SAGA
ShApes Geometry Algebra

BOOK OF ABSTRACTS

SAGA FALL SCHOOL

KOLYMPARI, GREECE

OCTOBER 4–8, 2010

MARIE CURIE ACTIONS
SEVENTH FRAMEWORK PROGRAMME
National and Kapodistrian University of Athens

CONTENTS

FOREWORD

The **Second Annual SAGA School** (http://erga.di.uoa.gr/sagaschool) takes place on 4–8 October 2010, at the OAC conference center, near Kolympari, Greece. It is preceded by the SAGA Midterm Review. Both events are in the framework of the Marie-Curie Initial Training Network SAGA: Shapes, Geometry, and Algebra (http://www.saga-network.eu), funded by the EU within FP7. They are organized by the Department of Informatics and Telecommunications of the University of Athens.

The School includes Tutorial presentations, Contributed talks, and a Poster session, corresponding to the three parts of this Book. The scientific subject of the School is Computer-Aided Geometric Design; more specifically, the School aims at supporting the exchange of ideas between the fields of Geometric Modeling, Algebraic Geometry, and Numerical Analysis. Topics of interest include: Geometry of curves and surfaces, Singularities: detection and analysis, Curves and surfaces classification, Intersection problems, Isogeometric Analysis, Rational/Implicit representation, Resultant constructions and implicitization, Approximate and/or certified methods, and Computer implementations of Algorithmic algebraic geometry.

The School includes 5 plenary presentations, each including a tutorial presentation, of which two are offered by SAGA members and the others by researchers from USA, Argentina, and Greece. There is one presentation and interactive session on communicating science by a free-lance science journalist from USA. Confirmed participants include 28 SAGA members, in addition to two affiliated scientists.

Besides SAGA members, the School includes external students and researchers in academia and research institutes. In fact, the Network supports the participation (and partial travel costs) of 20 students, 6 postdoctoral researchers and 2 senior researchers from Greece, France, Germany, Austria, Italy, USA, and Denmark (in order of number of participants). There are another two student and one researcher participants from Korea, USA, and Germany.

Last but not least, the program includes a half-day excursion to the Archelogical Museum of Chania and the Old City, concluding with dinner at the Old Port of Chania.

Scientific and Organizing Committee: Tor Dokken (SAGA Coordinator), Ioannis Emiris (Local Coordinator), Raffaele De Amicis, Laureano Gonzalez-Vega, Bert Juettler, Rimvydas Krasauskas, Dominique Laffret, Bernard Mourrain, Ragni Piene, Ewald Quak, Eigil Samset.

Local Organizing Committee: Ioannis Emiris, Vissarion Fisikopoulos, Tatjana Kalinka, Christos Konaxis, Angelos Mantzaflaris.

PART I. COURSES AND TUTORIALS

DEMONSTRATION OF TOPSOLID SOFTWARE

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The talk will focus on the features of TOPSOLID CAD to design example parts and then will focus on TOPSOLID CAM to describe the operations involved in machining the designed part. Results of an algorithm implemented for finishing operations (material left removal) are then shown.

IMPLICITIZATION TECHNIQUES: A FRUITFUL INTERACTION BETWEEN PRACTITIONERS AND ALGEBRAISTS

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We will recall different algebraic techniques for the implicitization of rational surfaces, with an emphasis on the use of linear syzygies. The theoretical approach for this method uses rather sophisticated algebraic techniques for its justification, introduced in this setting by Laurent Busé, Marc Chardin and Jean Pierre Jouanolou. Their work was inspired by the method of moving curves, proposed by Thomas Sederberg and Falai Chen. We will focus on exploiting the monomial structure of the polynomials defining the parametrization, following recent work with Nicolás Botbol and Marc Dohm, and we will concentrate on expressing the theoretical results into very concrete terms. We will also present implementations in the free computer algebra system Macaulay2, developed by Nicolás Botbol, Marc Dohm and Manuel Dubinsky.

