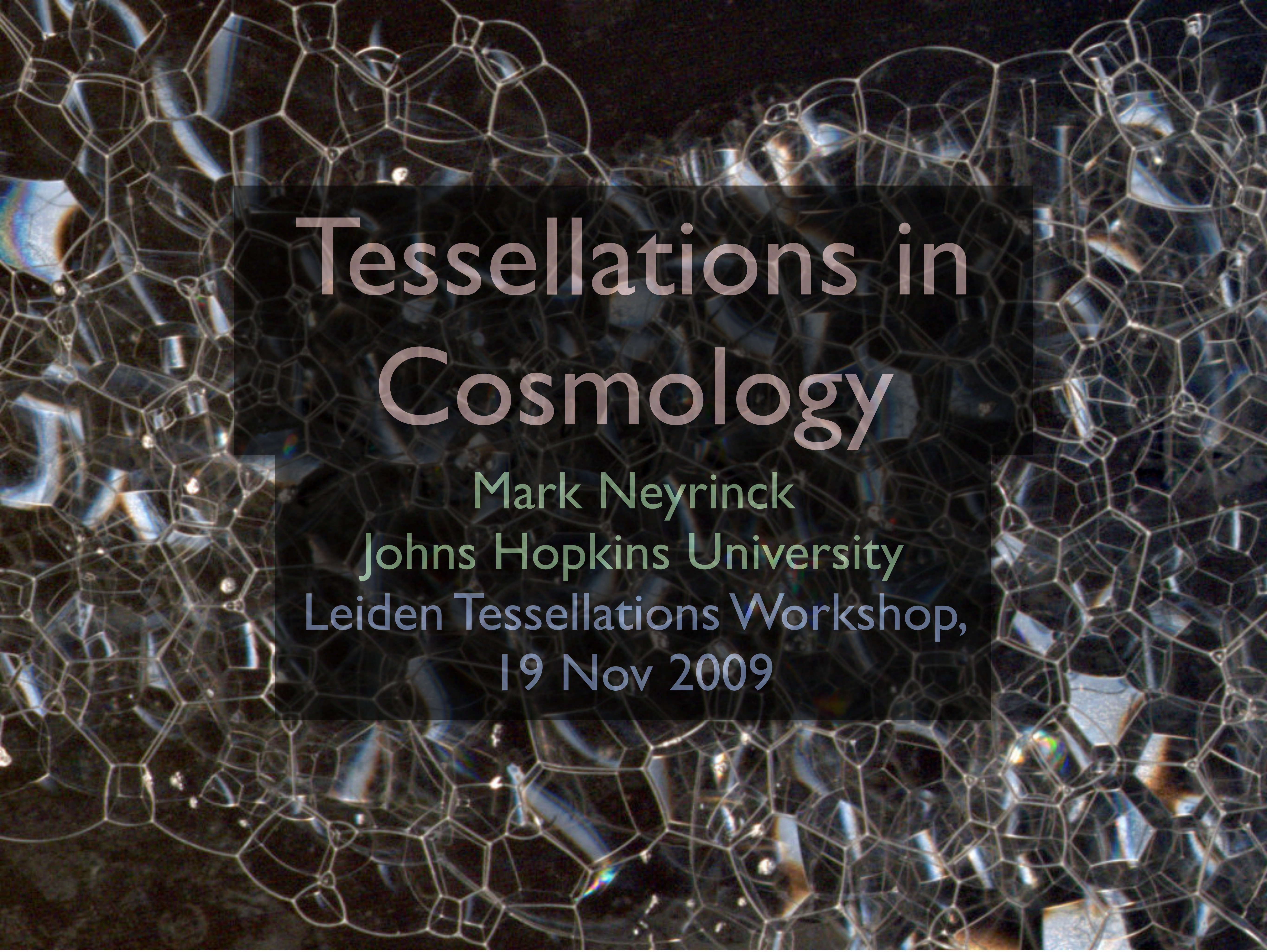


Tessellations in Cosmology

Mark Neyrinck
Johns Hopkins University
Leiden Tessellations Workshop,
19 Nov 2009

Outline

I. The large-scale structure of the Universe as a Voronoi foam

II. Tessellation methods of data analysis (that I've used)

  A. Density estimation

  B. Void, cluster finding -- superstructures and dark energy

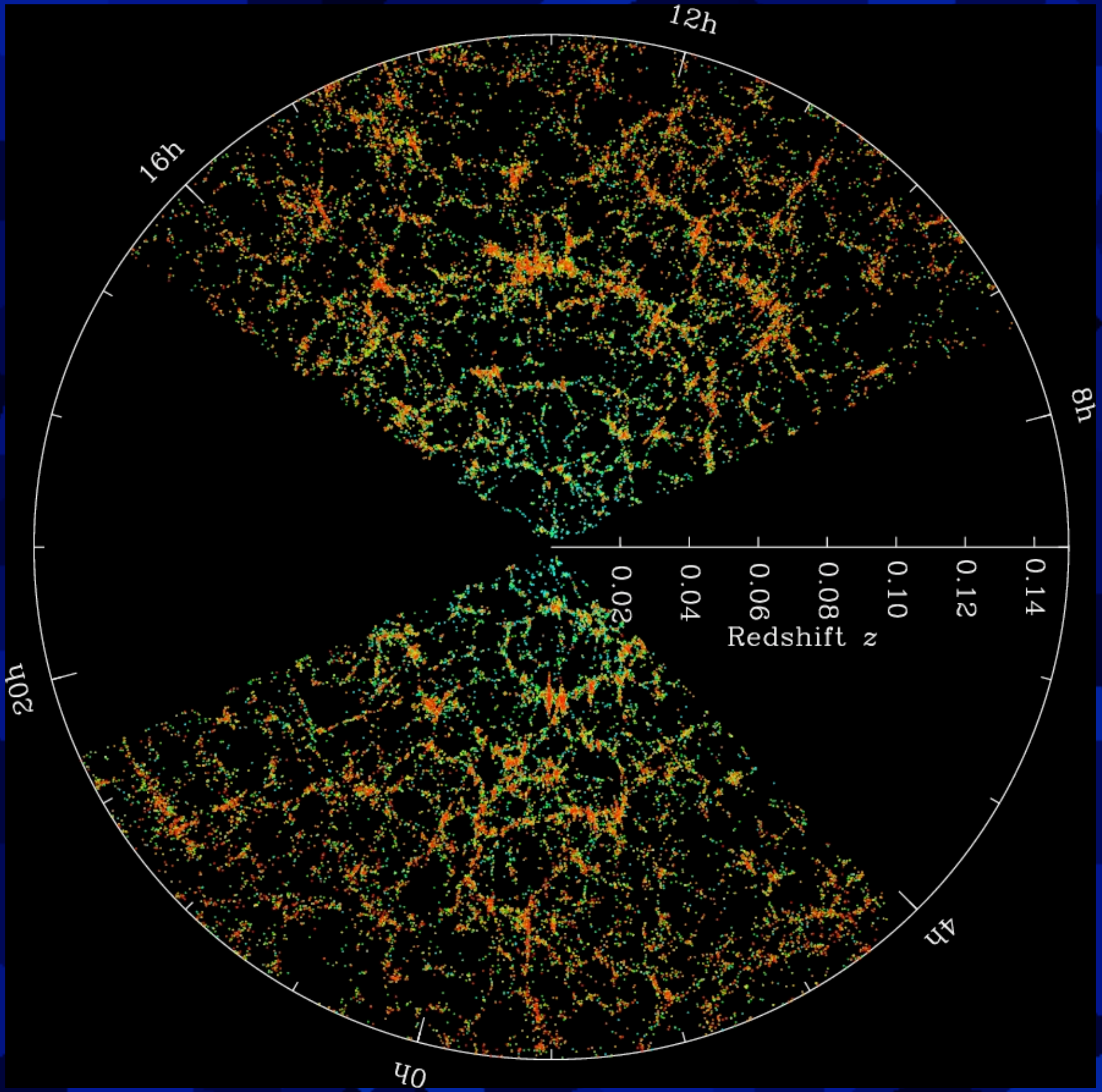
  C. Estimation of the cosmological potential

  D. Measuring divergences of a vector field

Large-scale structure

How large is large? Length scales of galaxy clustering, i.e. scales of intergalaxy separations.

Voids, filaments visible on moderately large scales. Why?

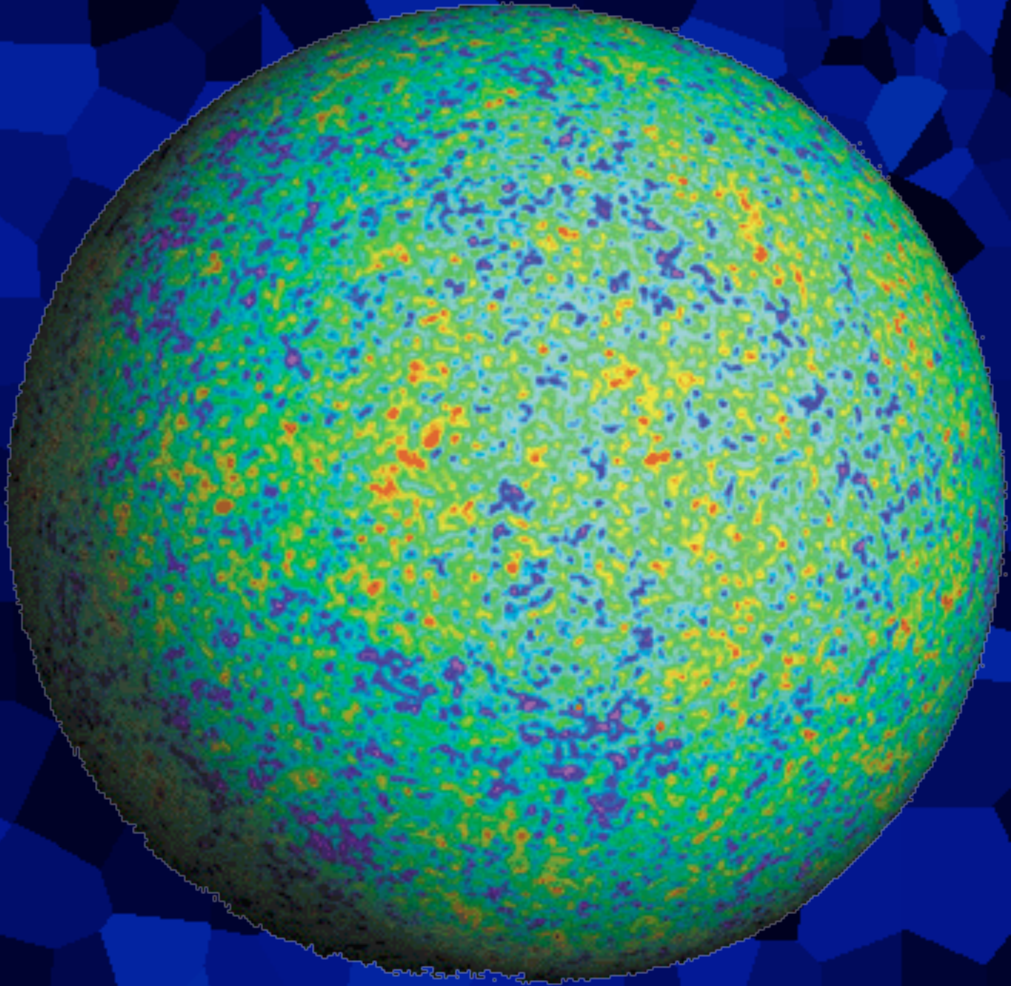


Sloan Digital Sky Survey

Large-scale structure

(simplest if really large, or really early)

Observations of the cosmic microwave background (WMAP, shown) tell us that the fluctuations in the early universe were ~10^{-5}, and Gaussian within measurement errors.



"Gaussian?" N-pt pdf depends only on 2-point correlation function, $\xi(r)$. Fourier amplitudes have all the useful information. 1-pt pdf on all smoothing scales is Gaussian.

