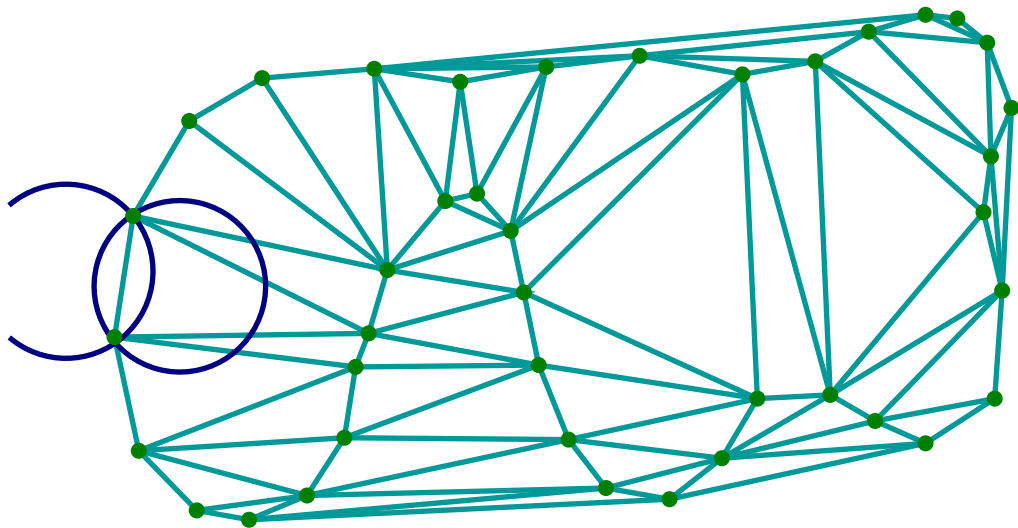
α forme



β squelette

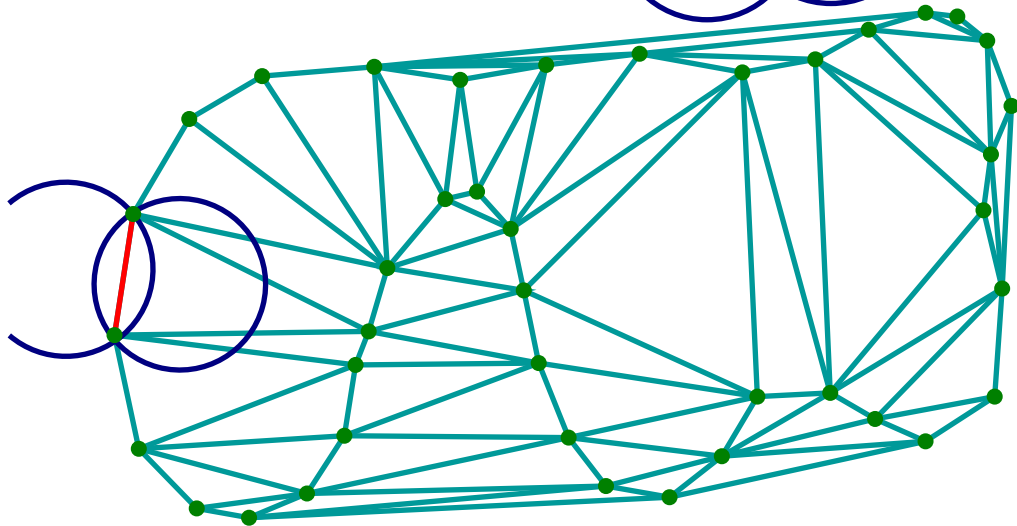
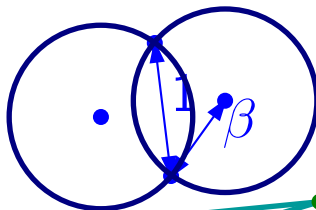