THE HISTORY OF CAD AND EXAMPLES OF ITS USE IN INDUSTRY

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What we now know as Computer Aided Design (CAD) and Computer Aided Manufacturing (CAM) can be traced back to the 1950s and the first uses of computers in design and manufacturing. The first interactive CAD-type system Sketchpad was demonstrated as early as 1963. However, the price of 3D graphics systems was prohibitive for use in all but the most advanced industries until the end of the 1970s. The CAD of the 1960s and early 1970s was dominated by batch systems using punched cards as input, and pen plotters as output to produce engineering drawings. CAD was used to automate the design of engineering drawing, and to generate NC-programs for controlling machines with numerical controls. The introduction of the Tektronix 4010 storage tube in 1972 drastically reduced the cost of graphics screens, however, they still priced very high compared to current graphics systems. This, however, allowed the input to be interactive, and the results to be displayed on a high resolution screen, the interactive 2D CAD systems for drawing systems emerged.

As the design drawings represented 3D shapes, the next step was 3D design systems. The first 3D systems were wire frame oriented, with the shape represented by a network of curves. However, the wire frame did not give a complete description of the geometry, so a further step was to describe the objects by the surfaces constituting the inner and outer hulls. As early as 1962 Pierre Bézier described what are now known as Bézier surfaces for the use in the automotive industry. However, commercial surface based CAD-systems were not widespread until the 1970s. The mathematical approaches were fragmented, and not until the deployment of B-splines in CAD-systems, did a unified approach to surface descriptions in CAD emerge. The theory of B-splines was established in the 1970s with the Cox de Boor algorithm in 1972, and the Oslo Algorithm in 1980.

In 1973 Ian Braid in his PhD thesis described the boundary structures for solid models with elementary shaped surfaces. Later NURBS have been added to these structures. In 1985 Alan Grayer, Ian Braid, and Charles Lang formed Three-Space Ltd which developed ACIS, which is the solid modelling kernel for many current CAD-systems.

The introduction of the Ethernet in the 1980s introduced simplified communication between computers, and gradually information exchange on paper was replaced by digital information exchange. Data exchange standards for CAD were already on the agenda in the 1970s, but were first developed as an international standard in the late 1980s and the early 1990s as the ISO 10303 STEP-standard. This standardization has allowed interoperability between CAD-systems from different vendors, and from CAD to other systems in product development and production. However, the introduction of the CAD data exchange standard also reduced the room for innovation in CAD. Consequently the CAD-systems in 2010, in principle are very similar to the CAD-systems of the 1990s. However, the technology is much more stable; the models handled more complex, and the 3D graphics of much higher quality.

The lecturer started to work with development of spline technology for CAD in 1978 at what is now known as SINTEF in Oslo, Norway. Thus he has experienced much of the above evolution, and also met many people from the earlier days of CAD. The talk will combine the general history of CAD and personal experiences of CAD related research though a three decade period.

THE EMERGING PIPE JUNCTION CHALLENGE

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As the intersection curve between two cylinders cannot be represented exactly using a rational parametrization, CAD-systems have to resort to approximations when representing pipe junctions.

In the early days of CAD when curves were plotted on paper this did not represent a problem, as the plotted curve itself was inaccurate as the plotting process involved approximation of the intersection curve by straight lines or circle segments. When the intersection curve was used for production purposes (e.g., numerically controlled machining) there was no problem as long as the tolerances of the curve approximation were kept within production tolerances.

When boundary structured (B-rep) based CAD with trimming was introduced from the end of the 1980s CAD models still were only used for design and production purposes so pipe joint approximations represented no large problem. B-rep based CAD allows that adjacent surface patches are not exactly matched; small gaps between surfaces are allowed provided they are within controlled tolerances. When such models are used in Finite Elements Analysis they have to go through a conversion process named gridding. In this process the gaps are removed, making the model watertight. Traditional Finite Elements are of low degree (typically degree 2 or lower), the cylinders and their intersection curves have consequently to be approximated. So also in this traditional analysis scenario the inaccurate pipe junctions of CAD poses no special problem.

However, the introduction of NURBS based isogeometric analysis in 2005 by Tom Hughes once more put the pipe junction challenge on the agenda. As the general intersection curve between two surfaces do not have a rational parameterization, some approximation strategy has to be employed for modelling pipe junctions for NURBS based isogeometric analysis. An additional complication is that B-rep type trimming is not employed in isogeometric analysis, so the pipe junction has to be built from regular volumetric blocks with 6 faces. The talk will address different aspects of the pipe junction challenge.

