

Algorithmic Geometry of Triangulations

Jean-Daniel Boissonnat
Geometrica, INRIA

`http://www-sop.inria.fr/geometrica`

Winter School, University of Nice Sophia Antipolis
January 26-30, 2015

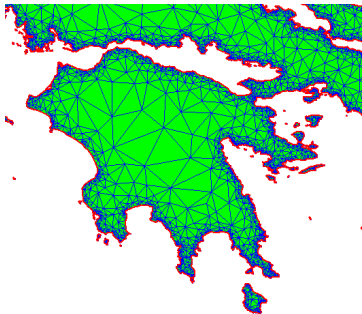
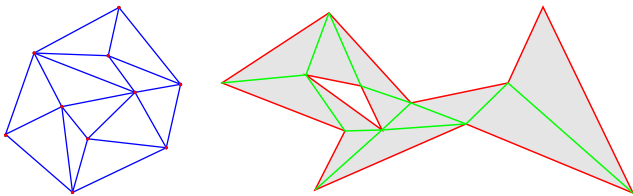
Algorithmic Geometry of Triangulations

Jean-Daniel Boissonnat
Geometrica, INRIA

<http://www-sop.inria.fr/geometrica>

Winter School, University of Nice Sophia Antipolis
January 26-30, 2015

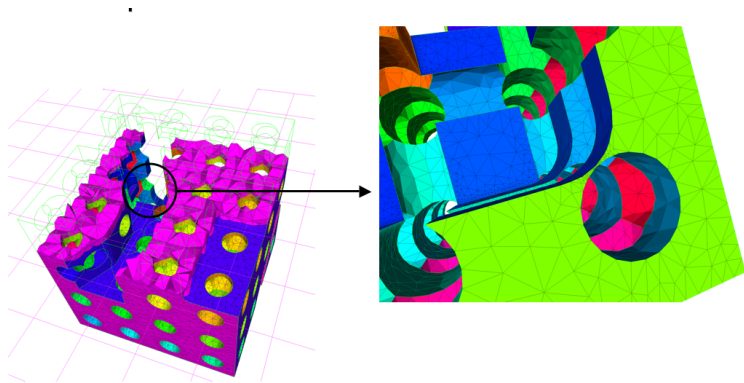
Triangulations and Meshes



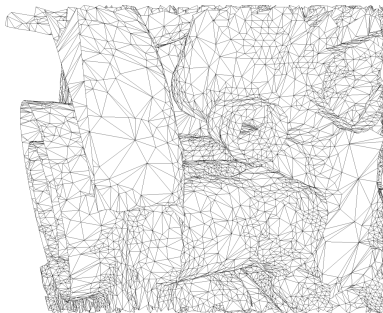
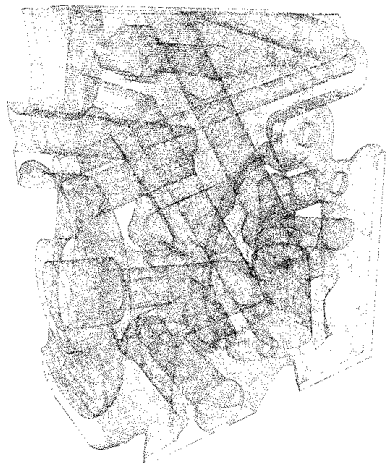
Mesh generation

Meshing with sharp features

A polyhedral example

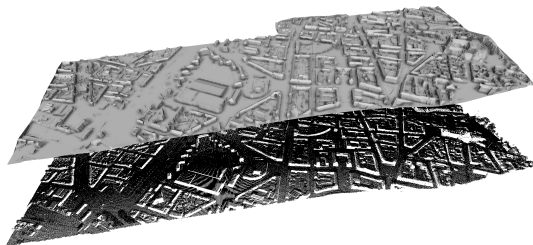
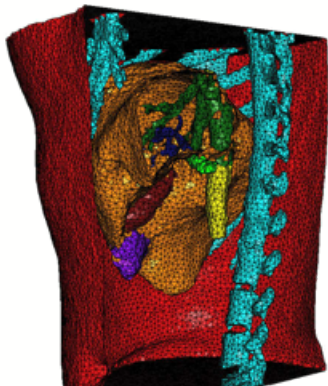


Reconstructing surfaces from point clouds



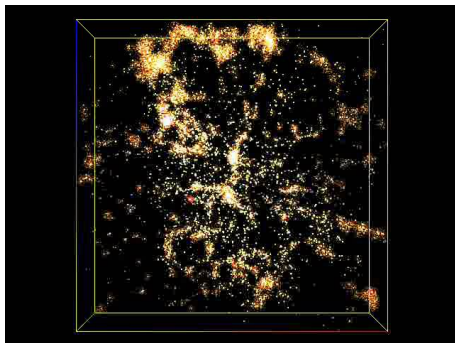
One can reconstruct a surface from 10^6 points within 1mn

[CGAL]



Geometric data analysis

Images, text, speech, neural signals, GPS traces,...



