ATELIER INTERNATIONAL SUR

LA THÉORIE (ALGÉBRIQUE et ANALYTIQUE) DES RÉSIDUS

ET SES APPLICATIONS

PARIS (IHP) du 18 au 20 MAI, 1995

La théorie des résidus est un domaine qui connaît une récente et intéressante activité. Plusieurs groupes de travail se sont penchés sur ce thème durant cette année. Cet atelier est donc l'occasion d'une présentation, confrontation et synthèse des idées sur le sujet, aussi bien d'un point de vue théorique que pratique.

Quelques points d'intérêt :

- Unification des différentes approches algébriques sur les résidus.
- Connexion entre les approches analytiques et algébriques.
- Complexes résiduels et leur constructions.
- Théorèmes de Briançon-Skoda.
- Calculs effectifs de sommes de résidus.
- Applications des résidus à la théorie de l'élimination, aux formules de représentations, aux résolutions de systèmes polynomiaux.

Orateurs invités :

Reinhold HÜBL, Craig HUNEKE, Ernst KUNZ, Joseph LIPMAN.

Organisateurs :

Marc CHARDIN, Centre de Mathématiques
Ecole polytechnique F-91128 Palaiseau (France)

Mohamed ELKADI, Dept. de Mathématiques,
Univ. de Nice Sophia-Antipolis, Parc Valrose, 06034 Nice (France)

Bernard MOURRAIN, INRIA, Projet SAFIR
2004, route des Lucioles F-06565 Valbonne (France)

Les organisateurs ont été aidés dans leur travail par les participants de deux séminaires "informels" sur ce sujet, l'un à Nice (Formes & Formules) et l'autre à Paris, et aussi par David EISENBUD et Jean-Pierre JOUANOLOU.

Cet atelier a reçu le soutien du Laboratoire GAGE, École polytechnique (Palaiseau), et du projet SAFIR, (Nice Sophia-Antipolis) à travers le programme HCM, SAC.

INTERNATIONAL WORKSHOP ON

ALGEBRAIC AND ANALYTIC THEORY OF RESIDUES

AND ITS APPLICATIONS

PARIS (IHP) 18-20 MAY 1995

Residues theory is a field which has known recent and interesting activities. Several working groups have focus on this theme during the last years. The workshop is the occasion of a presentation, confrontation and synthesis of ideas on this subjects as well from a theoretical point of view, as for its applications.

Some points of interest:

- Unification of the different algebraic approaches on residues.
- Residual complexes and their construction.
- Connection between the analytic and algebraic approaches.
- Briançon-Skoda theorems.
- Effective computation of sums of residues.
- Application of Residue Theory to elimination theory, representation formulas and polynomial systems solving.

Invited speakers:

Reinhold HÜBL, Craig HUNEKE, Ernst KUNZ , Joseph LIPMAN

Organizers:

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The organizers were (and are) helped in their task by the participants of two informal seminars on this subject, one in Nice and one in Paris, and by David EISENBUD and Jean-Pierre JOUANOLOU.

This workshop is supported by the Laboratoire GAGE, Ecole polytechnique (Palaiseau), the project SAFIR, (Nice Sophia-Antipolis) via the HCM programm SAC.

PROGRAMME - PROGRAM

JEUDI - THURSDAY

MATIN - MORNING

9h30 - 10h30: *Amnon YEKUTIELI*
Adeles and the De Rham-Residue Complex

11h - 12h: *Uwe STORCH*
Resultants for homogeneous regular sequences

APRES-MIDI - AFTERNOON

14h - 14h45: *Carlos BERENSTEIN*
On the residue formula of Jacobi and its applications

15h - 15h45: *Alain YGER*
Integral representation formulas and multidimensional residues

16h15 - 17h: *Alicia DICKENSTEIN*
Residues in Toric Varieties

17h15 - 18h: *Marie-Francoise ROY*
Quadratic forms and Bezoutians

VENDREDI - FRIDAY

MATIN - MORNING

9h30 - 10h30: *Craig HUNEKE*
Tight closure and theorems of Briancon-Skoda type

11h - 12h: *Joseph LIPMAN*
Formal Duality, Fundamental Class, and the Residue Theorem

APRES-MIDI - AFTERNOON

14h - 15h: *I-Chiau HUANG*
An explicit construction of residual complexes

15h15 - 16h: *Leovigildo ALONSO-TARRIO*
Formal Completion and Duality

16h30 - 17h15: *Mikael PASSARE*

Courants résiduels et faisceaux dualisants

17h30 - 18h15: *Teresa KRICK*

Elimination by arithmetic circuits. The duality tool

SAMEDI - SATURDAY

MATIN - MORNING

9h30 - 10h30: *Ernst KUNZ*

Generalization of a theorem of Chasles

11h - 12h: *Reinhold HÜBL*

Generalization of a theorem of Waring

APRES-MIDI - AFTERNOON

14h -14h30: Abdellah AL AMRANI

Fibrés projectifs tordus et classes de Chern

14h:45 -15h30: *Salomon OFMAN*

Application de l'algorithme de Gelfand-Leray-Shilov aux courants-résidus

15h:30 - 16h15: *Djilali BOUDIAF*

Interprétation des résidus composés à l'aide des courants

Interprétation des résidus composés à l'aide des courants résiduels

Djilali Boudiaf

Résumé

Soient X une variété analytique complexe de dimension n et $\mathcal{F} = \{Y_1, \ldots, Y_p\}$, $1 \leq p \leq n$, une famille ordonnée d'hypersurfaces complexes de X de réunion $\widetilde{Y}$ et d'intersection Y telle que chaque sous-famille $\mathcal{F}_i = \{Y_1, \ldots, Y_i\}$, $2 \leq i \leq p$, soit en position d'intersection complète; c'est-à-dire, $\dim_{\mathbb{C}} \cap \mathcal{F}_i = n - i$, où $\cap \mathcal{F}_i = Y_1 \cap \cdots \cap Y_i$. On désigne par $\mathcal{E}^{\cdot}_X(*\widetilde{Y})$ (resp. $'\mathcal{D}^{\cdot}_X$, $'\mathcal{D}^{\cdot}_{Y^\infty}$) le complexe (des faisceaux des germes) des formes semi-méromorphes sur X à pôles sur $\widetilde{Y}$ (resp. des courants sur X, à supports sur Y) et par $\Gamma_Y(X; '\mathcal{D}^{\cdot}_X)$ l'ensemble des sections globales de $'\mathcal{D}^{\cdot}_X$ à support dans Y. Suivant Herrera-Lieberman [5], pour le cas $p = 1$, on montre que le diagramme de cohomologie suivant est commutatif :

$$\begin{array}{ccc} H^q\Gamma(X; \mathcal{E}^{\cdot}_X(*\widetilde{Y})) & \underset{\simeq}{\xrightarrow{I(*\widetilde{Y})}} & H^q(X \backslash \widetilde{Y}; \mathbb{C}) \\ \overline{R^p_{\mathcal{F}}} \Big\downarrow & & \Big\downarrow res^p_{\mathcal{F}} \\ H^{q+p}\Gamma_Y(X; '\mathcal{D}^{\cdot}_X) & \underset{\tau_Y}{\xrightarrow{\simeq}} & H^{q+p}_Y(X; \mathbb{C}) \end{array}$$

où $I(*\widetilde{Y})$ est l'isomorphisme de Grothendieck [4], τ_Y est un isomorphisme, composé de l'isomorphisme de Poly [8]

$$H^{q+p}\Gamma_Y(X; '\mathcal{D}^{\cdot}_X) \simeq H_{2n-q-p}\Gamma_Y(X; '\mathcal{D}_{\cdot, X}) \longrightarrow H_{2n-q-p}(Y; \mathbb{C})$$

et de
$$H_{2n-q-p}(Y; \mathbb{C}) \longrightarrow H^{q+p}_Y(X; \mathbb{C}),$$

inverse de l'isomorphisme de dualité de Poincaré $\cap[X]$ obtenu par cap-produit par la classe fondamentale de X, $\overline{R^p_{\mathcal{F}}}$ est l'*homomorphisme résiduel* défini dans [1] à partir de l'opérateur de Coleff-Herrera $R^p_{\mathcal{F}} : \mathcal{E}^q_X(*\widetilde{Y}) \longrightarrow {'\mathcal{D}}^{q+p}_{Y^\infty}$ [2] et enfin $res^p_{\mathcal{F}}$ est l'*homomorphisme résidu composé* défini par Poly [9] à partir de travaux de Sorani [10]. C'est la généralisation de l'homomorphisme de Leray [6] et Norguet [7] : en effet, si les hypersurfaces Y_j sont *lisses* et *en position générale* alors l'homomorphisme $res^p_{\mathcal{F}}$ associe à toute classe de cohomologie $[\widetilde{\omega}]$ de $X \setminus \widetilde{Y}$ la classe (de cohomologie) *p-résidu composé* de Leray-Norguet notée $res^p_{\mathcal{F}}[\widetilde{\omega}]$. Si ω est une forme semi-méromorphe à pôles *simples* sur $\widetilde{Y}$ alors la *forme p-résidu composé* $res^p_{\mathcal{F}}[\omega]$ est définie et on a la relation

$$R^p_{\mathcal{F}}[\omega] = (2\pi i)^p I[Y] \wedge res^p_{\mathcal{F}}[\omega]$$

où le premier membre est le courant résiduel associé à ω et $I[Y]$ est le courant d'intégration de Lelong sur Y.