CAD IN THE AIRCRAFT INDUSTRY

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Historically, design of airplanes and aerospace products is accomplished in three distinct phases: Conceptual design, preliminary design, and detailed design. The purpose of conceptual design is to explore broad design alternatives and match them up with specific vehicle mission requirements. For example, should an airplane have a fuselage and two wing-mounted engines or should it be a flying wing configuration with engines mounted on top? What configuration best matches the overall engineering requirements? Once the broad concept has been identified, preliminary design can take place. In this phase, the purpose is to identify overall size and shape, determine specific engineering approaches for all of the components of the design, analyze airplane performance in considerably more detail than was possible in conceptual design, and optimize many attributes of the configuration as well as its overall performance. Finally, in detail design, every component of the airplane must be defined to a level of detail sufficient to enable its manufacture. Over the past 20 years or so, each phase has been characterized by vast differences in how much Computer-Aided Design (CAD) is used in each phase, with very little usage in conceptual design, some usage in preliminary design, while detailed design has been accomplished almost exclusively with CAD tools. As advanced aerospace designs become more integrated and engineering practice becomes more multidisciplinary, it becomes increasingly important to make heavy use of CAD not only in preliminary design, but often in conceptual design as well. There are several reasons for this, including the desire to perform higher fidelity engineering analysis at earlier stages of design, a need to perform analysis and optimization in increasingly automated ways, and a need to better integrate engineering knowledge directly into the design process. After providing an overview of the airplane design process, this talk will focus primarily on the last of these needs, namely the need to better integrate engineering knowledge directly into the design process.

This requirement leads to substantial changes in the process for lofting curves and surfaces within CAD in many cases, and particularly those engineering applications which are heavily motivated by the underlying physics of the design challenge.

COMPUTER AIDED SHIP DESIGN

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A bit of history - The ship-design problem as a constrained optimization problem: the case of minimizing the wave resistance - Ship-hull geometric modeling: constructing a parametric model - Designing the control-curve network: constructing fair 2D/3D curves under shape constraints - Conclusions and future work.

OVERVIEW OF THE CAD MARKET, VENDORS AND SOFTWARE/ HARDWARE REQUIREMENTS

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Approximate market size - Sectors requiring CAD/CAM (Automotive, Aircraft, Appliances, Energy) - Fields of application (Mechanical, Architecture, Wood) - Review of vendors - Operating systems, CPUs , graphics systems, Cloud computing.
Open discussion.

WRITING ABOUT SCIENCE/ A FREELANCER'S LIFE

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A neglected part of the training of many scientists is the art of communicating their work to the public. Science communication is important, because most of the money for scientific research ultimately comes from the public, and the taxpayers deserve to know what they are getting for their money. Writing for the public can also be fruitful for scientists. It forces them to confront the big questions that the public always wants answers to: Why am I doing this work? Why does it matter? Where did the questions that I am studying come from? Where is the field going? Popular science writing is very different from teaching science in the classroom, and even more different from writing technical articles or research articles. A science writer needs to think about making the story accurate, understandable, and entertaining. Accuracy comes naturally to scientists, but relatively few think about how to make their writing understandable to the general public, and most have no clue at all how to make their writing fun to read. In this first lecture, Dana Mackenzie will talk about some of the tricks of the trade for science writers, and some of the bad habits that academics need to unlearn when writing for the public for the first time. He will also tell how he changed careers from mathematician to math and science writer. He will also lead a group discussion, in which we will discuss the practical aspects of writing a popular science story and we will look at examples that the participants have brought with them. It should be interesting to compare popular science writing from different countries.

PART II. CONTRIBUTED PRESENTATIONS

WOODEN RECONSTRUCTION USING A QUASI-ISOPYCNIC MODEL APPROXIMATION

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During the last few years, we have seen an increasing use of methods for iso-surface or iso-line extraction in many branches, e.g. in medicine for visualization of MRI (Magnetic Resonance Imaging) or CT (Computed Tomography) data, meteorology or in geography. In this talk, we will describe method used to reconstruct wooden structures from tomography images. More specifically, we will show how a quasi-isopycnic model can be used to create an approximated spline representation that best fits the wooden inner structure.

ISOGEOMETRIC ANALYSIS AND SHAPE OPTIMISATION

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One of the attractive features of isogeometric analysis is the exact representation of the geometry. The geometry is furthermore given by a relative low number of control points and this makes isogeometric analysis an ideal basis for shape optimization. I will describe some of the results we have obtained and also some of the problems we have encountered.

One of the problems is the parametrization problem. The geometry of the shape is given by the boundary but isogeometric analysis requires a parametrization of the whole domain. So in every optimization cycle we need to extend a parametrization of the boundary of a domain to the whole domain.