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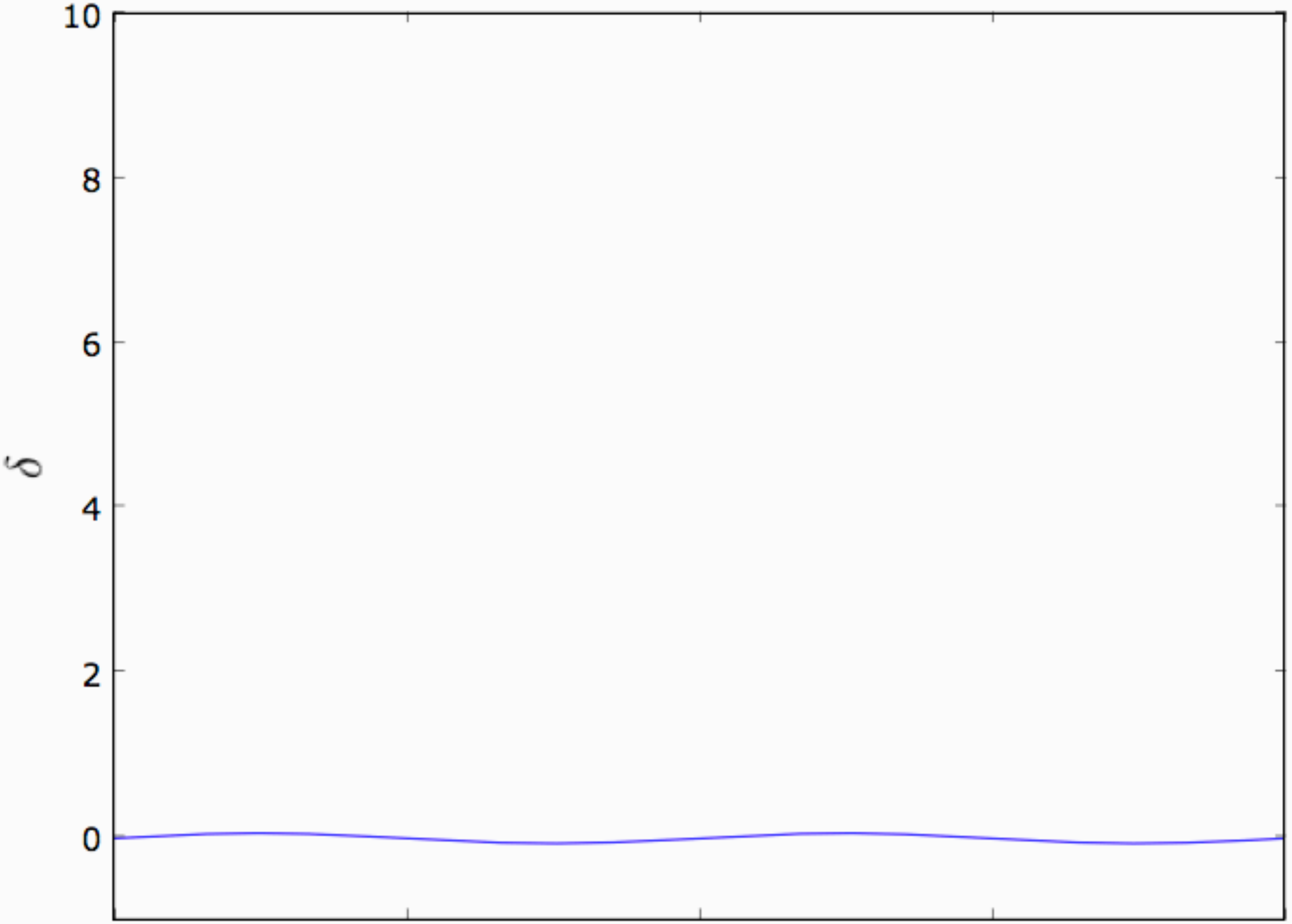
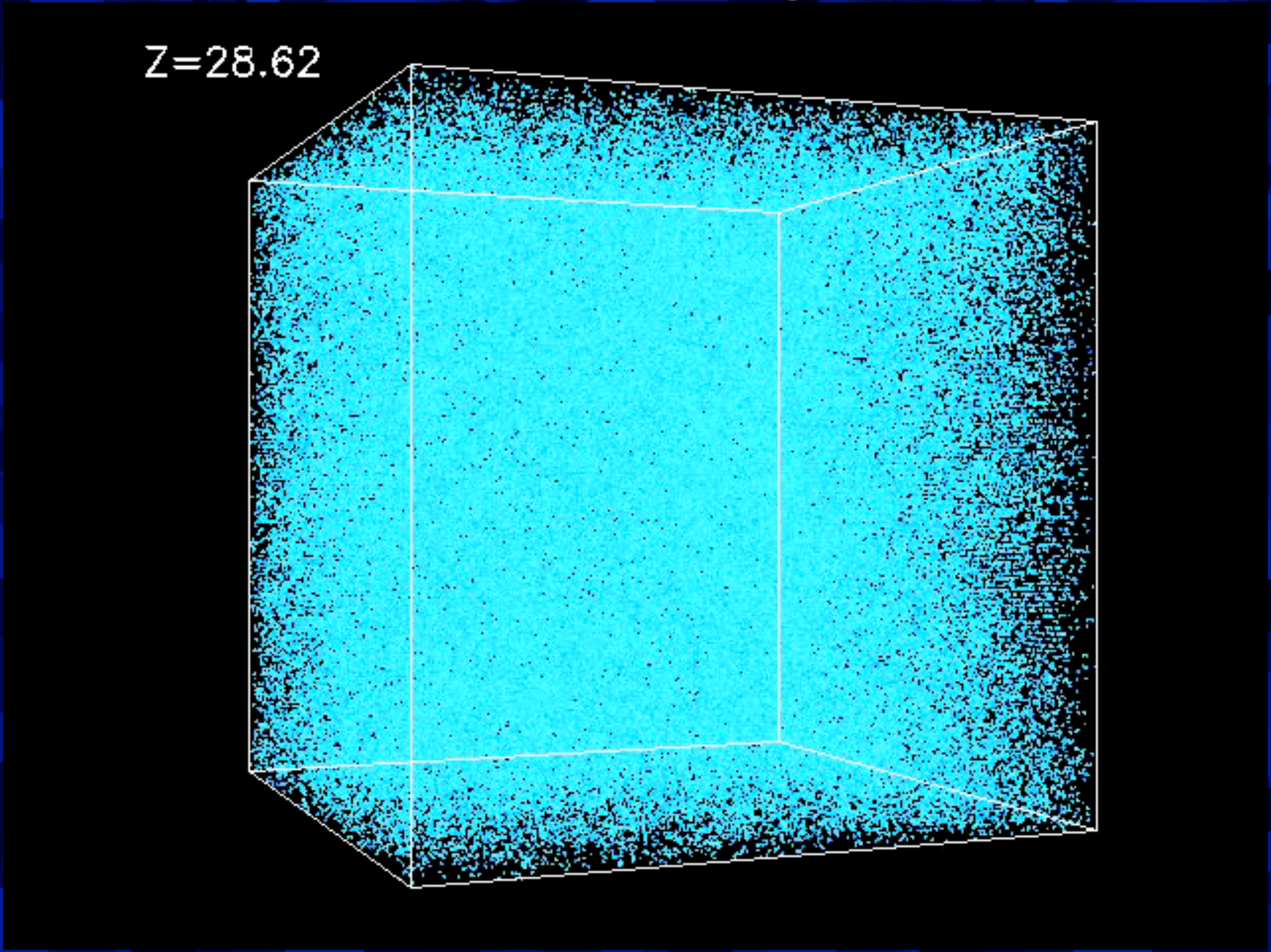


10
8
6
4
2
0
δ

1d → 3d



Kravtsov
& Klypin

The Cosmic Web

Qualitative description of cosmic web: matter flows away from initial density depressions, collects at walls, filaments, and clusters. Density depressions are the generators of a Voronoi diagram: Voronoi vertices are clusters of galaxies, edges are filaments, and faces are walls (van de Weygaert & Icke 1989). Not bad description of 2-pt correlation function. However, full simulations still necessary for high accuracy, investigation of dependence on cosmological parameters.

Mark Neyrinck, JHU

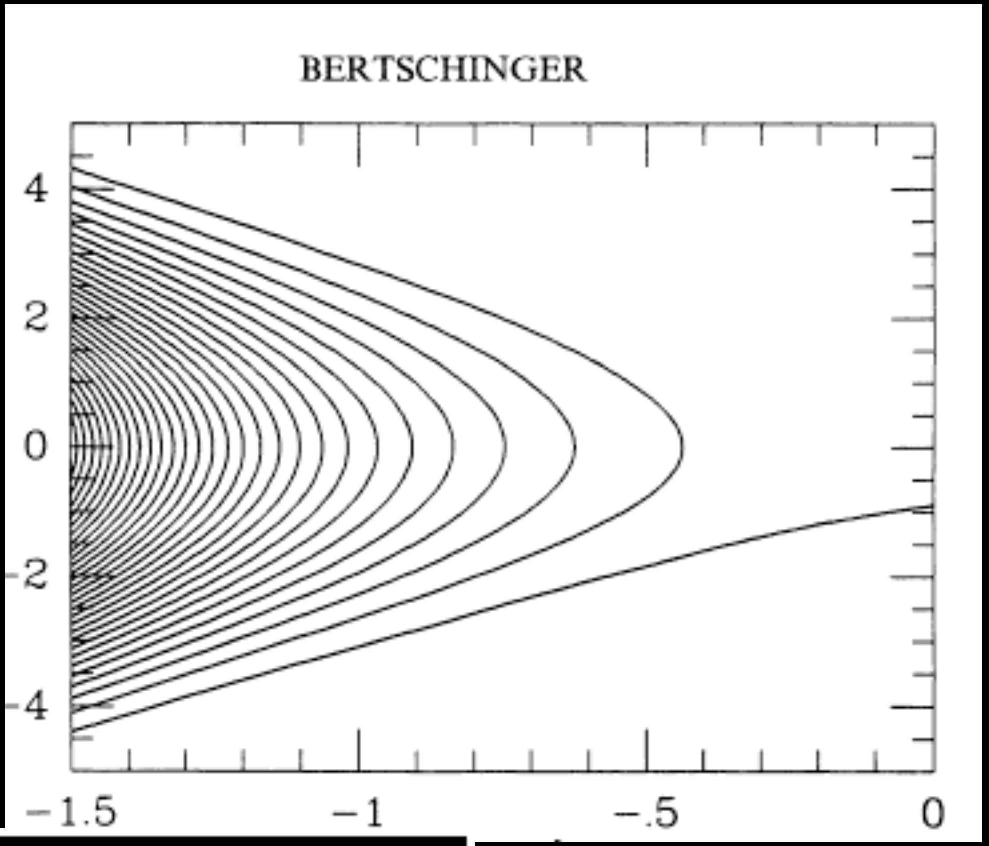
Leiden, 19 Nov 2009

The Cosmic Web

In fact 6D phase space, initial density depression zones likely curl into the interiors of non-linear structures.

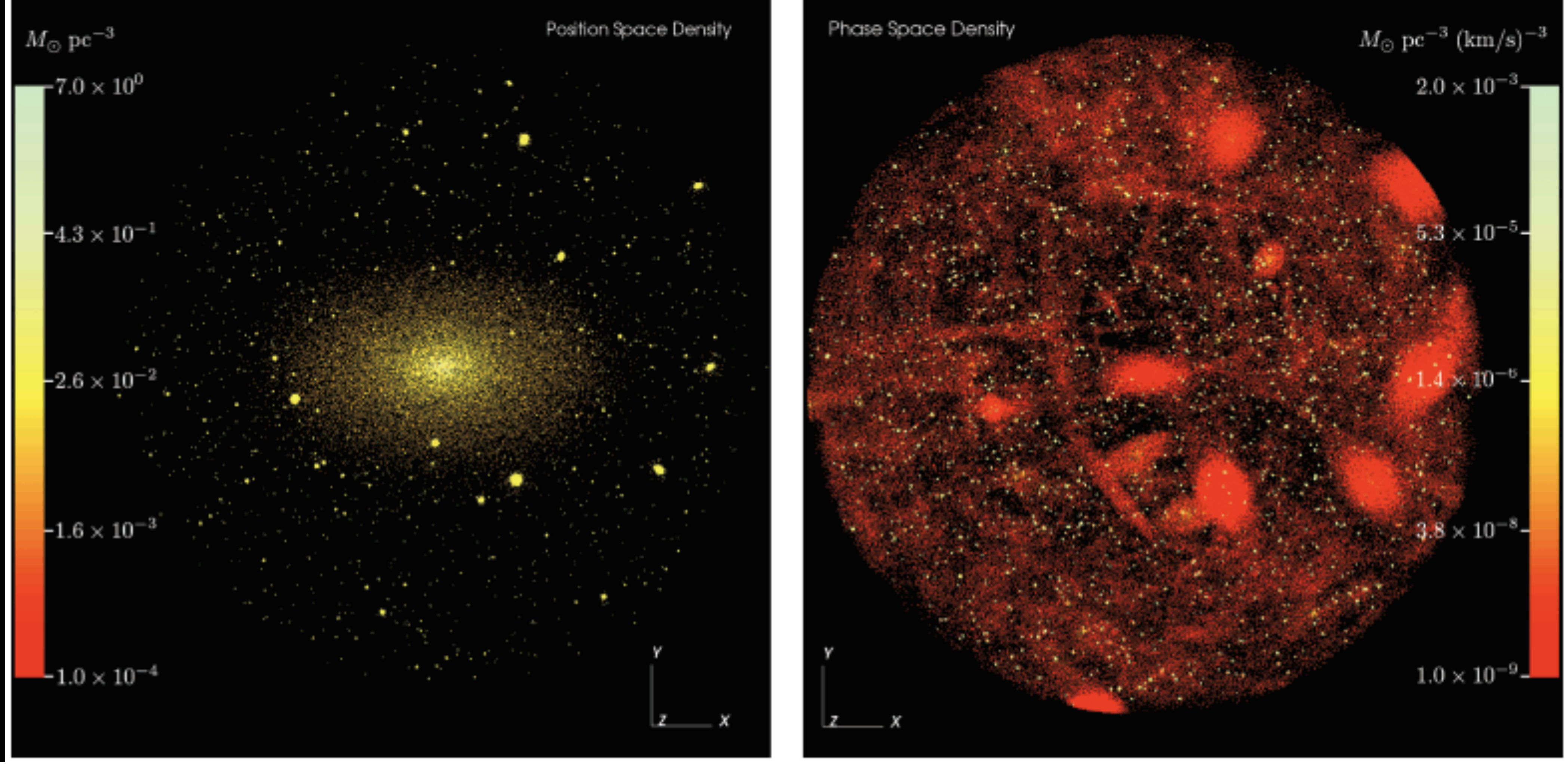
Phase space tessellated almost entirely into (non-Voronoi) regions defined by initial perturbations?