β squelette

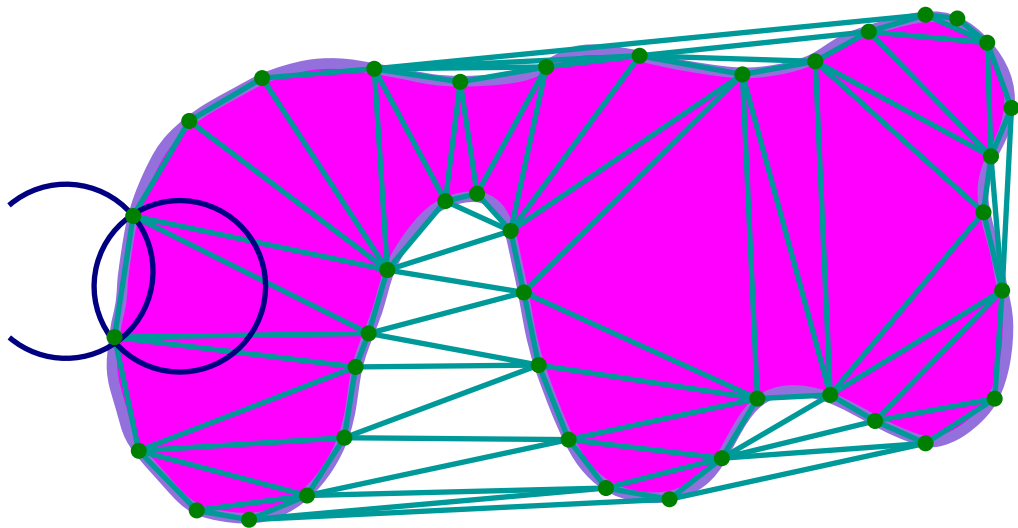


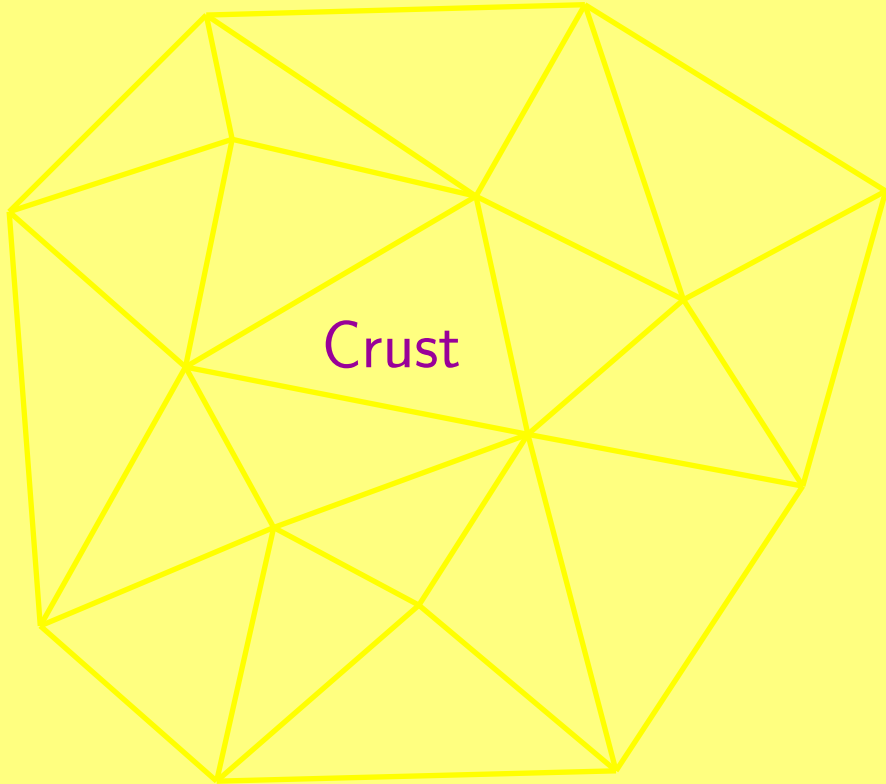
β squelette

2 cercles vides

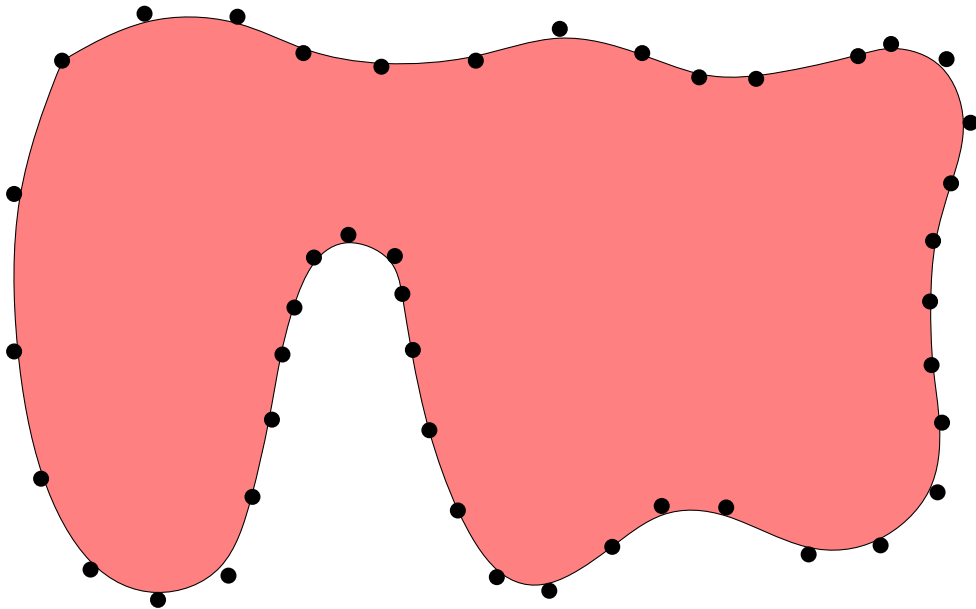


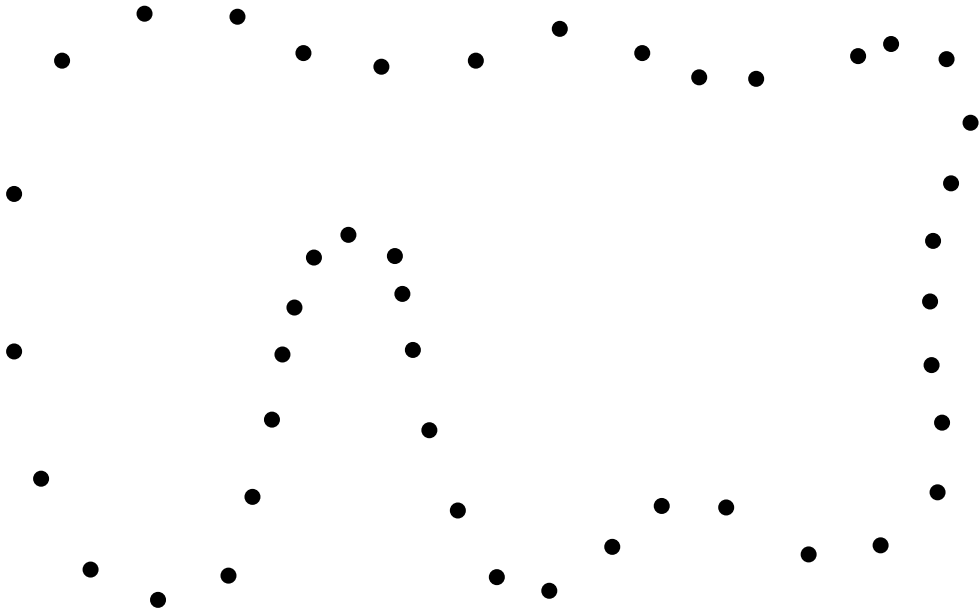
β squelette

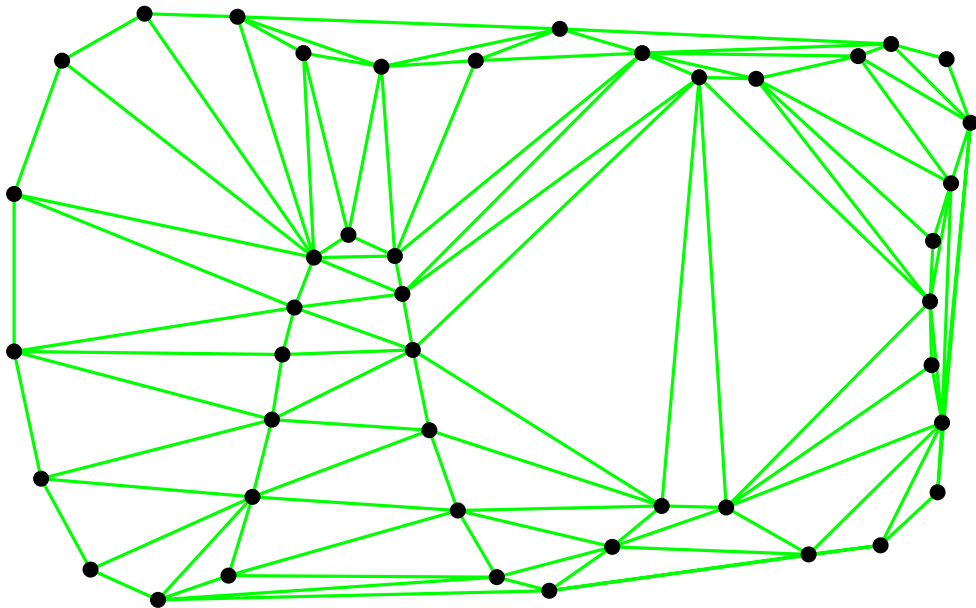


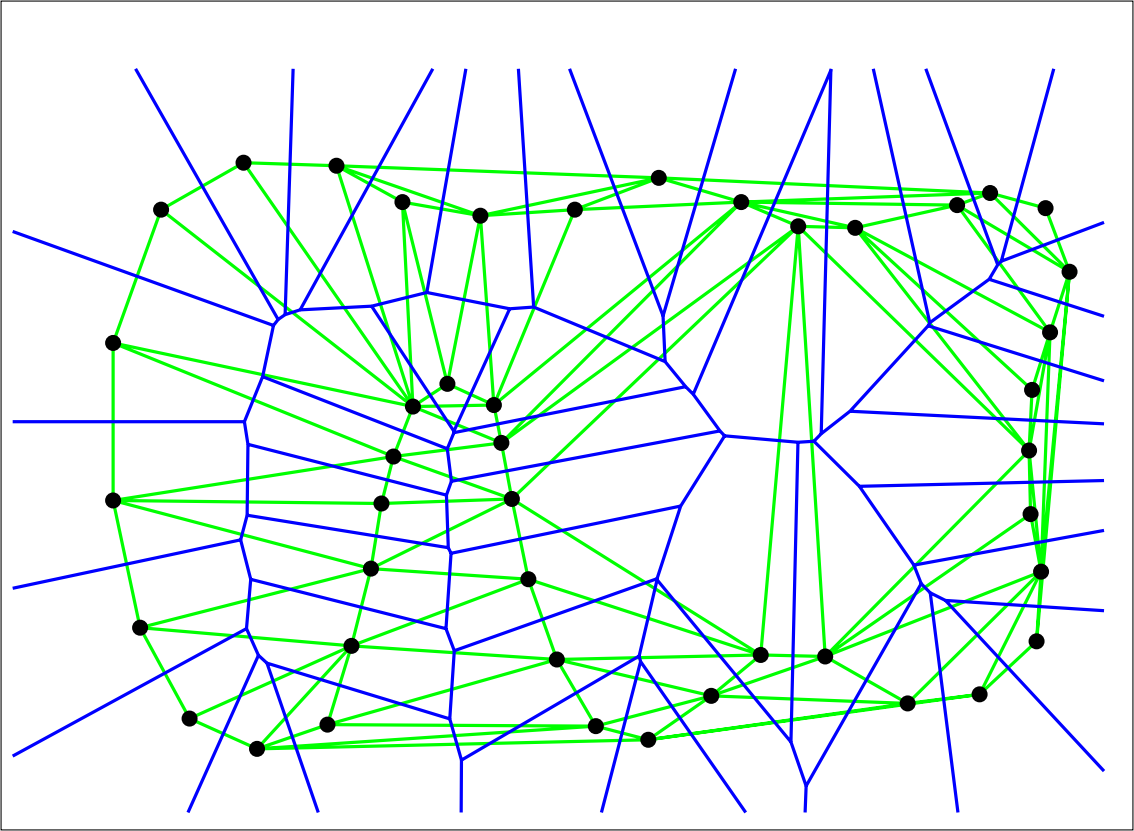


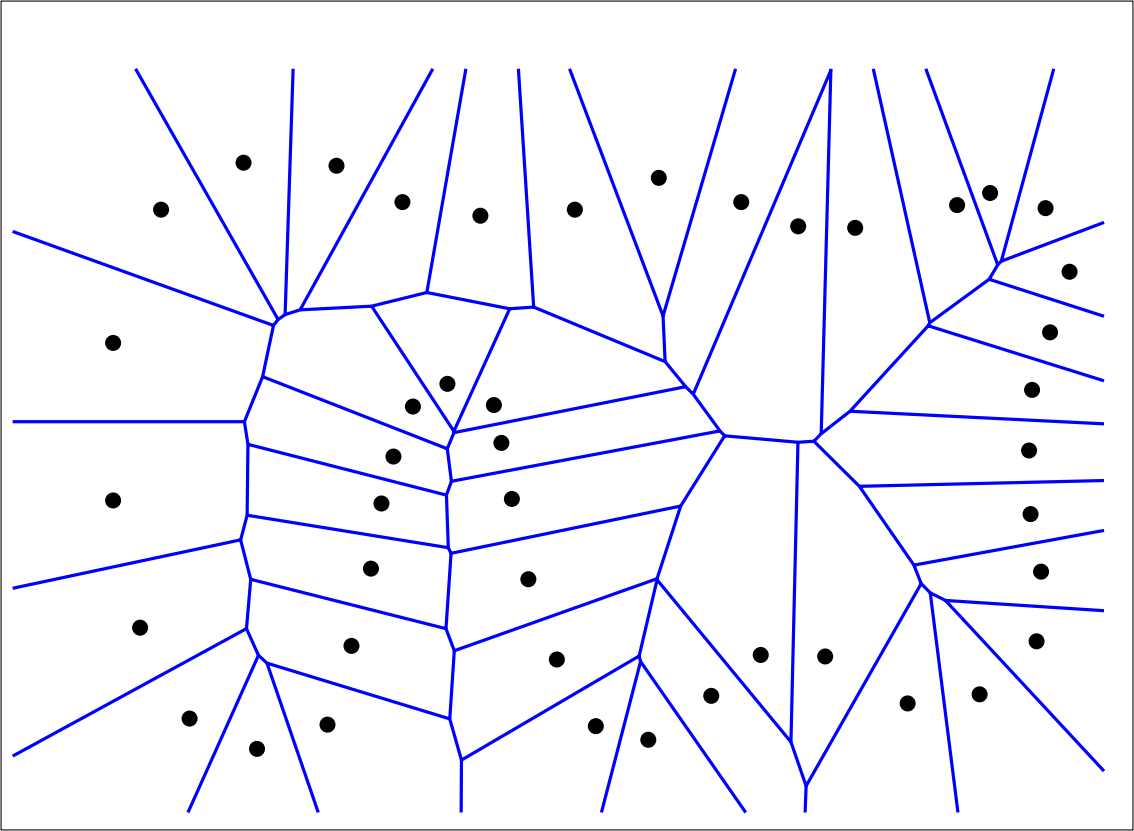
Crust

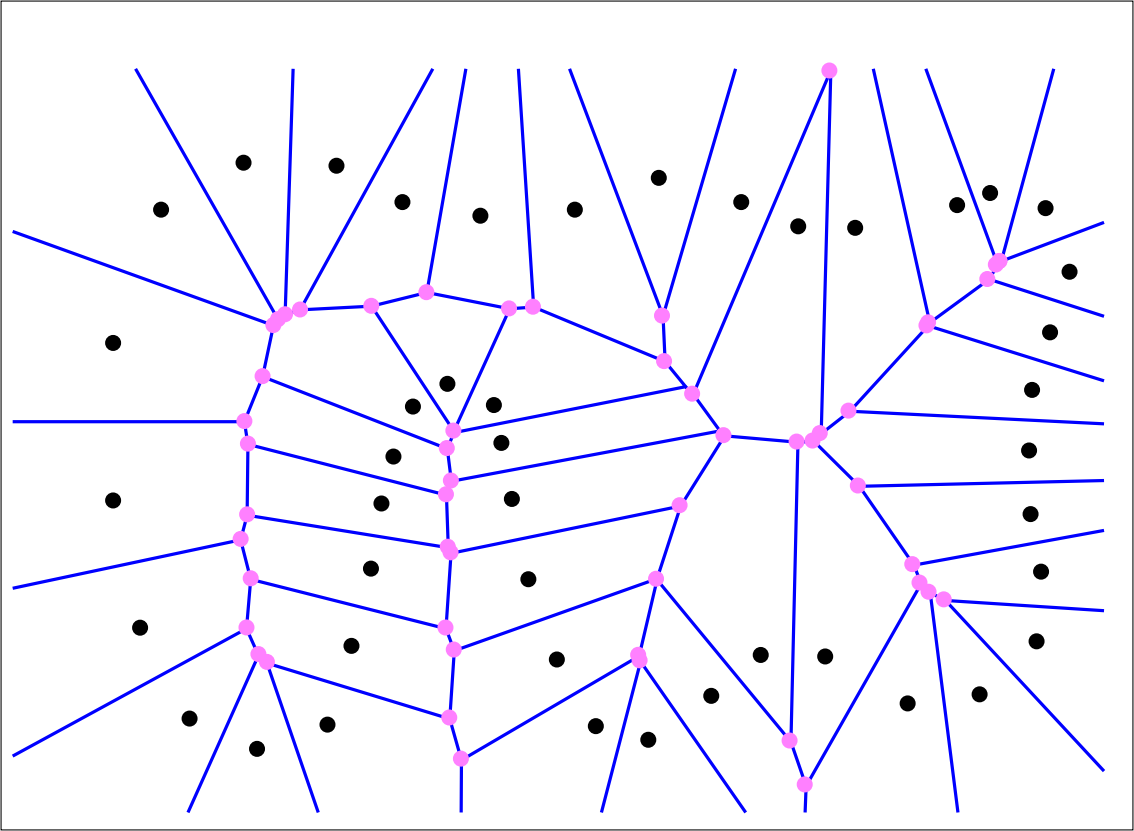


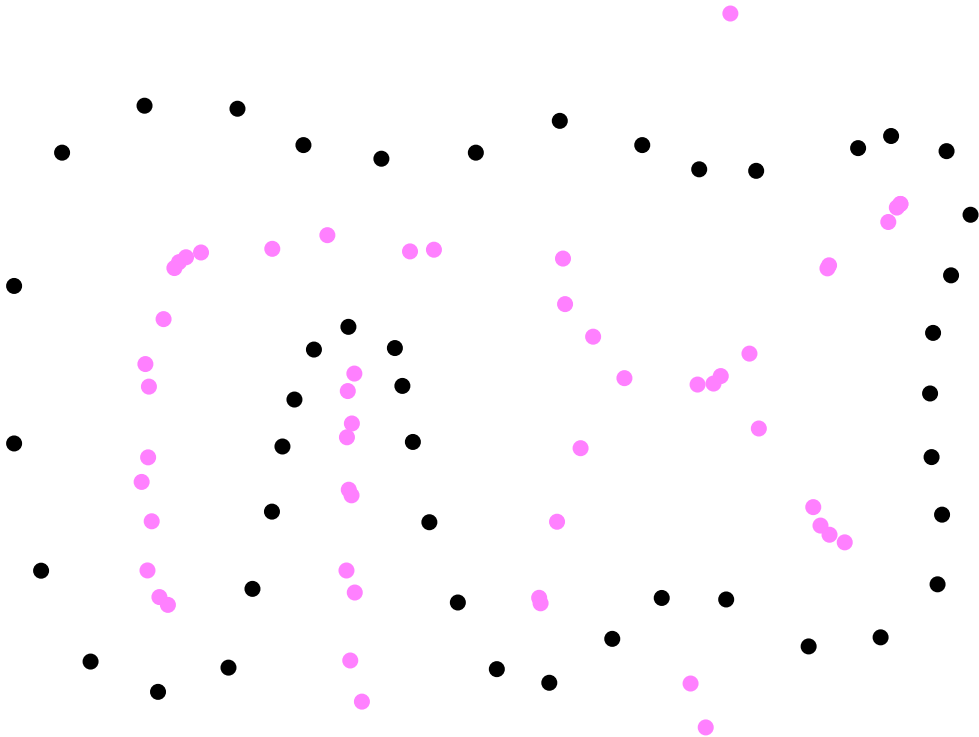


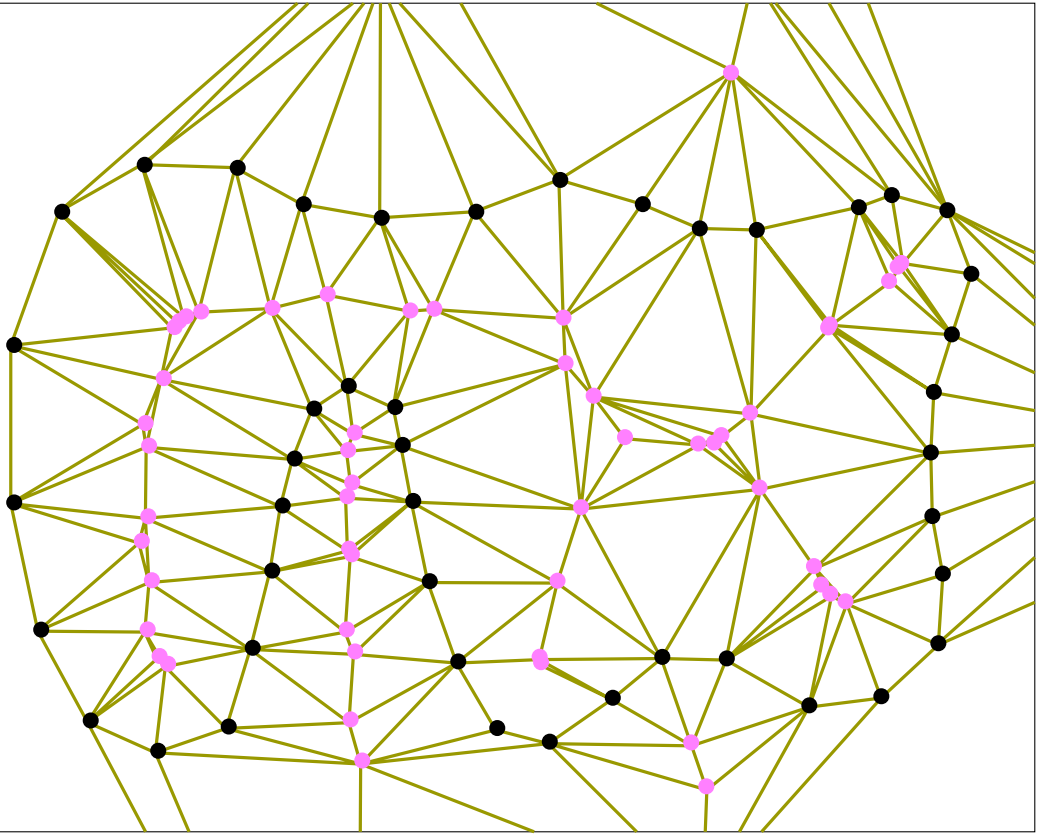


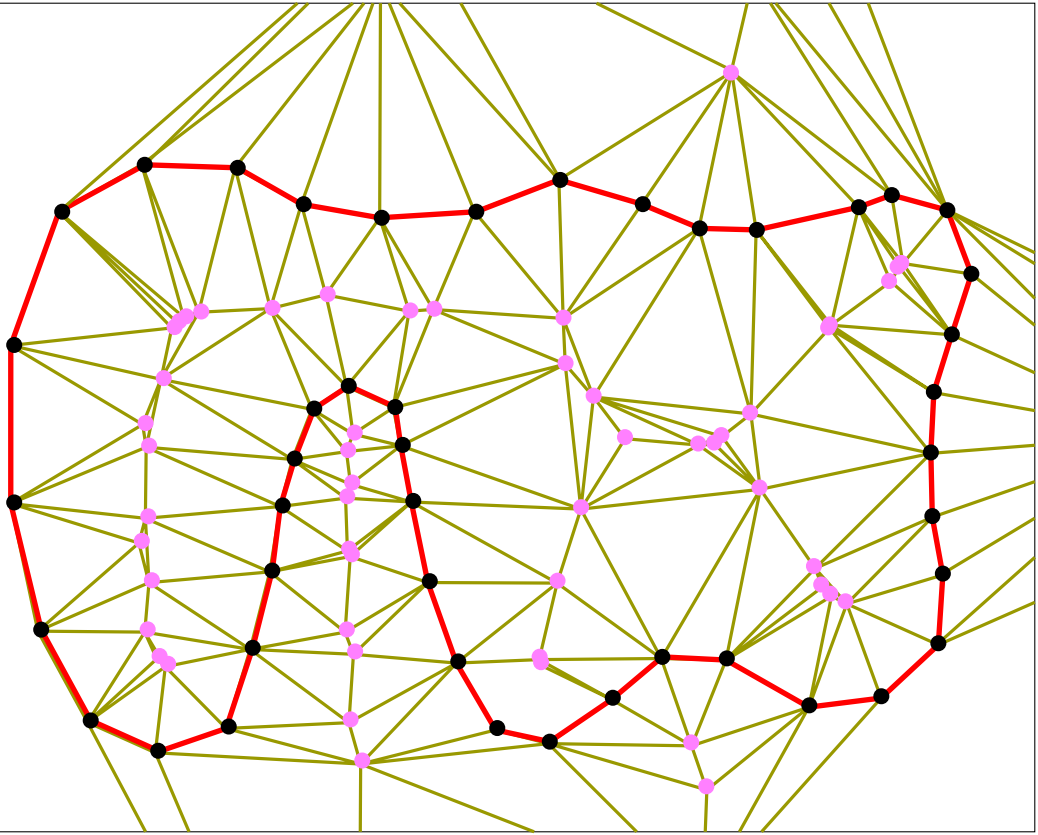


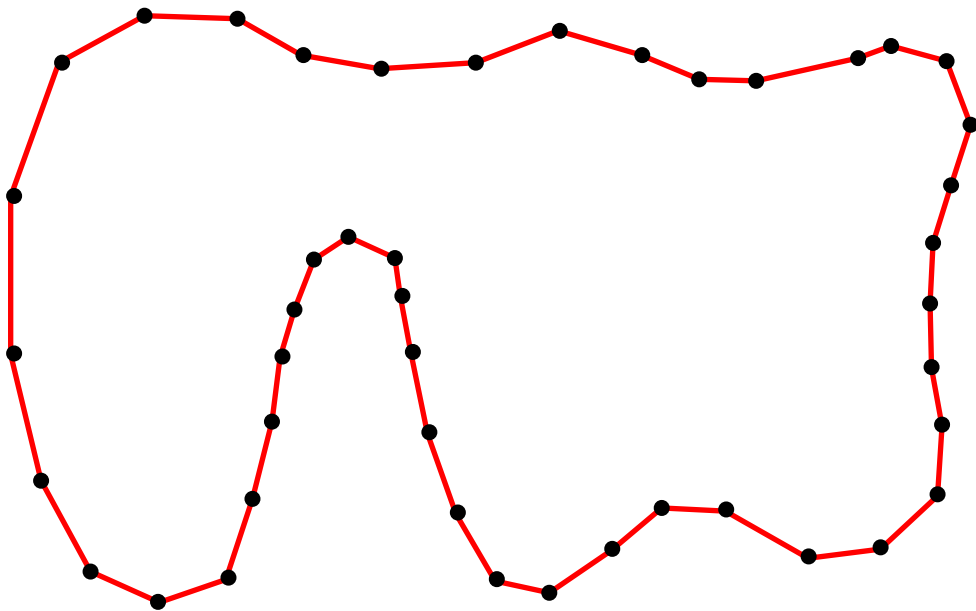


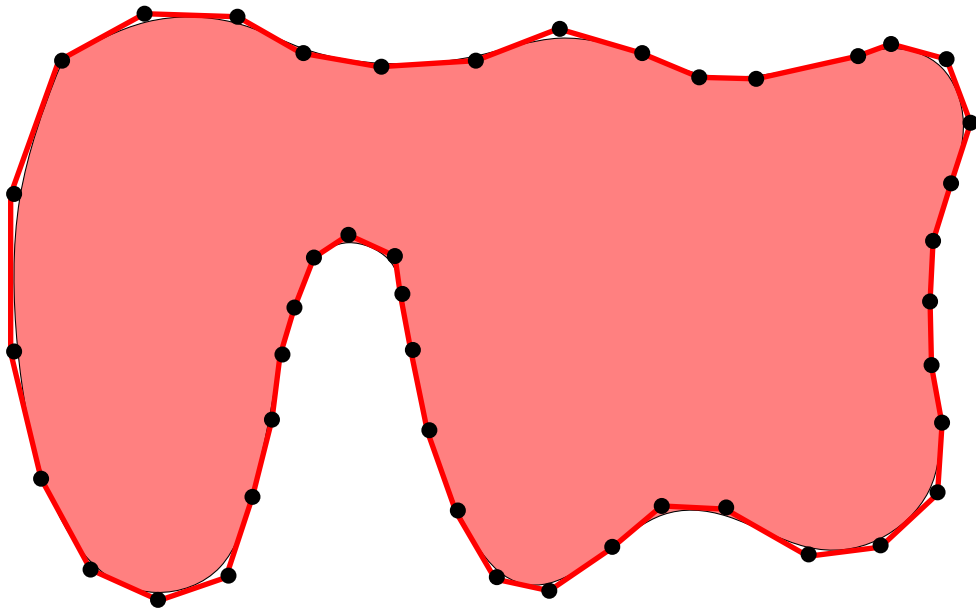




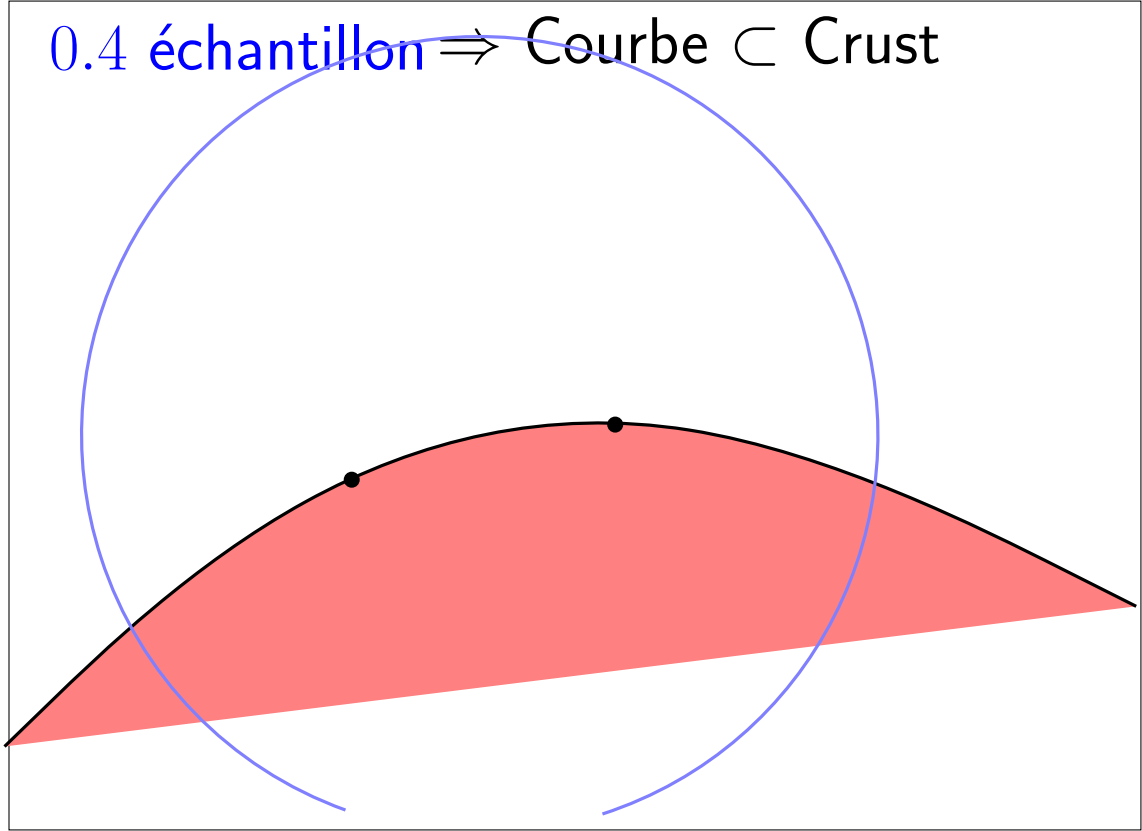






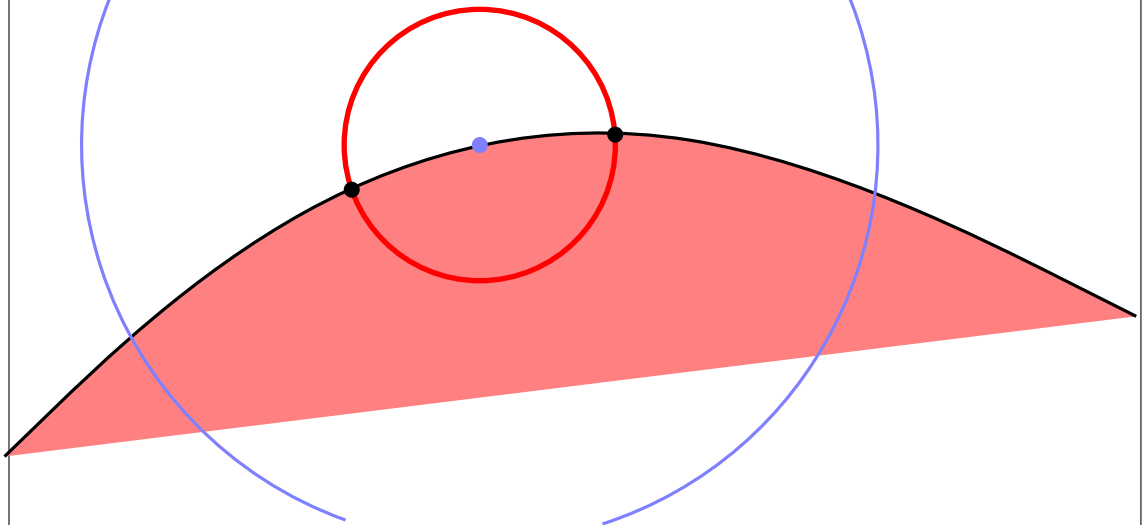


0.4 échantillon \Rightarrow Courbe \subset Crust



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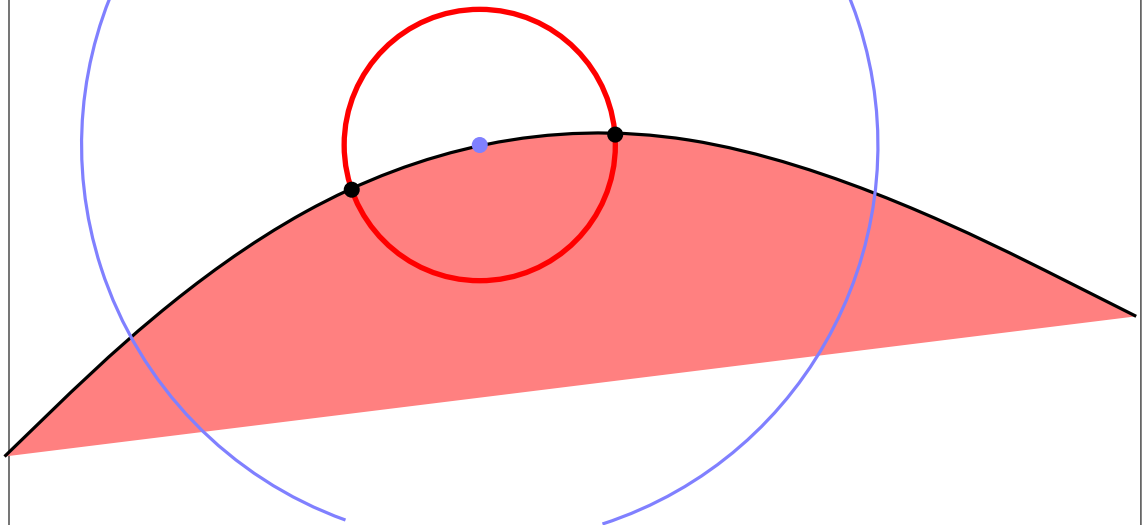
disque centré sur le bord
passant par échantillon