Another problem is «empty degrees of freedom». If the shape contains a straight line the control points can move without changing the geometry. In the context of shape optimization, the optimizer may then make a bad parametrization that introduces numerical errors that improve the objective function.

SHAPE-PRESERVING INTERPOLATION IN TWO AND THREE DIMENSIONS

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We are going to discuss the concept of shape-preserving interpolation in two and three-dimensions, and present a couple of methodologies for constructing shape-preserving splines that interpolate a given set of two- or three-dimensional points (along with appropriately defined boundary conditions). We will focus on potential drawbacks of existing notions of shape-preserving interpolation and suggest ways to improve them. Finally, we will dwell on open problems and possible extensions.

COMPUTING REFLECTIVE SYMMETRIES FOR B-REP NURBS VOLUMES

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Symmetry properties of components have many applications during a product development process, including shape transformations for modification purposes, Finite Element Analysis (FEA), model retrieval, etc. This paper presents an algorithm to generate 3D model symmetry planes using the B-Rep model of CAD volumes. In the framework of CAD software, 3D models are described as B-Rep volume models. Design processes of volume models strongly rely on extrusion and revolution primitives from sketches containing essentially straight line segments and circular arcs. Hence, the boundary surfaces considered are planes, cylinders, cones, tori and spheres. Global geometry properties help characterizing the symmetry planes that can initiate the global symmetry planes of the object. The intersection curves of these reference surfaces provide strong properties to characterize possible global symmetry planes of the object. Then, the algorithm starts from analyzing the symmetry properties of the faces followed by the intersection curves between them. Subsequently, the candidate symmetry planes set up contains all the possible global symmetry planes. Finally, the properties of neighboring surfaces and the use of intrinsic parameters of surfaces help determining robustly the global symmetry planes, whether there is a finite number or an infinite number. Computing all the symmetry planes can

be achieved with a complexity smaller than quadratic with respect to the number of faces of the B-Rep, which is much faster than algorithms processing facetted representations.

SOLVING THE CURVE/CURVE AND CURVE/SURFACE INTERSECTION PROBLEMS WITH MATHEMAGIX

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Computing the intersection between algebraic varieties is a fundamental task in Computer Aided Geometric Design. Several methods and approaches have been developed for that purpose. Some of them are based on matrix representations of the objects that allow to transform the computation of the intersection locus into generalized eigencomputations. The aim of our works is to compute the matrix representation of these objects and to show that similar algorithms can be implemented even if the matrix representation used are non square matrices. In this talk, we present some implementations of the software *Mathemagix* to solve the curve/curve and curve/surface intersection problems.

THE JOURNAL OF MATHEMATICS IN INDUSTRY

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The talk will describe the process of launching the new Journal of Mathematics in Industry. I shall explain its scientific concept, how it differs from journals with similar names, and also how Open Access publishing works with this journal.

DISTANCE COMPUTATION USING QUADRATICALLY SUPPORTED CURVES

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I will present the recent progress on the work made with Bert Juettler during my post-doc position in Linz. We are interested in the computation of the distance of two given curves. The idea is to approximate these curves by «fat» quadratically supported curves and circles, and then to compute the distance between the two approximation. This talk will discuss of the approximation process.

SOME REMARKS ON INTELLECTUAL PROPERTY RIGHTS

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In this complementary talk, I will give a short introduction concerning various intellectual property rights and their protection (through patents, etc.) that researchers should be aware of.

SOFTWARE DEVELOPMENT FOR INDUSTRIAL APPLICATIONS USING THE EXAMPLE OF A DIP PAINT SIMULATION

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It is a big challenge to develop dip paint simulation software for industrial applications, since the CAD data as input data for the computation is very difficult to handle and there a lots of requirements of the customer which need to be considered. For our software ALSIM and MERGE we had to consider following aspects: When dipping in a full Body in White it is necessary to avoid the becoming of gas bubbles. Usually these gas bubbles influence the e-coat quality and thickness. As a result the corrosion of an auto body starts much earlier. In the same way after dipping out of the bath it is important, that the liquid can drain off. Otherwise it may happen that liquid volumes get carried over to the next bath which leads to a fouling of the bath. It is also possible that liquid volumes come to the drying chamber where the volumes are forced to open. This also results into corrosion problems. Even today it happens that prototypes of Body in Whites after the dip paint process get sliced up to be inspected for the problem areas. This takes a lot of time and delays the whole development process of an auto body.