Large-scale filaments reach far into walls/ filaments/clusters in phase space?



v_x

log x

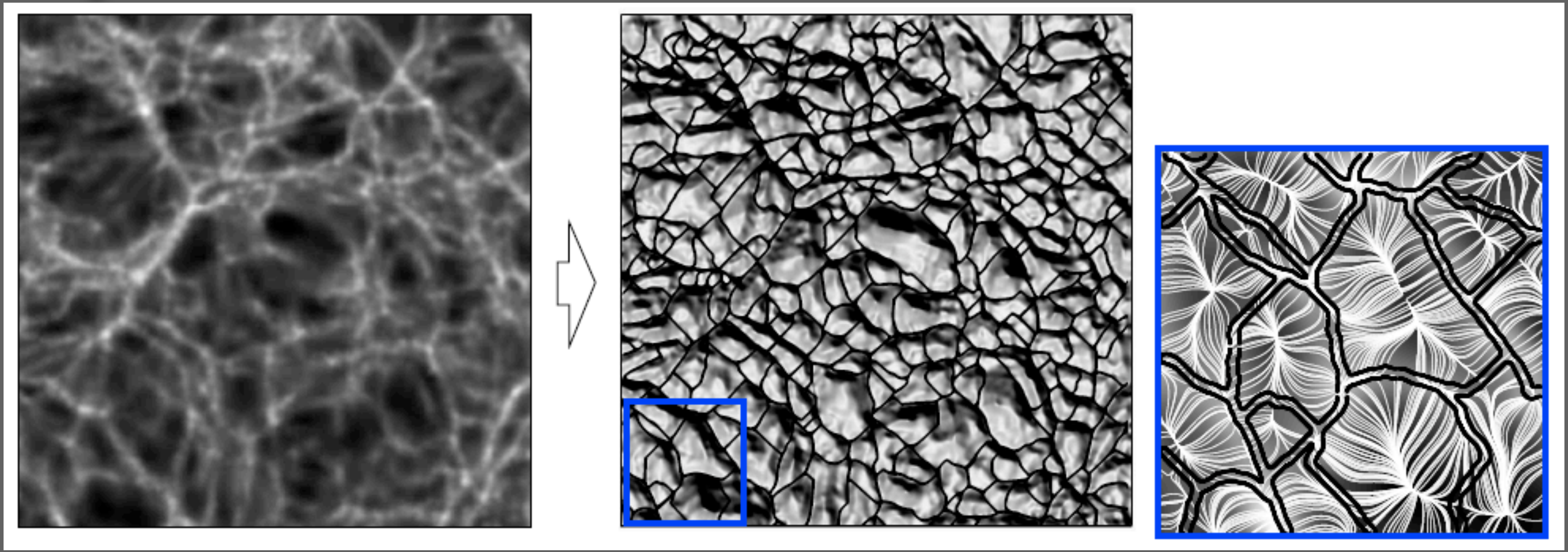


Zemp et al. 2009

Scales & Stats

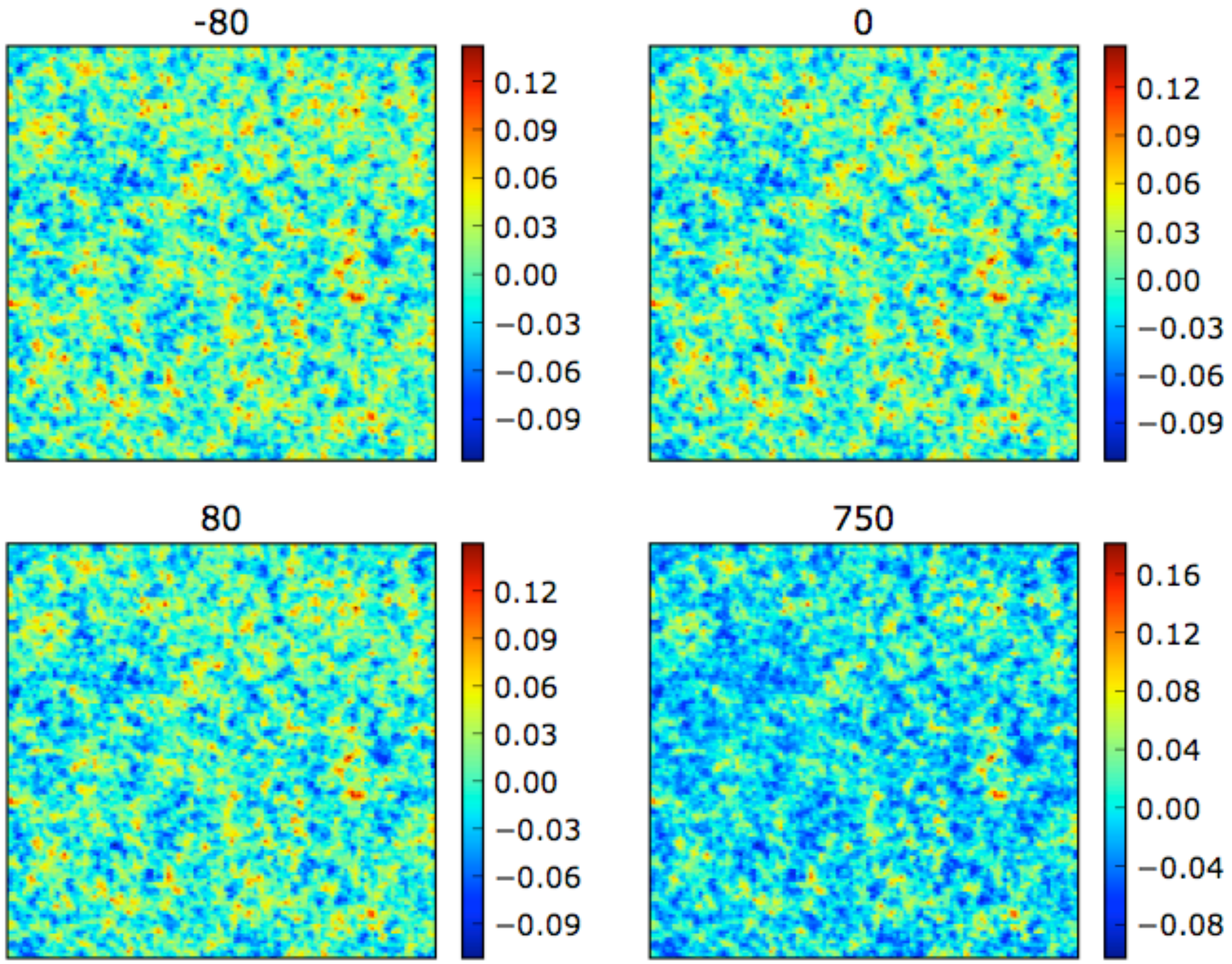
Scale	Structure	Best Statistic
~ Hubble length, size of observable universe, ~ 1 Gpc	essentially homogeneous	(power spectrum)
>~10 Mpc	Gaussian fluctuations	power spectrum
~4-10 Mpc	"lognormal" fluctuations	power spectrum of log density
<~10 Mpc	Cosmic Web	Void, filament, cluster stats
<~100 kpc	cluster, filament, galaxy structure	?

Spine/Skeleton of the Cosmic Web

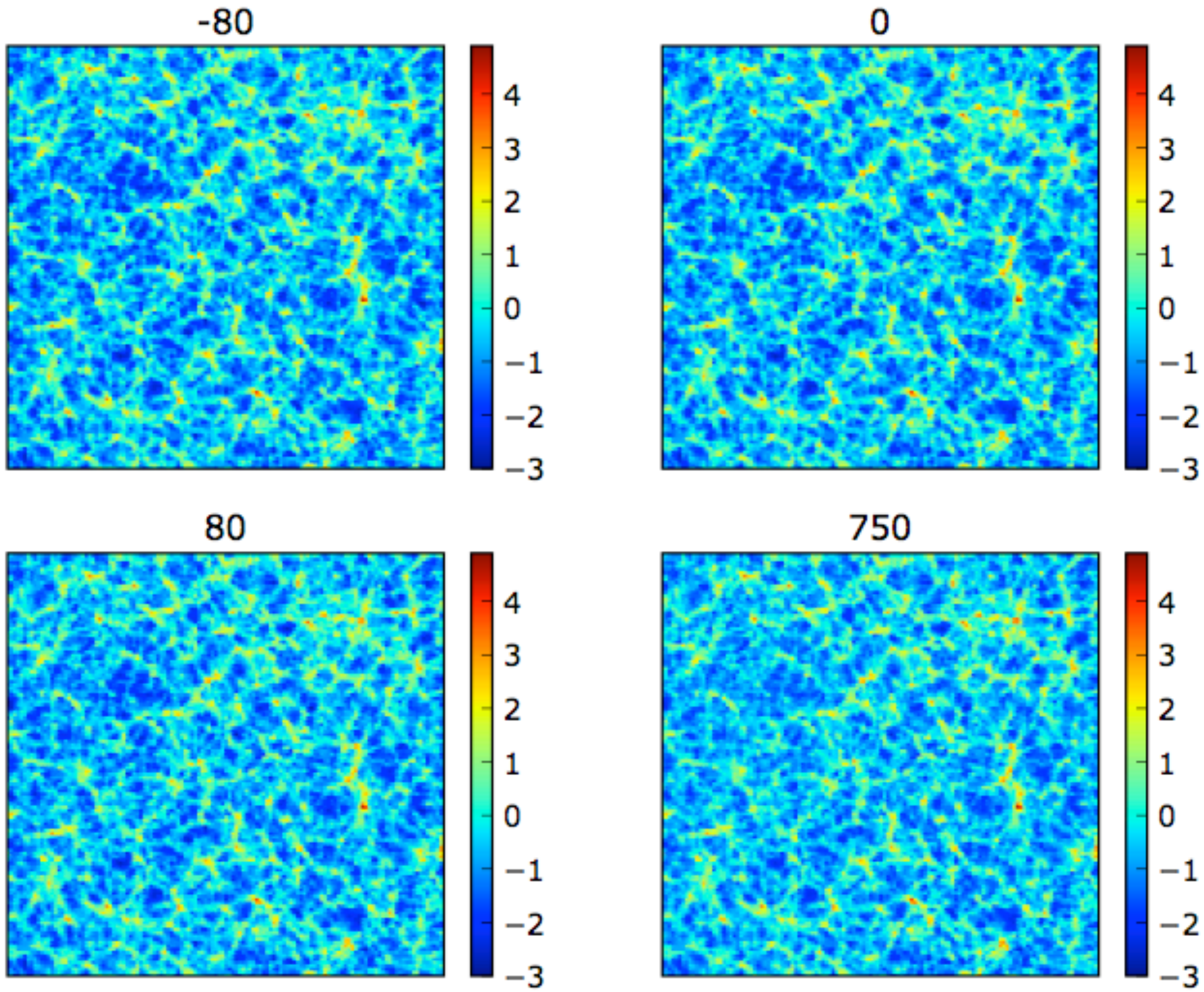


Total length of the spine depends somewhat on expansion history, matter content (Sousbie et al. 2006, Aragón-Calvo et al. 2008, Pogosyan et al. 2009)

Balance between voids/clusters depends on initial non-Gaussianity



Initial δ



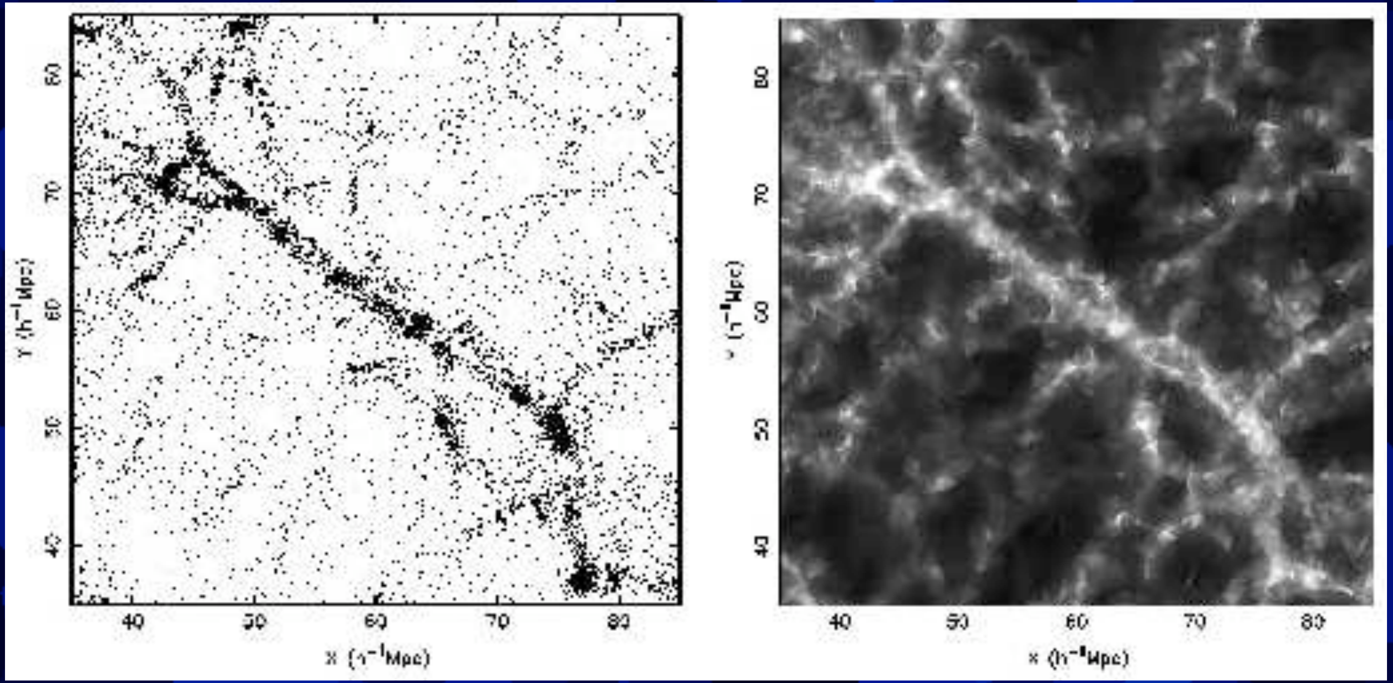
Final log(1+δ)

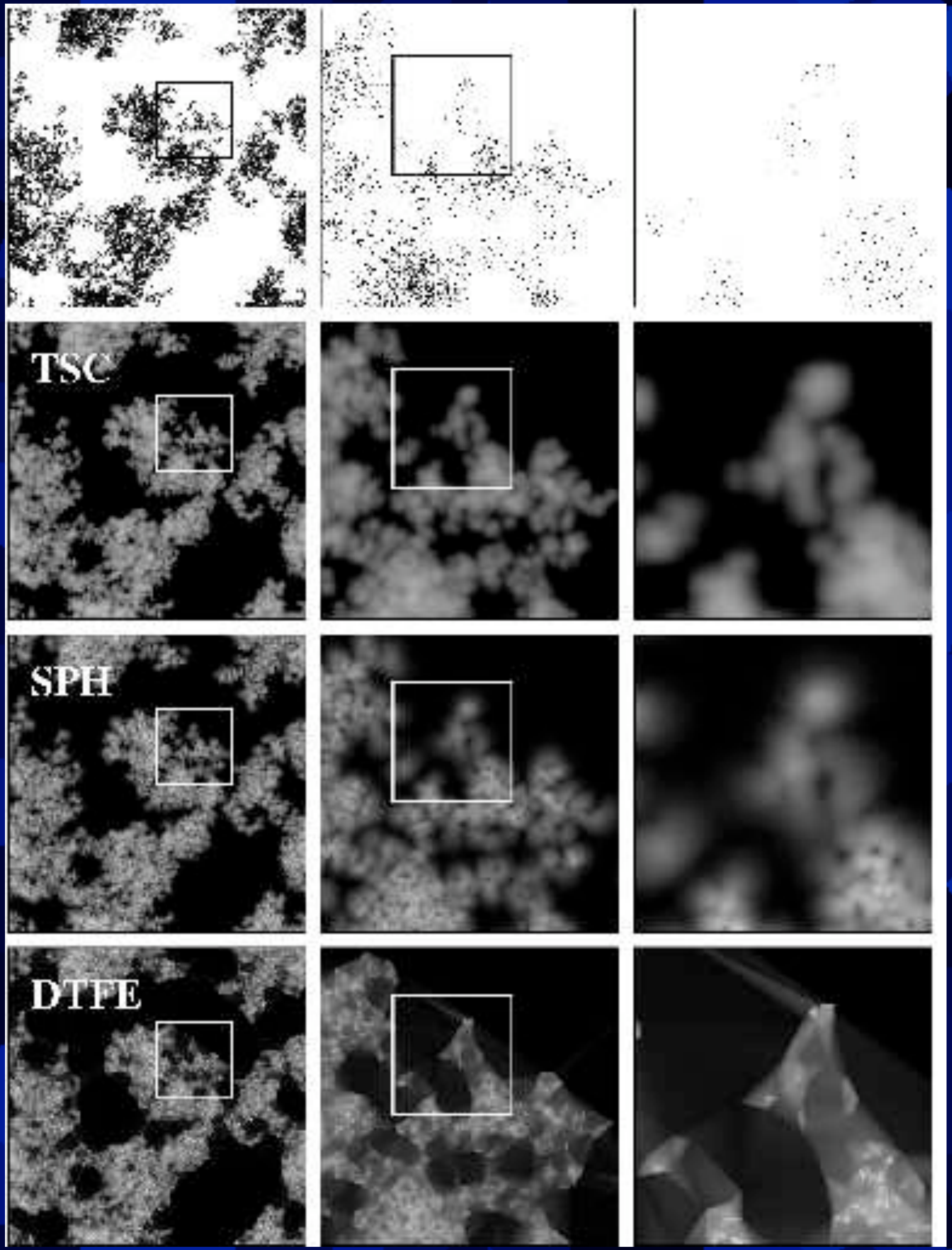
Simulations by Giannantonio, Porciani et al.

