Reduction of Exterior Differential Systems under Symmetry Pseudogroups

Juha Pohjanpelto

Department of Mathematics, Oregon State University, Corvallis, OR 97331, USA

We will discuss a symmetry-based method for constructing solutions to systems of differential equations founded on the reduction of exterior differential systems invariant under the action of an infinite dimensional pseudogroup. Any system of differential equations $\Delta = 0$ with a symmetry group $\mathcal{G}$ can be associated with an exterior differential system $\mathcal{I}$ invariant under $\mathcal{G}$ so that solutions of $\Delta = 0$ correspond to integral manifolds of $\mathcal{I}$. With the help of a moving frame, the exterior differential system gives rise to a reduced system on a given cross section to the action of $\mathcal{G}$. All integral manifolds of the original system $\mathcal{I}$ can then be reconstructed from those of the reduced system by the way of an equation of generalized Lie type for the symmetry group parameters. Accordingly, we obtain a two-step algorithm for finding integral manifolds for exterior differential systems akin to Vessiot's method of group foliation. As examples, applications of the reduction process to the construction of analytic solutions to non-linear partial differential equations will be presented.