SIMPLE APPROXIMATE VARIETIES FOR SETS OF EMPIRICAL DATA

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We present a symbolic-numeric approach for the analysis of a given set of noisy data, represented as a finite set X of limited precision points. Starting from X and a permitted tolerance ϵ on its coordinates, we present a method that automatically determines a low degree monic polynomial whose associated variety passes close to each point of X by less than the given tolerance ϵ.

MULTIVARIATE (AGGREGATE) SEPARATION BOUNDS

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We present new and close to optimal separation bounds, that is bounds on the minimum distance between any two isolated roots of a polynomial system. We call them DMM after Davenport, Mahler and Mignotte. These are the first bounds applicable to arbitrary positive dimensional systems. For the construction we exploit the structure of the system and the height of the sparse resultant by means of mixed volume, as well as recent advances on aggregate root bounds for univariate polynomials. DMM allows us compute improved bounds for the number of steps that a subdivision algorithm performs in any dimension.

This is joint work with Ioannis Emiris and Bernard Mourrain.

DIMENSION OF C^1 SPLINE SPACES ON THE PLANE

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Given a triangulated polygon Π in R^2 , we consider the space S^n (Π) of C^1 piecewise polynomial functions of degree n defined in Π. We wish to study two questions: what is the dimension of this space and what is a convenient basis for it. These two questions were consider by Gilbert Strang and he conjectured a formula for the dimension. In this talk I shall present some methods that have been used to calculate the dimension, such as a recent application of the theory of Gröbner basis and the method used by Morgan and Scott to construct a nodal basis which verify the conjecture for most of the values of n. For the values of n for which these are still open questions I will show some positive examples and one example for n = 2 where the formula does not give the correct dimension.

PART III. POSTERS

ZERO DISTRIBUTION ALGORITHMS FOR MATRIX POLYNOMIALS

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Laureano Gonzalez–Vega

Let $N(x)$ be a $m \times m$ non–singular matrix polynomial in one variable and complex coefficients. We look for an algorithm computing the number of complex zeros of $\det(N(x))$ inside some concrete regions of the complex plane $\mathbb{C}$ (such as the real line, the open unit disk, the open upper half plane or the open right half plane) in terms of the signature of a constant hermitian matrix B constructed from the coefficients of the polynomials in $N(x)$ (and avoiding the explicit computation of $\det(N(x))$).

The computation of the roots of $\det(N(x))$ is a question appearing very often when dealing with algebraic plane curves. For example, when computing the topology of the real curve $f(x, y) = 0$, $N(x)$ is the matrix whose determinant produces the discriminant of f with respect to y and we look for the real roots of $\det(N(x))$ or, when computing the intersection points between the curves $f(x, y) = 0$ and $g(x, y) = 0$, $N(x)$ is the matrix

whose determinant produces the resultant of f and g with respect to y (i.e. the Sylvester or the Bezout matrix of f and g with respect to y) and we look for the complex roots of $\det(N(x))$.

Following the results of Dym and Volok (H. Dym and D. Volok, Zero distribution of matrix polynomials, Linear Algebra and its Applications 425 (2007) 714–738) we introduce a new algorithm determining explicitly a constant matrix B whose signature produces the required information concerning the roots of $\det(N(x))$ when the polynomials in $N(x)$ have degree one.

We will show also the connection of this approach with the generalized eigenvalue problem and how it can be used when dealing with algebraic plane curves.

IMPLICIT EQUATION OF BIGRADED SURFACES

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We present a method for computing the implicit equation of a hypersurface given as the image of a rational map φ : P1 × P1 --> P3 , via syzygies. This method can be extended to a more general setting, putting instead of P1 × P1 any arithmetically Cohen-Macaulay toric variety. In previous articles by Busé-Dohm and Botbol-Dickenstein-Dohm, P1 × P1 is embedded in a projective space via a Segre-Veronesse embedding. The need of the embedding comes from the algebraic conditions of the method (which require the study of the regularity of the coordinate ring). The aim of this talk is to give an alternative to this approach: we study the implicitization problem directly, without any embedding. Indeed, we deal with the bihomogeneous structure of the coordinate ring of P1 × P1 . The main motivations for our change of perspective are that it is more natural to deal with the original grading, and that the embedding leads to an artificial homogenization process that makes the effective computation slower, as the number of variables to eliminate increases. Finally we show an implementation in Macaulay2 that exploits the bihomogenity, decreasing the sizes of the matrices the method computes. We also compare with a similar implementation by means of the Segre-Veronesse embedding.

