Algebraic Properties and Symbolic Aspects of Ordinary Integro-Differential Operators and Applications to Boundary Problems

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An integro-differential algebra combines a differential algebra with a suitable notion of an integral operator. By the fundamental theorem of calculus, the integral should be a right inverse of the derivation. We require additionally a version of integration by parts that allows us to define an "evaluation" in any integro-differential algebra. This is also our vantage point for treating initial and boundary conditions in an algebraic setting.

We discuss the construction of the algebra of ordinary integro-differential operators over an integro-differential algebra. We focus in particular on algebraic properties and algorithmic aspects of the integro-differential operators over the polynomial ring in one indeterminate over a field of characteristic zero with the usual derivation and integration. This algebra has also been studied recently by V. V. Bavula in a series of papers using the fact that it can be constructed as a generalized Weyl algebra.

Integro-differential operators over smooth or analytic functions provide an algebraic structure for computing with boundary problems for linear ODEs as well as their solution operators (Green's operators). Our implementation is based on the fact that every integro-differential operator can be written uniquely as a sum of a differential, an integral, and a boundary operator, and we illustrate it with some sample computations.

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