Abstracts of the talks given at the SAGA Winter School, Auron, 15-19 March 2010

Monday 15 March

Rolling ball blends between natural quadrics

Heidi Dahl, Vilnius University

Abstract: The majority of mechanical parts in CAD are constructed using natural quadrics (i.e. spheres, right circular cylinders and cones) and blends between them. We consider all positions of pairs of natural quadrics and fixed radius rolling ball blends between them. We present the algorithm for minimal degree Bezier representations of such blends when they are rational surfaces.

Computing distance between objects by using supported functions

Adrien Poteaux, Johannes Kepler University, Linz

Abstract: We will present a work in progress of the SAGA project. The aim of this work is to compute the minimal distance between two objects, or between an object and a point. The first point of the work is to compute an envelope of the object by supported functions. Then, we compute the minimum distance between the supported functions. The talk will be about the first point.

Geometric Design of the Plaited Road Interchange

Rimvydas Krasauskas, Vilnius University

Abstract: We will present a short introduction to geometric principles of road design (using splines composed of linear, circular and clothoid segments) with the list of the most popular interchange types. Then we will focus on the recently invented "Plaited" road interchange with 3, 4 and 5 directions. Several optimization problems for this case will be considered: symmetric and non-symmetric with variations of number of lanes, speed limits, and different directions of in/outcomming roads. Finally we will formulate a few general problems related to combinatorics and topology of arbitrary possible road interchange.

Demonstration of Topsolid Software for design and machining of parts

Jayasimha Bagalkote, MISSLER Software

Abstract: The talk is intended to demonstrate the use of Topsolid'Design to design basic parts and Topsolid'Cam to machine the parts already designed. This will be followed by presentation of the problem to be solved in an operation called as *material left machining*. A suggested algorithm to solve a part of the problem is then sketched.

Mathemagix

Jaris Van Der Hoeven, CNRS-Ecole Polytechnique

Installation and first steps in Mathemagix

Jérémy Berthomieu, Ecole Polytechnique and Bernard Mourrain, INRIA, Galaad

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Tuesday 16 March

The calculation of the degree of an approximate common divisor of two polynomials

Joab Winkler, Sheffield University

Abstract:
The calculation of the degree of an approximate greatest common divisor (AGCD) of two inexact polynomials f(y) and g(y) is a non-trivial computation because it reduces to the estimation of the rank loss of a resultant matrix R(f,g). This computation is usually performed by placing a threshold on the small singular values of R(f,g), but this method suffers from disadvantages because the numerical rank of R(f,g) may not be defned, or the noise level imposed on the coeffcients of f(y) and g(y) may not be known, or it may only be known approximately. This talk addresses this problem by considering three methods for estimating the degree of an AGCD of f(y) and g(y), such that an estimate of the noise level is not assumed. The first method assigns probability distributions to the zero and non-zero singular values, and then uses the principle of maximum likelihood to calculate the most likely rank of R(f,g). The second method involves the calculation of the angle between two subspaces that are apparent from the structure of the Sylvester resultant matrix S(f,g), which is one type of resultant matrix. The third method uses the theory of subresultant matrices, which are derived from S(f,g) by the deletion of some of its rows and columns. The three methods are compared computationally on non-trivial polynomials(high degree, many multiple roots).

From extended companion matrices to approximate GCD

Olivier Ruatta, XLIM UMR 6172 Université de Limoges - CNRS.

Abstract: We propose a new matricial approach to approximate GCD. This approach relies on extended companion matrices. If f and g are two polynomials with complex coefficients, we say that a matrix M is an extended companion of g with respect to f if the eigenvalues of M are the values of the evaluations of g at the roots of f. We give some instance of such matrices and explain how to use it to compute an approximate gcd.

A Symbolic-Numeric Approach toward Polynomial Algebra

Wen-shin Lee, University of Antwerp.