MEAN VALUE SURFACE FITTING

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Floater's mean value coordinates generalize barycentric coordinates to arbitrary polygons. This makes them ideal for applications such as image warping and surface parameterization. Langer and Seidel introduce another application by considering the smooth 2-manifold that results from using mean value coordinates to interpolate a non-planar control polygon. These 'mean value surfaces' could be useful in animation, compression and visualization, if paired with a good algorithm for data fitting. To explore the viability of this application, we consider the problem of fitting a mean value surface to a set of point samples in R^3. Our results show that for some datasets, the resulting representation is more flexible than polynomial surfaces and able to fit the data with lower error whilst using a comparable number of degrees of freedom. There are several open problems, however, surrounding the initialization and robustness of this fitting process.

AN ENUMERATION ALGORITHM OF CLASSES OF REGULAR TRIANGULATIONS

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Motivated by resultant computations we study the convex polytope, called the Secondary polytope, whose vertices correspond to all regular triangulations of a given point set A in R^d. A triangulation of A is called regular if it can be obtained as the projection of the lower hull of the lifted A, for some lifting to R^{d+1}. For this polytope there is an output-sensitive algorithm for enumerating its vertices. We describe equivalence classes of the vertices of this polytope. Our goal is output-sensitive algorithms for enumerating one representative from each class. To this end, we offer algorithmic characterizations of the polytope's edges that connect two different classes and we use them to construct an enumeration algorithm of these classes. Some applications of this theory are in polynomial equation solving and in the implicitization of parametric (hyper)surfaces.

WACHSPRESS VARIETIES

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We examine an algebraic variety parametrized by barycentric coordinates of a polygon. We will make use of a polynomial that we can associate to any polygon called the adjoint to find generators of the ideal of this variety. We also observe that this variety can be seen as the blowup of certain points in the projective plane.

IMPLICITIZATON OF RATIONAL PARAMETRIC CURVES AND SURFACES USING PREDICTED SUPPORT

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In order to change the representation of geometrical objects presented in parametric form, to implicit form, predicting the support of the implicit equation proved to be useful, since it reduces implicitization to linear algebra. In this work, we compare support prediction methods that apply sparse elimination theory to

calculate the implicit equation's Newton Polytope. These methods are suitable for polynomial and rational parameterizations and are based on computing mixed subdivisions of the input Newton Polytopes and regular triangulations of point sets defined by Cayley's trick.

SINGLE-LIFTING MACAULAY-TYPE FORMULAE OF GENERALIZED UNMIXED SPARSE RESULTANTS

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Resultants are defined in the sparse (or toric) context in order to exploit the structure of the polynomials as expressed by their Newton polytopes. Since determinantal formulae are not always possible, the most efficient general method for computing resultants is by rational formulae. This is made possible by Macaulay's famous determinantal formula in the dense homogeneous case, extended by D'Andrea to the sparse case. However, the latter requires a lifting of the Newton polytopes, defined recursively on the dimension. Our main contribution is a single lifting function of the Newton polytopes, which avoids recursion, and yields a simpler method for computing Macaulay-type formulae of sparse resultants. We focus on the case of generalized unmixed systems, where all Newton polytopes are scaled copies of each other, and sketch how our approach extends to mixed systems of up to four polynomials, as well as those whose Newton polytopes have a sufficiently different face structure. In the mixed subdivision used to construct the matrices, our algorithm defines significantly fewer cells than D'Andrea's, though the formulae are same in both cases. We discuss asymptotic complexity bounds and illustrate our results by fully studying a bivariate example.

APPROXIMATE VOLUMES OF TREMENDOUS CONSTRUCTIVE SOLID GEOMETRY MODELS

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The input to the Monte Calro algorithm for solving the neutron transport equation is a hierarchy of millions of Constructive Solid Geometry nodes. The volume of each node must be approximated accurately because they are used for computing physical properties such as pressure and temperature. Exact, mesh-

based and stochastic algorithms for volume calculation have been considered but can be too slow in practice. Exact algorithms resolve details that result in negligible volume, mesh-based algorithms create and maintain more topological structure than is necessary and stochastic algorithms do not take full advantage of the input representation. We propose an algorithm that does not resolve small features, or topology. Instead, we compute a set of trimmed surface patches that approximate a node's boundary and use the decomposability of volume calculation to compute the volume. In addition, we exploit the hierarchical structure of the model to reduce computation. I will illustrate the input to the Monte Carlo algorithm and describe our volume calculation algorithm. In addition, I will discuss the engineering decisions used in our implementation and provide experimental results on realistic inputs.