II. Tessellation data analysis

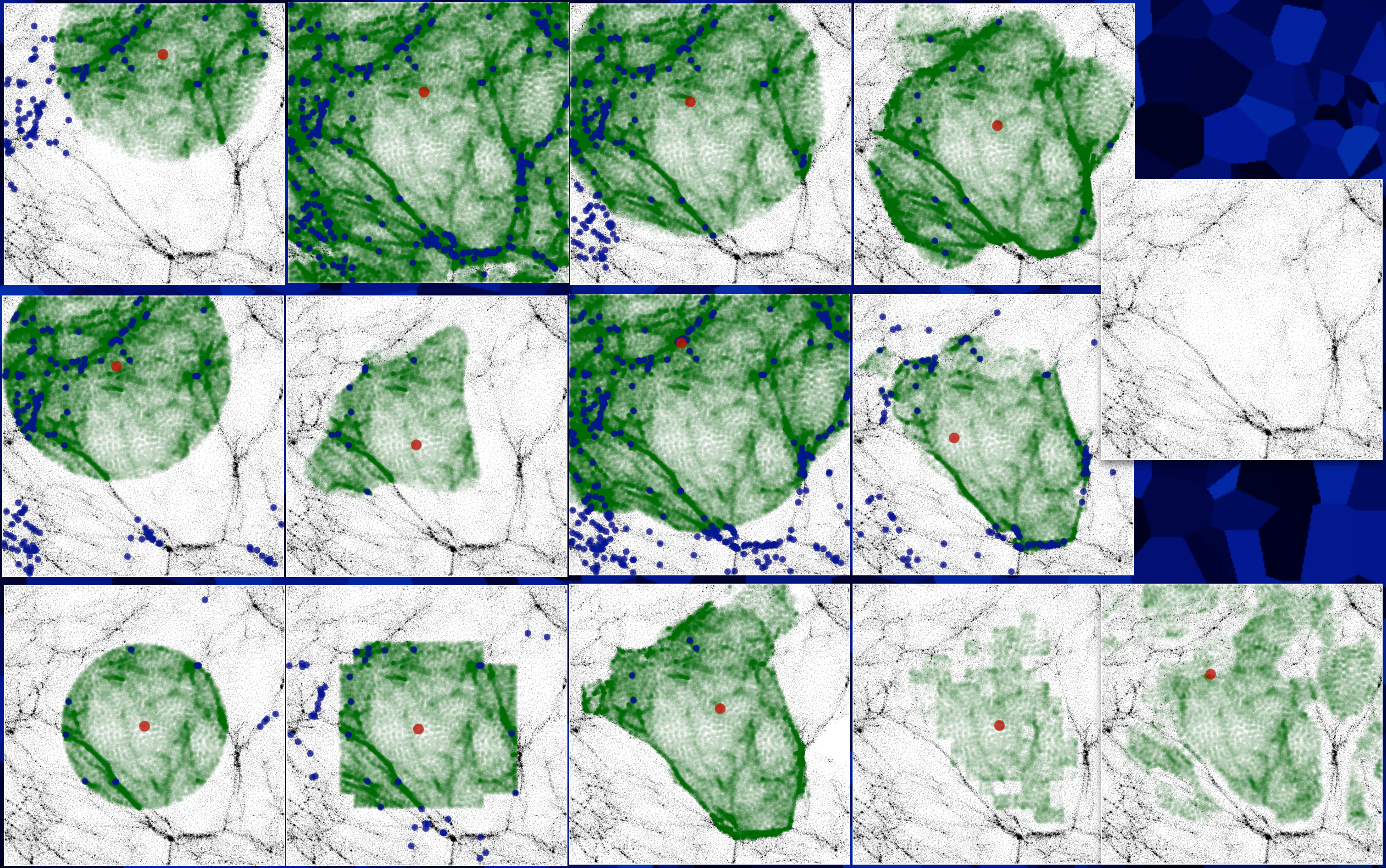
Galaxies are assumed to be points from the view of large-scale structure. Tessellation methods naturally suited.

Delaunay, Voronoi Tessellation Field Estimators (DTFE, VTFE) VTFE measures densities exactly at galaxies; DTFE relatively easily to interpolate. (van de Weygaert, Schaap, Platen ...)





Void, cluster finding



Void, cluster finding:

Philosophy similar to Erwin Platen's Watershed Void Finder:

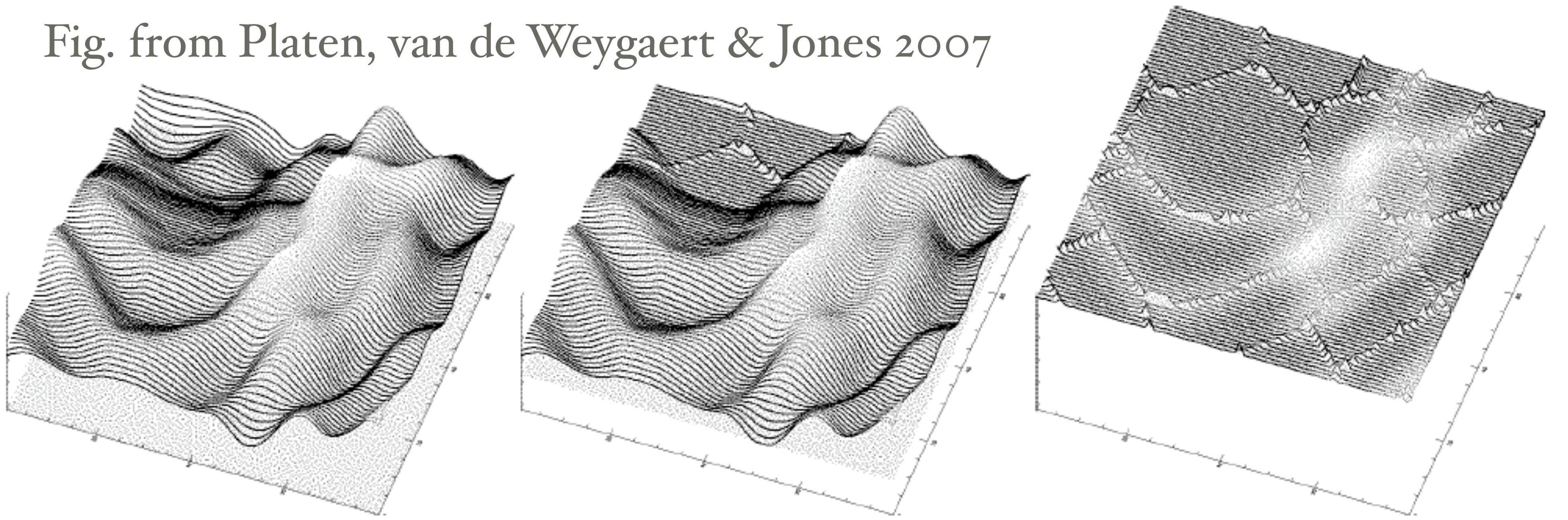
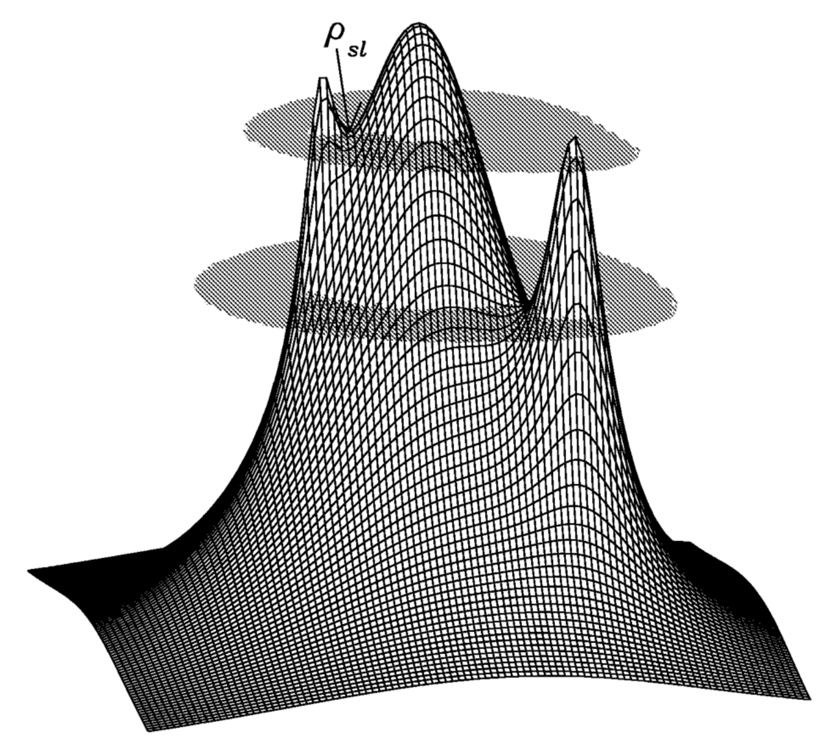
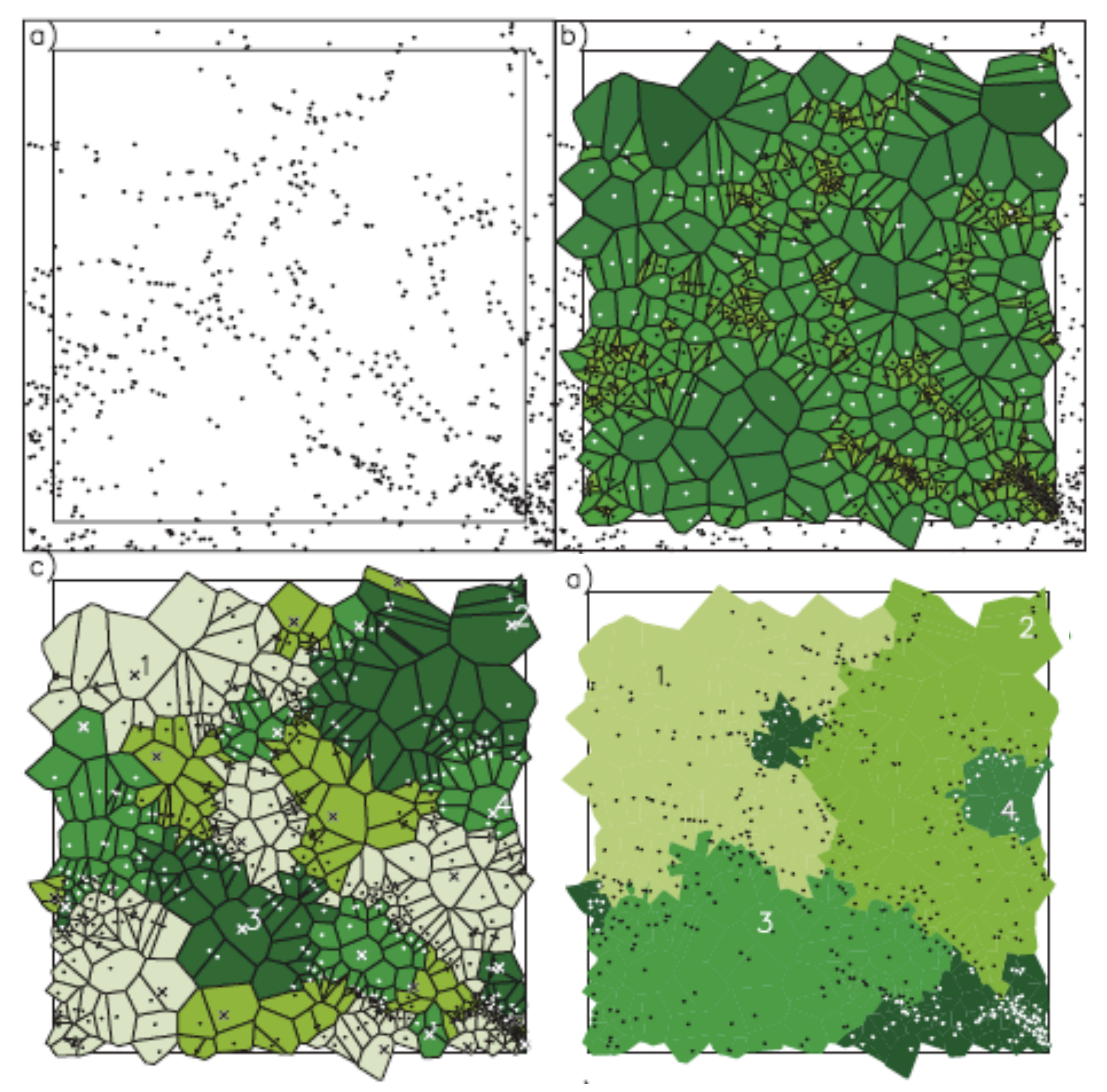