0.4 échantillon \Rightarrow Courbe \subset Crust

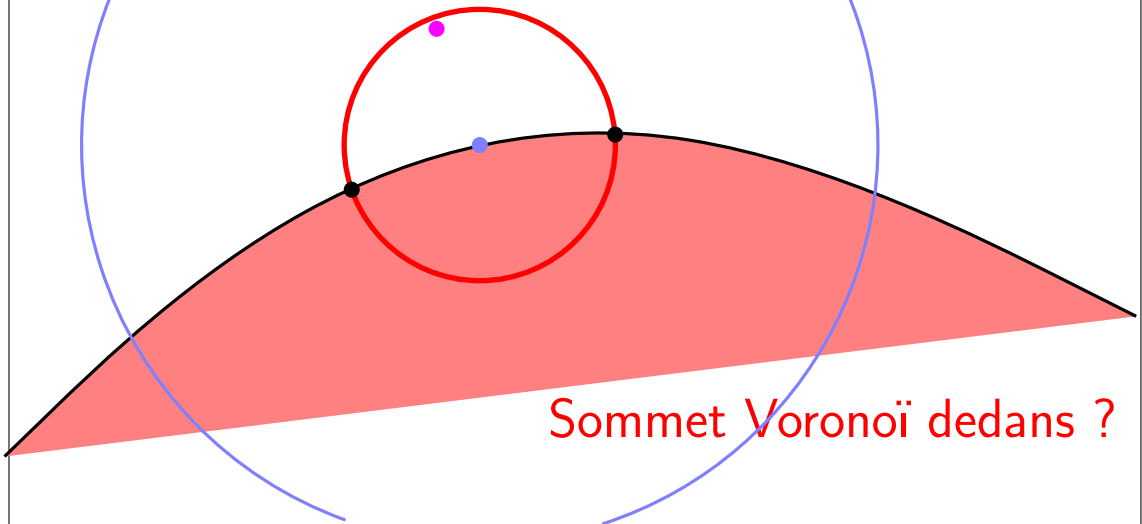
disque centré sur le bord
passant par échantillon

rayon l_{fs}



0.4 échantillon \Rightarrow Courbe \subset Crust

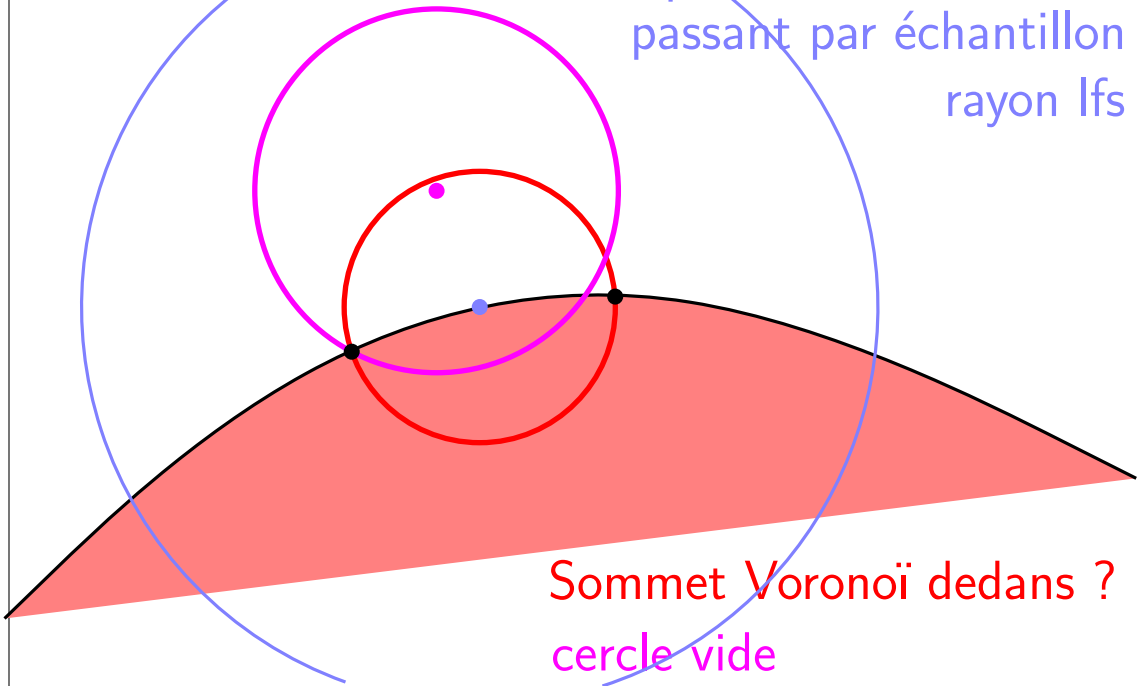
disque centré sur le bord
passant par échantillon
rayon l_{fs}



Sommet Voronoï dedans ?

0.4 échantillon \Rightarrow Courbe \subset Crust

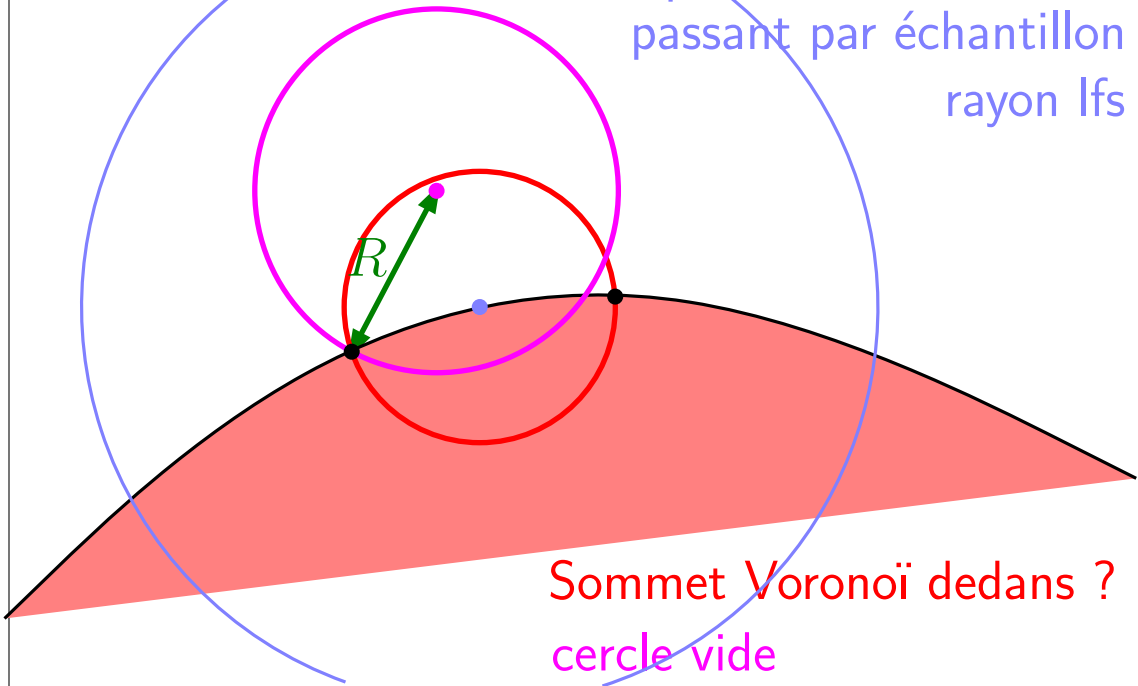
disque centré sur le bord
passant par échantillon
rayon l_{fs}



Sommet Voronoï dedans ?
cercle vide

0.4 échantillon \Rightarrow Courbe \subset Crust

disque centré sur le bord
passant par échantillon
rayon lfs



Sommet Voronoï dedans ?
cercle vide

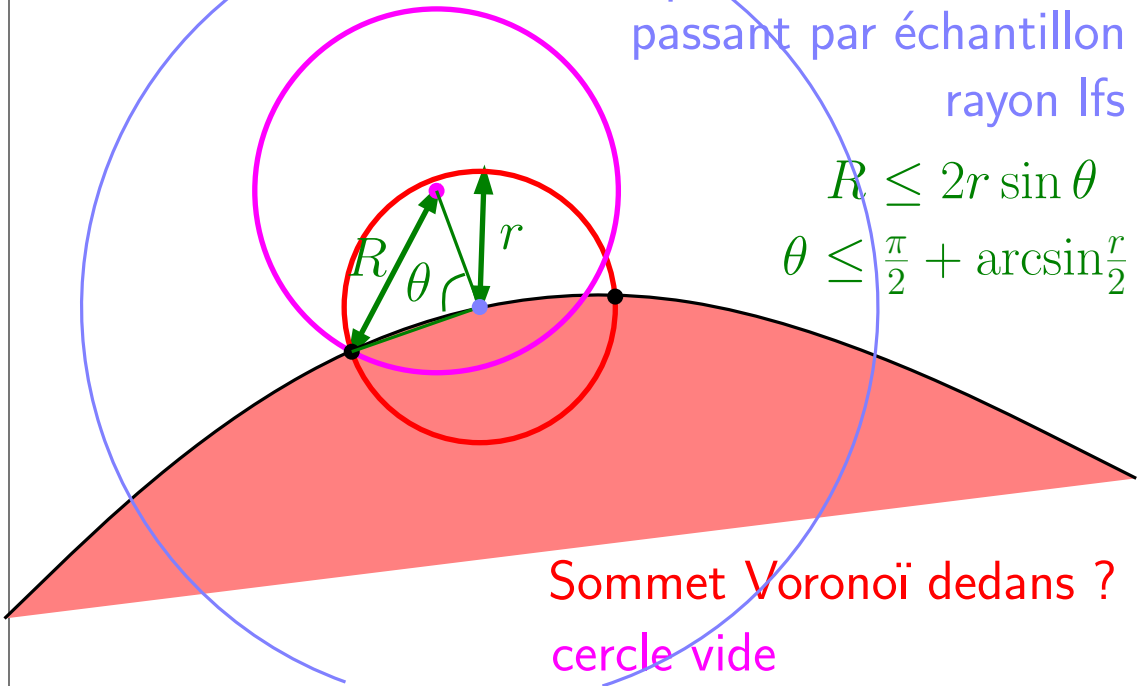
0.4 échantillon \Rightarrow Courbe \subset Crust

disque centré sur le bord
passant par échantillon

rayon lfs

$$R \leq 2r \sin \theta$$

$$\theta \leq \frac{\pi}{2} + \arcsin \frac{r}{2}$$

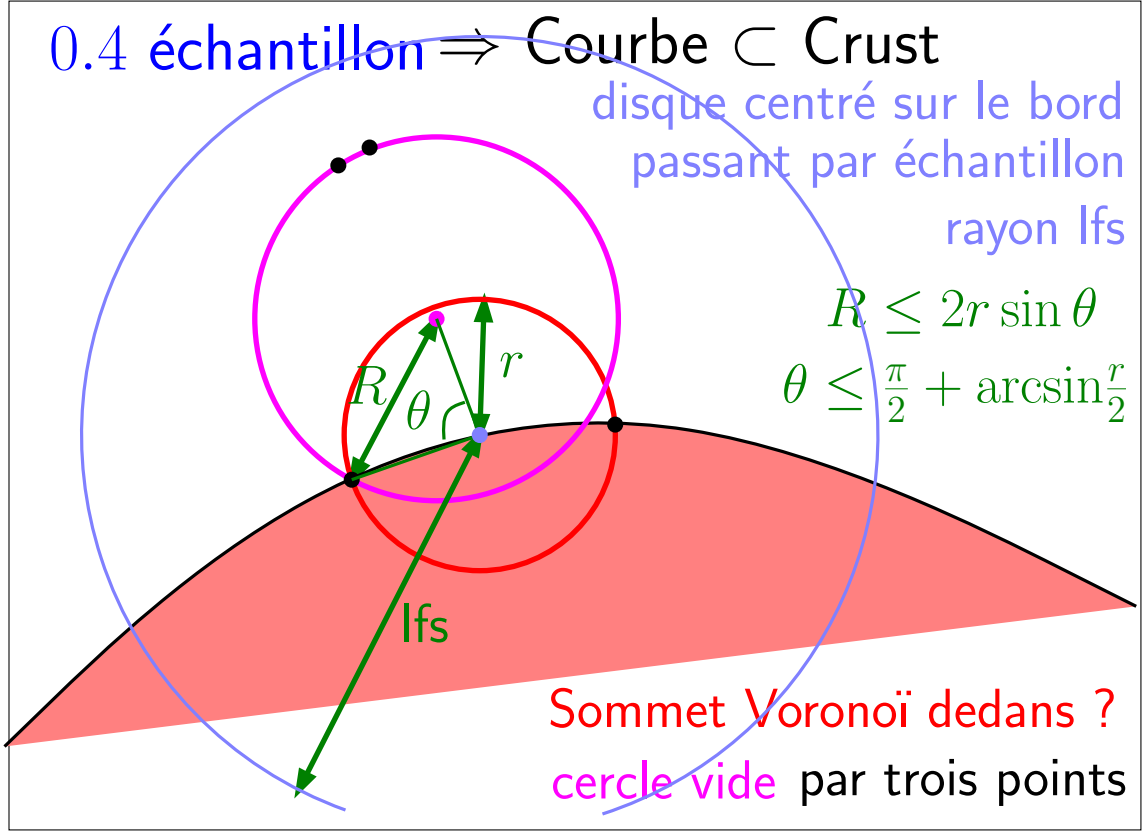


Sommet Voronoï dedans ?
cercle vide

0.4 échantillon \Rightarrow Courbe \subset Crust

disque centré sur le bord
passant par échantillon
rayon lfs

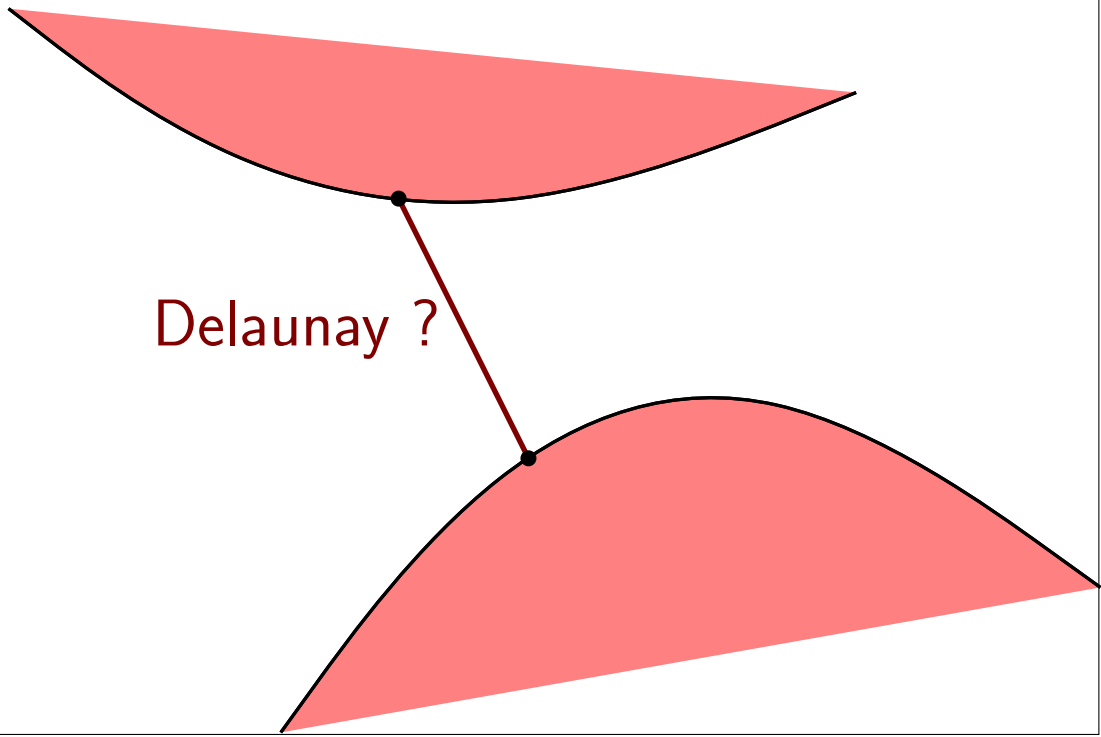
$$R \leq 2r \sin \theta$$
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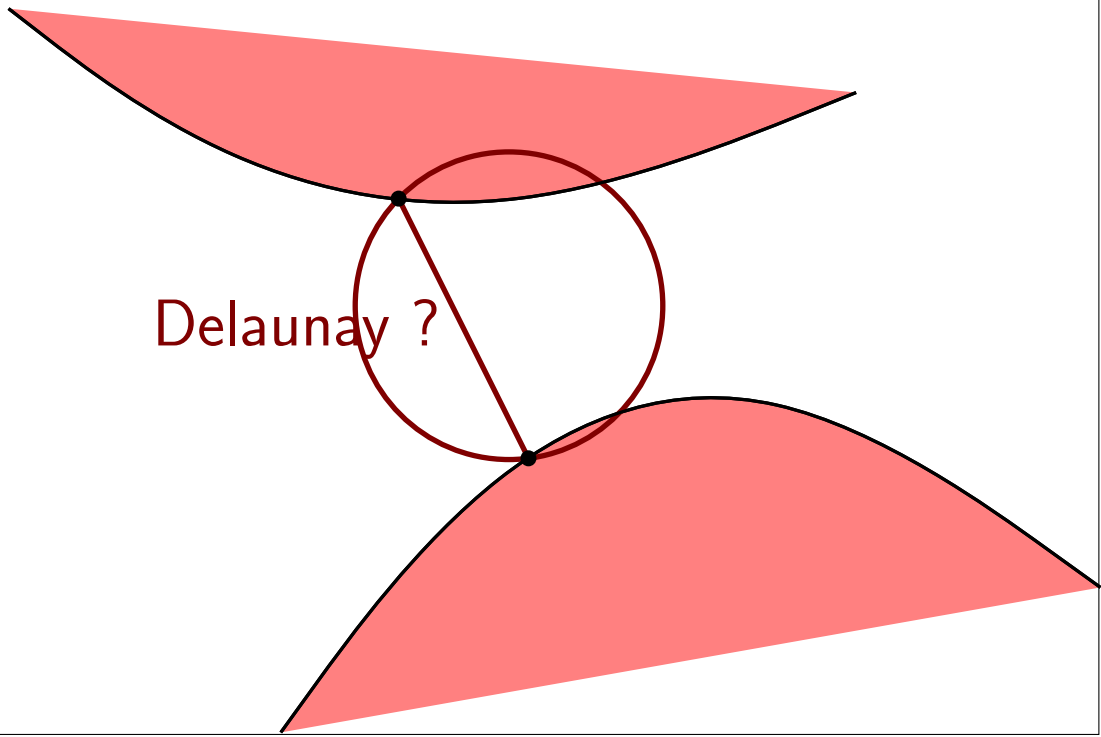
Sommet Voronoï dedans ?
cercle vide par trois points

0.25 échantillon \Rightarrow Courbe = Crust

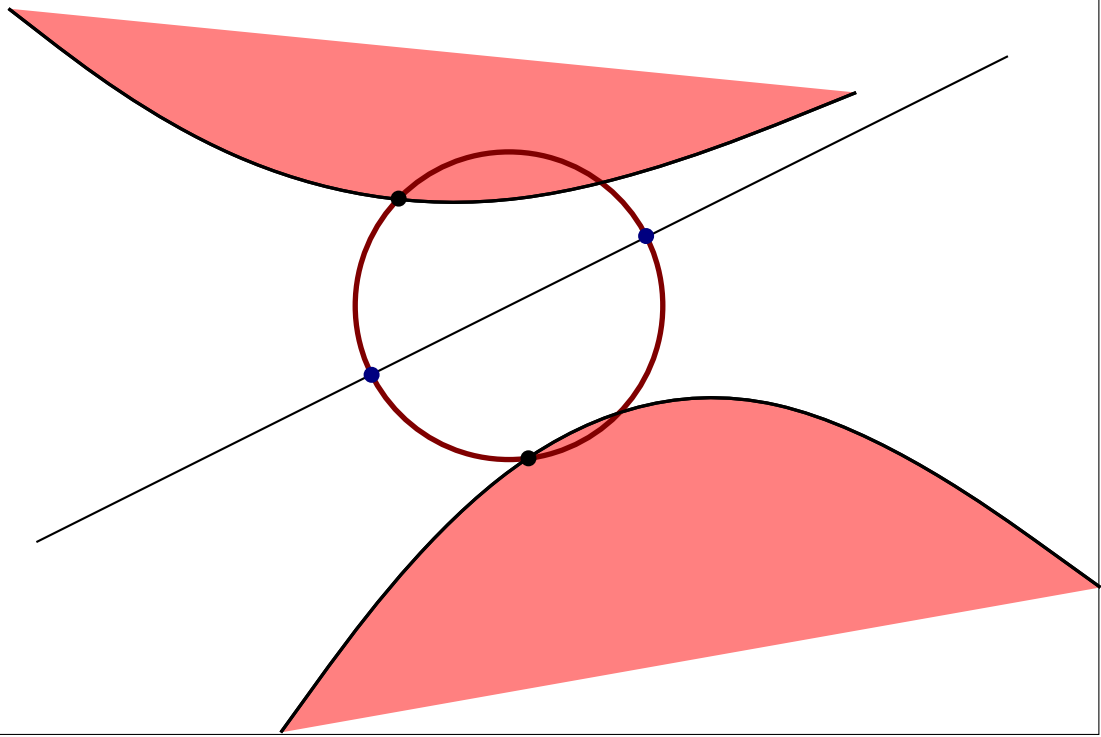
0.25 échantillon \Rightarrow Courbe = Crust