GENERATING G^1 BEZIER SURFACE USING T-SHAPED DEGENERATED BOUNDARY CURVES

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There are methods for generating G^1 Bezier surface using boundary curves. Usually, when three boundary curves meet on a point (valence 3 case), we can make three Bezier surfaces with G^1 continuity. However, if two curves are collinear then the surface will be degenerated surface and the three boundary curves have T-shape. In this case, we cannot use the previous G^1 surface generating methods because we cannot calculate the normal vector on the point of degenerated surface. In this paper, we suggest the G^1 Bezier surface generating method for T-shaped degenerated boundary curves.

COMPUTING THE TOPOLOGY OF AN IMPLICIT OR PARAMETRIC PLANE CURVE WHOSE DEFINING EQUATIONS CAN ONLY BE EVALUATED

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The problem of computing the graph (even topologically) of an implicitly defined algebraic plane curve has received special attention from the CAGD and Symbolic Computation communities, independently. Such a problem is important in CAGD because it is a basic subproblem which appear very often

in practice. For the Symbolic Computation community, on the other hand, such a problem has been the motivation for many achievements in the study of subresultants, symbolic real root counting, and infinitesimal computations, etc. In many practical problems, one is interested in computing the topology of an algebraic curve whose parametric or implicit equations have rather high degrees, numerous terms and very large coefficients. Applying the currently available methods for computing the topology of the curve, with such polynomial equations, requires a big amount of memory space and a significant amount of computing time. This is one of the main reasons why, in this work, we assume that neither the implicit equation, nor the parametrization of the given curve are known. Instead, sufficient number of points in the curve and the corresponding values of the equations and derivatives are known, and the degree is also given or can be deduced. The method presented here allows the computation of the topology of the curve using only this data. The method also works, if one has the implicit equation (or the parametrization) of the curve, together with an efficient procedure for computing the required points and the corresponding values of the derivatives.

GEOMETRIC PREDICATES AS ARRANGEMENTS OF HYPERSURFACES: APPLICATION TO COMPARISON OF ALGEBRAIC NUMBERS

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A lot of geometric predicates can be formulated as an arrangement of hypersurfaces (algebraic varieties) in a high-dimensional space, where each cell of the arrangement corresponds to an outcome of the predicate, and an evaluation of the predicate maps to point-location queries in this arrangement. To do this successfully, the arrangement has to be decomposed by the aid of subsidiary hypersurfaces, the degree of which plays a fundamental role in the algebraic complexity of the predicate, with respect to the input coefficients. For example, the widely used predicate of root comparison of quadratic polynomials can be mapped to an arrangement of lines and a parabola. For cubics, it becomes an arrangement of planes and a quartic surface, when a monic polynomial of degree d is represented as a point in R^d. Minimizing the degree of the subsidiary equations is an outstanding open problem.

THIN SHELLS FOR SHAPE ANALYSIS AND GEOMETRIC OPTIMIZATION

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We consider thin shells of constant thickness that can be parametrized over a middle surface and consist of homogenous and isotropic material. Under certain assumptions, including the Kirchhoff-Love assumption, elastic deformations of such a shell can be described by geometric properties of the middle surface. We consider two different discretizations on thin shells: an efficient discretization based on polyhedral surfaces and a new discretization based on Catmull-Clarks subdivision surfaces. Applications to shape analysis and geometric optimization are demonstrated.

DUPIN CYCLIDE REPRESENTATION USING QUATERNIONS

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We propose new bilinear quaternionic representation of Dupin cyclide patches. The standard formula for Bezier parametrization is used with quaternion control points and weights, that directly correspond to 4 corner points and their tangent vectors. Similar construction can be employed for triangular or quadrangular spherical patches. This approach is useful for C^1-spline composed of cyclide patches built on cyclic quadrangular meshes (examples will be provided). These results can be generalized to 3D volumes, and to higher ambient spaces (e.g. in context of Laguerre geometry) by replacing quaternions with certain geometric algebra.