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Residues in Toric Varieties

Eduardo Cattani, David Cox and Alicia Dickenstein

Toric residues provide a tool for the study of certain homogeneous ideals of the homogeneous coordinate ring of a toric variety —such as those appearing in the description of the Hodge structure of their hypersurfaces [BC]. They were introduced in [C2], where some of their properties were described in the special case when all of the divisors involved were linearly equivalent. The main results of the present work are: an extension of the Isomorphism Theorem of [C2] to the case of non-equivalent ample divisors, a global transformation law for toric residues, and a theorem expressing the toric residue as a sum of local (Grothendieck) residues.

Assume that X is a complete simplicial toric variety of dimension n. S denotes the homogeneous coordinate ring of X, which is the polynomial ring $S = \mathbf{C}[x_1, \ldots, x_{n+r}]$. Here, the variables x_i correspond to the generators of the 1-dimensional cones in the fan which determines X, and hence to torus-invariant irreducible divisors D_i of X. S is graded by declaring that the monomial $\Pi_{i=1}^{n+r} x_i^{d_i}$ has degree $[\sum_{i=1}^{n+r} d_i\, D_i]$ in the Chow group $A_{n-1}(X)$ ([C1]).

We let $\beta = \sum_{i=1}^{n+r} \deg(x_i) \in A_{n-1}(X)$ denote the anticanonical class on X. Then, given homogeneous polynomials $F_i \in S_{\alpha_i}$ for $i = 0, \ldots, n$, we define their *critical degree* to be $\rho = (\sum_{i=0}^{n} \alpha_i) - \beta \in A_{n-1}(X)$. Each $H \in S_\rho$ determines a meromorphic n-form $\omega_F(H) = \dfrac{H\,\Omega}{F_0 \cdots F_n}$ on X ([BC]).

If in addition the F_i don't vanish simultaneously on X, then relative to the open cover $U_i = \{x \in X : F_i(x) \neq 0\}$ of X, this gives a Čech class $[\omega_F(H)] \in H^n(X, \Omega_X^n)$. The *toric residue*

$$\mathrm{Res}_F : S_\rho / \langle F_0, \ldots, F_n \rangle_\rho \longrightarrow \mathbf{C}$$

is given by the formula

$$\mathrm{Res}_F(H) = \mathrm{Tr}([\omega_F(H)]),$$

where $\mathrm{Tr} : H^n(X, \Omega_X^n) \to \mathbf{C}$ is the trace map.

Our first main result is the following *Global Transformation Law*:

Theorem 1. *Let $F_i \in S_{\alpha_i}$ and $G_i \in S_{\beta_i}$ for $i = 0, \ldots, n$. Suppose*

$$G_j = \sum_{i=0}^{n} A_{ij}\, F_i,$$

where A_{ij} is homogeneous of degree $\beta_j - \alpha_i$, and assume the G_i don't vanish simultaneously on X. Let ρ be the critical degree for $F_0, \ldots, F_n$. Then, for each $H \in S_\rho$, $H \det(A_{ij})$ is of the critical degree for $G_0, \ldots, G_n$, and

$$\mathrm{Res}_F(H) = \mathrm{Res}_G(H \det(A_{ij})).$$

The proof uses a Čech cochain argument. One application of this transformation law is that in certain cases, we can describe explicit elements of S_ρ with nonzero residue. For this purpose, assume X is complete and its fan Σ contains a n-dimensional *simplicial* cone σ. Then denote the variables of the coordinate ring as $x_1, \ldots, x_n, z_1, \ldots, z_r$, where $x_1, \ldots, x_n$ correspond to the 1-dimensional cones of σ. Also suppose that $\alpha_0, \ldots, \alpha_n$ are $\mathbf{Q}$-ample classes, which means that some multiple is Cartier and ample. In this situation, each F_j can be written in the form

$$F_j = A_{0j}\, z_1 \cdots z_r + \sum_{i=1}^{n} A_{ij}\, x_i.$$

Then the $(n+1)\times(n+1)$-determinant $\Delta_\sigma = \det(A_{ij})$ is in S_ρ and has the following important property:

Theorem 2. *Assume X is complete and $\sigma \in \Sigma$ is simplicial and n-dimensional. Suppose that $F_i \in S_{\alpha_i}$ for $i = 0, \ldots, n$, where α_i is $\mathbf{Q}$-ample and the F_i don't vanish simultaneously on X. Then*

$$\mathrm{Res}_F(\Delta_\sigma) = \pm 1.$$

We also prove the following *Residue Isomorphism Theorem*:

Theorem 3. *Let X be complete and simplicial, and assume that $F_i \in S_{\alpha_i}$ for $i = 0, \ldots, n$, where α_i is ample and the F_i don't vanish simultaneously on X. Then:*

(i) The toric residue map $\mathrm{Res}_F : S_\rho/\langle F_0, \ldots, F_n\rangle_\rho \to \mathbf{C}$ is an isomorphism.

(ii) For each variable x_i, $0 \le i \le n+r$, we have $x_i \cdot S_\rho \subset \langle F_0, \ldots, F_n\rangle$.

In the case when all the α_i are equal to a fixed ample divisor α, this theorem follows from the fact that $F_0, \ldots, F_n$ are a regular sequence in the Cohen-Macaulay ring $S_{*\alpha} = \oplus_{k\ge 0} S_{k\alpha}$ [C2,§3]. In the general case, the proof relies on the use of the Cayley trick and results of Batyrev and Cox [BC] concerning the cohomology of projective hypersurfaces in toric varieties, to show that

$$\dim(S_\rho/\langle F_0, \ldots, F_n\rangle_\rho) = 1$$

when X is simplicial and the divisors $F_i = 0$ are ample with empty intersection. Then, the first (and main) part of the Residue Isomorphism Theorem follows immediately from Theorem 2, and the second part is a consequence of the first using Theorem 2 and Cramer's Rule.

As a corollary of Theorems 2 and 3, we get a simple algorithm for computing toric residues in terms of normal forms.

We also show that for simplicial toric varieties, the toric residue may be computed as a sum of local Grothendieck residues. The toric setting is not essential here and, in fact, it is convenient to work with the more general notion of a V-manifold or orbifold (see [Sa]). The proof of the following local/global theorem is based on the theory of residual currents ([CH]).

Theorem 4. *Let X be a complete simplicial toric variety of dimension n, and let $F_0, \ldots, F_n$ be homogeneous polynomials which don't vanish simultaneously on X. If $H \in S_\rho$, where ρ is the critical degree and $D_{\hat{k}} = \{x \in X : F_i(x) = 0,\ i \ne k\}$ is finite, then the toric residue is given by*

$$\mathrm{Res}_F(H) = (-1)^k \sum_{x\in D_{\hat{k}}} \mathrm{Res}_{k,x}\left(\frac{H\,\Omega}{F_0 \cdots F_n}\right).$$

Note that the finiteness condition holds automatically whenever the divisor $\{F_k = 0\}$ is $\mathbf{Q}$-ample. Under appropriate conditions, Theorem 4 gives a framework for the study of sums of local residues —both in the affine and toric cases— as a global residue defined in a suitable toric compactification. It is possible, for example, to interpret in this light the results of [CDS] which correspond to the case when the toric variety under consideration is a weighted projective space.

Finally, we show that, in the equal degree case, the toric residue equals a single local residue at the origin of the affine cone of X. This generalizes the observation in [PS] that toric residues on $\mathbf{P}^n$ can be written as a residue at the origin in $\mathbf{C}^{n+1}$.

*

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Generalization of a Theorem of Waring

Reinhold Hübl

In this talk we will present results obtained jointly with E. Kunz.

In the 19^{th} century many beautiful theorems have been proved about the intersection of two plane curves. Many of these results can be obtained by rather explicit calculations, using the equations of the curves. In some cases however a proof can also be given using residues of differential forms and the residue theorem on curves as already introduced by Serre, and these proofs very often generalize to curves in higher-dimesional spaces. As an example of this method we will present a generalization of a theorem of Waring (c.f. [Co], p. 166).

Let K be a field of characteristic 0. Recall that for a zero-dimensional subscheme $S \subseteq \mathbb{A}^n_K$ of degree N with support $|S| = \{P_1, \ldots, P_r\}$, $P_i = (a^i_1, \ldots, a^i_n)$ the centroid $\Sigma(S)$ of S is defined to be the vector sum

$$\Sigma(S) := \frac{1}{N} \sum_{i=1}^{r} lg(\mathcal{O}_{S,P_i}) \cdot (a^i_1, \ldots, a^i_n)$$

in $\mathbb{A}^n_K$. For a reduced and irreducible curve $\Gamma \subseteq \mathbb{A}^n_K$ an asymptote to Γ is defined to be a tangent $A_{P,i}(\Gamma)$ to an analytic branch R_i of the projective closure $\overline{\Gamma}$ at a point $P \in \overline{\Gamma}$ at infinity. The asymptote cycle $A(\Gamma)$ of Γ is defined to be the formal sum over all asymptotes of Γ, counted with multiplicities. For an arbitrary curve $C \subseteq \mathbb{A}^n_K$ with reduced and irreducible components $C_1, \ldots, C_t$ the asymptote cycle $A(C)$ of C is given by

$$A(C) = \sum_{i=1}^{t} lg(\mathcal{O}_{C,C_i}) \cdot A(C_i)$$

In this situation we have the following higher-dimensional analogue of Waring's theorem:

Theorem. *Let $C \subseteq \mathbb{A}^n_K$ be a curve such that none of its asymptotes is contained in the hyperplane at infinitiy and let $H \subseteq \mathbb{A}^n_K$ be a hyperplane having no common points with C at infinity. Then*

$$\Sigma(C \cap H) = \Sigma(A(C) \cap H)$$

This theorem is proved by expressing centroids in terms of residues, and by showing that we get the same expression if we replace C by its asymptote cycle.