Fig. from Platen, van de Weygaert & Jones 2007

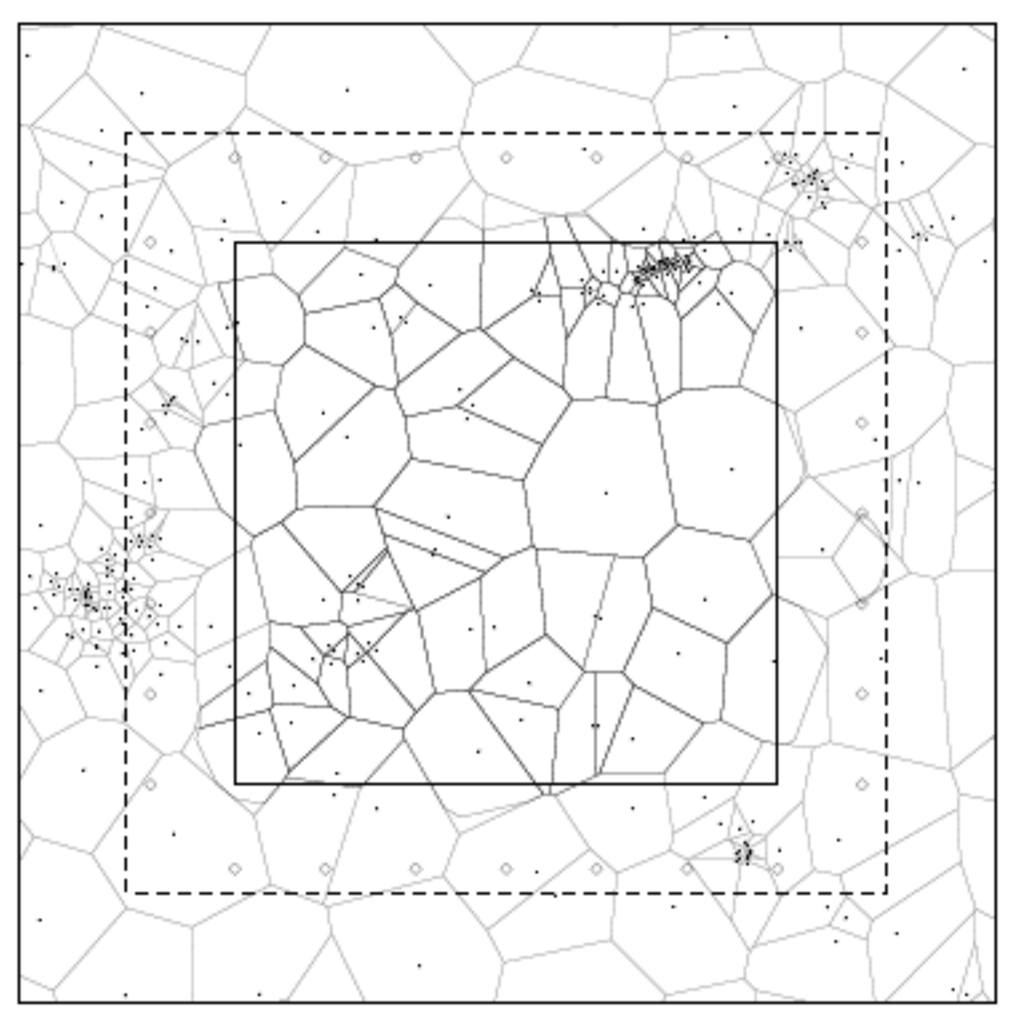
- Parameter-free (although so far for practical use I've used a density contrast/significance threshold)
- ZOBOV void/VOBOZ (Neyrinck, Gnedin & Hamilton 2005) cluster: catchment basin in the density field.
- "Water" flows along steepest gradients to Voronoi neighbors
- Freely available: http://skysrv.pha.jhu.edu/~neyrinck/zobov

- Measures probability (based on density contrast) that each void/cluster arose from discreteness noise. Spurious vloidsters joined with deeper ones.





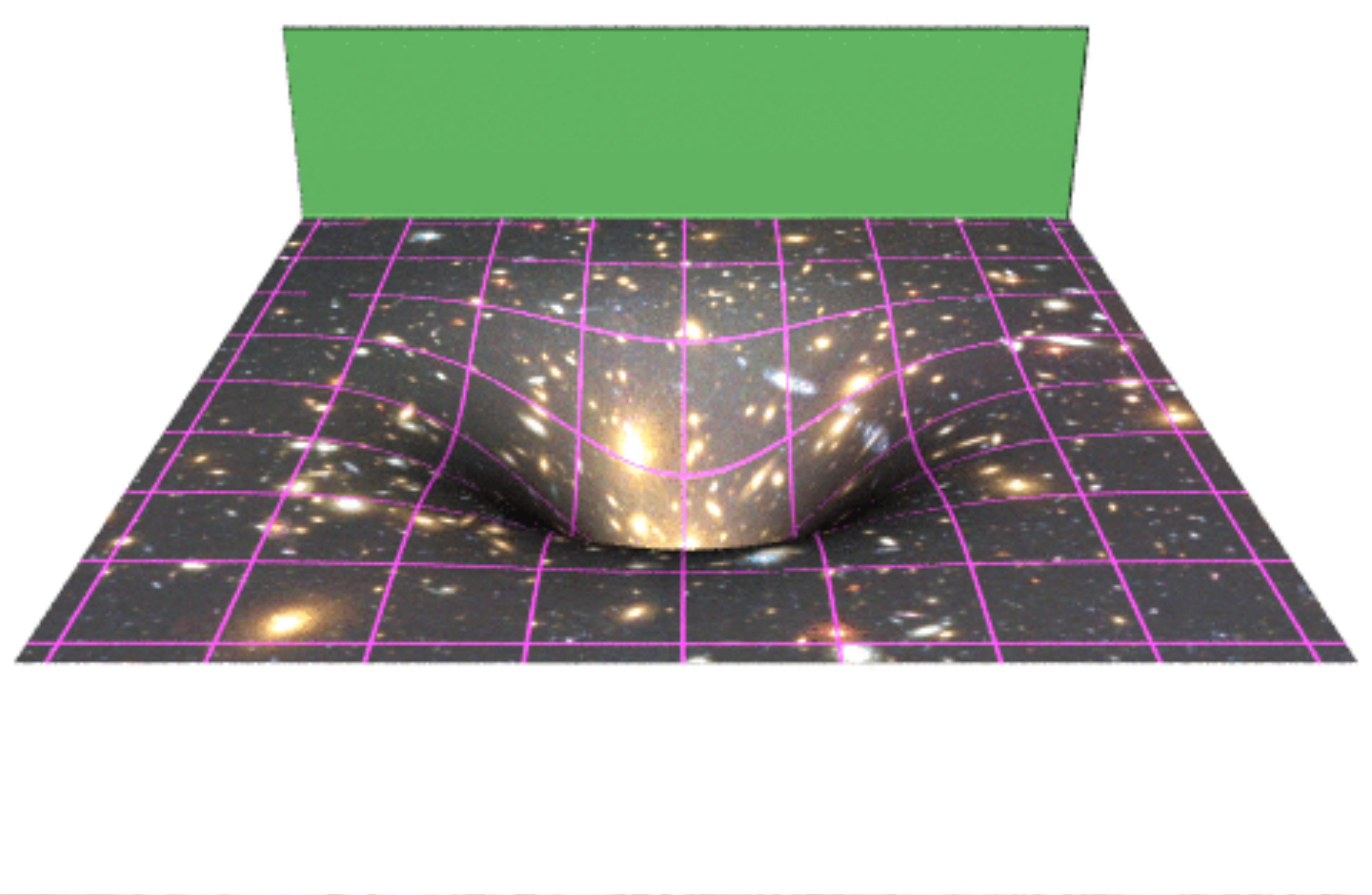
Maybe-interesting way to tessellate subboxes (could be applied to periodic boxes too): deploy “guard points” in a buffer around a region of interest that, if encountered as Voronoi adjacencies, signal that the buffer should be extended.



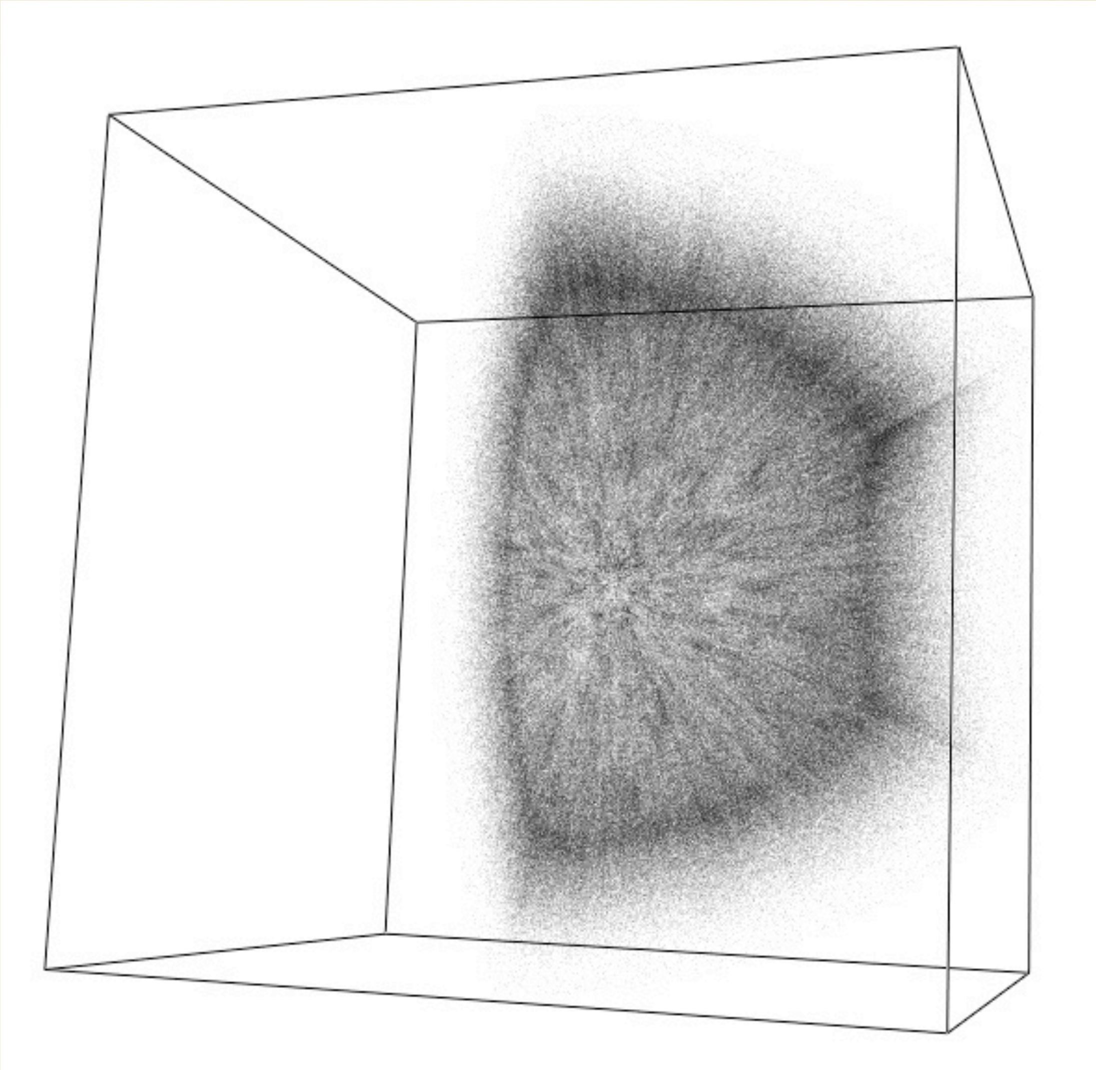
Application of $^{\mathrm{V}}_{\mathrm{Z}}\mathrm{OBO}^{\mathrm{Z}}_{\mathrm{V}}$:

Integrated Sachs-Wolfe (ISW) effect: unusually direct sign of "dark energy"/ cosmic acceleration

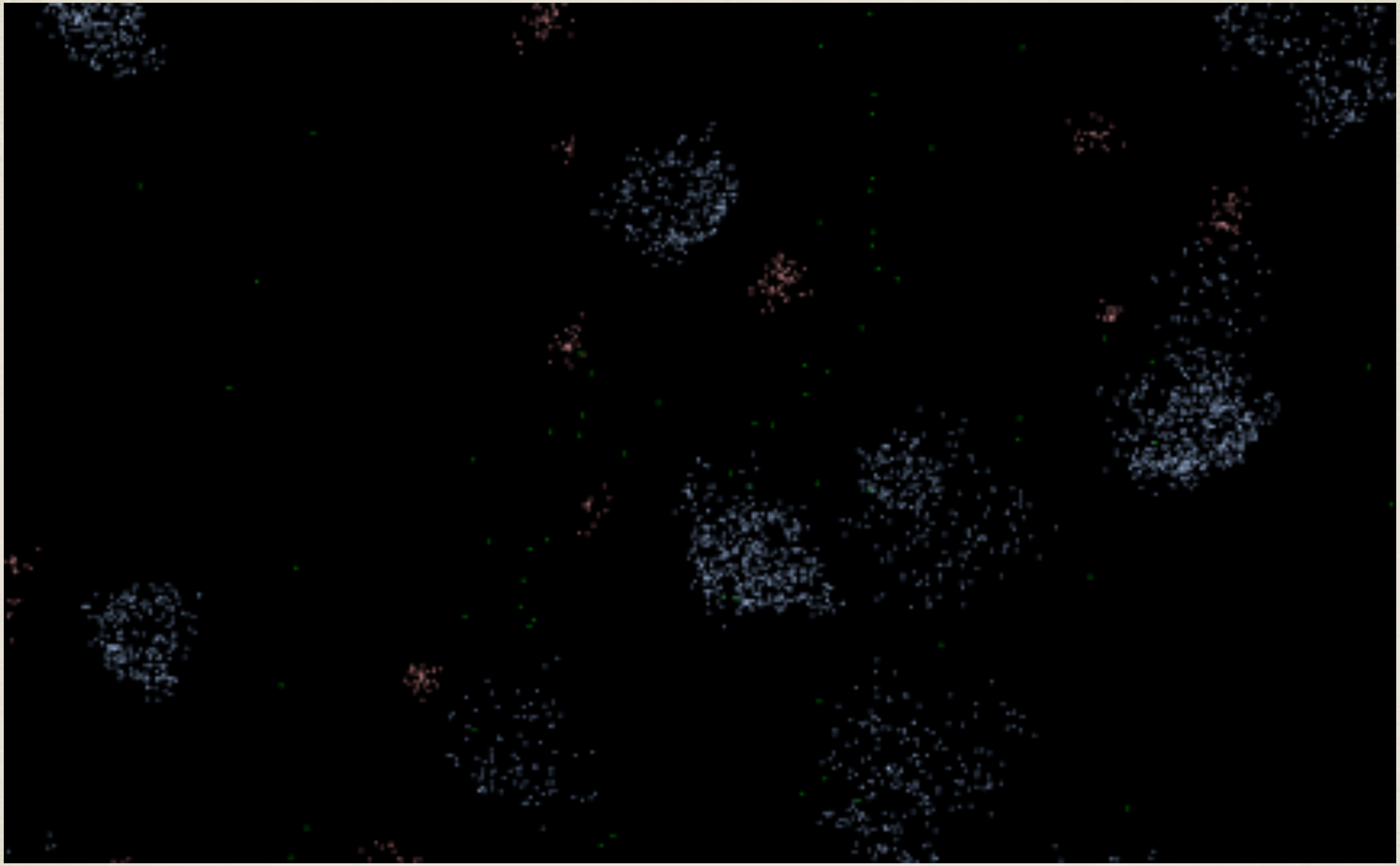
$$\frac{\Delta T_{\mathrm{ISW}}}{T_{\mathrm{CMB}}} = -2 \int d\tau \frac{d\Phi}{d\tau}$$