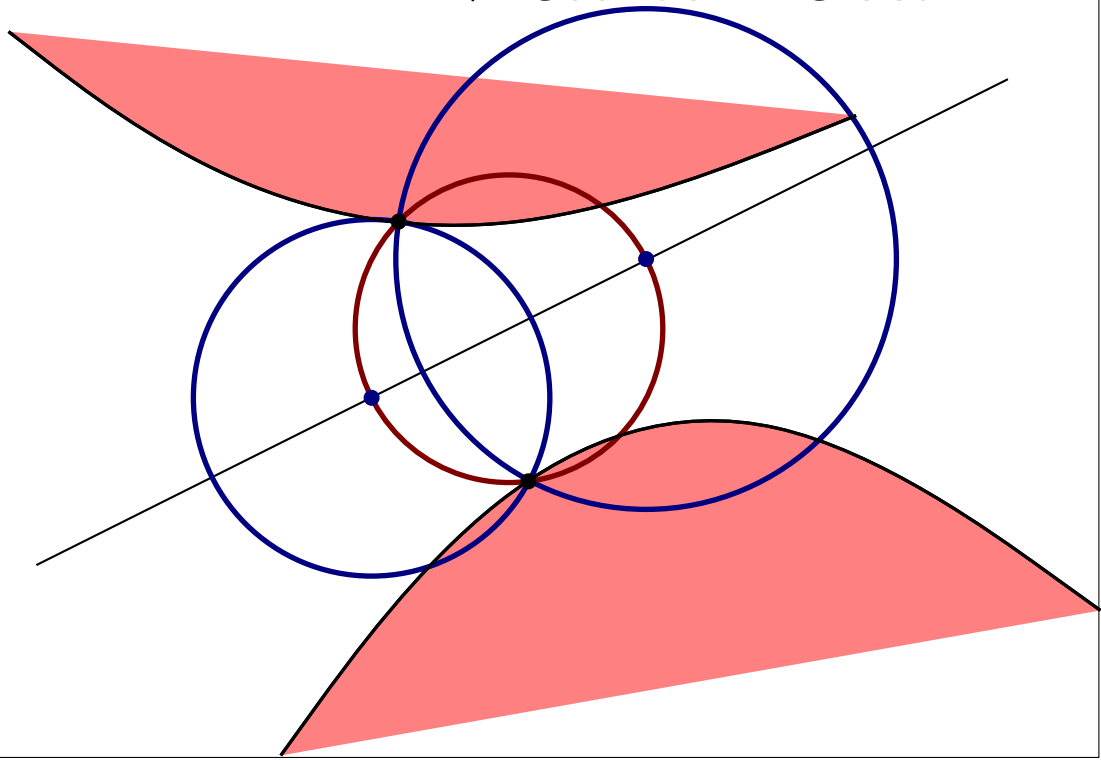
0.25 échantillon \Rightarrow Courbe = Crust



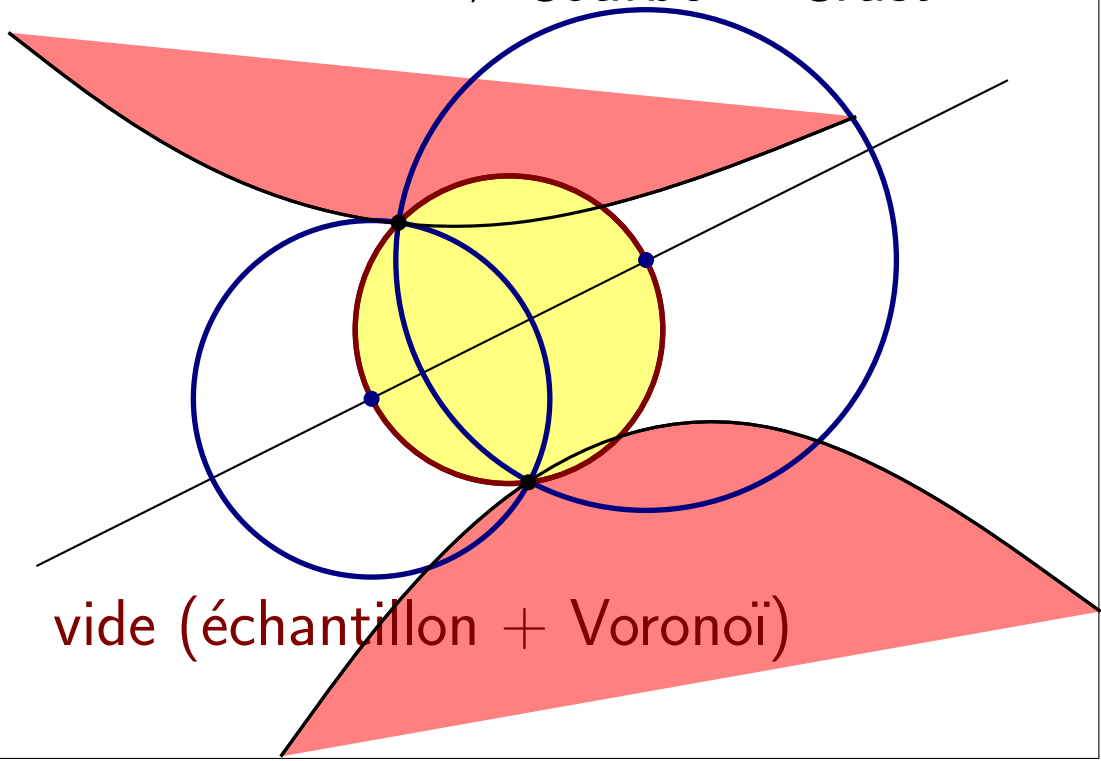
0.25 échantillon \Rightarrow Courbe = Crust



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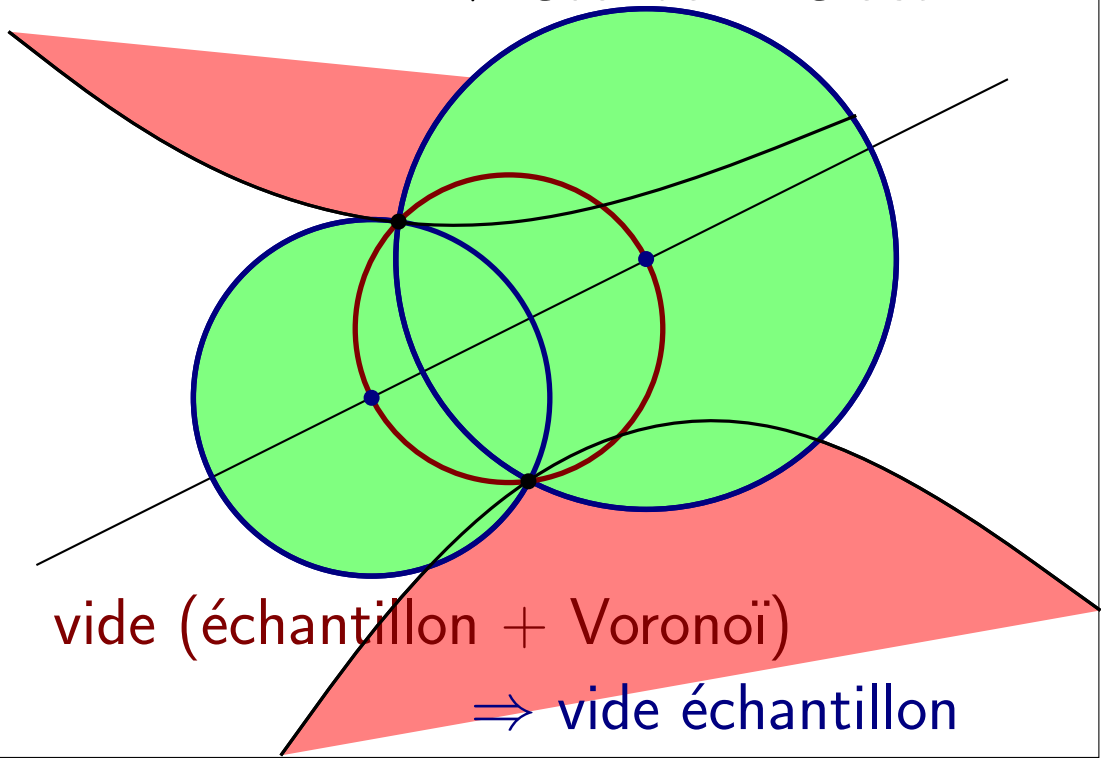


0.25 échantillon \Rightarrow Courbe = Crust



vide (échantillon + Voronoï)

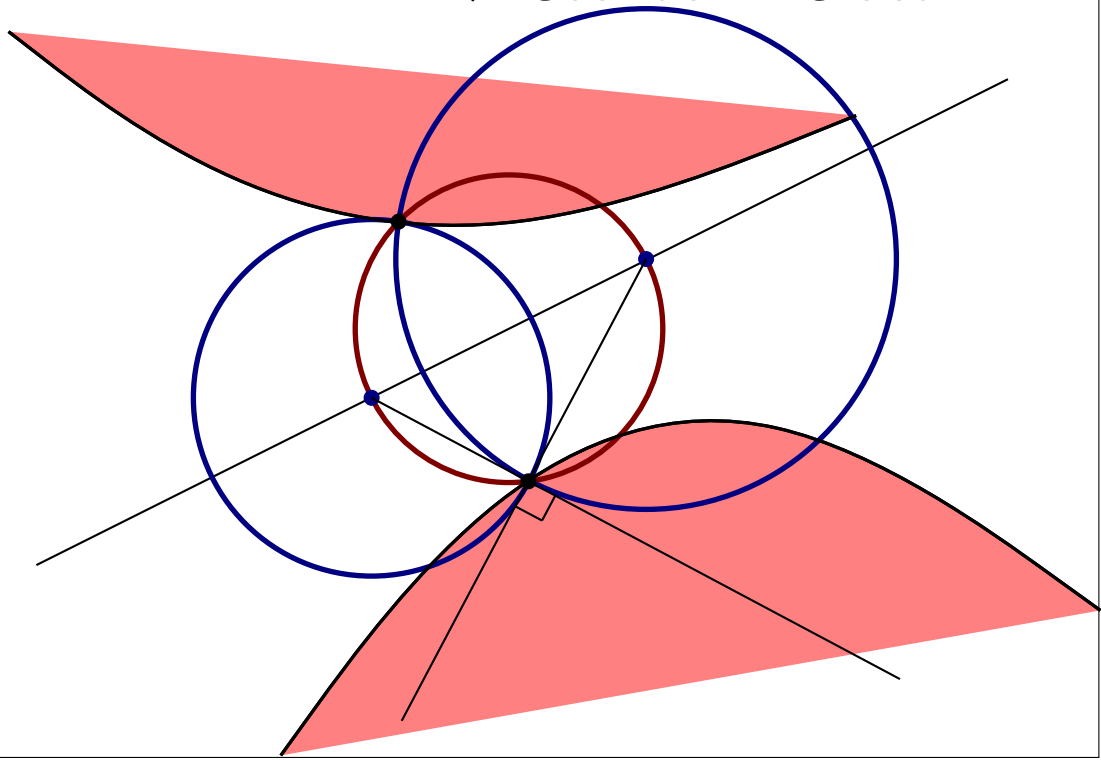
0.25 échantillon \Rightarrow Courbe = Crust



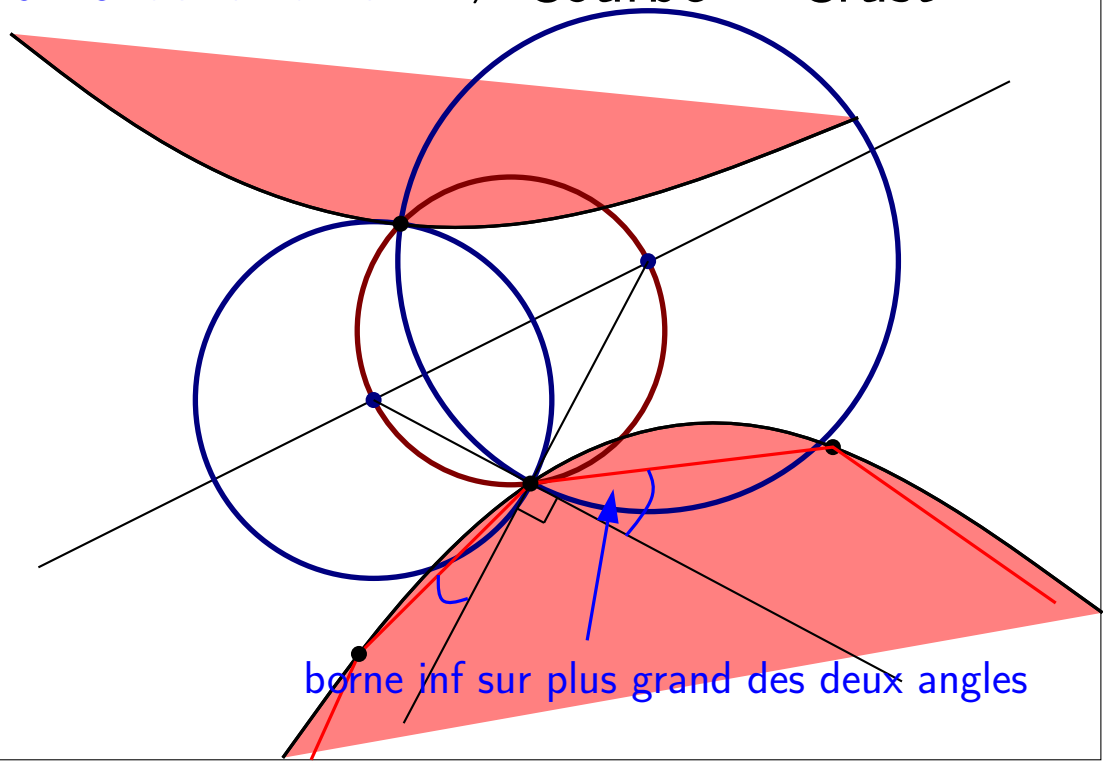
vide (échantillon + Voronoï)

\Rightarrow vide échantillon

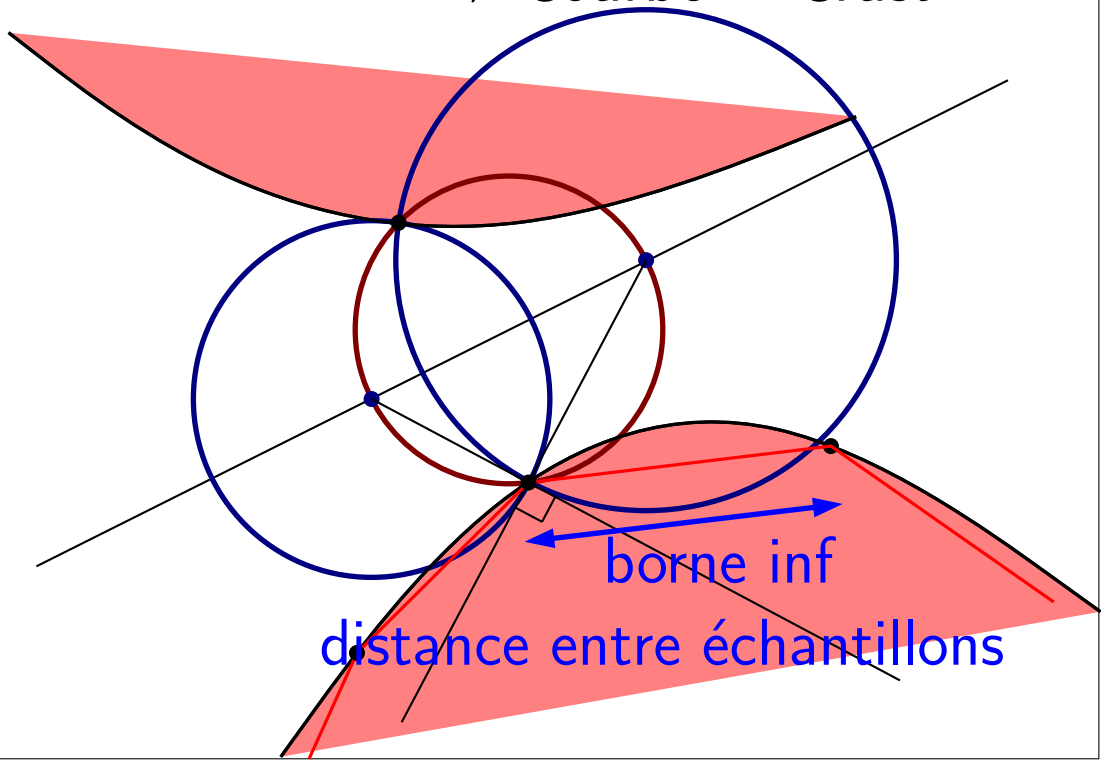
0.25 échantillon \Rightarrow Courbe = Crust



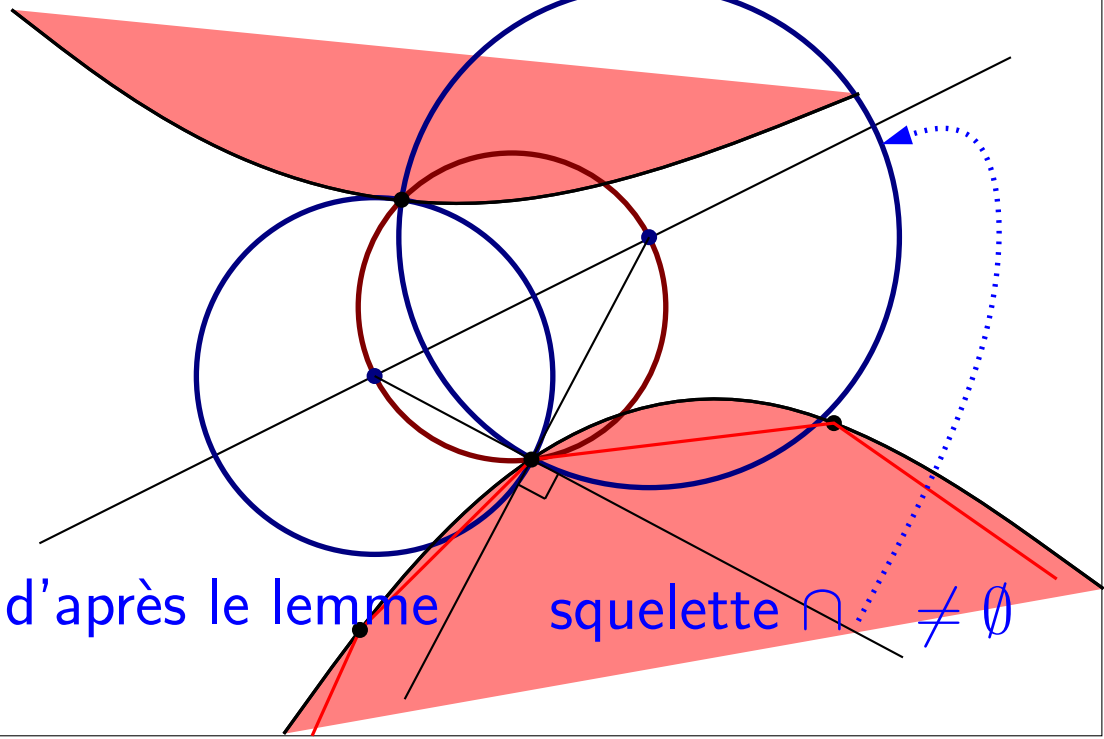
0.25 échantillon \Rightarrow Courbe = Crust