As an immediate consequence of Waring's theorem we conclude the following result which in the case of plane curves already was known to Newton.

Theorem. *For a curve C as above let H run through a family of parallel hyperplanes which are not asymptotic to C. Then all centroids $\Sigma(C \cap H)$ are on a line.*

Such a line is called a **diameter** of C. A closed point $M \in \mathbb{A}^n_K$ is called a **center** of C if it lies on all diameters. Waring's theorem also allows a classification of those curves that have centers (in the sense defined above).

*

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Tight closure and theorems of Briançon-Skoda type

Craig Huneke

A theorem of Briançon and Skoda, later generalized by Lipman and Sathaye, and still later by Lipman and Teissier, states that if R is a local ring which is a rational singularity of dimension d, then the integral closure of the dth power of an arbitrary ideal I, denoted $\overline{I^d}$, is contained in the ideal I. Recall the definition of the integral closure.

Definition Let I be an ideal of a Noetherian ring R. An element x is in the *integral closure* of I, $\overline{I}$, if x satisfies an equation of the form $x^k + a_1x^{k-1} + \cdots + a_k = 0$ where $a_i \in I^i$.

A more general version of the theorem of Lipman and Teissier states:

Theorem.. *Let (R, m) be a d-dimensional local ring which is a rational singularity. Let I be any ideal of R. Then for any $w \geq 0$*

$$\overline{I^{d+w}} \subseteq I^{w+1}.$$

A tight closure version was given by Hochster and Huneke in 1990. The definition of tight closure in positive characteristic is:

Definition Let I be an ideal of R, and $x \in R$. An element x is said to be in the tight closure of I if there exists an element c, not in any minimal prime of R, such that for all large $q = p^e$, $cx^q \in I^{[q]}$, where $I^{[q]}$ is the ideal generated by the qth powers of all elements of I. We denote the tight closure of I by I^*.

A definition can also be given for all Noetherian rings containing a field by using reduction to characteristic p.

The theorem of Hochster and Huneke states,

Theorem. *Let R be a d-dimensional Noetherian ring containing a field. Let I be any ideal generated by n elements. For all $w \geq 0$*

$$\overline{I^{n+w}} \subseteq (I^{w+1})^*.$$

In particular for all ideals I of R,

$$\overline{I^{d+w}} \subseteq (I^{w+1})^*.$$

The *test ideal* of the ring R is the intersection of $I : I^*$, where the intersection runs over all ideals I of R. We denote the test ideal by $\tau(R)$. If R is excellent reduced and has an isolated singularity, the test ideal is either the whole ring, or is m-primary, where m is the maximal ideal of R. In general if R_c is regular, c has a power which is a test element. This leads to the following theorem for an excellent reduced local ring (R, m) of dimensional d with isolated singularity: if m^t is contained in the test ideal, then for every ideal I, the integral closure of I^{d+t} is contained in I.

Of particular interest, then, is the power t of the maximal ideal m such that $m^t \subseteq \tau(R)$. If R is graded over an algebraically closed field of characteristic 0, a conjecture is that one can take $t = a + 1$, where a is the a-invariant of R, namely the highest degree of a nonzero element in the top local cohomology of R. This conjecture has been verified by Karen Smith and myself for graded rings which are complete intersections (with isolated singularity). It turns out that this conjecture is equivalent in the Gorenstein case to another conjecture which I will discuss which is in turn a generalization of the Kodaira vanishing theorem.

The talk will focus on the relationship between the Briançon-Skoda type theorems, the test ideal and theory of tight closure, and the Kodaira vanishing theorem.

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Elimination by arithmetic circuits. The duality tool.

Teresa Krick

Le but de l'exposé est de décrire une méthode informatique, le circuit arithmétique, qui permet de fournir des calculs d'évaluation pour les polynômes apparaissant dans les formules de représentation lors de la résolution des systèmes polynomiaux.

Plus précisément, on traitera le problème du Théorème des Zéros Effectif et celui de l'appartenance et la représentation pour des intersections complètes dans l'anneau de polynômes $K[X_1, \ldots, X_n]$:

Soit $I = (f_1, \ldots, f_s) \subseteq K[X_1, \ldots, X_n]$ *un idéal dont la variété des zéros dans la clôture algébrique de* K *est vide (resp. de dimension pure* $n - s$*, et dans ce cas soit* $f \in I$*).*
Posons $d := \max\{\deg f_i \ (, \deg f)\}$.
Alors il existe un circuit arithmétique de taille $sd^{O(n)}$ *et de profondeur* $O(n \log d)$ *qui permet d'évaluer des polynômes* $a_1, \ldots, a_s \in K[X_1, \ldots, X_n]$ *vérifiant :*

$$1 = a_1 f_1 + \ldots + a_s f_s \qquad (\text{resp.} \quad f = a_1 f_1 + \ldots + a_s f_s)$$

Ce résultat permet de fournir dans le cas de l'appartenance à une intersection complète la borne optimale $d^{O(n)}$ *pour les degrés des polynômes* $a_1, \ldots, a_s$ *et de retrouver cette même borne pour le Nullstellensatz (due initialement à Brownawell, Caniglia-Galligo-Heintz et Kollár).*
Si de plus les polynômes $\{f_1, \ldots, f_s\}$ *sont à coefficients entiers rationnels, on retrouve également pour le Nullstellensatz les résultats sur la hauteur des polynômes (rationnels)* $a_1, \ldots, a_s$ *de Berenstein-Yger, et on obtient (simultanément à Elkadi) le même type de bornes pour l'appartenance à une intersection complète : si* H *majore tous les coefficients des polynômes* f_i *et* f *(en valeur absolue), alors* $H^{d^{O(n)}}$ *majore numérateurs et dénominateurs de tous les coefficients des* g_i.

Les calculs d'évaluation avaient été appliqués dans le cadre de l'élimination dès 1977 par Heintz, Morgenstern, Schnorr, Sieveking et d'autres. Ils ont ensuite été repris par les deux premiers, Giusti et le groupe Fitchas pour les questions ci-dessus.

Dans notre contexte, c'est l'outil de la dualité (la trace) et celui de l'évitation des divisions de Strassen qui, se prêtant bien à une traduction "circuit arithmétique", entraîne les résultats mentionnés.

On se propose aussi de discuter la possibilité d'obtenir, grâce à cette même philosophie, des résultats d'un ordre complètement nouveau pour les deux problèmes considérés : on se place maintenant dans le cadre d'une suite régulière $f_1, \ldots, f_r$ et éventuellement f_{r+1} non diviseur de zéro modulo $(f_1, \ldots, f_r)$. Les entrées f_i (et f) sont représentées, de même que le seront les sorties g_i, par des circuits arithmétiques permettant de les évaluer. Les paramètres mesurant l'entrée sont d, n, la taille et la profondeur du circuit permettant d'évaluer l'entrée et les degrés géométriques affines des variétés définies par les polynômes $f_1, \ldots, f_i$ $(1 \leq i \leq r)$. Dans ces conditions les taille et profondeur du circuit permettant de fournir les calculs d'évaluation des polynômes g_i sont maintenant polynomiales en tous les paramètres considérés ! (Notons que ceci est impossible si l'on donne l'entrée et la sortie par représentation dense, et aussi semblerait-il —selon une borne inférieure de Heintz-Morgenstern— si on utilise les circuits arithmétiques mais on n'introduit pas le degré géométrique en tant que paramètre de mesure de l'entrée). Une conséquence de ce fait est que les degrés des polynômes g_i est polynomial en d, n et les degrés géométriques affines des variétés définies par $f_1, \ldots, f_i$. Ceci est un travail en cours de Fitchas et Giusti.

Des majorations pour les degrés de même type, polynomiales en tous les paramètres considérés, peuvent également être obtenues pour les degrés des générateurs du radical d'un idéal polynomial décrit par une suite régulière (sous certaines conditions) comme le montre un résultat récent d'Armendariz et Solernó.

Elimination by arithmetic circuits. The duality tool.