Abstract: Recent advances in Pade approximation theory reveal unforeseen links to polynomial algebra. The connection between the detection of poles of univariate rational approximants and the sparse interpolation of multivariate polynomials leads to a deterministic algorithm for sparse polynomial interpolation in floating point arithmetic. This numerical sparse interpolation requires neither the number of terms nor the partial degrees of the target polynomial to be supplied as input and is extended to the sparse interpolation of multivariate rational functions. By further exploring the correspondence between the poles and the sparsity of a polynomial, algorithms for computing approximate sparse representations are being developed. These algorithms seek the approximation by a polynomial or rational function that is sparse when represented in a basis after a shift or linear transform.

A detailed study of a multivariate algorithm for the detection of pole curves and surfaces provides us with a new tool to extract algebraic structure from numerical partial data. Among the examples are numerical factorization, GCD, and irreducibility.

Beside to report our current research under this framework, we plan to comment on a latest research direction that is connected to resultant theory and motivated by the observation of a curious analogy among the formally orthogonal Hadamard polynomials, Jacobi's method for approximating dominant roots, and the generating polynomial of a linear recurrence sequence.

This is joint work with Annie Cuyt.

Matrix representation of rational space curves and the intersection problem by means of matrix representation

Thang Luu Ba, INRIA Galaad

Abstract: Let $f_0, f_1, ..., f_n$ be n homogeneous polynomials in $\mathbb{K}[s,t]$ of the same degree $d \geq 1$ such that $\gcd(f_0, f_1, ..., f_n) \in \mathbb{K}$ and consider the rational map

$$\begin{array}{rcl} \phi : \mathbb{P}^1_{\mathbb{K}} & \to & \mathbb{P}^n_{\mathbb{K}} \\ (s:t) & \mapsto & (f_0 : f_1 : ... : f_n)(s,t). \end{array}$$

The image of ϕ is an algebraic curve $\mathcal{C}$ in $\mathbb{P}^n_{\mathbb{K}}$ which is usually called a *rational* curve. A matrix $\mathtt{M}(\mathbf{f})$ with entries in $\mathbb{K}[x_0, x_1, ..., x_n]$ is said to be a representation of a given rational space curve $\mathcal{C}$ if

(i) $\mathtt{M}(\mathbf{f})$ is generically full rank,

(ii) the rank of $\mathtt{M}(\mathbf{f})$ drops exactly on the curve $\mathcal{C}$.

At first, we present how to obtain a new matrix representations of this space rational parameterized curves. And after, we develop the problem Curve/Curve intersection in $\mathbb{P}^n_{\mathbb{K}}$ which are motivated by recent research in this topic.

Implicitization of parametric Curves and Surfaces using Resultant Polytopes

Tatjana Kalinka, National Kapodistrian University of Athens

Abstract:

In order to change the representation of geometrical objects, presented in parametric form, to implicit form, predicting the support of the implicit equation proved to be useful, since it reduces implicitization to linear algebra, it exploits sparseness, and allows for approximate implicitization. In this work, we use the implicit support, found by calculating the Resultant Polytope, and apply it to the implicitization of 2d (curves) and 3d (surfaces) geometrical objects.

Approximate Implicitization of Triangular Bézier Surfaces

Oliver J. D. Barrowclough, SINTEF

Abstract:

We discuss how Dokken's method of approximate implicitization can be applied to triangular Bézier surfaces in both the original and weak forms, and how the properties of these surfaces make them ideal candidates for Dokken's method. The weak form is presented in two ways: first utilizing the fact that the integral of the triangular Bernstein basis functions of degree n are equal, and second by numerical integration. The necessary multiplication of Bernstein bases in both these methods are shown to contain certain symmetries that can be utilised to improve efficiency of the algorithm, and some of these multiplication processes can be run in parallel. Explicit examples are presented to compare the relative pros and cons of the different methods and their respective approximation errors.

Fast arithmetic for polynomials and matrices in Mathemagix

Grégoire Lecerf, CNRS, Université de Versailles

Abstract:
We will present the implementations of the elementary operations with polynomials and matrices available in the C++ libraries of Mathemagix. This includes most of the classical methods for univariate polynomials and series, but also very recent techniques with several variables. Most of the algorithms can easily benefit from parallelization and vectorization features now widely spread in all recent platforms. We will show applications to polynomial factorization and polynomial system solving.