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d'après le lemme

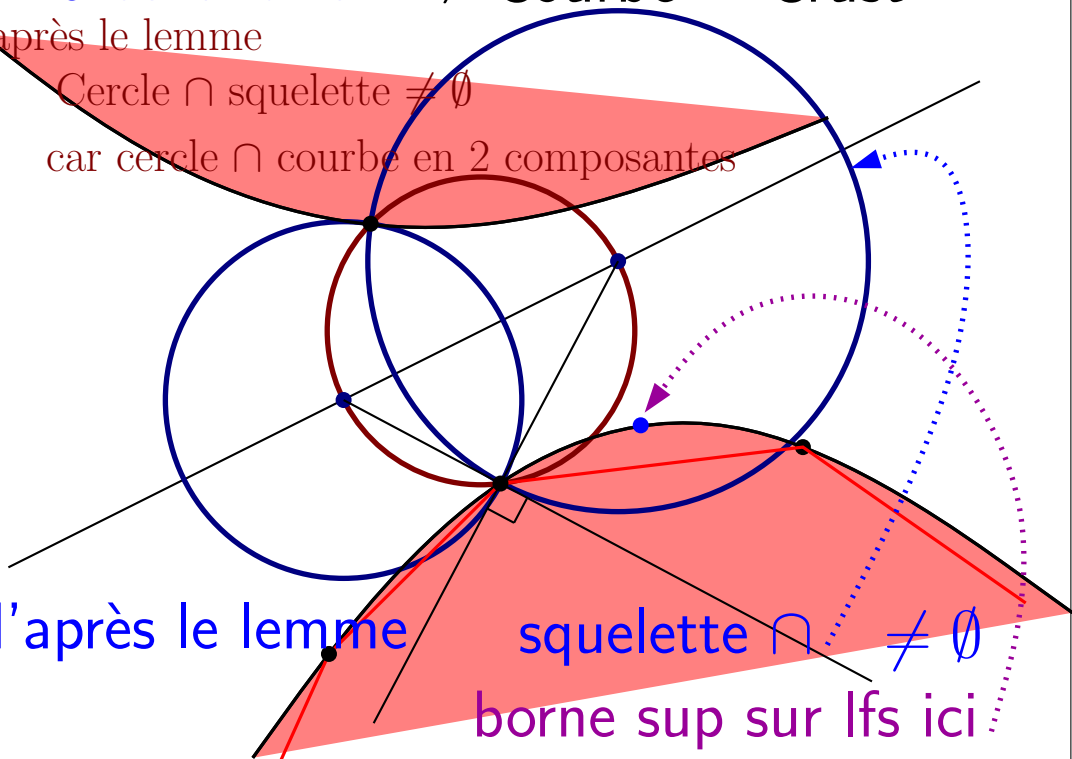
Cercle \cap squelette $\neq \emptyset$

car cercle \cap courbe en 2 composantes

d'après le lemme

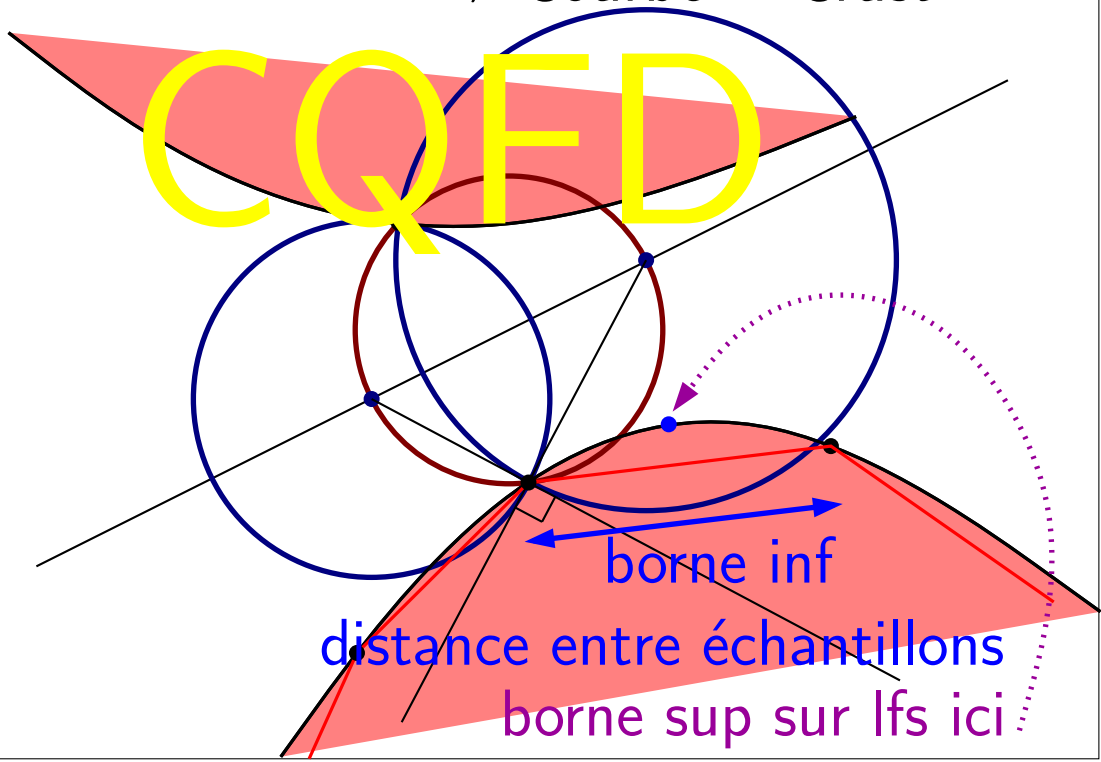
squelette \cap $\neq \emptyset$

borne sup sur lfs ici



0.25 échantillon \Rightarrow Courbe = Crust

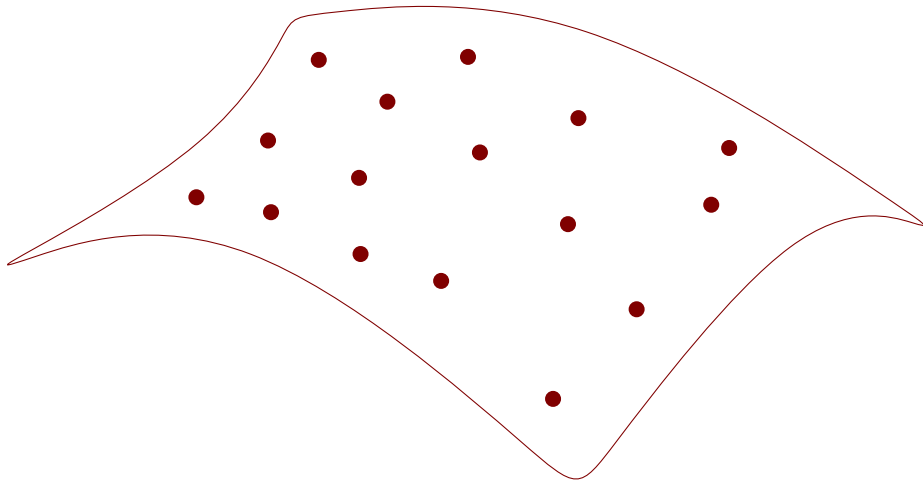
COQFD



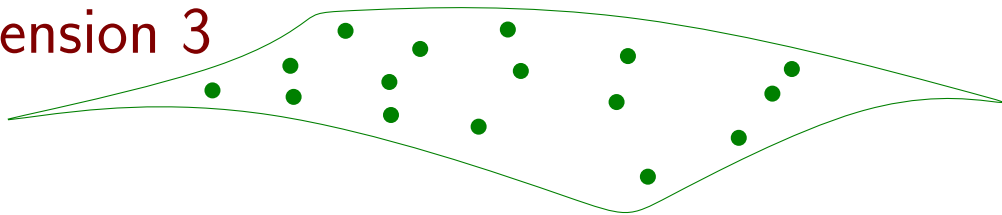
distance entre échantillons

borne sup sur lfs ici

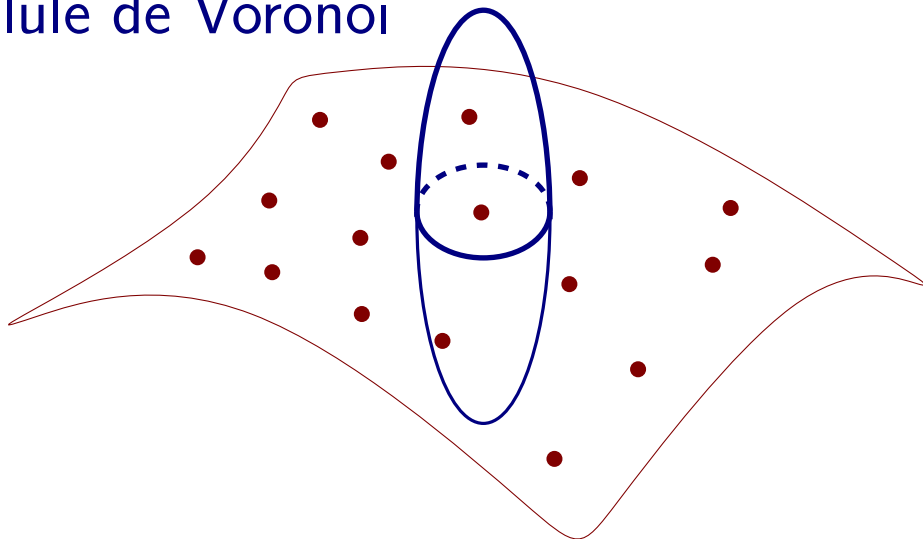
Dimension 3



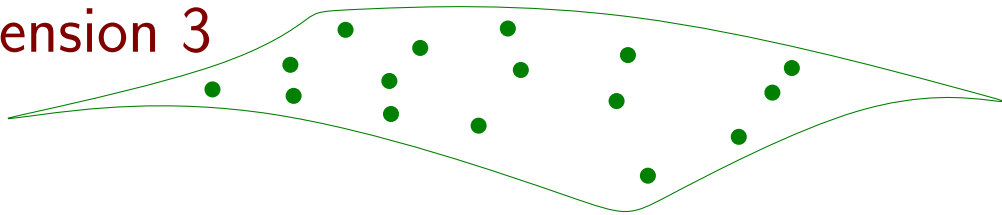
Dimension 3



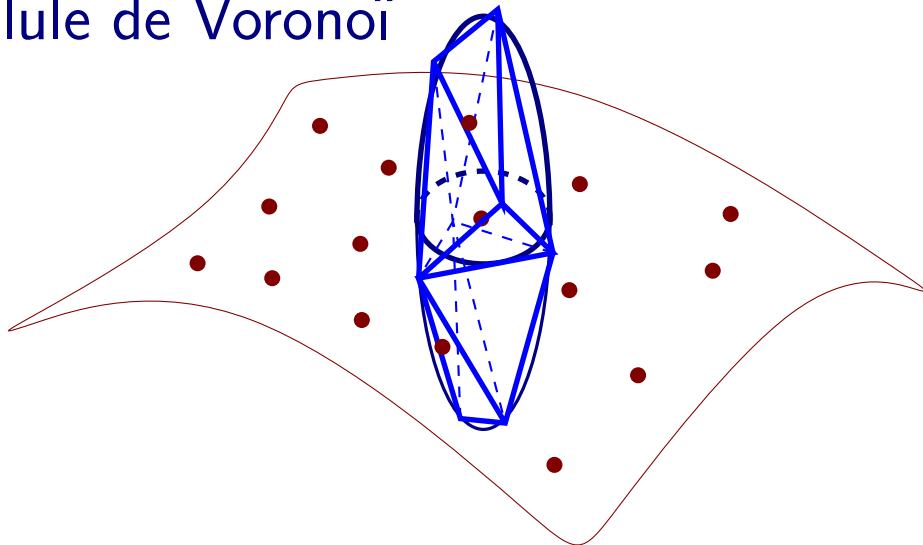
cellule de Voronoï



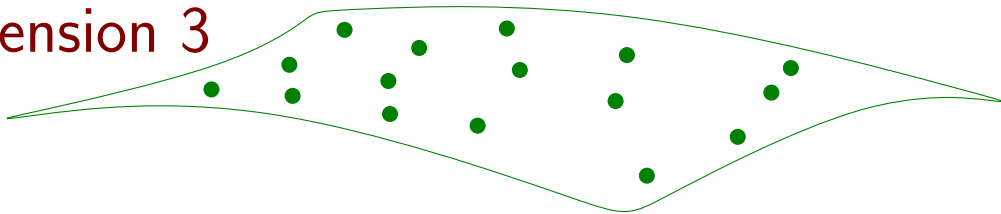
Dimension 3



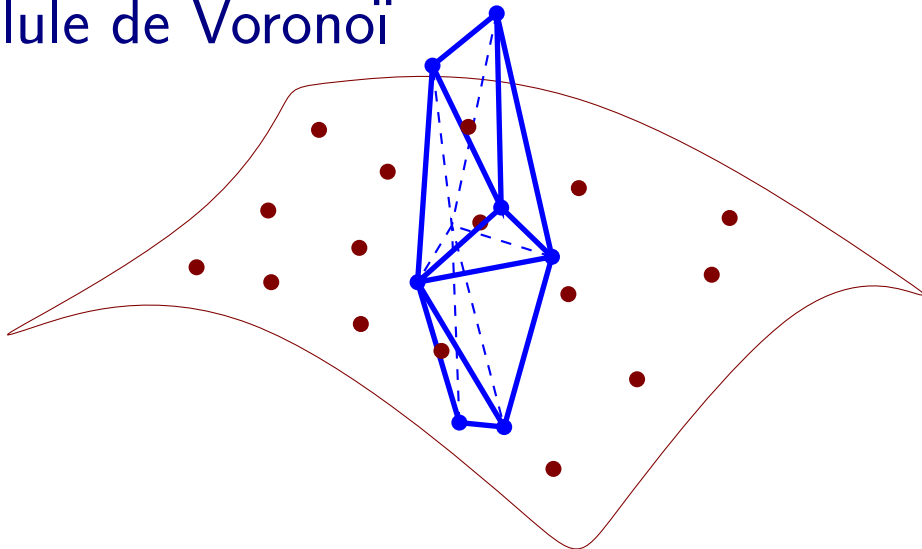
cellule de Voronoï



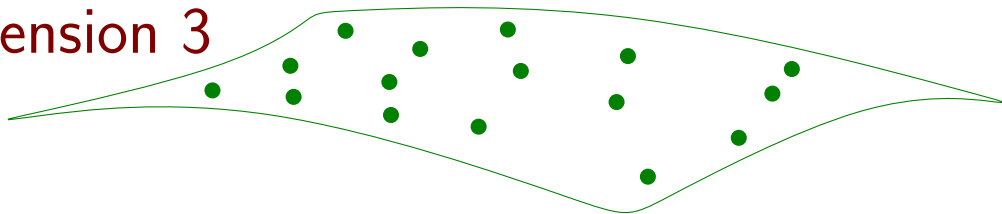
Dimension 3



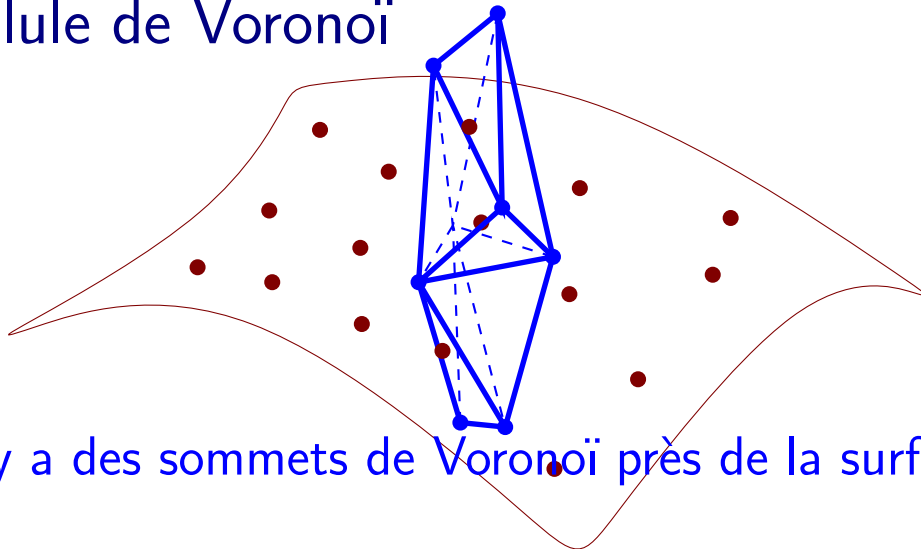
cellule de Voronoï



Dimension 3

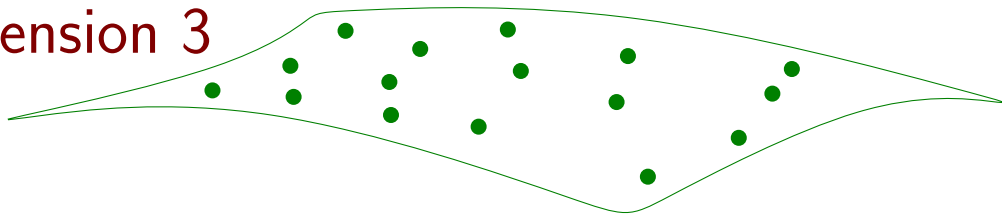


cellule de Voronoï

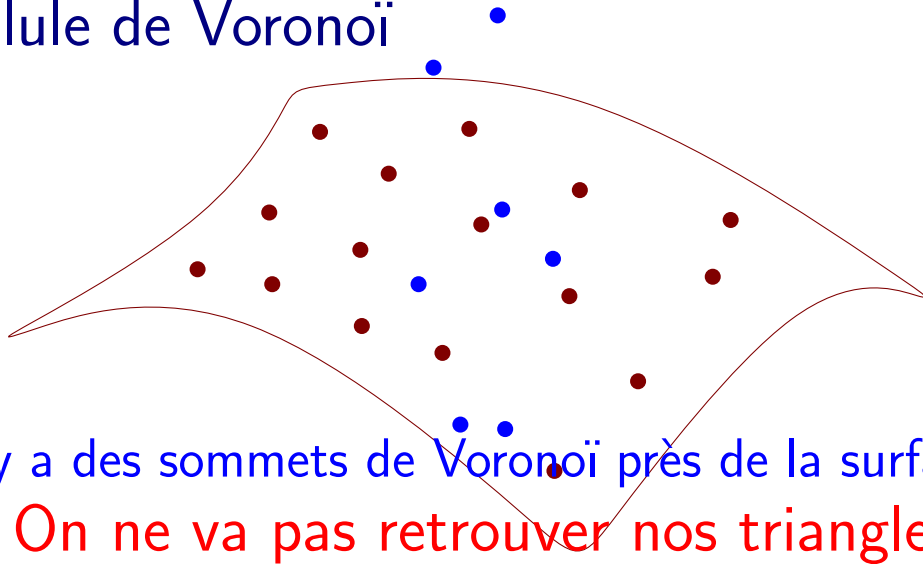


il y a des sommets de Voronoï près de la surface

Dimension 3



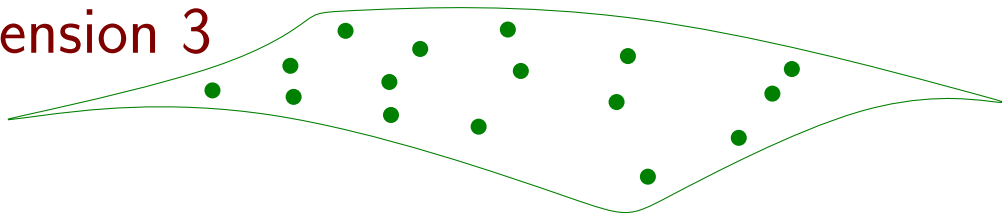
cellule de Voronoï



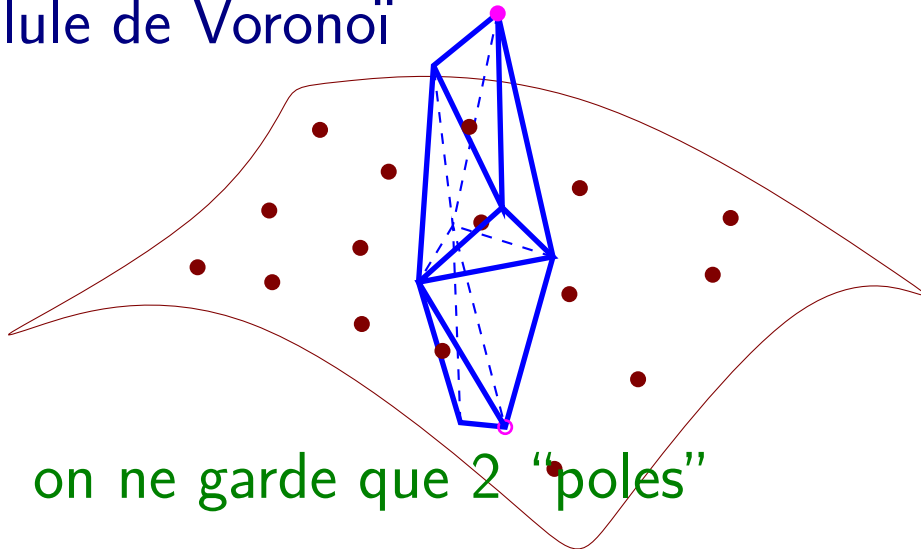
il y a des sommets de Voronoï près de la surface