The aim of the talk is to describe a computational method, the *arithmetic circuit* (or *straight line program*) which allows one to evaluate certain representations of polynomials which appear when one solves polynomial systems. Roughly speaking, a *division-free non-scalar straight line program* is a division-free algorithm which computes the evaluation of a multivariate polynomial $f \in K[X_1, \ldots, X_n]$ at any point in K^n, introducing if necessary some auxiliary fixed constants of K. The *size* of the algorithm is measured by taking K-linear operations for free, and its *depth* is the maximal number of intermediate polynomials recursively linked (see [5]). We'll consider here the effective Nullstellensatz and the membership and representation problems within complete intersection ideals in the polynomial ring $K[X_1, \ldots, X_n]$, where K is a field of characteristic zero:

Let $I = (f_1, \ldots, f_s) \subseteq K[X_1, \ldots, X_n]$ be an ideal which defines the empty variety over the algebraic closure of K (resp. of pure dimension n-s, and in this case, let $f \in I$). Set $d := \max\{\deg f_i\}$ (resp. $d := \max\{\deg f_i, \deg f)\}$).
Under these conditions there exists a straight line program of size $sd^{O(n)}$ and depth $O(n \log d)$ which computes the evaluation of polynomials $a_1, \ldots, a_s \in K[X_1, \ldots, X_n]$ satisfying:

$$1 = a_1 f_1 + \ldots + a_s f_s \qquad (resp. \quad f = a_1 f_1 + \ldots + a_s f_s).$$

This result ([4]) allows one to obtain the optimal bound $d^{O(n)}$ for the degrees of the polynomials $a_1, \ldots, a_s$ in the membership problem for complete intersections, and to *recover this same bound for the Nullstellensatz (initially due to Brownawell, Caniglia-Galligo-Heintz and Kollár).*

Moreover, when the polynomials $f_1, \ldots, f_s$ (and f) have integer coefficients, one also recovers for the Nullstellensatz the bounds for the heights of the (rational) polynomials $a_1, \ldots, a_s$ due to Berenstein-Yger ([2]), and gets —simultaneous to Elkadi's result ([3])— the same type of bound for the membership problem for a complete intersection: if H bounds from above the absolute value of all the coefficients of the polynomials f_i (and f), then $H^{d^{O(n)}}$ bounds the numerators and denominators of all the coefficients of the a_i ([10]).

These evaluation programs had been applied in the elimination framework around 1977 by Heintz, Morgenstern, Schnorr, Sieveking and others. They have been reconsidered more recently for the questions stated above by the first two researchers, Giusti and the Fitchas group (see [6,8,4,9,10,7]). In our context, it is the trace duality tool ([11]) and the avoidance of divisions of Strassen ([13]) which, admitting a nice straight line program "translation", produces the results mentioned here.

Also, we would like to discuss the possibility of applying the same philosophy and obtaining results of a completely new style for the same two questions: Assume that $f_1, \ldots, f_r$ form a regular sequence ($r \leq n$) and that f_{r+1} is not a zero-divisor modulo $(f_1, \ldots, f_r)$. Moreover, suppose that the ideals $(f_1, \ldots, f_i)$ are reduced for $1 \leq i \leq r$. We consider the Nullstellensatz for $(f_1, \ldots, f_{r+1})$ or the membership of f in $(f_1, \ldots, f_{r+1})$. The inputs $f_1, \ldots, f_{r+1}$ (and eventually f) will be given no longer by their dense representation but by straight line programs. The output will also be a straight line program which computes $a_1, \ldots, a_{r+1}$. The parameters measuring the input are d, n, the size (and the depth) of the straight line program producing the input and the <u>affine geometric degrees</u> of the varieties defined by $(f_1, \ldots, f_i)$ for $1 \leq i \leq r$. Under these conditions, the size (and depth) of the straight line program producing the output $a_1, \ldots, a_{r+1}$ is <u>polynomial in all the aforementioned parameters</u>! (Note that this is impossible if the input and output are given in the dense representation, and also, it seems —due to a lower bound by Heintz-Morgenstern ([9])— if one considers straight line programs but does **not** introduce the geometric degrees as a parameter measuring the input.) This is work in progress by Fitchas-Giusti. A consequence of this fact is that the degrees of the polynomials a_i are polynomial in d, n and the degrees of the varieties defined by $(f_1, \ldots, f_i)$, as it implicitly appears in [12].

Upper bounds of the same type, polynomial in all the aforementioned parameters, can also be obtained for the degrees of the generators of the radical of a polynomial ideal described by a regular sequence (under certain conditions) as shown in a recent result by Armendariz-Solernó ([1]).

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Generalization of a theorem of Chasles

Ernst Kunz

(Joint work with R.Waldi)

Let $Z = \sum_{i=1}^{r} \nu_i[P_i]$ be an effective 0–cycle in $\mathbb{A}_K^n$ (i.e. $\nu_i \in \mathbb{N}_+$, $P_i = (a_1^i, \dots, a_n^i) \in \mathbb{A}_K^n$ closed points). Assume $\operatorname{Char} K = 0$ and let $N := \sum_{i=1}^{r} \nu_i$ be the degree of Z. The vector sum

$$\Sigma(Z) := \frac{1}{N} \sum_{i=1}^{r} \nu_i \cdot (a_1^i, \dots, a_n^i) \in \mathbb{A}_K^n$$

is called the **centroid** of Z.

Let $C \subset \mathbb{A}_K^2$ be a smooth curve of degree $d > 0$ which has d distinct points at infinity. Let L be the linear system of lines parallel to a given line not asymptotic to C and let $Z_L := \Sigma \mu_P[P]$ where $P \in C$ are the points with tangents $T_P(C)$ belonging to L and $\mu_P + 1 = \mu_P(C, T_P(C))$ (intersection multiplicity). The **theorem of Chasles** states that the centroid $\Sigma(Z_L)$ is independent of L (see [T] and [C], p.167). It is called the **tangential center** of C.

The theorem is a corollary of the residue theorem as stated in [K]. In fact, if C is the zero-set of a polynomial $F \in K[X_1, X_2]$ of degree d, then $\Sigma(Z_L)$ turns out to be the centroid of the critical scheme $\operatorname{Spec}(K[X_1, X_2]/(\frac{\partial F}{\partial X_1}, \frac{\partial F}{\partial X_2}))$.

The theorem be generalized to reduced curves in higher dimensional affine spaces and systems L of parallel hyperplanes if the curve is not the union of pairwise disjoint lines. The system L defines a noetherian normalization $K[x] \subset K[C]$ and Z_L is the cycle on C defined by the Dedekind different of $K[C]/K[x]$. Again $\Sigma(Z_L)$ is independent of L. Our proof is by generic projection to the plane.

If the curve is a union of pairwise disjoint lines there is obviously no tangential center.

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Formal Duality, Fundamental Class, and the Residue Theorem

JOSEPH LIPMAN[1]

Grothendieck Duality is based on the existence for certain proper scheme maps $f\,X \to Y$ of a right adjoint $f^!$ of the derived functor $\mathbf{R}f_*$—or at least of its restriction to complexes with quasi-coherent homology—and the compatibility of $f^!$ with flat base change. The Grothendieck residue arose as a localized aspect of the adjunction map $\int_f\ \mathbf{R}f_*f^! \to \mathbf{1}$ [H1][p. 195–199]. Approaching (local) residues via (global) duality, while ultimately essential to a full understanding, is rather indirect and abstract, crying out for down-to-earth commutative-algebra constructions. Over the past twenty five years several such treatments of residues and their relationships have been worked out, in terms of differential forms, or Hochschild homology, or topological local fields, ... and associated trace maps, see e.g., [HK1], [Y], [S], and their references.

In [HK1] Hübl and Kunz consider certain equidimensional ring maps $R \to S$ together with an S-ideal J such that S/J is finite over R. Under their assumptions, there is a finitely generated module $\omega_{S/R}$ consisting of meromorphic differential d-forms of S/R (d=fibre dimension), the module of *regular* differential d-forms, implicit in Grothendieck's duality theory, but first explicated by Kunz (see [KW]). At primes where S/R is smooth, e.g., $\omega_{S/R}$ coincides with the relative holomorphic differentials $\Omega^d_{S/R}$. Hübl and Kunz define, concretely, a residue symbol, and also a *local integral* $\int_{S/R,J}: H^d_J(\omega_{S/R}) \to R$ given, roughly, by fibrewise summation of residues (see [HK][3.7]). [HK][3.4] gives the following version of Local Duality (where ˆ denotes J-adic completion): *The pair* $(\hat\omega_{S/R}, \int_{\hat S/R, J\hat S})$ *represents the functor* $\mathrm{Hom}_R(H^d_J(E), R)$ *of* $\hat S$*-modules* E.

They then globalize, considering a finite-type map $f\,X \to Y$ of e.g., reduced excellent noetherian schemes, with f generically smooth and equidimensional of fibre-dimension d, together with a closed subscheme $Z \subset X$ finite over Y, whose inverse image over any affine open subset, say $U = \mathrm{Spec}(R)$, is contained in an affine open subset of $f^{-1}U$, say $V = \mathrm{Spec}(S)$. The regular differentials glue together into a coherent sheaf ω_f on X whose module of sections over any such V is $\omega_{S/R}$. There is then a unique map $\int_{f,Z}: R^d_Z f_*\omega_f \to \mathcal{O}_Y$ which over any affine U is the sheafification of $\int_{S/R,J}$, where S and R are as just stated and $J \subset S$ is a defining ideal of $Z \cap V$. (Here $R^d_Z f_*$, the relative local cohomology supported in Z, is a derived functor of $f_*\Gamma_Z$, Γ_Z being the sheafified functor of sections with support in Z.) These maps behave functorially with respect to inclusions $Z \subset Z'$.

The basic local-global relationship is summarized in the Residue Theorem[2], which for *proper* f, asserts the existence of a canonical map $\int_f^d\ R^d f_*\omega_f \to \mathcal{O}_Y$ such that:

(i) The corresponding map (via duality) $\omega_f \to H^{-d}f^!\mathcal{O}_Y$ is an *isomorphism,* (i.e., $(\omega_f, \int_f^d)$ represents the functor $\mathrm{Hom}(R^d f_*\mathcal{E}, \mathcal{O}_Y)$ of quasi-coherent $\mathcal{O}_X$-modules $\mathcal{E}$);

(ii) The composition of $\int_f^d$ with $R^d_Z f_*\omega \xrightarrow{\text{nat'l}} R^d f_*\omega$ (any Z as above) is $\int_{f,Z}$.