Geometrisation : Data = points + distances between points

Hypothesis : Data lie close to a structure of
“small” intrinsic dimension

Problem : Infer the structure from the data

Image manifolds

An image with 10 million pixels

→ a point in a **space of 10 million dimensions!**



camera : 3 dof
light : 2 dof

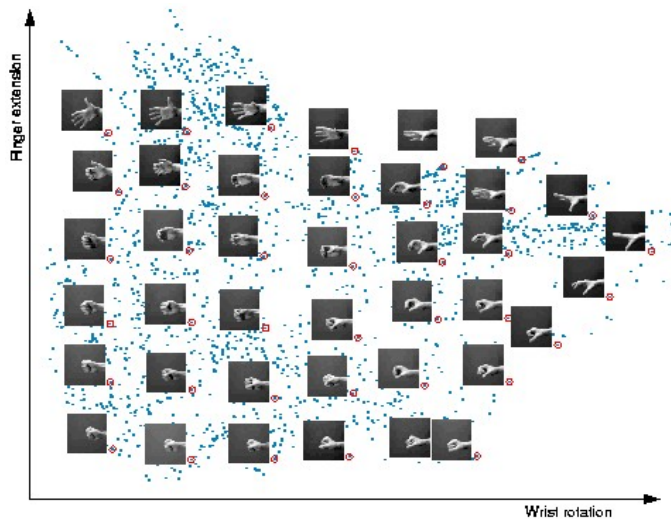
The image-points lie close to a structure of **intrinsic dimension 5** embedded in this **huge ambient space**

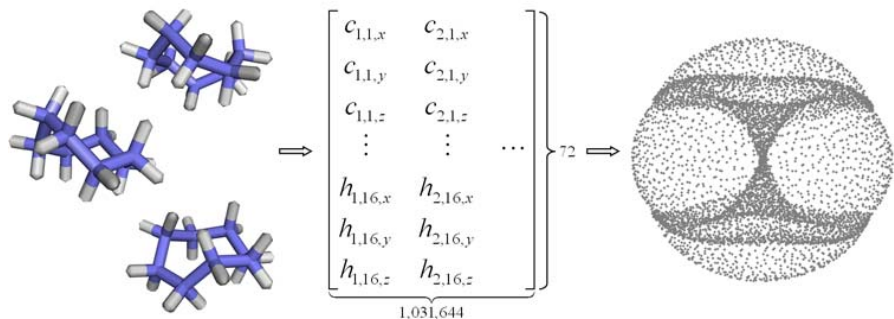
Motion capture



Typically $N = 100$, $D = 100^3$, $d \leq 15$

Dimensionality reduction





- Each conformation is represented as a point in \mathbb{R}^{72} (\mathbb{R}^{24} when neglecting the H atoms)
- The intrinsic dimension of the conformation space is 2
- The geometry of C_8H_{16} is highly nonlinear

Course overview

Algorithmic geometry of triangulations

1 **Simplicial complexes in metric spaces**

Representation of simplicial complexes (D. Mazauric)

2 **Delaunay-type complexes**

Exercises (A.C. De Vitis)

3 **Union of balls and α -complexes**

Exercises (M. Rouxel-Labbé)

4 **Mesh generation**

Surface reconstruction (P. Alliez)

5 **Reconstruction of submanifolds**

Exam !

Further reading

CGALmesh :

- <https://www-sop.inria.fr/geometrica/software/cgalmesh/>

Recent theses at Geometrica Sophia Antipolis

- Triangulation of manifolds : A. Ghosh (2012)
- Data structures for computational topology : C. Maria (2014)

Course Notes

www-sop.inria.fr/geometrica/courses/supports/CGL-poly.pdf

Colloquium J. Morgenstern

www-sop.inria.fr/colloquium

Vin de Silva : Point-clouds, sensor networks, and persistence:
algebraic topology in the 21st century

26/3/2009

Projects

- **European project Computational Geometric Learning (CGL)**
`cgl.uni-jena.de/Home/WebHome`
- **ANR TopData**
Geometry meets statistics `http://geometrica.saclay.inria.fr/collaborations/TopData/Home.html`
- **ERC Sdvanced Grant GUDHI**
Geometry Understanding in Higher Dimensions
`https://project.inria.fr/gudhi/`
- **On the industrial side**
Californian Startup : `www.ayasdi.com`