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Wednesday 17 March

Optimal Delaunay Triangulation

Wenping Wang, Hong Kong University

Abstract: After a brief review of the concepts and previous works on Centroidal Voronoi Tessellation (CVT), I shall discuss our recent works on computation and applications of CVT: 1) GPU accelerated computation of CVT; 2) Extension of CVT to computing anisotropic triangulations; 3) Applying CVT to modeling minimal surfaces. Finally, I shall present a comparative study between CVT and ODT (optimal Delaunay triangulation), which is a promising alternative to CVT for generating high quality tetrahedral meshes.

The Genus Computation Problem: Symbolic-Numeric Solutions and Beyond

Madalina Hodorog, RICAM (Johan Radom Institute for Computational and Applied Mathematics)

Abstract: We report on a method for computing the genus of a plane complex algebraic curve. The method is based on knot theory and on the topology of singular points of plane complex algebraic curves, which is determined by the algebraic link of the singularity. The method states exactly how to compute the algebraic link of a singularity of a plane complex algebraic curve: (i) we take a plane complex algebraic curve considered as a real two-dimensional subset in the 4-dimensional space, which has a singularity in the origin; (ii) we consider the sphere centered in the origin and of a small radius; (iii) we intersect this sphere with the curve obtaining a real set in the 4-dimensional space, which we then project in the 3-dimensional space using the stereographic projection. For sufficiently small radius, we know that the image of this intersection set is an algebraic link (i.e. the algebraic link of the singularity). The algebraic link is uniquely identified by its corresponding Alexander polynomial, which is used to compute the delta-invariant of the singularity. The main formula for computing the genus requires the delta-invariant of each singularity of the plane complex algebraic curve. We base our method on several important results from algebraic topology proven by Milnor for the general case of algebraic varieties over infinite fields. We propose a symbolic-numeric algorithm, which is expected to give good results for complex algebraic curves whose defining polynomials have numeric coefficients. We split the main algorithm into several interdependent subalgorithms. We base some of our

subalgorithms on general algorithms from computational geometry (e.g. Bentley-Ottman), which we adapt for solving our particular problems. Whenever required, we design our own computational subalgorithms for solving the specific problems (e.g. computation of the Alexander polynomial). We use for the implementation of our algorithm the Axel algebraic geometric modeler, developed at INRIA, Sophia-Antipolis.
Keywords : Plane complex algebraic curve, genus computation, stereographic projection, algebraic link, Alexander polynomial.

Polynomial Knot Diagrams

P.-V. Koseleff, UPMC-Paris 6 and Inria-Salsa.

Abstract: It is known that every (non compact) knot admits a polynomial parametrization : $x = P(t)$, $y = Q(t)$, $z = R(t)$.

Let us given a polynomial parametrization, we explain how to compute the knot diagram, from which we can compute some invariants and try to identify the knot.

When K is a rational (or 2-bridge) knot with N crossings, we describe an explicit parametrization of K with degrees $(3, b, c)$ and $b + c = 3N$. We explain the specific tools we are developing in order to show that these parametrization have minimal degrees.

Keywords: Knot theory, continued fractions, interpolation, polynomial space curve.

Visualizing Real Curves and Surfaces with Singularities

Oliver Labs, Saarland University, Germany.

Abstract: We give an overview of the geometry of real algebraic plane curves and surfaces in three-space with an emphasis on their singular points. On the way, we will use several visualization tools to illustrate the current difficulties in producing good and correct visualizations. At the end of the talk we give a brief indication of recent applications in the emerging field of convex algebraic geometry.