[1]Partially supported by the National Security Agency.

[2]stated at several levels of generality in various places in the literature, a recent one—including references to others—being [HS].

Thus we have, via differentials and residues, a canonical realization of, and compatibility between, local and global duality.

In the talk, we will outline new proofs of the two parts of the Residue Theorem.

For (ii), going from concrete back to abstract, we replace ω_f by $H^{-d}f^!\mathcal{O}_Y$ (thereby begging the question of (i)) and $\int_{f,Z}$ by the homology of a more general, but abstract, *formal integral,* i.e., a map of the form $\mathbf{R}f_*f^! \to \mathbf{1}$ where f is now a proper map of *formal* noetherian schemes, with respect to which a *generalized version of Grothendieck Duality* obtains.

This Formal Duality theorem contains, e.g., formal duality à la Hartshorne [H2][p. 48], and a related local-global duality theorem of [L1][p. 188] which includes the classical local duality theorem. Modulo (i), the central Theorem 10.2 in [L2][p.87] and the above Local Duality are also special cases. Formal Duality turns (ii) into a simple statement about functoriality of the formal integral (with respect to the canonical map from the formal completion of X along Z to X itself).

As for (i), one can formalize the above Local Duality to see that the concrete and abstract integrals both provide representations of the same functor; and so deduce the existence of a *local* isomorphism $\omega_{S/R} \xrightarrow{\sim} H^{-d}f^!\mathcal{O}_Y|_V$ (see above). The remaining question is: *Does this local isomorphism comes from a global one?*

In [HS], the question is answered affirmatively by means of a fairly complex pasting argument (generalizing [L2][§§6,9], where Y is the Spec of a perfect field), reducing ultimately via Zariski's Main Theorem and traces to the case $X = \mathbb{P}^d_Y$, which is treated in [HK2].

We will indicate a more direct approach. The basic relation of holomorphic differential d-forms to $f^!$ is encapsulated in a canonical derived category map

$$C_f\,\Omega^d_f \to f^!\mathcal{O}_Y\,,$$

the *fundamental class* of f. For smooth f, C_f is a well-known isomorphism [V][p. 397, Thm. 3]). The characteristic 0 case (with singularities) was studied by Angéniol and El Zein [AE] by means of a theorem of Bott on Grassmannians. We will define C_f quite generally via simple derived-category formalism; and state a "trace property" relating $C_{f\circ g}$ and C_f ($g\ X' \to X$ a finite map), which should provide an answer to the preceding question.

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ALGORITHME DE GELFAND-LERAY-SHILOV ET COURANTS-RÉSIDUS

Salomon Ofman

(RÉSUMÉ)

La théorie des résidus a son origine dans les travaux de Cauchy et surtout pour le multi-résidu chez Poincaré. Une théorie satisfaisante basée sur la dualité est donnée par J. Leray dans [L] dont l'aspect cohomologique a été montrée par F. Norguet [N] ; cependant, cette situation est essentiellement formulée dans le cadre des formes semi-méromorphes d'ensemble polaire réunion d'hypersurfaces lisses. Une généralisation de cette théorie au cas des formes semi-méromorphes quelconques est faite par F. Coleff et N. Herrera avec les courants-résidus. Cependant, alors qu'on a un algorithme simple pour calculer le résidu dans le cadre de la théorie de Leray (c'est l'algorithme de Gelfand-Leray-Shilov ou G.L.S.), cela est beaucoup moins aisé dans celui de Coleff-Herrera. C'est une motivation pour étudier la dépendance du résidu en fonction d'un paramètre afin de ramener certains problèmes sur les courants-résidus à des problèmes de résidus cohomologiques.

Cependant, cette dépendance n'est certainement pas “continue” comme le met en évidence l'exemple très simple ci-dessous en une variable :

Soit D (respectivement ∂) le disque unité (respectivement le cercle unité) de $\mathbf{C}$, 1 la fonction égale à 1 dans $\mathbf{C}$; si ϕ est une forme semi-méromorphe dans D d'ensemble polaire C, soit $Res_C\phi$ le courant-résidu au sens de Coleff-Herrera. Pour tout $t \in D — \{0\}$, on a en posant $C_t = \{-t, t\}$

$$(1/2i\pi)\langle Res_{C_t}\bar{z}dz/(z^2-t), 1\rangle \in \partial$$

tandis que

$$(1/2i\pi)\langle Res_{\{0\}}\bar{z}dz/z^2, 1\rangle = 0.$$

Par contre en utilisant le théorème de Stockes pour les ensemble semi-analytiques, on a (cf. [O])

Théorème 1. *Soit ψ une forme différentielle d-fermée de dimension p dans une variété analytique Y, $F_T = (f_{1,t_1}, \ldots, f_{p,t_p})$ une famille de p-uples de fonctions méromorphes d'ensembles polaires respectifs $C_T = (c_{1,t_1}, \ldots, c_{p,t_p})$; si lorsque T tend vers 0 la famille F_T converge (dans un sens naturel) vers le p-uple F, on a $\lim\limits_{T\to 0} Res_{F_T}[\psi/F_T] = Res_F[\psi/F]$.*

En utilisant les propriétés fondamentales des courants-résidus des formes semi-méromorphes on peut, pour certains calculs, se ramener par des tronquages adéquats au cas du théorème précédent puis à celui des résidus cohomologiques de Leray. L'algorithme G.L.S. donne alors une formule explicite pour l'obtention du résidu

$$((2i\pi)^p/K!)\int\limits_{c_1\cap\ldots\cap c_p} \rho\partial^{|K|}\psi/\partial u_1^{k_1}\cdots\partial u_p^{k_p} = \langle Res_F[dU\wedge\psi/U^{K+1}], \rho\rangle$$

où $\mathbf{1}$ désigne le p-uple formé de 1, $u_i = 0$ une équation minimale de c_i $(i = 1, \dots, p)$, $U = (u_1, \dots, u_p)$ et ρ une fonction C^∞ à support compact.

Un exemple d'application est une démonstration très simple et une généralisation de la formule de transformation (cf. [D]) sous des hypothèses convenables (essentiellement les ensembles d'annulation des fonctions f_i forment une intersection complète) : soit alors ψ une forme différentielle C^∞ dans une variété analytique complexe Y, $F = (f_1, \dots, f_p)$ et $G = (g_1, \dots, g_p)$ deux p-uples de fonctions méromorphes sur Y avec $G = M \cdot F$ où M est une matrice de fonctions holomorphes

Théorème 2. *Pour tout p-uple d'entiers I on a*

$$Res_G[\psi/G^I] = (\sum_{p \le |L| \le |I|} Res_F[C(\det M, K, L)\psi/F^L])$$

les $C(\det M, K, L)$ *étant des "constantes" convenables (qui dans le cas lisse sont celles intervenant dans la formule de changement de variables des dérivations de fonctions).*

Un cas particulier de cette égalité est utilisé dans [O] pour la construction de la transformation de Radon générale sur les cycles analytiques les variétés analytiques.

PETIT GLOSSAIRE

Admissible (trajectoire) : c'est une fonction δ défini sur un voisinage de $0 \in \mathbf{R}_>$ à valeurs dans $(\mathbf{R}_>)^p$ vérifiant, lorsque ε tant vers 0, $\delta_p(\varepsilon) = 0$ et $\delta_i(\varepsilon)$ converge vers 0 plus vite que toute puissance de $\delta_{i+1}(\varepsilon)$ $(i \in \{1, \dots, p-1\})$.

Courant-résidu : soit ϕ une forme différentielle semi-méromorphe dans un polydisque $D \subset \mathbf{C}^n$ d'ensemble polaire contenu dans la réunion des hypersurfaces $c_1, \dots, c_p$ d'équations respectives $u_i = 0$. Pour toute forme différentielle C^∞ à support compact ρ et toute trajectoire admissible δ la limite $\int_{\bigcap_{i=1}^{p} \{z; |u_i(z)| = \delta_i(\varepsilon)\}} [\phi \wedge \rho]$ notée $\langle Res_U \phi, \rho \rangle$ (ou encore $\langle Res_C \phi, \rho \rangle$) existe lorsque ε tend vers 0, est indépendant du choix des équations u_i et dépend continûment de ρ (U et C désignant respectivement les p-uples $(u_1, \dots, u_p)$ et $(c_1, \dots, c_p)$). Ainsi $Res_U \phi$ définit un courant-résidu sur D et par partition de l'unité, on peut les définir sur toute variété analytique.

Dimension (d'une forme) : si ψ est une forme différentielle de degré k sur une variété Y, sa dimension est $\dim Y - k$.

Semi-méromorphe (forme) : une forme différentielle est semi-méromorphe si elle s'écrit localement comme quotient d'une forme C^∞ par une fonction holomorphe.

GELFAND-LERAY-SHILOV ALGORITHM AND RESIDUES-CURRENTS

(ABSTRACT)

The origins of the theory of residues are in Cauchy's works and especially for multi-residues in Poincaré's. A satisfactory theory based on duality was given by J. Leray [L] and the cohomological aspect was shown by F. Norguet [N] ; nevertheless, this situation is essentially formulated for semi-meromorphic differential forms whose polar set have smooth irreducible parts. The generalization to any semi-meromorphic forms was done by F. Coleff and N. Herrera with the residues-currents. But, whereas we have a convenient algorithm to compute the residues inside Leray's theory (the so-called Gelfand-Leray-Shilov or G.L.S. algorithm), this is much less easy for the Colefff-Herrera's one. This is a motivation to study how the residues are depending of a parameter in order to solve some problems on the residues-currents using the tools of cohomological residues.

