Symbolic Computation of Lax Pairs of Integrable Nonlinear Partial Difference Equations on Quad-Graphs

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The presentation deals with two-dimensional nonlinear partial difference equations (PΔEs) which are completely integrable, i.e., they admit a Lax representation.

Based on work by Nijhoff, Bobenko and Suris, a method to compute Lax pairs will be presented. The method is algorithmic and can be implemented in the syntax of computer algebra systems, such as MATHEMATICA and MAPLE.

A MATHEMATICA program will be demonstrated that automatically computes Lax pairs for *scalar* PΔEs on quad-graphs, including lattice versions of the potential Korteweg-de Vries (KdV) equations, the modified KdV and sine-Gordon equations, as well as lattices derived by Adler, Bobenko, and Suris.

The generalization of the symbolic code to nonlinear *systems* of integrable PΔEs on quad-graphs is work in progress. Examples of Lax pairs of systems of PΔEs will be shown, including the potential KdV and nonlinear Schrödinger lattices, and various Boussinesq-type lattices.