On the Isotopic Meshing of an Implicit Algebraic Surface

Daouda Niang Diatta, University of Limoges

Abstract:

I present a complete algorithm for computing the topology of an algebraic surface S given by a squarefree polynomial in $\mathbb{Q}[X, Y, Z]$. The algorithm involves only subresultant computations and entirely relies on rational manipulation. We extend a previous work, on the topology of non-reduced algebraic space curves, and apply it to the polar curve or apparent contour of the surface S. This gives us rational parametrizations of the components of the polar curve, which are used to lift the topology of the projection of the polar curve. We deduce the connection of the two-dimensional components above the cell defined by the projection of the polar curve. A complexity analysis of the algorithm is provided leading to a bound in $\widetilde{\mathcal{O}_B}(d^{21}\tau)$ for the complexity of the computation of the topology of an implicit algebraic surface defined by integer coefficients polynomial of degree d and coefficients size τ.

Gotools

Tor Dokken, SINTEF.

Abstract:

GoTools is the third generation of spline based geometry toolkits from SINTEF. Its predecessors were APS B-spline Library implemented in FORTRAN (1978-1990), SISL - SINTEF Spline Library (1989-) in c. The evolution of GoTools started in the middle of the 1990s to take care of c++ code developed in different projects. Originally GoTools addressed challenges of curves and surfaces related to CAD. Following the IST FET-Open GAIA-project (2001-2005) GoTools was extended with approximate implicitization of

rational parametric surfaces. In 2008 became the core of the projects within SINTEF addressing Isogeometric analysis. Now GoTools is extended with tri-parametric (tri-variate) spline volumes addressing the expected needs of analysis. The talk will give an overview of GoTools and it modules, and give some information on how to get started using GoTools.

Geometric Modeling with Axel

Angelos Mantzaflaris, Galaad, INRIA

Abstract:
In this lecture we give an introduction to the Axel modeling environment. Axel is a QT-based application which aims in porting algebra-effective tools to CAD technology.

Using simple OpenGL primitives such as points, lines or triangles, Axel effectively visualizes and manipulates point clouds, meshes, parametric curves/surfaces, implicit curves/surfaces or volumes.

The computational support behind Axel is embedded in different Mathemagix modules, which bring the heavy machinery of computational algebra to the frame of geometric modeling. This includes effective algebraic representations, multivariate polynomial solvers, certified local computations, treatment of singular cases.

Main tasks supported by Axel include computing the topology of algebraic curves, the arrangement of several algebraic curves, the topology of surfaces, the (self-)intersection of parametric surfaces.

A way to extend or add functionalities is the plugin system. Among currently available plugins we find a plugin for 3D polytopes, an algebraic singularity analysis plugin, isogeometic analysis plugins, a tree reconstruction plugin and GPU ray caster plugin.

We demonstrate the use of the framework by giving some details on recent on-going work on manipulating semi-algebraic subsets in Axel.

Thursday 18 March

From Waring problem to tensor rank through secant varieties

Alessandra Bernardi, Centro Internazionale Ricerca Matematica - Fondazione Bruno Kessler

Abstract: A Number Theory problem, posed by Waring in 1770, asks which is the minimum integer r such that the generic integer can be written as the sum of r d-th powers of r integers. Another problem, originated from this, thook shape in Algebraic Geometry and it generated a research area that studies the dimensions of secant varieties of varieties parameterizing certain kind of homogeneus polynomials. The most known case, since nowadays it is the only one completely solved, is the computation of the dimension of secant varieties of Veronese varieties (due to J. Alexander and A. Hirschowitz - 1995). The study of secant varieties of varieties parameterizing homogeneous polynomials have a great interest in the litterature and it was extended to secant varieties of varieties parameterizing tensors that is strictly connected to the notion of rank of a generic tensor. The knowledge of the rank of a tensor and its decomposition in terms of tensors with symmetric properties, are two of the most acheived areas of modern interest in many applications (telecommunications, phylogenetics...). In this seminar I will present classical aspects and tecniques for the study of the dimension of secant varieties, starting from linear algebra to open questions on the rank of tensors, through known results and some of the tecniques used today to study secant varieties and tensor rank from an Algebraic Geometry point of view.

Effective representation of Hilbert scheme

Jérôme Brachat, INRIA.

