From algebra and algebraic geometry to differential equations

Franz Winkler,
Research Institute in Symbolic Computation,
Linz, Austria

An algebraic ordinary differential equation (AODE) is of the form $F(x, y, y', ..., y^{(n)}) = 0$, where F is an $(n + 2)$-variate polynomial over a differential field K. Solving AODEs is an open problem. There are currently no general methods for solving such non-linear differential equations. We describe an algorithm for determining the rational solvability of AODEs of order 1, and, in the positive case, finding a general rational solution. Moreover, we introduce a group of affine transformations preserving rational solvability. The orbits of this group provide a classification of AODEs of order 1, where the classes consist of equations having the same computational complexity with respect to the computation of general rational solutions.