Reduced Forms of Linear Differential Systems

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A linear differential system $[A] : Y' = AY$, with $A \in \mathrm{Mat}(n, \overline{k})$ is said to be in reduced form if $A \in \mathfrak{g}(\overline{k})$ where $\mathfrak{g}$ is the Lie algebra of the differential Galois group $G$ of $[A]$.
In this talk, we will first explain why this notion is natural and desirable. A classical result of Kolchin and Kovacic shows that any linear differential system admits reduced forms; we will propose a procedure to achieve this reduction constructively (when the Galois group is reductive and unimodular). The key ingredient is the following result : when $G$ is reductive and unimodular, the system $[A]$ is in reduced form if and only if all of its invariants (rational solutions of appropriate symmetric powers) have constant coefficients (instead of rational functions). When $G$ is non-reductive, we give a similar characterization via the semi-invariants of $G$.