However this dependence is not "continuous" as we can see in the very simple following example in one variable :

Let D (resp. ∂) be the unit disk (resp. the unit circle) in $\mathbf{C}$, 1 the function equals to 1 in $\mathbf{C}$ and $C_t = \{-t, t\}$. For any $t \in D — \{0\}$, we obtain

$$(1/2i\pi)\langle Res_{C_t}\bar{z}dz/(z^2-t), 1\rangle \in \partial$$

whereas

$$(1/2i\pi)\langle Res_{\{0\}}\bar{z}dz/z^2, 1\rangle = 0.$$

Let Y an analytic manifold of dimension Y and ψ a d-closed differential of dimension p in Y. Using Stockes' theorem for semi-analytic sets we have however (cf. [O])

Theorem 1. *Let $F_T = (f_{1,t_1}, \ldots, f_{p,t_p})$ a p-uples family of meromorphic functions of respective polar sets $C_T = (c_{1,t_1}, \ldots, c_{p,t_p})$; if $\lim\limits_{T\to 0} F_T = F$ (in a natural way), we have*

$$\lim_{T\to 0} Res_{F_T}[\psi/F_T] = Res_F[\psi/F].$$

Using some fundamental properties of the residues-currents and appropriate truncating, it is possible reduce some problems to the case of the previous theorem and then to the one of Leray's cohomological residues. Then the G.L.S. algorithm gives an explicit formula to obtain the residu :

$$((2i\pi)^p/K!)\int_{c_1\cap\ldots\cap c_p} \rho\partial^{|K|}\psi/\partial u_1^{k_1}\cdots\partial u_p^{k_p} = \langle Res_F[dU\wedge\psi/U^{K+\mathbf{1}}], \rho\rangle$$

where $\mathbf{1}$ is the p-uple $1, \ldots, p$, $u_i = 0$ a minimal equation of c_i $(i = 1, \ldots, p)$, $U = (u_1, \ldots, u_p)$ and ρ a function C^∞ with compact support.

An application is a very simple demonstration and generalization of the transform formula (cf. [D]) under some suitable (essentially the zero sets of the functions f_i is a complete intersection) : Let $F = (f_1, \ldots, f_p)$ and $G = (g_1, \ldots, g_p)$ be two p-uples of meromorphic functions in Y and $G = M \cdot F$ where M is a matrix of holomorphic functions in Y

Theorem 2. *For any p-uple of integers I we have*

$$Res_G[\psi/G^I] = (\sum_{p\leq|L|\leq|I|} Res_F[C(\det M, K, L)\psi/F^L])$$

where the $C(\det M, K, L)$ are suitable "constants" (in the smooth case they are the ones appearing in the formula of change of variables for the derivatives of functions).

A particular case of this equality is used in [O] for the construction of the Radon transformation for the analytic cycles of analytic manifolds.

SMALL GLOSSARY

Admissible (trajectory) : it is an function δ defined in a neighborhood of $0 \in \mathbf{R}_>$ with values in $(\mathbf{R}_>)^p$ such that when ε tends to 0, $\delta_p(\varepsilon) = 0$ and $\delta_i(\varepsilon)$ tends to 0 faster than any power of $\delta_{i+1}(\varepsilon)$ $(i \in \{1, \ldots, p-1\})$.

Residu-current : Let ϕ be a semi-meromorphic differential form in the polydisc $D \subset \mathbf{C}^n$ of polar set contained in the union of hypersurfaces $c_1, \ldots, c_p$ of respective equations $u_i = 0$. For any C^∞ differential form ρ with compact support and any admissible trajectory δ, the limit $\int_{\bigcap_{i=1}^{p} \{z; |u_i(z)| = \delta_i(\varepsilon)\}} [\phi \wedge \rho]$ denoted by $\langle Res_U \phi, \rho \rangle$ (or $\langle Res_C \phi, \rho \rangle$) exists when ε tends to 0, does not depend of the choice of the equations u_i and depends continuously of ρ (U and C are respectively the p-uples $(u_1, \ldots, u_p)$ and $(c_1, \ldots, c_p)$). Then $Res_U \phi$ defines a current on D the residu-current ; by a partition of the unity, we can define them on any analytic manifold.

Dimension (of a form) : if ψ is a differential form of degree k on Y, its dimension is $\dim Y - k$.

Semi-meromorphic (form) : a differential form is semi-meromorphic if it is locally a quotient of a C^∞ form by an holomorphic function.

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Quadratic forms and Bezoutians

MARIE-FRANÇOISE ROY

In the univariate situation, quadratic forms based on Bezoutians and residues can be defined and their signatures are the Cauchy indices of rational functions. Hermite quadratic form counting the number of real roots can be recovered this way. The multivariate situation is as follows. Multivariate Bezoutians and residues give in the complete intersection case a general construction of quadratic forms. Their signature give interesting informations in some particular cases: topological degree (Eisenbud-Levin's results), real root counting (Hermite method). The general case is not clearly understood yet and we do not know what is the multivariate generalization of the Cauchy index.

From a computational point of view, the quadratic forms so obtained have small coefficients (using dual basis shortens the data). In general Bezoutians and residues have been shown useful in order to control the size of coefficients in several important problems of computer algebra and a better knowledge of the algebraic approach should help us to understand these phenomena more deeply.

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Formal completion and duality

Leovigildo Alonso Tarrío

In this talk we will explain the results in [AJL] and some consequences for the algebraic theory of residues.

Let X be a noetherian separated scheme. Let $\mathbf{D}(X)$ be the derived category of the category of sheaves of modules over X and $\mathbf{D}_{qc}(X)$ the full subcategory of complexes with quasi-coherent homology. For any closed subscheme Z of X, we can consider the endofunctors of $\mathcal{O}_X$−Mod defined over the objects by

$$\varGamma'_Z\mathcal{F} = \varinjlim_{n>0} \mathcal{H}om(\mathcal{O}_X/\mathcal{I}^n, \mathcal{F}) \text{ and}$$

$$\varLambda_Z\mathcal{F} = \varprojlim_{n>0} \mathcal{O}_X/\mathcal{I}^n \otimes \mathcal{F},$$

where $\mathcal{I}$ is a coherent $\mathcal{O}_X$−ideal such that Z is the support of $\mathcal{O}_X/\mathcal{I}$.

The functor $\varGamma'_Z$ is a left exact subfunctor of $\varGamma_Z$, and it can be derived on the right using injective resolutions. Moreover the natural map $\mathbf{R}\varGamma'_Z\mathcal{F} \to \mathbf{R}\varGamma_Z\mathcal{F}$ is an isomorphism if $\mathcal{F} \in \mathbf{D}_{qc}(X)$. The functor $\varLambda_Z$ is not right exact, but it has a left derived functor $\mathbf{L}\varLambda_Z : \mathbf{D}_{qc}(X) \to \mathbf{D}(X)$ describable via quasi-coherent flat resolutions.

This two operations are related by an adjunction formula

$$\mathbf{R}\mathcal{H}om(\mathbf{R}\varGamma'_Z\mathcal{E}, \mathcal{F}) \simeq \mathbf{R}\mathcal{H}om(\mathcal{E}, \mathbf{L}\varLambda_Z\mathcal{F}) \qquad (\mathcal{E} \in \mathbf{D}(X),\, \mathcal{F} \in \mathbf{D}_{qc}(X)),$$

which is a sheafified derived-category version of [GM, Thm. 2.5]. (We note that this is proved in [AJL] for a general quasi-compact separated scheme X with a mild restriction over Z.) The proof reduces to the case $\mathcal{E} = \mathcal{O}_X$, establishing an isomorphism between $\mathbf{R}\mathcal{H}om(\mathbf{R}\varGamma_Z\mathcal{O}_X, -)$—whose homology can be called *local homology*—and $\mathbf{L}\varLambda_Z$, as functors from $\mathbf{D}_{qc}(X)$ to $\mathbf{D}(X)$.

Some consequences of this result are *local duality* as in [Gr, p. 85, Thm. 6.3] and [H1, p. 280, cor. 6.5], a more general *local-global duality* [L1, Theorem in p. 188], and Hartshorne's *affine duality* in [H2, p. 152, Thm. 4.1].

Furthermore, when $\mathcal{F} \in \mathbf{D}^+_c(Y)$ (coherent homology) the previous formula becomes

$$\mathbf{R}\mathcal{H}om(\mathbf{R}\varGamma'_Z\mathcal{E}, \mathcal{F}) \simeq \kappa_*\mathbf{R}\mathcal{H}om(\kappa^*\mathcal{E}, \kappa^*\mathcal{F}),^*$$

where $\kappa : \widehat{X}_{/Z} \to X$ is the canonical map.

Let $f : X \to Y$ be a proper map of noetherian schemes, where Z and W are closed subsets of X and Y respectively such that $f(Z) \subset W$. The natural map $\rho : \mathbf{R}f_*\mathbf{R}\varGamma_Z f^! \to \mathbf{R}\varGamma_W$ (*abstract residue*) obtained via the trace map of Grothendieck duality, together with $(*)$, induces a functorial isomorphism

$$\kappa_*\mathbf{R}\mathcal{H}om(\kappa^*\mathcal{E}, \kappa^* f^!\mathcal{F}) \simeq \mathbf{R}f_*\mathbf{R}\mathcal{H}om(\mathbf{R}f_*\varGamma_Z\mathcal{E}, \mathbf{R}\varGamma_W\mathcal{F}),$$

for $\mathcal{E} \in \mathbf{D}_{qc}(X)$, and $\mathcal{F} \in \mathbf{D}^+_c(Y)$.