Abstract:

In this presentation we will examine the effective representation of the punctual Hilbert scheme. We give new equations, which are simpler than Bayer and Iarrobino-Kanev equations. These new Plücker-like equations define the Hilbert scheme as a subscheme of a single Grassmannian and are of degree two in the Plücker coordinates. Following a dual point of view, we prove that the punctual Hilbert functor Hilb can be represented by the scheme Hilb defined by these relations and the well-known Plücker relations on the Grassmanian. This explicit complete set of defining equations for Hilb are deduced from the commutation relations characterising border bases and from generating equations.

Rational cuspidal plane curves

Karoline Moe, Institute of Mathematics, University of Oslo.

Abstract: How many and what kind of cusps can a rational cuspidal plane curve have? This talk will address the above question. In particular, we will look at rational cuspidal curves of degree three and four. We will present explicit constructions of some of these curves using two different methods; projections and Cremona transformations. We will also discuss a surpris- ing conjecture regarding the number of cusps of a rational cuspidal plane curve. We will link this conjecture to the question of whether all cusps of a rational cuspidal plane curve can have real coordinates.

Counting singular curves on surfaces

Nikolay Qviller, Institute of Mathematics, University of Oslo

Abstract: A classical problem in algebraic geometry is to count plane curves of given degree d satisfying certain requirements; for instance, we may impose a given number of double points, r. Increasing this number lowers the dimension of the curve family considered. We may further reduce the dimension by requiring that the curves pass through preassigned points in general position in the plane, until we have a 0-dimensional (i.e., finite) family. The number $N_r(d)$ of such curves can be obtained, as a function of d and r, by a combination of modern intersection theory and classical combinatorics, with the appearance of the well-known Bell polynomials. Interest in this kind of problems has recently been revived, as the generating functions of such curve numbers appear naturally in the context of physics' mirror symmetry. There exist several conjectures regarding the shape of the function $N_r(d)$. We will discuss and present basic evidence of some of these.

G^1 Bézier surface generation given boundary curve network with T-junctions

Tae-wan Kim, Seoul National University

Abstract:

T-junction usually appears in modeling process using given curve network. However, no T-shape patch topology is allowed in current CAD/CAM system. The existing G^1 surface generation methods are only for the n-sided patches (n-valence). Therefore the designer designs the curve network without a T-junction, or subdivides it into n-side patches to avoid a T-shape topology. In this talk we suggest the generation method of novel piecewise G^1 Bézier surfaces with T-junctions, which combines the coplanar G^1 continuity condition with de Casteljau algorithm to satisfy the vertex enclosure constraint. Here we need no subdivision or triangulation of the domain, and the curve network is kept unchanged.

Parameterization of computational domain and B-spline fitting using approximate implicitization

Thien Nguyen, Johannes Kepler University, Linz

Abstract: Since its introduction in 2005 by Hughes et al., Isogeometric Analysis (IGA for short) has been proved to be a promising framework for bridging the gap between computer aided design (CAD) and finite

element analysis (FEA). The computational domain is described exactly by using NURBS representations, instead of triangular or quadrilateral meshes used in the classical FEA. However, the existing CAD systems do not provide any functionalities for extending boundary representations of models in a general way to volume representations.

Given a (NURBS) boundary representations of an object, our research aims at finding techniques for automatically generating a transformation which maps the object to the unit cube or unit square, in order to obtain the parameterization of the object. Our approach is based on approximate implicitization. Consequently, we first change the representation of the boundaries to the zero level sets of several scalar-valued functions. These functions are then used to generate the parameterization of the object. Finally, standard least-squares methods are used to fit a NURBS representation to the object.

Isogeometric analysis with Axel

Gang Xu, INRIA Galaad

Abstract: In this talk, the isogeometric part in Axel will be introduced, including the isogeometric solver and optimizer for the heat conduction problem, optimal parameterization of the computational domain in IGA, visualization of the IGA results, and the ongoing work on simplex spline plugin for IGA problem. Several examples will be presented to show how to use this toolbox for isgeometric application.