On ne va pas retrouver nos triangles

Dimension 3

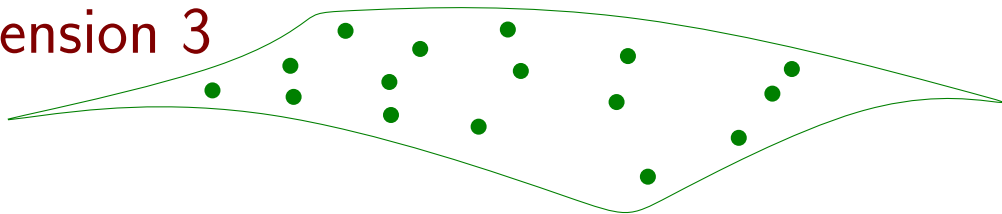


cellule de Voronoï

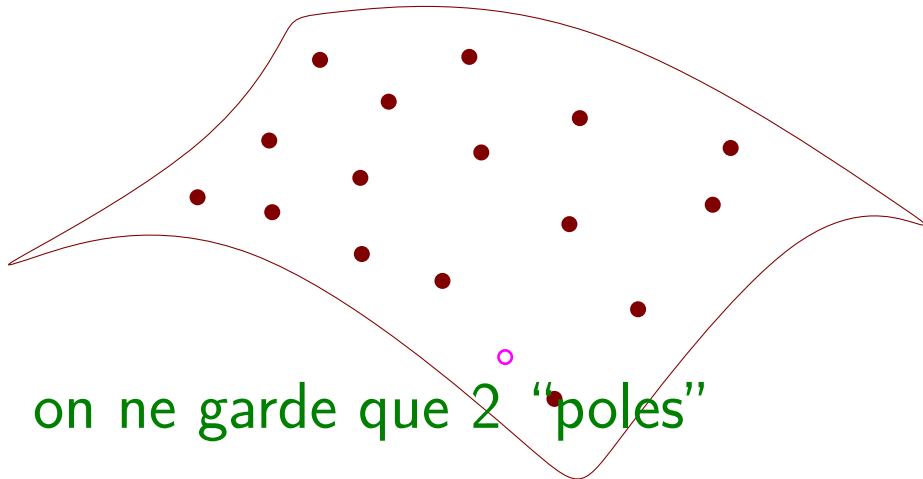


on ne garde que 2 "poles"

Dimension 3

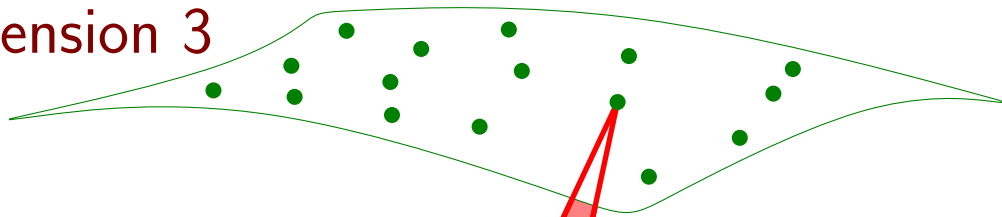


cellule de Voronoï

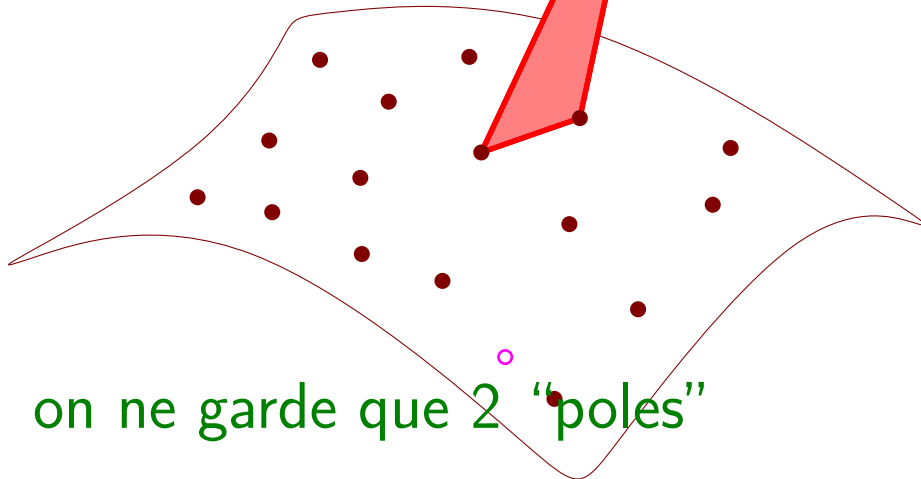


on ne garde que 2 "poles"

Dimension 3

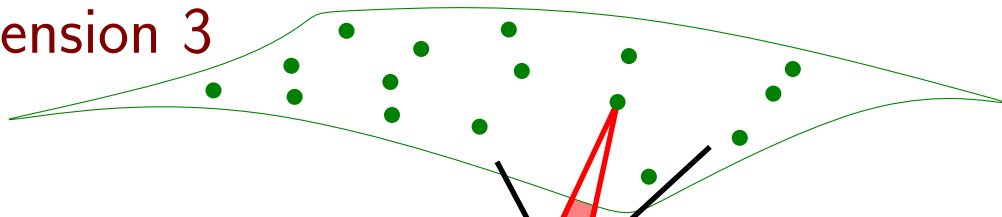


cellule de Voronoï

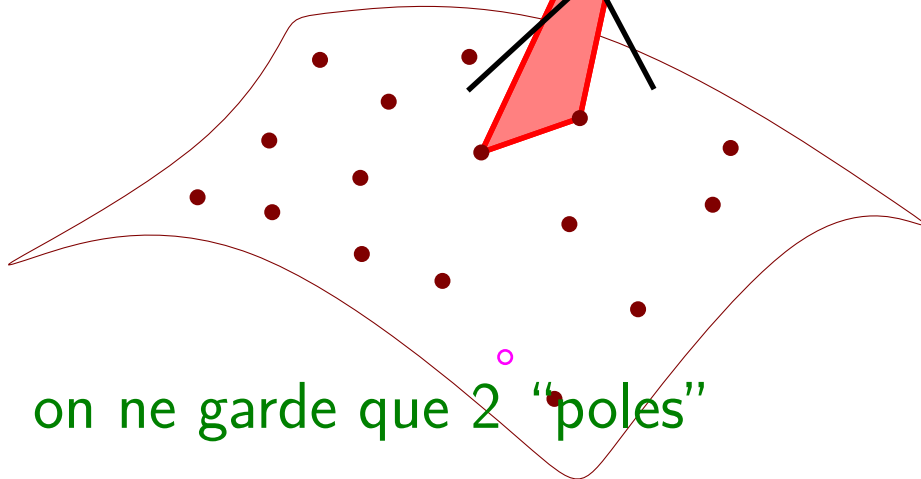


on ne garde que 2 "poles"

Dimension 3

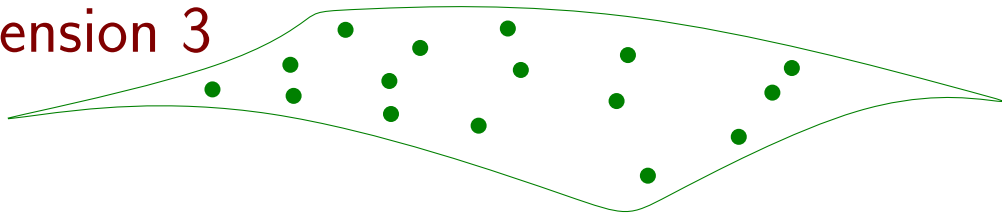


cellule de Voronoï

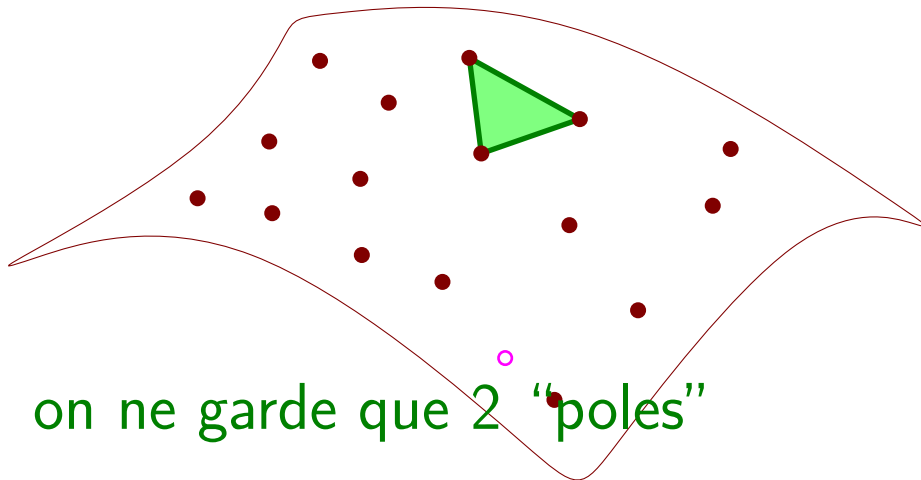


on ne garde que 2 "poles"

Dimension 3

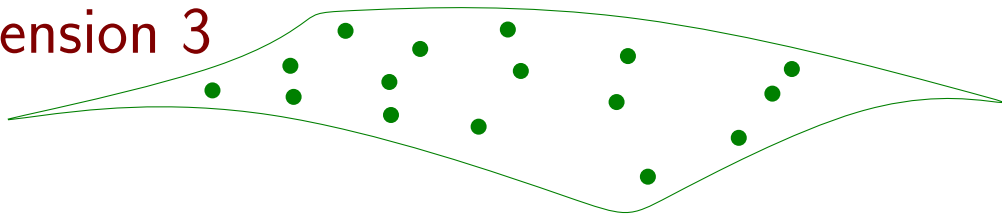


cellule de Voronoï

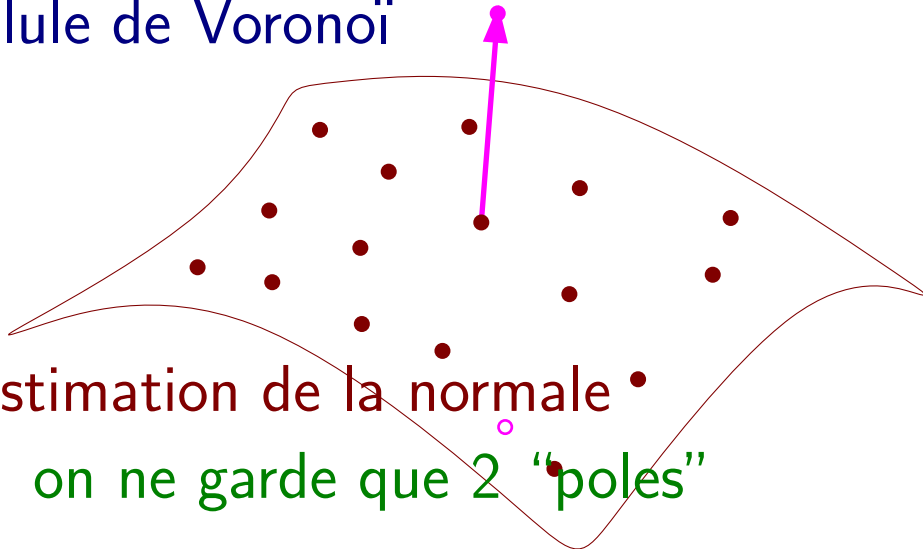


on ne garde que 2 "poles"

Dimension 3



cellule de Voronoï

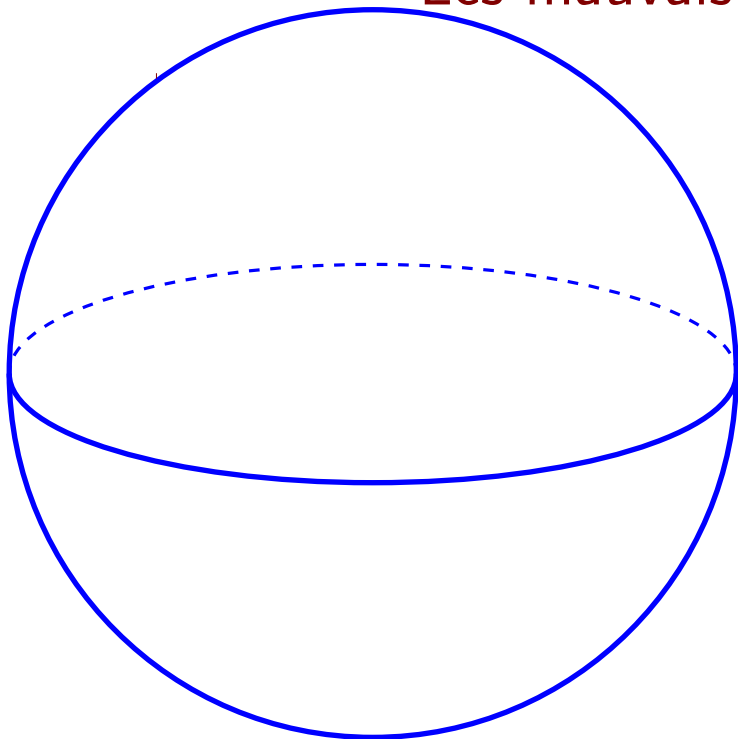


Estimation de la normale

on ne garde que 2 "poles"

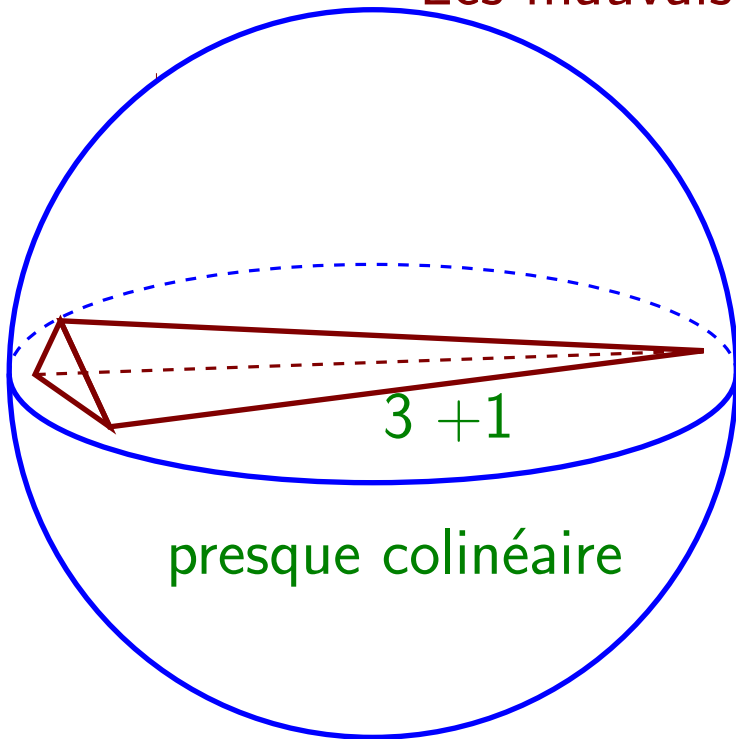
Dimension 3

Les mauvais tétraèdres



Dimension 3

Les mauvais tétraèdres

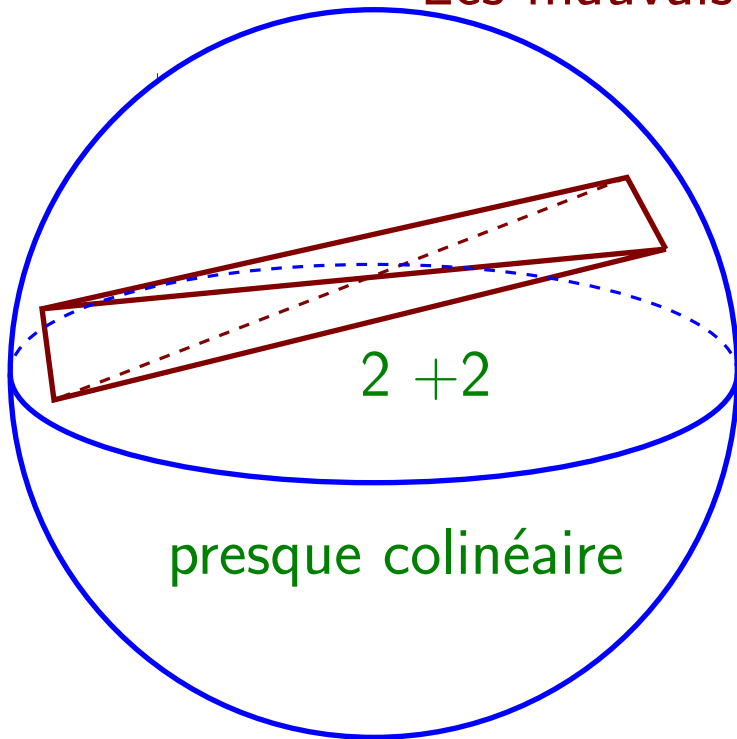


3 + 1

presque colinéaire

Dimension 3

Les mauvais tétraèdres

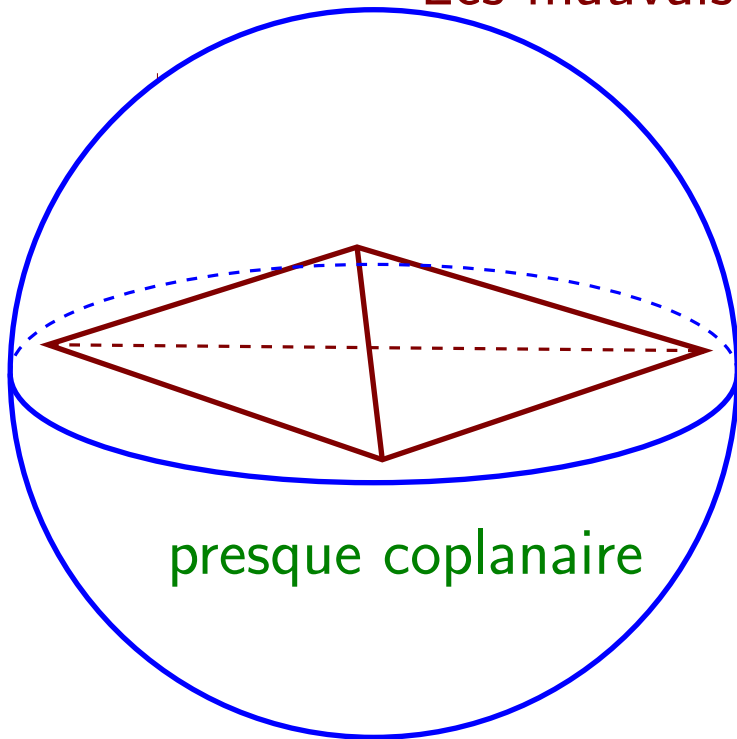


2 + 2

presque colinéaire

Dimension 3

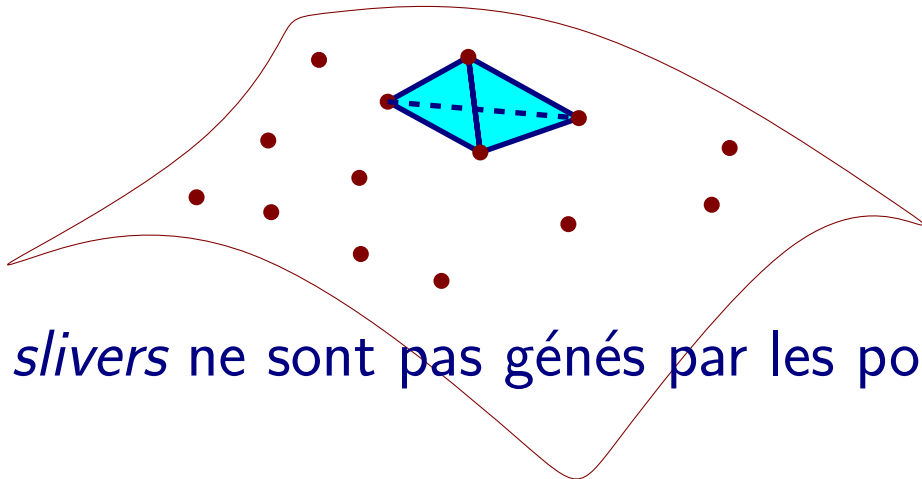
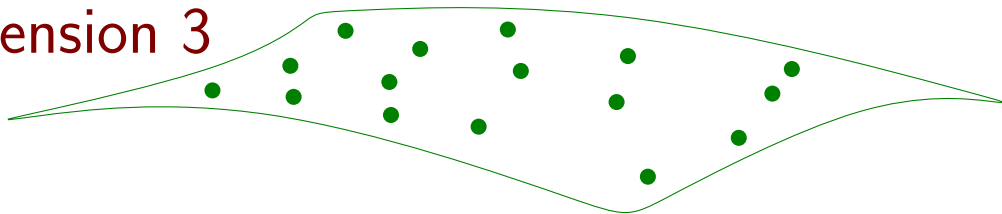
Les mauvais tétraèdres



