Motions of curves in projective plane inducing the Kaup-Kupershmidt hierarchy

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Interrelations between hierarchies of integrable non linear evolution equations and local motions of curves in homogenous spaces have been widely investigated in the last decades. K-S. Chu and C. Qu gave a complete account of integrable hierarchies originated by local motions of plane curves with respect to several transformation groups of R^2. In particular, they prove that the fifth order Kaup-Kupershmidt equation is related to a local motion in centro-affine geometry and that modified versions of the Kaup-Kupershmidt equations are related to motions of curves in projective plane. In this talk we will explain how to construct geometrical flows of curves in projective plane having the equations of the Kaup-Kupershmidt hierarchy as their analytical counterparts (for every order). We analyze the congruence curves of the flows and we investigate in more details the congruence curves defined by the cnoidal traveling wave solution of the fifth order Kaup-Kupershmidt equation. More generally, we show that all critical curves of the projective arc-length functional are congruence curves of the flow. We also exhibit new examples of traveling wave solutions in terms of Wieretsrass $\wp$-functions. Our approach to the problem is mostly based on symbolical and numerical computations performed with the software Mathematica 8.