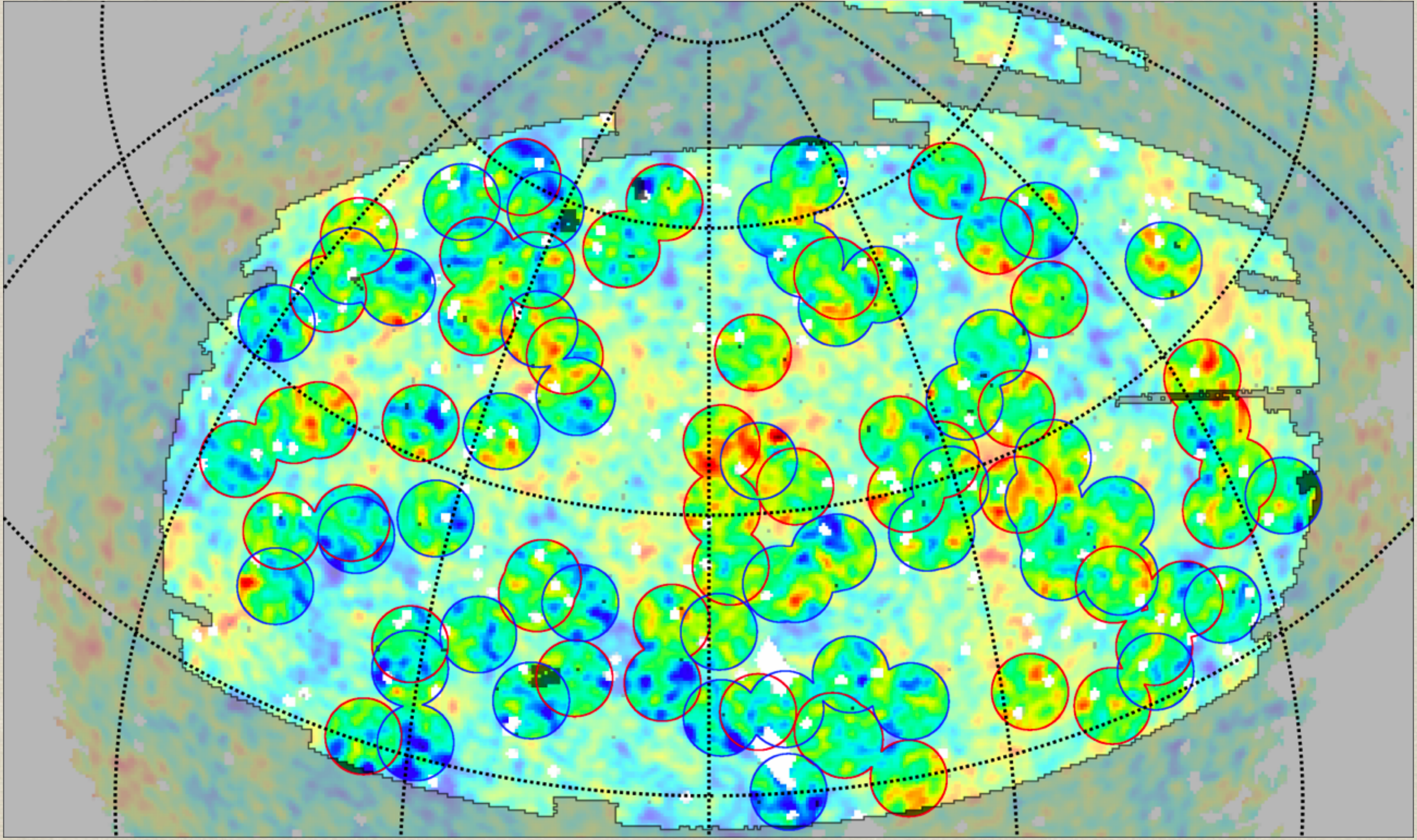
ISW effect has been measured using a cross-correlation function, but we detected it directly with voids/clusters of galaxies.



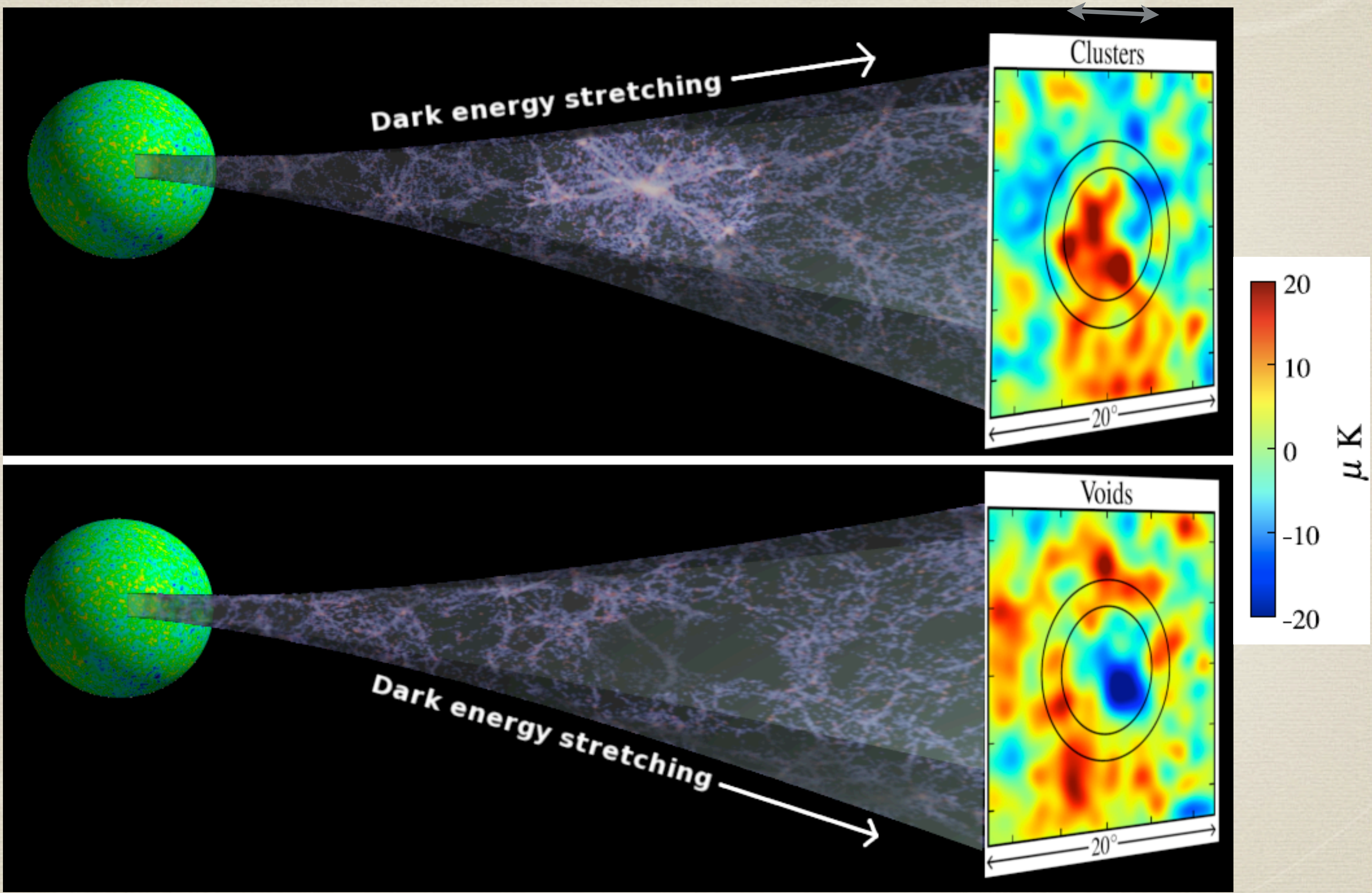
Used very bright "Luminous Red Galaxies" (LRG's), sample a vast volume of ~4 Gpc^3



CMB behind supervoids & superclusters



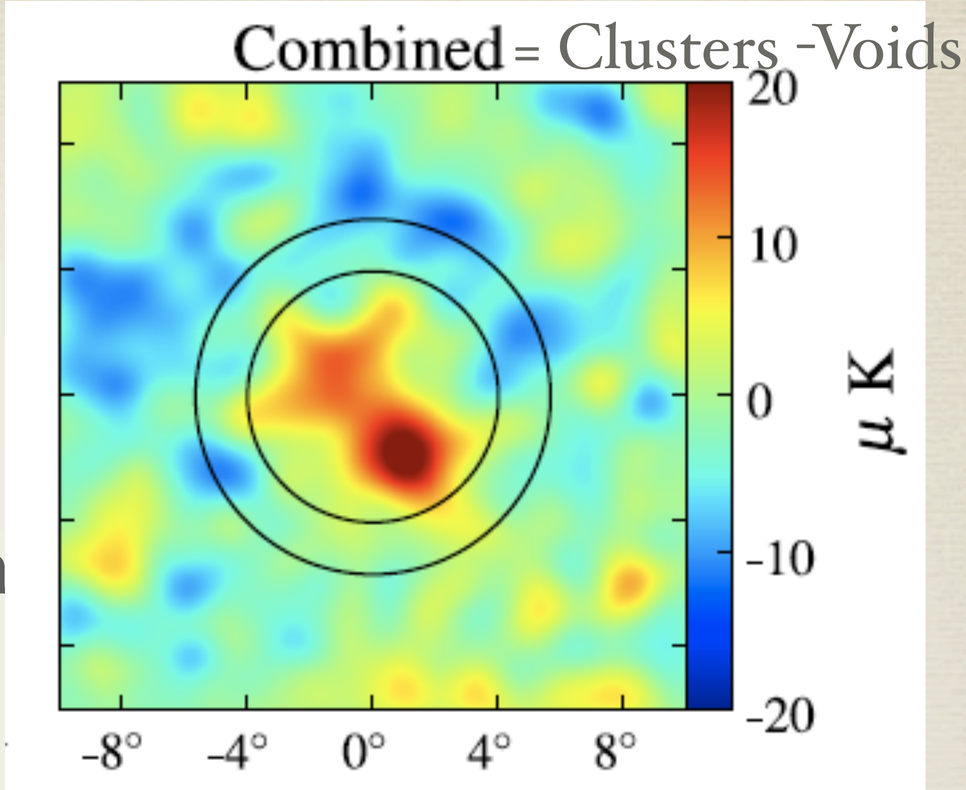
Stacked (average) signals:


~ 100 Mpc/h

Statistical significance & details:

Granett, Neyrinck & Szapudi, ApJL, 0805.3695

- ~4σ detection from SDSS Luminous Red Galaxies
- Used top 50 supervoids, 50 superclusters (roughly a 3-σ detection threshold). Too few wouldn't beat down "noise" (primordial CMB); too many would introduce dubious structures.



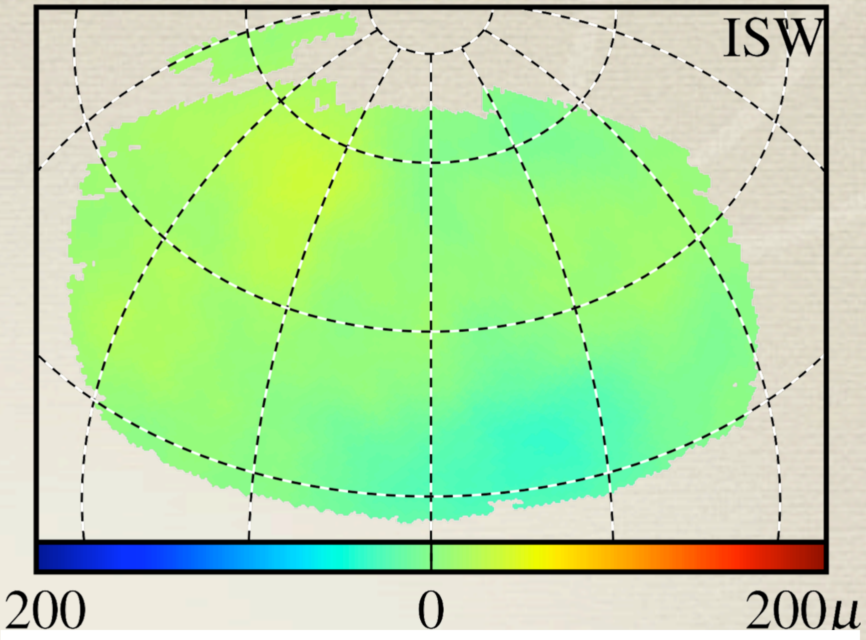
Measuring the (linear theory) signal:

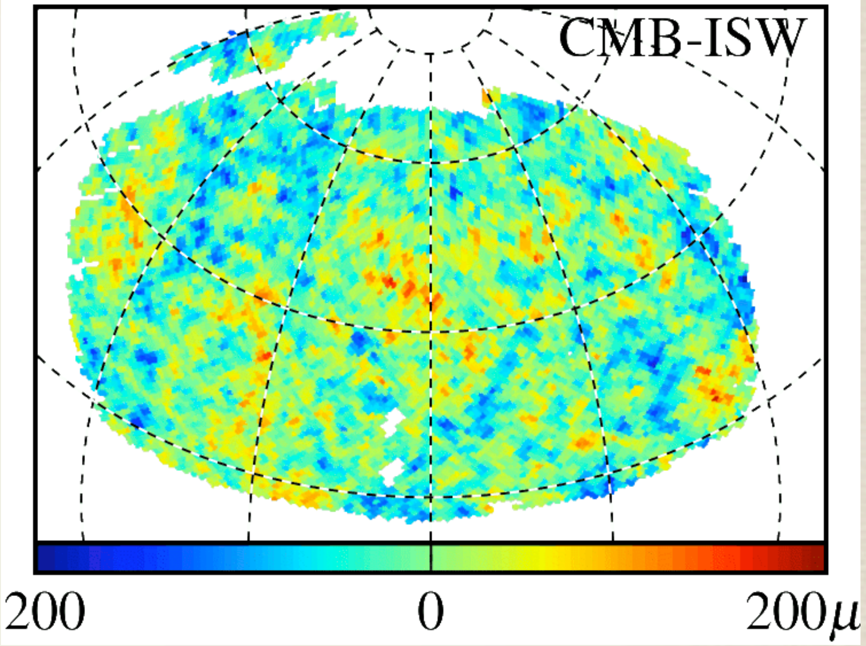
Granett, Neyrinck & Szapudi, ApJ 701, 414

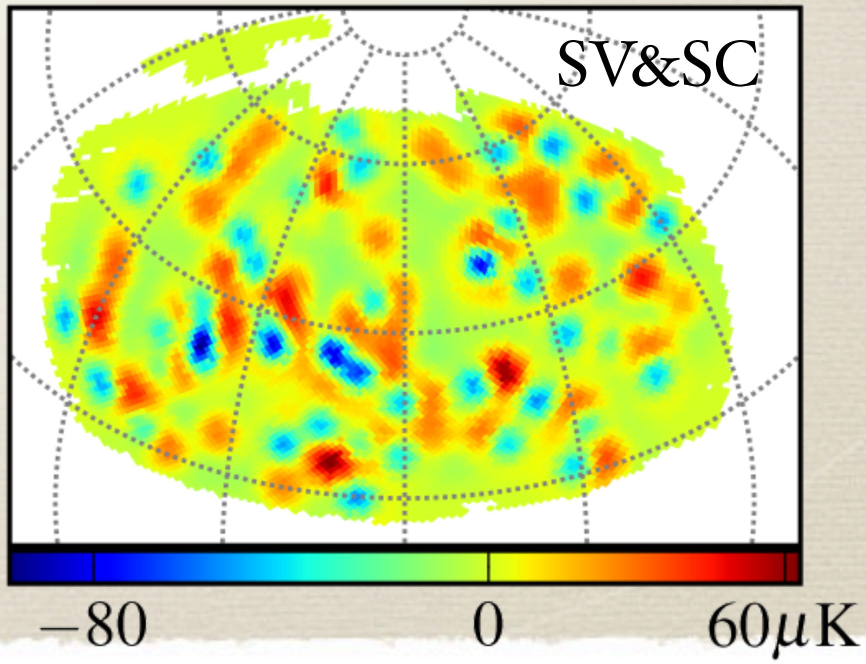
$$\Phi(\mathbf{r}) = -\frac{G\rho_{\mathrm{cr},0}\Omega_m}{c^2}(1+z_{\mathrm{med}})\sum_i \frac{\delta_i V_i}{|\mathbf{r}-\mathbf{r}_i|},$$

$$= -\frac{G\rho_{\mathrm{cr},0}\Omega_m}{c^2}(1+z_{\mathrm{med}})\sum_i \frac{\overline{V}(z_i)-V_i}{|\mathbf{r}-\mathbf{r}_i|}.$$

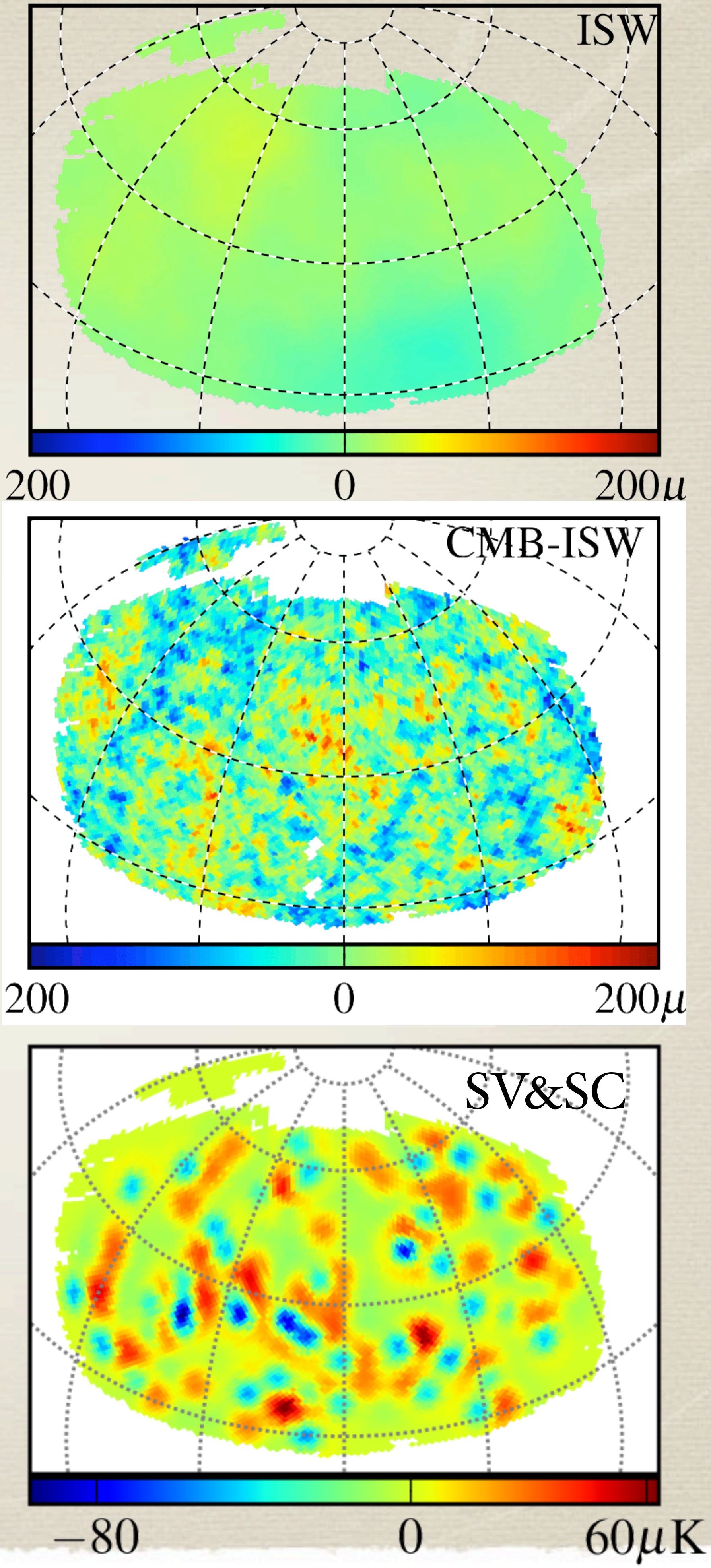
- Gravitational potential estimated from the 3D galaxy distribution
- Benefit of Voronoi tessellation: direct sum over galaxies, allowing true survey geometry to be used (unlike e.g. FFT methods)
- Small drawback: some galaxies on edges cannot be used





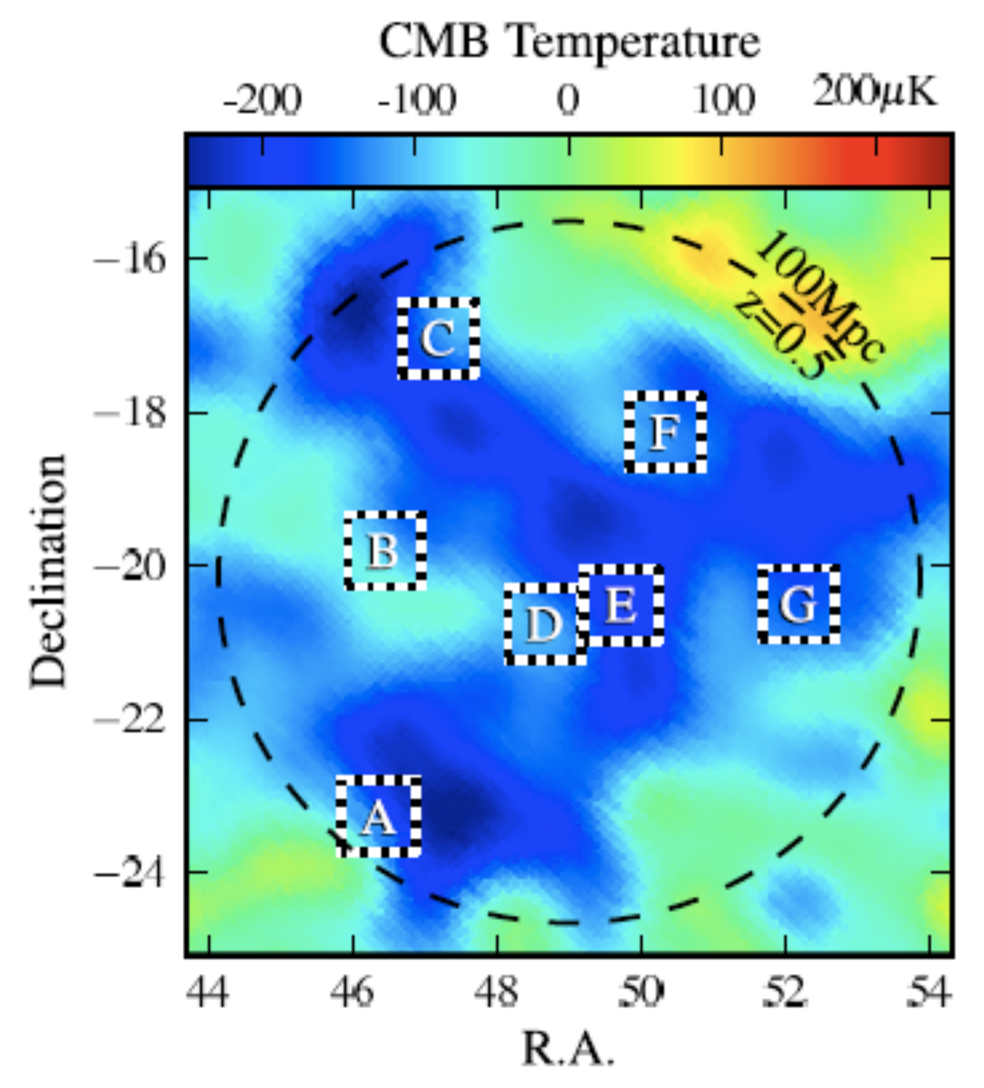


- Somewhat puzzling results: the signal we measured from supervoids and superclusters was hardly there in the potential reconstruction. Speculations:
  - large-scale voids, clusters more “powerful” (larger, deeper, more expansive, more nonlinear) on large scales than in the current cosmological paradigm?
  - Something dampens both visible light and microwaves that is not taken into account?



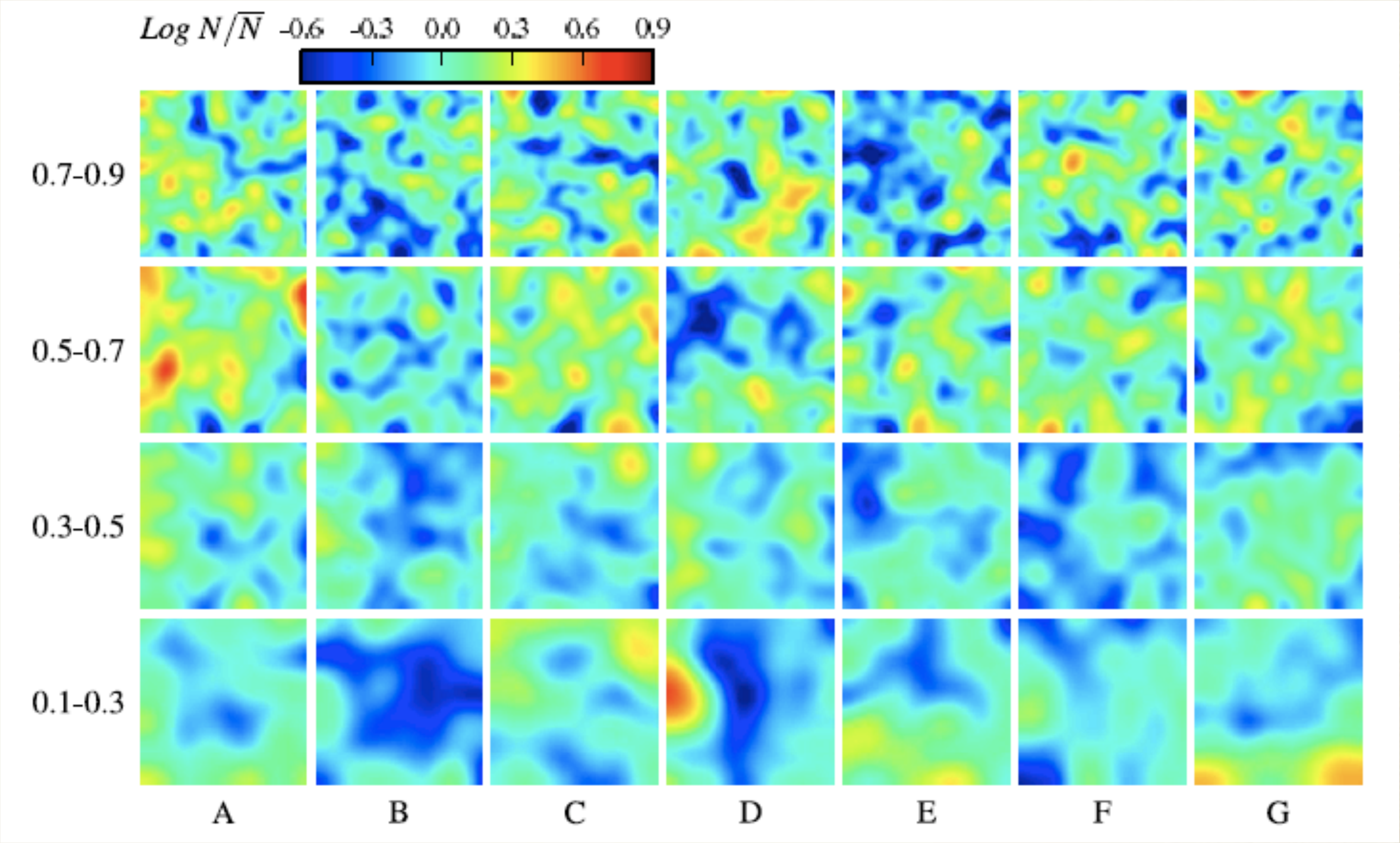
CMB Cold Spot

- A ~3-sigma cold, large fluctuation in the assumed Gaussian field of the CMB (e.g. Vielva et al. 2004)
- One possible explanation is a supervoid hinted at in radio galaxy observations (Rudnick et al. 2007), that got some news coverage
- No good galaxy surveys cover it (it's in the South), so we observed a few fields in it in Hawai`i.
  (Granett, Szapudi & Neyrinck; arXiv:0911.2223)