sliver

presque coplanaire

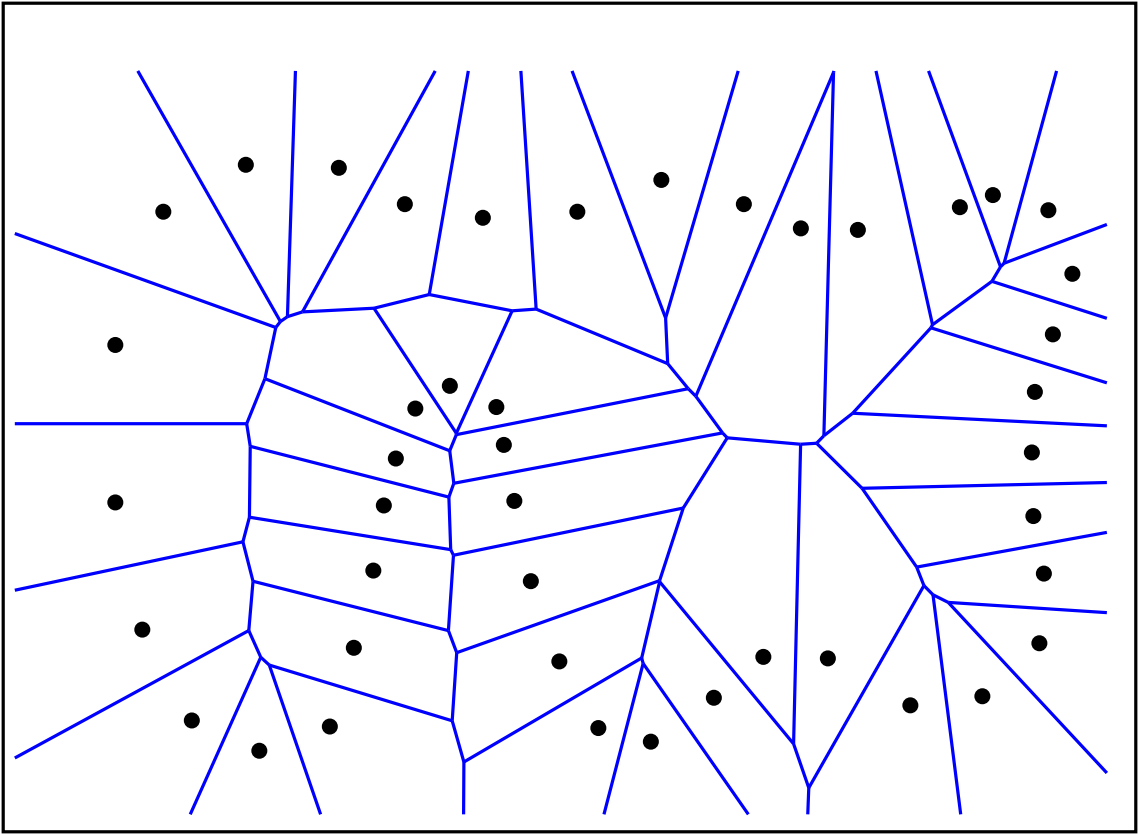
Dimension 3

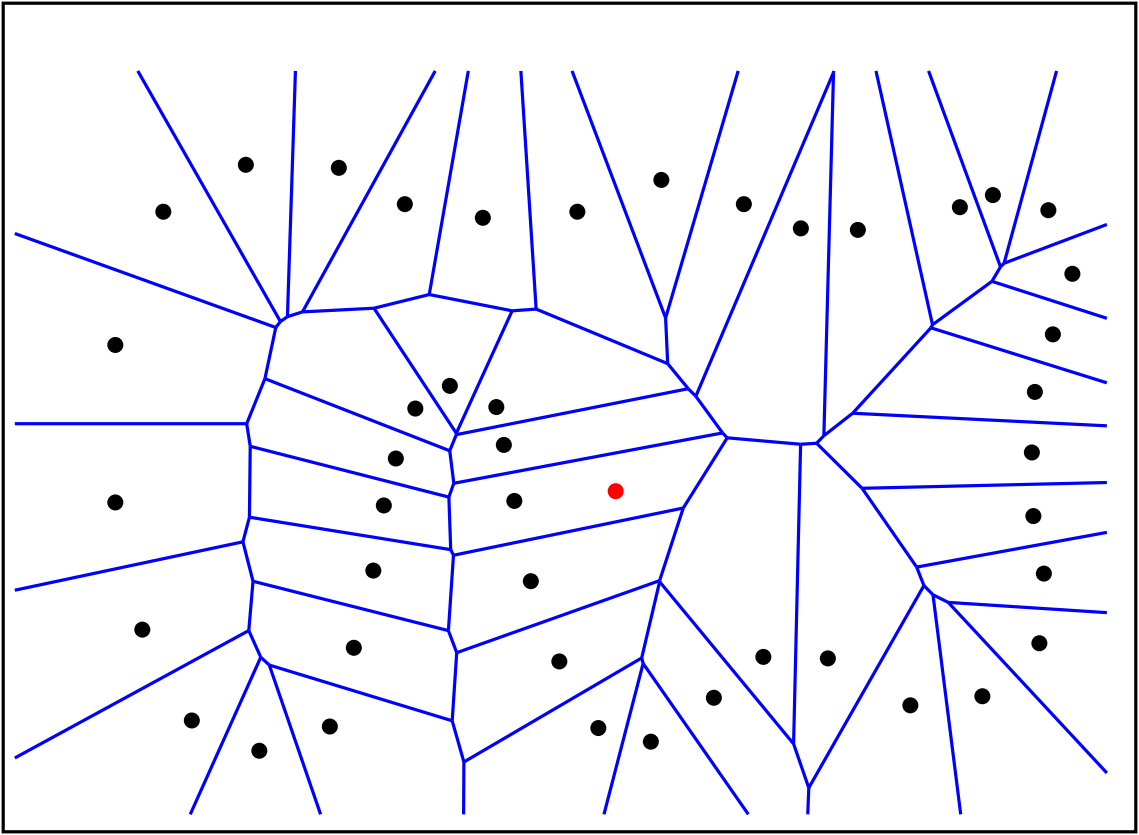


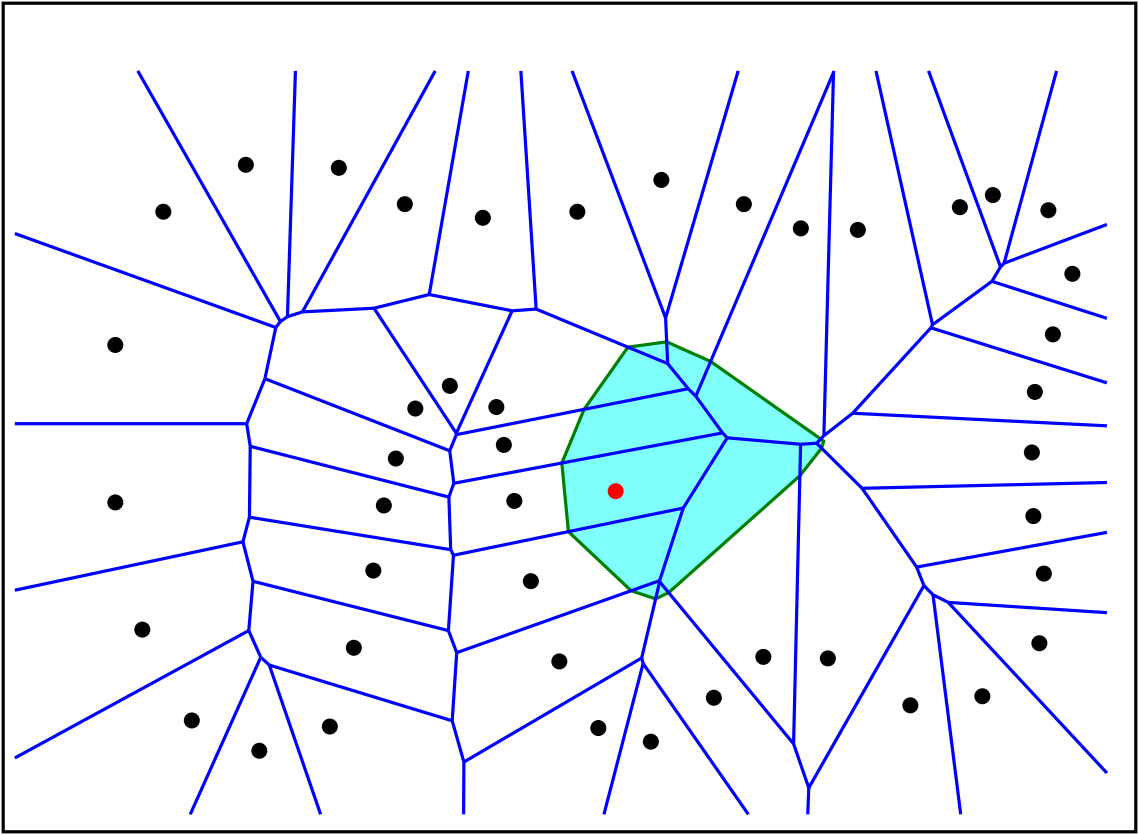
Les *slivers* ne sont pas gênés par les poles

The image features a complex geometric diagram composed of numerous yellow lines on a yellow background. The lines form a series of interconnected triangles and polygons, creating a dense, web-like structure. The overall shape is roughly circular but irregular. In the center of this structure, the text "Voisins naturels" is written in a purple, sans-serif font. The text is positioned horizontally and is clearly legible against the yellow background.

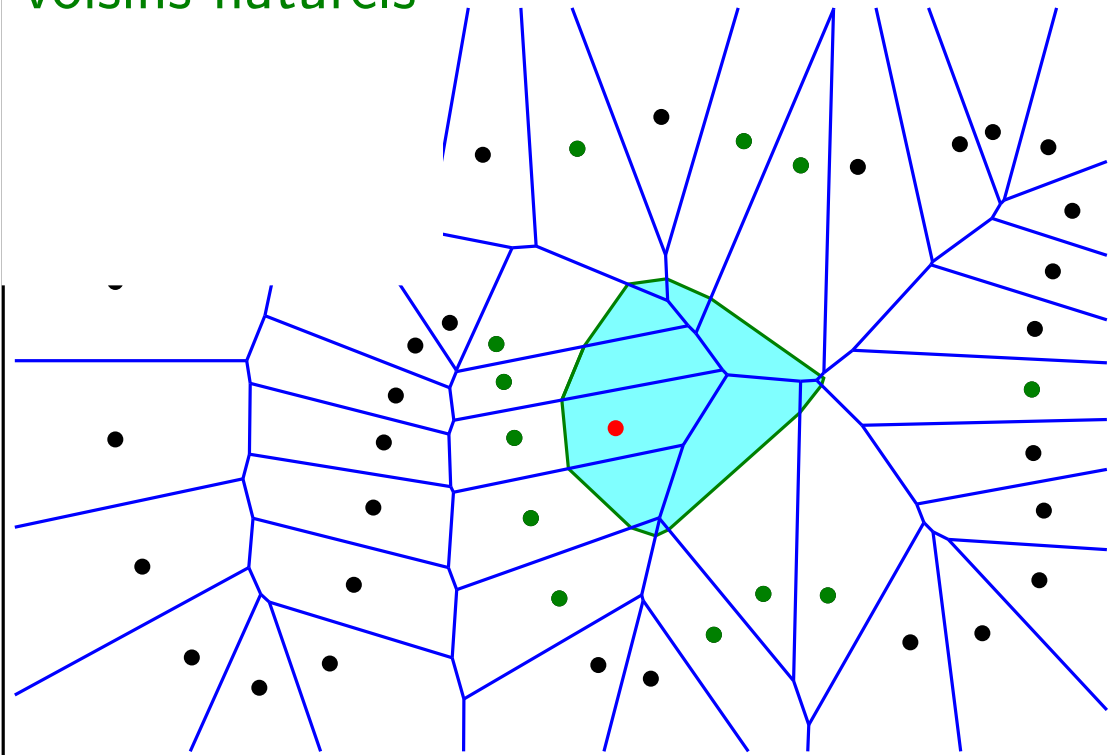
Voisins naturels





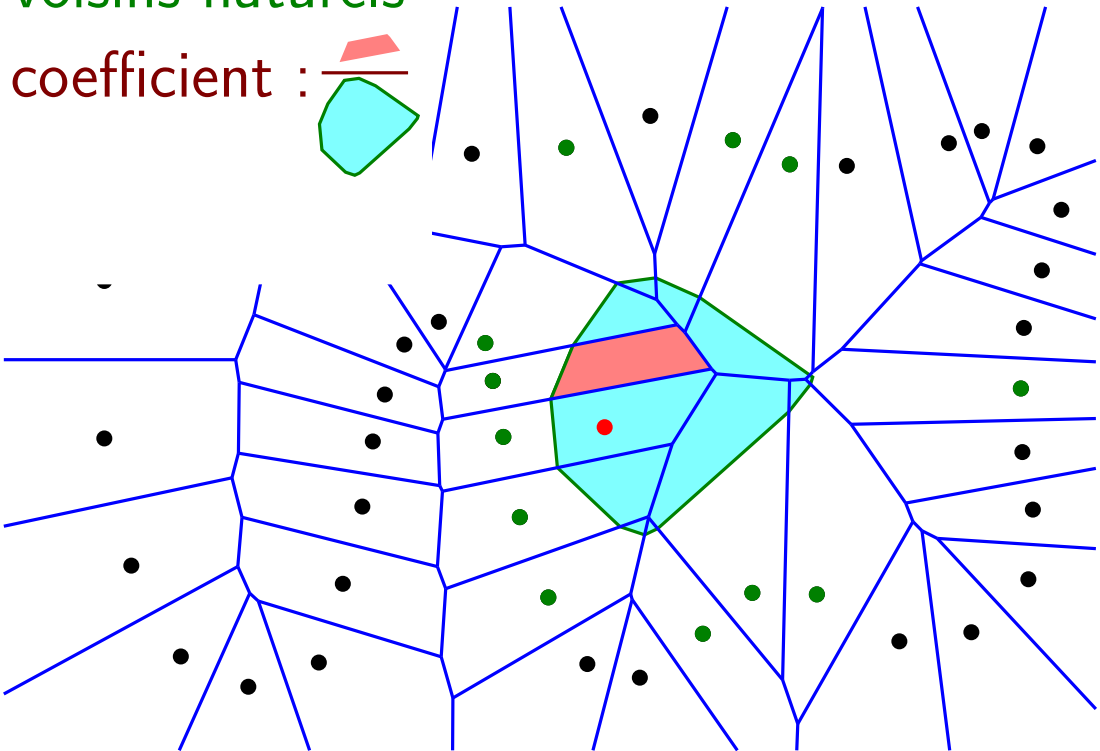
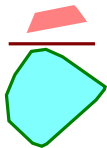