This isomorphism depends only on the formal completion of X (resp. Y) with respect to Z (resp. W). (The underlying reason is that it is possible to establish a Grothendieck duality theory for proper maps of noetherian formal schemes.) As an application we can recover [HK1, p. 73, Thm. 4.3] and [H3, p. 48, Prop. 5.2].

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Adeles and the De Rham-Residue Complex

Amnon Yekutieli

Introduction

Let me begin by recalling some notions from complex geometry. These will serve as models for the algebro-geometric constructions which will follow. Let M be an n-dimensional complex manifold. On it we have the sheaf $\mathcal{A}^{p,q}$ of smooth (p,q)-forms, and the sheaf $\mathcal{D}^{p,q}$ of (p,q)-currents. By definition for any $U \subset M$ open

$$\mathcal{D}^{p,q}(U) := \text{dual space of } \mathcal{A}_{\mathrm{c}}^{n-p,n-q}(U).$$

$\mathcal{A}^{\cdot}$ is a DGA (differential graded algebra) and $\mathcal{D}^{\cdot}$ is a DG $\mathcal{A}^{\cdot}$-module. The map

$$\mathcal{A}^{\cdot} \to \mathcal{D}^{\cdot},\ \alpha \mapsto (\beta \mapsto \int_U \alpha \wedge \beta)$$

is a quasi-isomorphism.

Observe that $\mathcal{A}^{\cdot}$ pulls back under any morphism $f : M \to N$ and $\mathcal{D}^{\cdot}$ pushes forward when f is proper.

De Rham-Residue Complex

This is the analog of the Dolbeault complex of currents. Suppose X is a finite type scheme over a perfect field k. The De Rham-Residue Complex appeared before in work of Hartshorne and El-Zein, under another name (the canonical, or Cousin resolution of $\Omega^{\cdot}_{X/k}$).

First consider the residue complex $\mathcal{K}^{\cdot}_X$. It has an explicit construction, which goes like this. For a point $x \in X$ let $\mathcal{O}_{X,(x)} := \widehat{\mathcal{O}}_{X,x}$, the complete local ring. By the theory of Beilinson completion algebras (BCAs) there is a dual module $\mathcal{K}(\mathcal{O}_{X,(x)})$. Set

$$\mathcal{K}_X(x) := \mathcal{K}(\mathcal{O}_{X,(x)}),\ \mathcal{K}^{\cdot}_X := \bigoplus_{x \in X} \mathcal{K}_X(x).$$

If (x,y) is a saturated chain of points (immediate specialization) then there is a BCA $\mathcal{O}_{X,(x,y)}$ and a map

$$\delta_{(x,y)} : \mathcal{K}(\mathcal{O}_{X,(x)}) \to \mathcal{K}(\mathcal{O}_{X,(x,y)}) \to \mathcal{K}(\mathcal{O}_{X,(y)}).$$

Putting $\delta_X := \sum \delta_{(x,y)}$ we get our complex.

Theorem 0.1 *Given a differential operator* (DO) $D : \mathcal{M} \to \mathcal{N}$ of $\mathcal{O}_X$-modules, there is a functorial DO

$$D^{\vee} : \mathcal{N}^{\vee} = \mathcal{H}om(\mathcal{N}, \mathcal{K}^{\cdot}_X) \to \mathcal{M}^{\vee}$$

which is a map of complexes.

Set $\mathcal{F}^{p,q}_X := \mathcal{H}om(\Omega^{-p}_{X/k}, \mathcal{K}^q_X)$.

Corollary 0.2 *$\mathcal{F}_X^{\cdot}$ is a complex, with operator* $\mathrm{D} := \mathrm{d}^\vee \pm \delta_X$.

Theorem 0.3 *If $f : X \to Y$ is proper, then there is a homomorphism of complexes* $\mathrm{Tr}_f : f_* \mathcal{F}_X^{\cdot} \to \mathcal{F}_Y^{\cdot}$.

The fundamental class C_X is an easily defined global section of $\mathcal{F}_X^{\cdot}$.

Adele-De Rham complex

This is our analog of the Dolbeault complex of smooth forms. Given a quasi-coherent sheaf on X, let $\underline{A}^q_{\mathrm{red}}(\mathcal{M})$ be the sheaf of degree q reduced Beilinson adeles. For an open set U, $\Gamma(U, \underline{A}^q_{\mathrm{red}}(\mathcal{M})) \subset \prod_\xi \mathcal{M}_\xi$, where ξ runs over the length q chains of points in U and $\mathcal{M}_\xi$ is the Beilinson completion.

Set $\mathcal{A}_X^{p,q} := \underline{A}^q_{\mathrm{red}}(\Omega^p_{X/k})$. $\mathcal{A}_X^{\cdot}$ is a DGA, $\mathrm{D} := \mathrm{d} \pm \partial$, and $\Omega^{\cdot}_{X/k} \to \mathcal{A}_X^{\cdot}$ is a quasi-isomorphism.

Variance: for any morphism $f : X \to Y$ there is a DGA homomorphism $\mathcal{A}_Y^{\cdot} \to f_* \mathcal{A}_X^{\cdot}$.

Theorem 0.4 *$\mathcal{F}_X^{\cdot}$ is a right DG $\mathcal{A}_X^{\cdot}$-module.*

Sketch of proof: one has $\mathcal{A}_X^{\cdot} = \Omega^{\cdot}_{X/k} \otimes \underline{A}^{\cdot}_{\mathrm{red}}(\mathcal{O}_X)$. The action of $\Omega^{\cdot}_{X/k}$ is obvious. Suppose $a = (a_\xi)$ is an adele. Fix a chain $\xi = (x_0, \ldots, x_q)$ and $\phi \in \mathcal{K}(x)$. What is $\phi \cdot a_\xi$? It is nonzero only if ξ is saturated and $x = x_0$. Then there are a BCA $\mathcal{O}_{X,\xi}$ and homomorphisms

$$\mathcal{K}(\mathcal{O}_{X,(x_0)}) \to \mathcal{K}(\mathcal{O}_{X,\xi}) \xrightarrow{\mathrm{Tr}} \mathcal{K}(\mathcal{O}_{X,(x_q)}),$$

and we set $\phi \cdot a_\xi = \mathrm{Tr}(a_\xi \phi)$.

(Co)homology: for X smooth the maps $\Omega^{\cdot}_{X/k} \to \mathcal{A}_X^{\cdot} \to \mathcal{F}_X^{\cdot}$, $1 \mapsto 1 \mapsto \mathrm{C}_X$, are quasi-isomorphisms. Otherwise (in char. 0) we consider a smooth formal embedding $X \subset \mathfrak{X}$; for example take an embedding $X \subset Y$, Y smooth, and set $\mathfrak{X} := Y_{/X}$, the formal completion. Then $\mathrm{H}^i_{\mathrm{DR}}(X) = \mathrm{H}^i \Gamma(X, \mathcal{A}_{\mathfrak{X}}^{\cdot})$ and $\mathrm{H}_i^{\mathrm{DR}}(X) = \mathrm{H}^{-i} \Gamma(X, \mathcal{F}_{\mathfrak{X}}^{\cdot})$.

Example: Chern Character

(Some of this is joint with R. Hübl.) There is an adelic Chern-Weil theory. It uses the Thom-Sullivan adeles $\tilde{\mathcal{A}}_X^{\cdot}$. Here X is smooth and char$k = 0$. By "integration on the simplex" we get a map of complexes $\int_\Delta : \tilde{\mathcal{A}}_X^{\cdot} \to \mathcal{A}_X^{\cdot}$. Let $\mathcal{E}$ be a lomcally free sheaf. On $\tilde{\mathcal{A}}_X^0 \otimes \mathcal{E}$ one can put a connection ∇, and the curvature $R = \nabla^2 \in \tilde{\mathcal{A}}_X^2 \otimes \mathcal{E}nd(\mathcal{E})$. Set

$$\mathrm{ch}(\mathcal{E}, \nabla) := \int_\Delta \mathrm{tr} \exp\, R \in \mathcal{A}_X^{\cdot}.$$

This defines the usual Chern character $\mathrm{ch} : \mathrm{K}^0 X \to \mathrm{H}^{\cdot}_{\mathrm{DR}}(X)$.