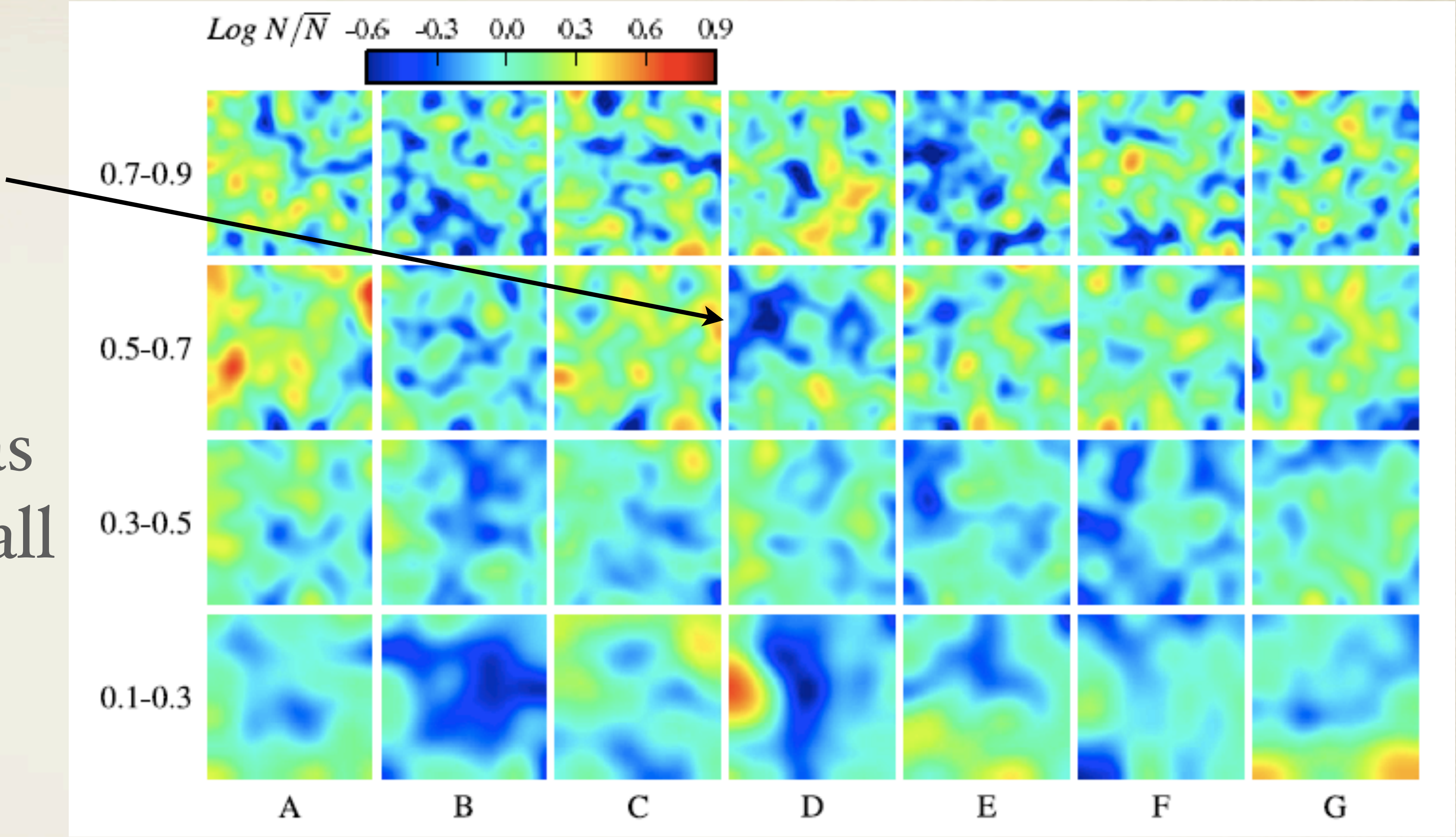
CMB Cold Spot

- Somewhat unfortunate geometry of each pencil beam: 1° square. ~3 Gpc long!
- To use a Voronoi tessellation, we distorted the pencil beam into a unit cube. (Still, 1/2 of galaxies thrown out)



- Treats 3 dimensions equally; not a bad approximation since the distance errors are very large.

CMB Cold Spot

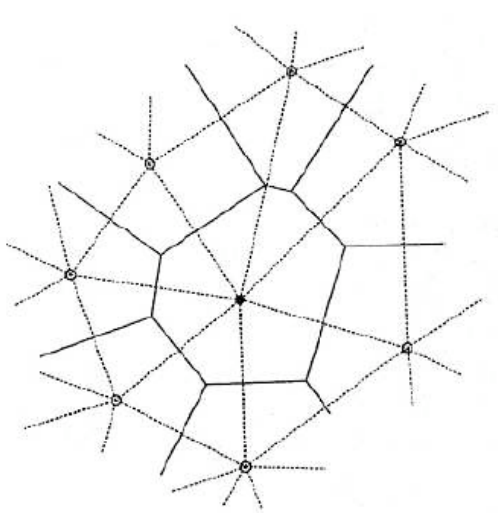


- ZOBOV did detect the void in field D, as the deepest void in all fields (these + 15 control fields). But (probably) not a supervoid!
- a hint of underclustering/underdensity in the cold spot, but nothing dramatic

Measuring div(***q***)

- *q* = Lagrangian displacement of particles between initial, final coordinates
- Used Delaunay-based estimator for div(***q***):

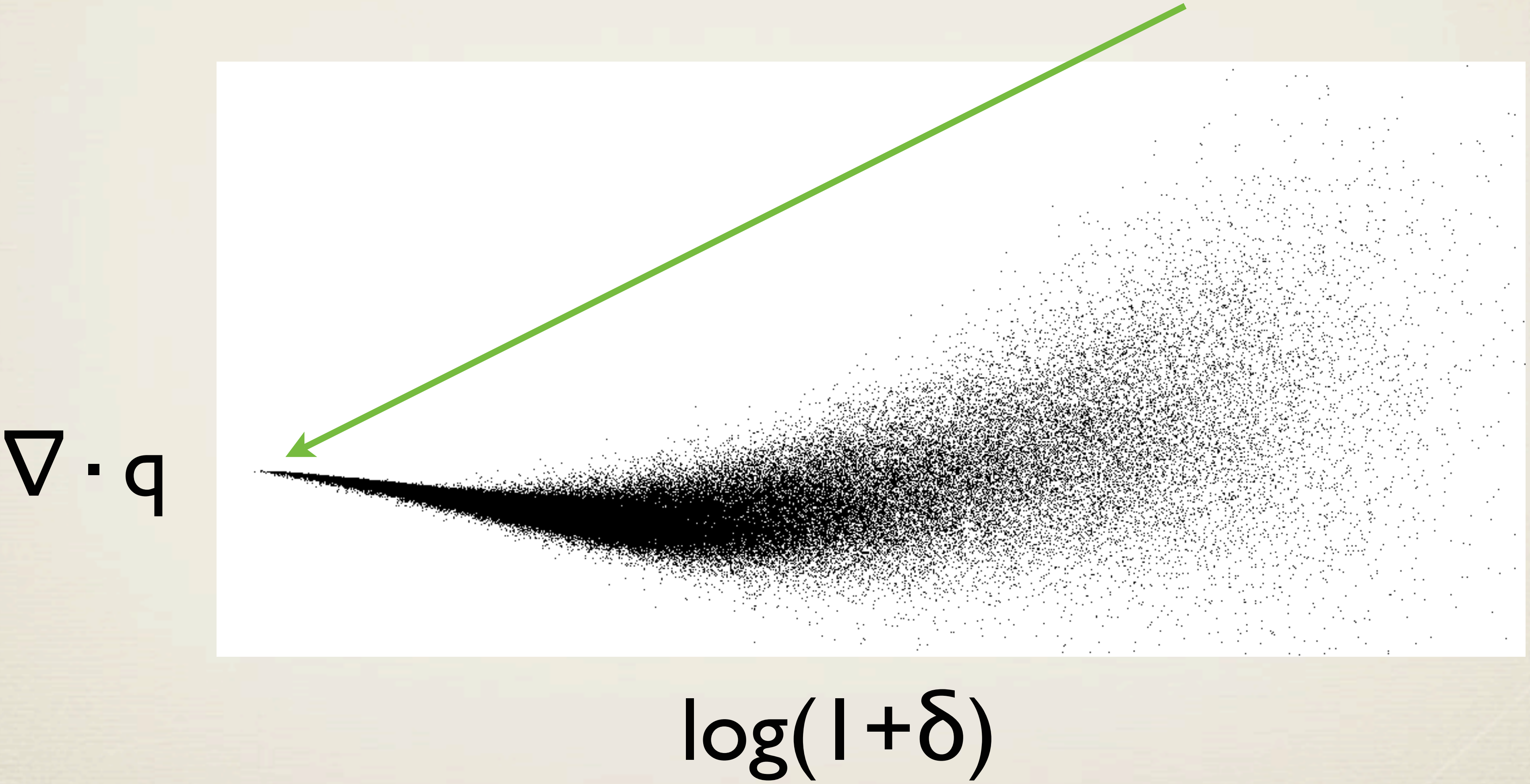
$$\nabla \cdot \mathbf{q} = \frac{1}{\sum_i V_i} \sum_i A_i \mathbf{q}_i \cdot \hat{\mathbf{n}}_i$$

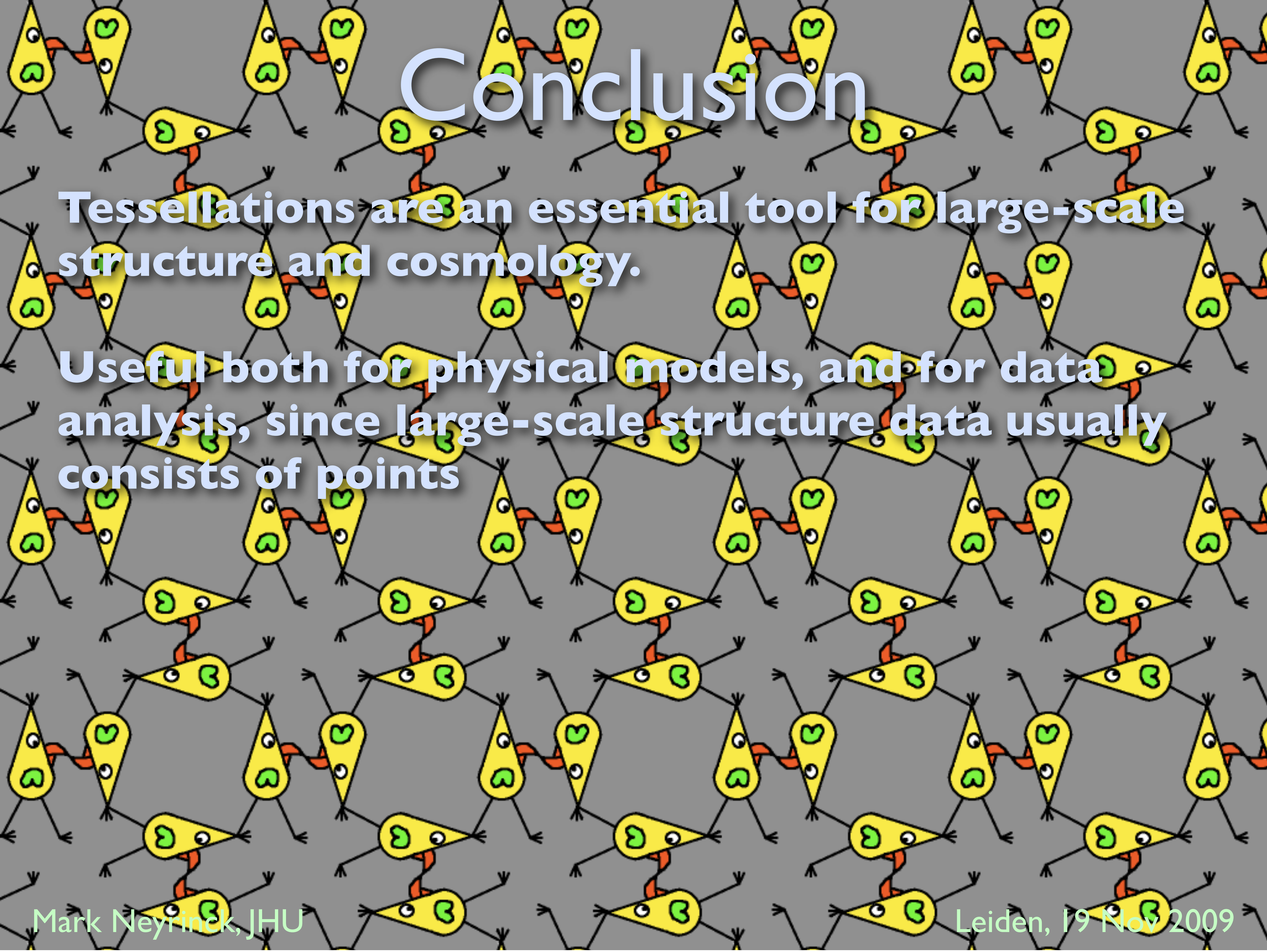


- Voronoi-based estimators less sensitive to changes in the tessellation, but this seems ok given a snapshot.
- Strange property: doesn't depend on ***q*** at the point in question (like measuring a derivative using *x*+*h*, *x*-*h*)

Measuring div(***q***)

- Preliminary results: scatter plot of particles.
- I wouldn’t know how to get low-density tail (that has the nice relationship) without tessellations.





Conclusion

Tessellations are an essential tool for large-scale structure and cosmology.

Useful both for physical models, and for data analysis, since large-scale structure data usually consists of points