Algorithmic Geometry of Triangulations

Jean-Daniel Boissonnat
Geometrica, INRIA
http://www-sop.inria.fr/geometrica

Winter School, University of Nice Sophia Antipolis
January 26-30, 2015
Algorithmic Geometry of Triangulations

Jean-Daniel Boissonnat
Geometrica, INRIA
http://www-sop.inria.fr/geometrica

Winter School, University of Nice Sophia Antipolis
January 26-30, 2015
Triangulations and Meshes
Mesh generation

Meshing with sharp features
A polyhedral example
Reconstructing surfaces from point clouds

One can reconstruct a surface from $10^6$ points within 1mn \[\text{[CGAL]}\]
CGAL-mesh

GeometryFactory, Acute3D

Meshing 3D multi-domains
Input from segmented 3D medical images [IRCAD]
PA-MY (INRIA Geometrica)

ARC-ADT-AE-2010 21 / 36
Geometric data analysis
Images, text, speech, neural signals, GPS traces,...

Geometrisation: Data = points + distances between points

Hypothesis: Data lie close to a structure of "small" intrinsic dimension

Problem: Infer the structure from the data
Image manifolds

An image with 10 million pixels
→ a point in a space of 10 million dimensions!

The image-points lie close to a structure of intrinsic dimension 5 embedded in this huge ambient space

camera : 3 dof
light : 2 dof
Motion capture

Typically $N = 100$, $D = 100^3$, $d \leq 15$
Dimensionality reduction

Isomap results: hands

Winter School: Algorithmic Geometry of Triangulations
Sophia Antipolis
Conformation spaces of molecules e.g. $C_8H_{16}$

- Each conformation is represented as a point in $\mathbb{R}^{72}$ ($\mathbb{R}^{24}$ when neglecting the $H$ atoms)
- The intrinsic dimension of the conformation space is 2
- The geometry of $C_8H_{16}$ is highly nonlinear
Course overview

Algorithmic geometry of triangulations

1 Simplicial complexes in metric spaces
   Representation of simplicial complexes (D. Mazauric)

2 Delaunay-type complexes
   Exercises (A.C. De Vitis)

3 Union of balls and $\alpha$-complexes
   Exercises (M. Rouxel-Labbé)

4 Mesh generation
   Surface reconstruction (P. Alliez)

5 Reconstruction of submanifolds
   Exam!
Further reading

CGALmesh:

- https://www-sop.inria.fr/geometrica/software/cgalmesh/

Recent theses at Geometrica Sophia Antipolis

- Triangulation of manifolds: A. Ghosh (2012)
- Data structures for computational topology: C. Maria (2014)

Course Notes

www-sop.inria.fr/geometrica/courses/supports/CGL-poly.pdf

Colloquium J. Morgenstern

Projects

- European project Computational Geometric Learning (CGL)
  cgl.uni-jena.de/Home/WebHome

- ANR TopData
  Geometry meets statistics http://geometrica.saclay.inria.fr/collaborations/TopData/Home.html

- ERC Sdvanced Grant GUDHI
  Geometry Understanding in Higher Dimensions
  https://project.inria.fr/gudhi/

- On the industrial side
  Californian Startup: www.ayasdi.com