Theorem 0.5 *Suppose $Z \subset X$ is integral of codimension m and $\cdots \mathcal{E}_1 \to \mathcal{E}_0 \to \mathcal{O}_Z \to 0$ is a lomcally free resolution. Then there are adelic connections ∇_i on $\mathcal{E}_i$ s.t.*

$$\mathrm{C}_X \cdot \mathrm{ch}(\mathcal{E}_\cdot; \nabla_\cdot) \in \Gamma_Z \mathcal{F}_X^{\cdot}$$

($\mathrm{ch}(\mathcal{E}_\cdot; \nabla_\cdot)$ is the alternating sum) and moreover

$$\mathrm{C}_X \cdot \mathrm{ch}(\mathcal{E}_\cdot; \nabla_\cdot)_{2m} = \mathrm{C}_Z \in \mathcal{F}_X^{\cdot}.$$

*

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Integral representation formulas and Multidimensional residues

Alain Yger

Integral representation formulas of the Bochner-Martinelli or Andreotti-Norguet type provide some sharp estimates for effectivity questions ([1] [2]). For example, if $(P_1, \ldots, P_n)$ is a proper map from $\mathbf{C}^n$ to $\mathbf{C}^n$, with Lojasiewicz exponent δ, and $Q \in \mathbf{C}[X_1, \ldots, X_n]$, the degree of the polynomial map

$$w = (w_1, \ldots, w_n) \rightarrow \left[\begin{array}{c} Q dX_1 \wedge \ldots \wedge dX_n \\ P_1 - w_1, \ldots, P_n - w_n \end{array} \right]$$

is at most

$$\left[\frac{deg(Q) + D - (n-1)\delta + n}{\delta} \right] - n$$

where

$$D = \max_{1 \leq j \leq n} \left[\sum_{k \neq j} deg(P_k) \right].$$

When the Lojasiewicz exponent δ is strictly a negative number, one can show that if $P_1, \ldots, P_m$ are polynomials such that

$$\|P(X)\| \geq \gamma \|X\|^{\delta} \qquad , \qquad \|x\| \gg 1$$

and that the sheaf $\mathcal{I}$ corresponding to the ideal $({}^hP_1, \ldots {}^hP_m)$ satisfies $depth(\mathcal{I}) \geq 2$, then, for any $Q \in \sqrt{(P_1, \ldots, P_m)}$,

$$Q^{\nu} = \sum_{j=1}^{m} P_j Q_j \,,$$

$$deg(P_j Q_j) \leq \nu deg Q + n(|\delta| + \max_{1 \leq j \leq m} deg(P_j)),$$

where ν denotes the maximum of local Noether exponents at all common zeroes of the P_j's. We will show in this lecture the role of to Briançon-Skoda theorem, together with Jacobi-Kronecker formula, in order to get economic solutions for the algebraic Nullstellensätze, where the estimates depend on the affine degree (or on the Lojasiewicz exponent) rather than on the projective degree, as in Kollár's approach.

The second aim of this lecture is to emphazise the role of the Bochner-Martinelli or Andreotti-Norguet formulas in the analytic theory of multidimensional residues. Recall that, in the local situation where $f_1, \ldots, f_n$ are germs of holomorphic functions defining the origin as an isolated zero, one can define the local residue symbols (for $m \in \mathbf{N}^n$) as

$$\left[\begin{array}{c} h d\zeta_1 \wedge \cdots \wedge d\zeta_n \\ f_1^{m_1+1}, \ldots, f_n^{m_n+1} \end{array} \right] = \frac{(-1)^{(n(n-1)/2}(n-1+|m|)!}{m!(2i\pi)^n} \int_{\|\zeta\|=\epsilon} h s^m \left(\sum_{k=1}^{n} (-1)^{k-1} s_k \bigwedge_{j \neq k} ds_j \right) \wedge d\zeta, \tag{1}$$

where $s=(s_1,\dots,s_n)$ denotes any n-uplet of $\mathcal{C}^1$ functions in some neighborhhod of $\{\|\zeta\|=\epsilon\}$ such that $< s, f >:= \sum s_j f_j \equiv 1$ in this neighborhhood (here $s^m := s_1^{m_1}\cdots s_n^{m_n}$, $|m| := \sum m_j$, $m! := m_1!\cdots m_n!$). From Kronecker's formula, one knows that any germ h in $\mathcal{O}_n$ can be expanded as

$$h(z) = \sum_{m\in\mathbf{N}^n} \begin{bmatrix} h\Delta(z,\cdot)d\zeta_1\wedge\cdots\wedge d\zeta_n \\ f_1^{m_1+1},\dots,f_n^{m_n+1}\end{bmatrix} f^m(z), \tag{2}$$

where Δ denotes the determinant of a matrix of germs $[g_{i,j}(z,\zeta)]$ such that

$$f_i(z)-f_i(\zeta) = \sum_{j=1}^n g_{i,j}(z,\zeta)(z_j-\zeta_j),\ i=1,\dots,n.$$

One can write (1) using ideas inspired by analytic theory of currents; if ϕ is any $(n,0)$ test form $\phi d\zeta$ with compact support, such that $\phi\equiv 1$ near the origin, then, if we chose $s=\overline{f}/\|f\|^2$, (1) can be rewritten (see for example [5]) as

$$\begin{bmatrix} hd\zeta_1\wedge\cdots\wedge d\zeta_n \\ f_1^{m_1+1},\dots,f_n^{m_n+1}\end{bmatrix} =$$
$$\left[\lambda\frac{(-1)^{(n(n-1)/2}(n-1+|m|)!}{m!(2i\pi)^n}\int_{\mathbf{C}^n} h\|f\|^{2(\lambda-n-|m|)}\overline{f}^m\overline{df}\wedge\phi d\zeta\right]_{\lambda=0}$$

where $\overline{df} := \overline{df_1}\wedge\cdots\wedge\overline{df_n}$, the notation meaning that one considers the analytic continuation of the function of λ (clearly defined for $Re(\lambda) >> 0$) and takes its value at $\lambda=0$. The $(0,n)$ current defined by the right hand side of (2) makes sense even if $(f_1,\dots,f_n)$ do not define the origin as an isolated zero. In fact, the function of λ in the right hand side of (2) is, for any $(n,0)$ test form ϕ, a meromorphic function with poles in $\mathbf{Q}^-$; its value at the origin defines the action of a $(0,n)$ current which is, as a current, annihilated by the ideals $\overline{I^n}$, $\overline{I}^n$ ($\overline{I}$ being the integral closure of $(f_1,\dots,f_n)$ and $\overline{I^n}$ the integral closure of I^n). We will show how currents of this form play a role in a substitute for Kronecker's formula (2) in the case when $f_1,\dots,f_n$ do not define the origin as an isolated zero. We will even assume that the number

$$Q(\zeta,X) = \frac{\sum_{k=1}^m \overline{f}_k(\zeta)dX_k}{\|f(\zeta)\|^2}.$$

Let us write, for $1\le p\le inf(n,m)$ the formal expression

$$\lambda\frac{\overline{\partial}\|f(\zeta)\|^2}{\|f(\zeta)\|^2}\|f(\zeta)\|^{2\lambda}Q(\zeta,X)\wedge(\overline{\partial}_\zeta Q(\zeta,X))^{p-1}$$

as

$$\sum_{1\le i_1<\cdots<i_p\le n} T_{i_1,\dots,i_p}(\zeta;\lambda)\wedge\bigwedge_{l=1}^p dX_{i_l}$$

All functions $\lambda\mapsto T_{i_1,\dots,i_p}(\cdot;\lambda)$ correspond to $(0,p)$-current-valued meromorphic functions of λ, all with poles in $\mathbf{Q}^-$, with values at the origin $(0,p)$ currents T_J^p, $J\subset\{1,\dots,m\}$, $\#J=p$, annihilated (as currents) by the ideals $\overline{I}^p$ or $\overline{I^p}$, where I denotes the ideal $(f_1,\dots,f_m)$ and the notation bar means as before one takes the integral closure. Assume that $m\le n$. We can represent the germs h in $\mathcal{O}_n$ modulo the ideal as follows. Take representants of $f_1,\dots,f_m$ defined in some neighborhood ω of the origin; let ϕ be a $\mathcal{C}^\infty$ test function, with compact support in ω, equal to 1 near some open subset ω' such that $0 \subset \omega' \subset\subset \omega$; let σ be a $\mathbf{C}^n$-valued function defined in a neighborhood of $\omega' \times Supp(\overline{\partial}\phi)$ and such that $< \sigma, \zeta - z > \equiv 1$ for $z \in \omega'$ and $\zeta \in Supp(\overline{\partial}\phi)$

$$\Sigma(z,\zeta) := \sum_{j=1}^{n} \sigma_j(z,\zeta) d\zeta_j,$$

$$g_i(z,\zeta) := \sum_{j=1}^{n} g_{i,j}(z,\zeta) d\zeta_j, \quad i = 1,\ldots,m.$$

Then, if h is some holomorphic function in ω, then (see [6], section 5), one can write h in ω' as

$$\begin{aligned} h(z) &= \tfrac{-1}{(2i\pi)^n} \textstyle\sum_{p=1}^{m} \sum_{\substack{J\subset\{1,\ldots,m\} \\ \#J=p}} \left\langle T_J^p, h\overline{\partial}\phi \wedge \left[\Sigma \wedge (\overline{\partial}_\zeta \Sigma)^{n-1-p} \wedge \bigwedge_{j\in J} g_j\right](z,\cdot)\right\rangle \\ &+ \textstyle\sum_{j=1}^{m} h_j(z) f_j(z), \end{aligned} \tag{3}$$

where $h_1, \ldots, h_m$ are holomorphic functions in ω'. The algebraic understanding of these techniques (all based on the approach of multidimensional residues with the Bochner-Martinelli or Andreotti-Norguet formulas) remains to be cleared. In particular, one would like to interpret the role of all currents T_J^p, $J \subset \{1,\ldots,m\}$, $\#J = p$, involved in the division formula (3). We would like also to point out that the transformation law (in its usual version [3] or generalized version [5]) can be also obtained (this was done in a recent work by J. Y. Boyer) as an easy consequence of Bochner-Martinelli and Norguet formulas. The crucial role of this transformation law in the algebraic theory of residues is a further reason for such tools to be understood from the algebraic point of view.

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