voisins naturels



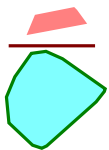
voisins naturels

coefficient : $\frac{\text{red}}{\text{cyan}}$

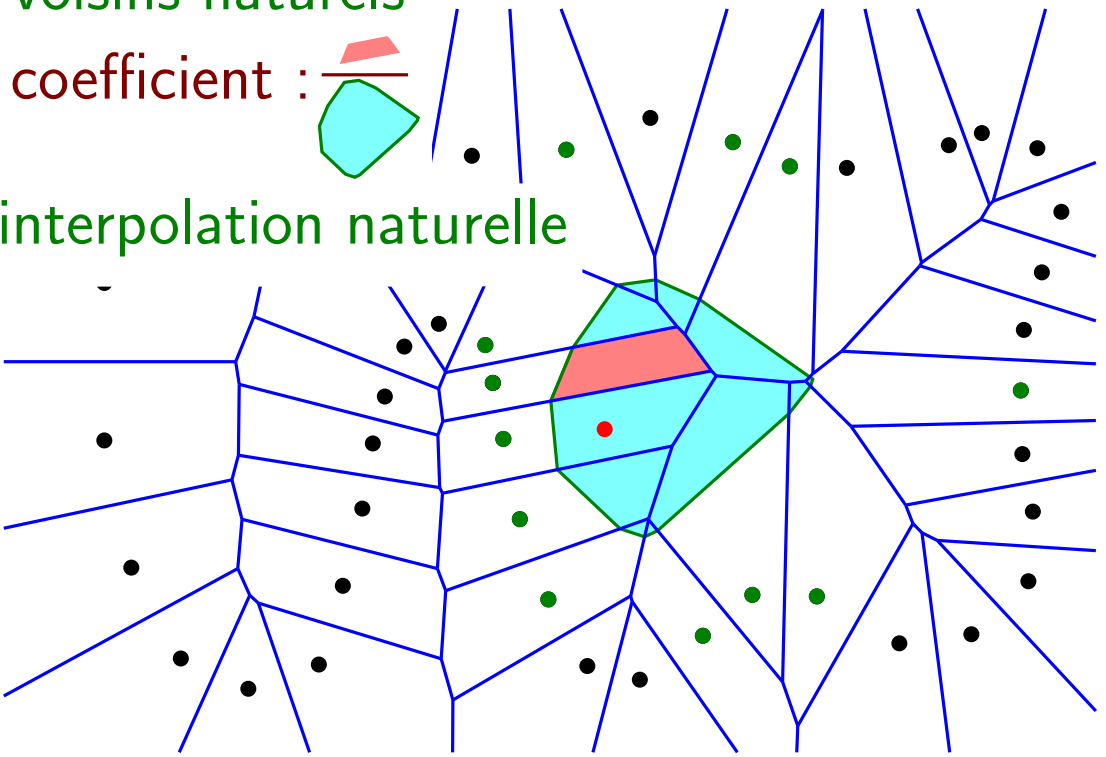


voisins naturels

coefficient : $\frac{\text{triangle}}{\text{polygone}}$

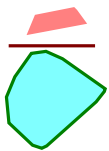


interpolation naturelle

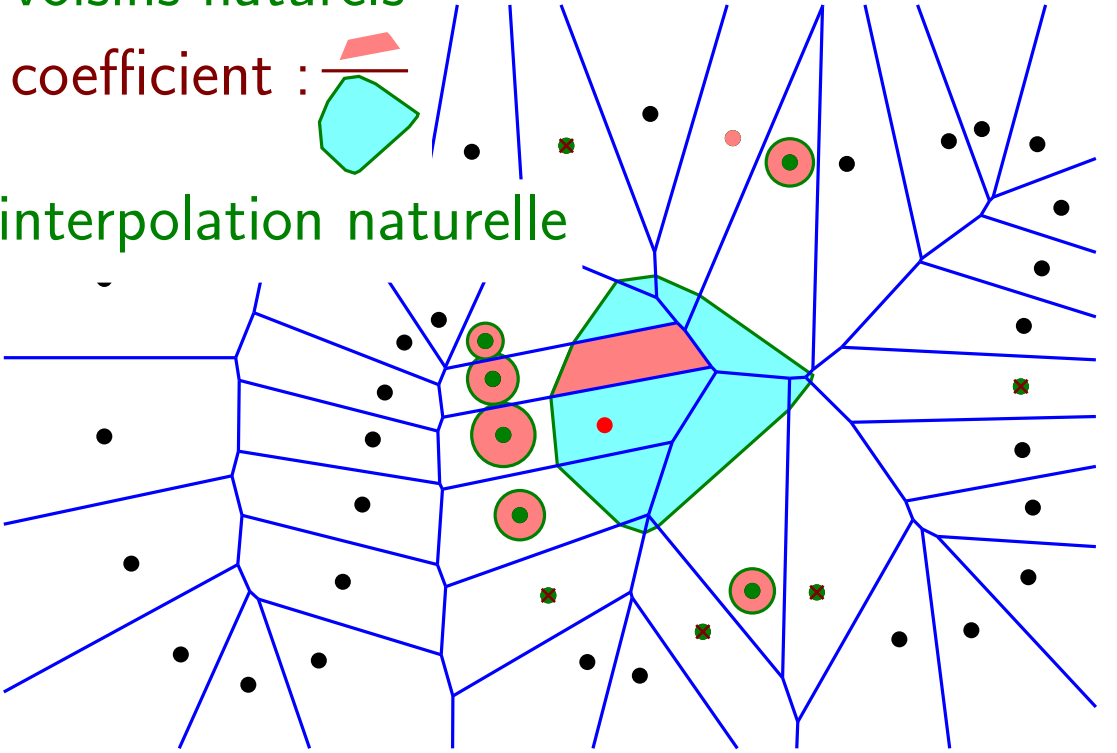


voisins naturels

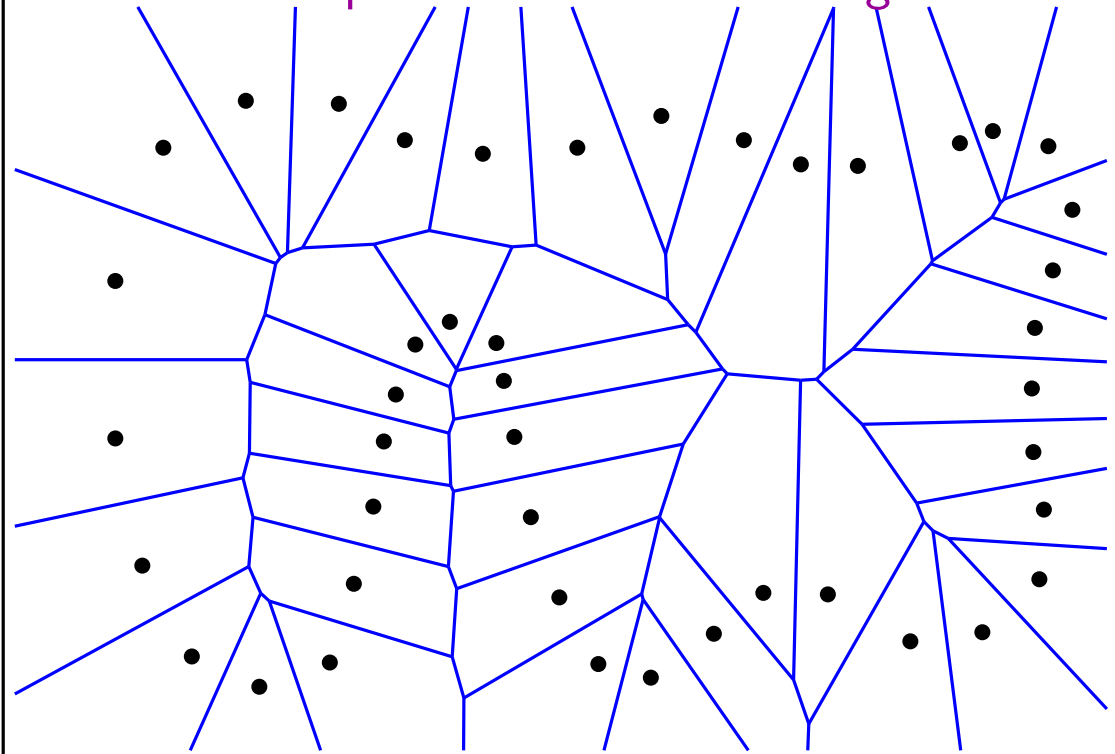
coefficient : $\frac{\text{red polygon}}{\text{cyan polygon}}$



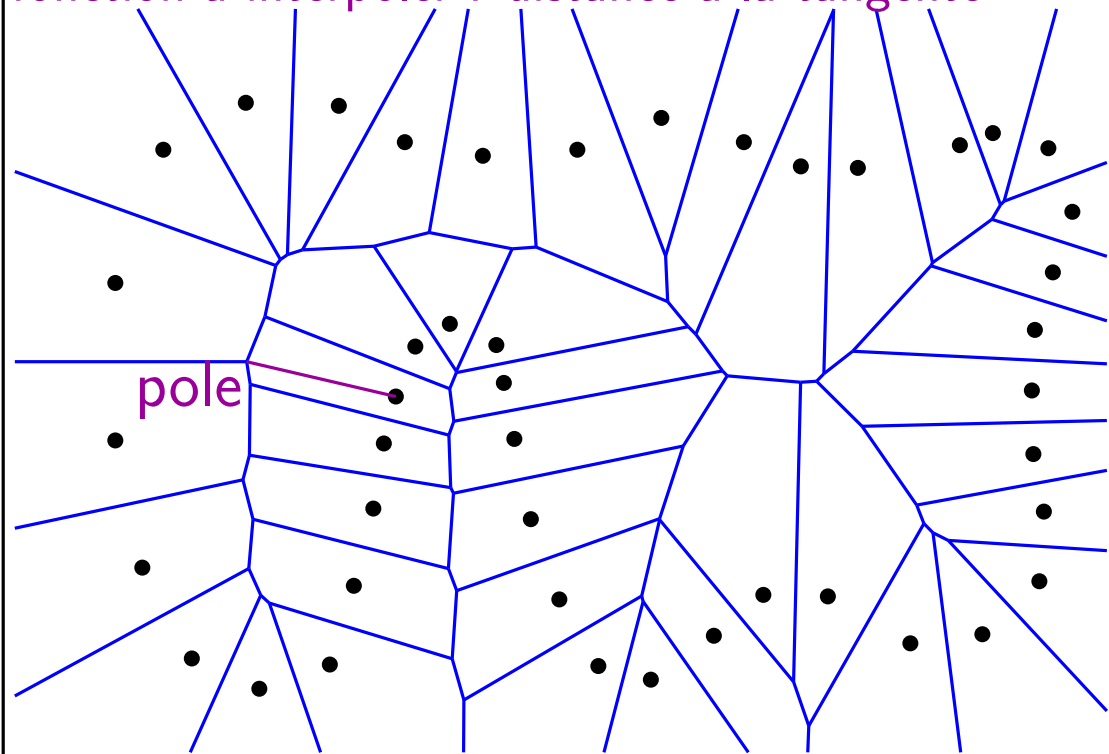
interpolation naturelle



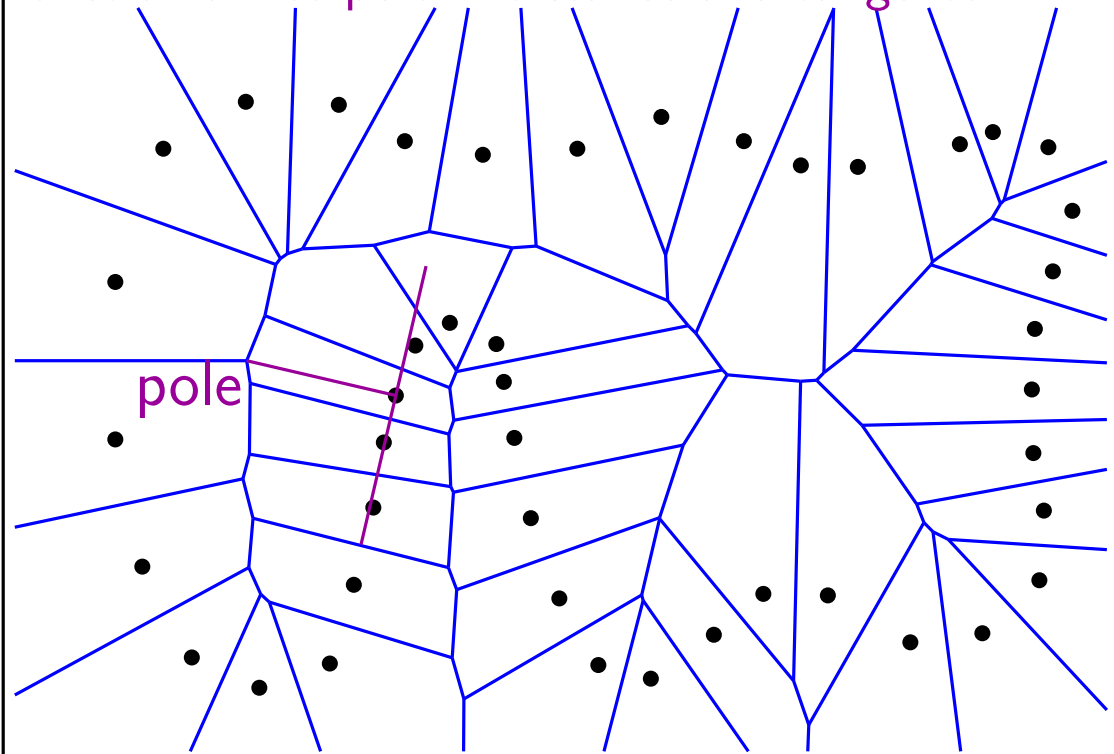
fonction à interpoler : distance à la tangente



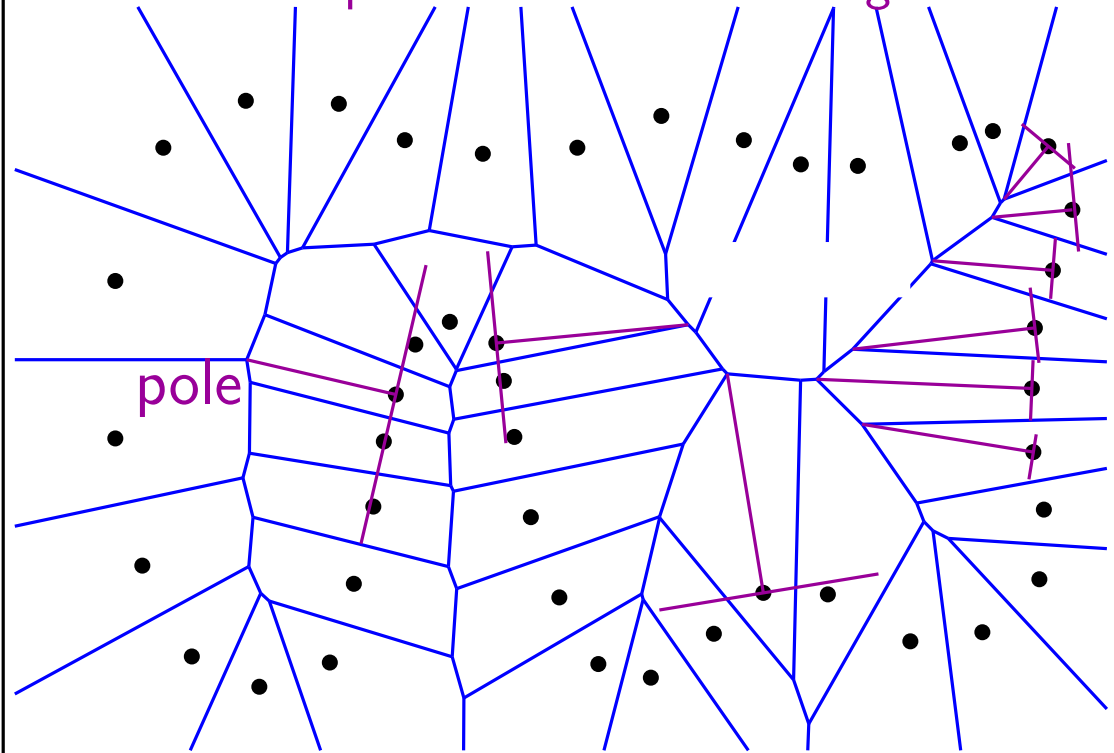
fonction à interpoler : distance à la tangente



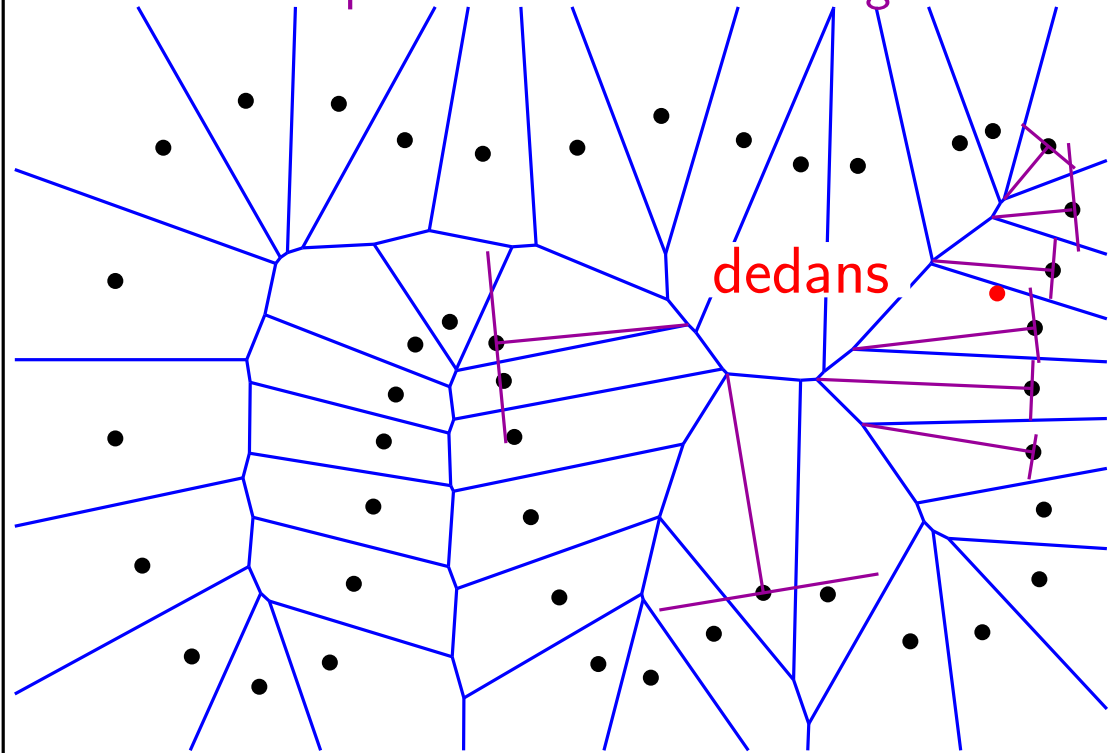
fonction à interpoler : distance à la tangente



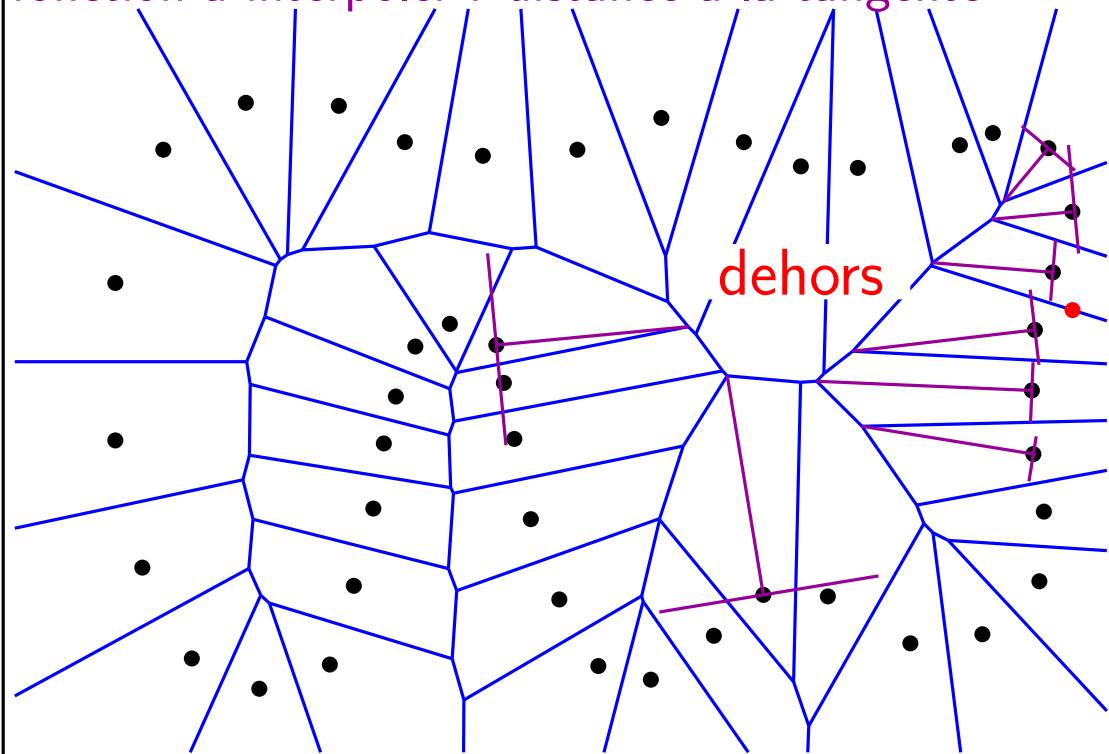
fonction à interpoler : distance à la tangente



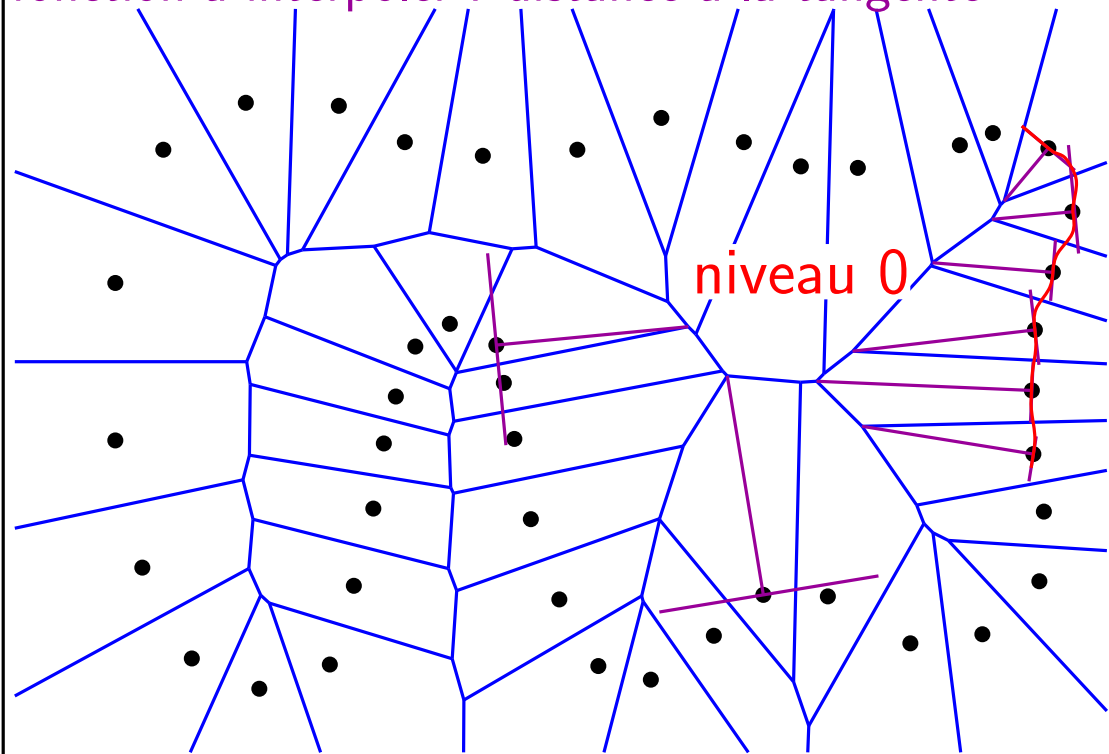
fonction à interpoler : distance à la tangente



fonction à interpoler : distance à la tangente



fonction à interpoler : distance à la tangente





C'est tout pour aujourd'hui