Friday 19 March

Real Solving

Elias Tsigaridas, University of Athens

Abstract:

We present recent algorithmic, complexity and implementation results for the problem of real root isolation of univariate polynomials and polynomials systems. For the univariate case we will present the subdivision algorithms, the CF algorithm, constant arithmetic complexity algorithms for degree ≤ 5, and new results about the real roots of random polynomials and and average case complexity. For the bivariate case we present three variants of a projection based algorithm that achieves the best known bit complexity bound, and we also sketch how they allow us to compute the topology of a real plane algebraic curves. Finally, we present a recent extension of the CF algorithm to the multivariate case, and ongoing work on multivariate separation bounds.

Stable border bases for ideals of points

Maria Laura Torrente, Università di Genova

Abstract:

We present a method and the corresponding algorithm for computing structurally stable border bases of ideals of points whose coordinates are affected by errors. The decision to use border bases to describe vanishing ideals of sets of approximate points is due to two main reasons: border bases have always been considered a numerically stable tool (see [3], [4], [5], [6]) furthermore, it is easy to study their structure, that is the support of their polynomials, as it is completely determined once a suitable order ideal has been chosen. If X is a set of empirical points, representing real-world measurements, then typically the coordinates of its points are known only imprecisely. Roughly speaking, if $\widetilde{X}$ is another set of points, each differing by less than the uncertainty from the corresponding element of X, then the two sets can be considered as equivalent. Nevertheless, it can happen that their vanishing ideals have very different bases–this is a well known phenomenon in Grobner basis theory. In order to emphasize the numerical equivalence of the points sets X and $\widetilde{X}$, we provide a common characterization of the their vanishing ideals $I(X)$ and $I(\widetilde{X})$. The method that we present (see [1]) computes a polynomial basis B of $I(X)$ which exhibits structural stability:

namely, there exists a basis $\widetilde{B}$ of the perturbed ideal $I(\widetilde{X})$ sharing the same structure as B, and whose coefficients differ only slightly, provided that $\widetilde{X}$! differs from X by only a small amount (up to some limit). The corresponding algorithm is implemented using the C++ language and the CoCoALib, and is available at [2].

Linear Algebra and Polynomial System Solving

Philippe Trébuchet, LIP6, Paris

Abstract: In this tutorial session, the usual technics for solving linear and nonlinear systems of equations will be presented. We will start this session by the implementation of the well known Gauss/Seidel iterative method for linear systems. A comparison with the results obtain via direct methods, either using LU decomposition or QR decomposition will be undertaken. Next experimentations involving eigenvalues/eigenvectors computations will be undertaken : the error estimators provided by the library Lapack will be compared to what is effectively computed by the different procedures in different precision.

Next, once used to the linear algebra tools, polynomial system solving tools will be presented : of short presentation of the border bases technics will made and somme classical results about zero dimensional systems recalled. We will present the different functions that are necessary for constructing the operators characterized by the well known results recalled and experiments on computing their eigenelements in different precision undertaken. As an illustration of this way of solving polynomial systems an algorithmic geometry problem will be treated.

Zero distribution algorithms for matrix polynomials

Ibrahim Adamou, Universidad de Cantabrira (SPAIN) - Université Abdou Moumuni (NIGER).

Abstract: Let N(x) be a given m x m non-singular matrix polynomial in one variable. We look for an algorithm computing the number of zeros of det(N(x)) inside some regions (the open unit disk; the open upper half plane and the open right half plane) of the complex plane C in term of signature of certain constant hermitian matrix X constructed from the coefficients of the matrix polynomial N(x).

In this talk we consider only the open unit disk case. When deg(N(x))=1 we will use the generalized eigenvalues of the matrix pencil N(x) in order to compute the roots of det(N(x)) in the open unit disk without computing explicitly det(N(x)) and through the signature of a constant matrix X before mentioned. At the end of this presentation we will introduce a conjecture for the explicit form of the matrix X in some particular cases.