A constructive version of Fitting's theorem on isomorphisms and equivalences of linear systems

Thomas Cluzeau[a], Alban Quadrat[b]

[a]Université de Limoges ; CNRS ; XLIM UMR 6172, DMI, 123 avenue Albert Thomas, 87060 Limoges cedex, France. `cluzeau@ensil.unilim.fr`

[b]INRIA Saclay-Île-de-France, DISCO Project, Supelec, L2S, 3 rue Joliot Curie, 91192 Gif-sur-Yvette cedex, France. `Alban.Quadrat@inria.fr`

Within the algebraic analysis approach to linear systems theory, a linear functional system can be studied by means of its associated finitely presented left module. Testing whether two linear systems/modules are isomorphic (the so-called equivalence problem) is an important issue in systems/module theory. In this talk, we explicitly characterize the conditions for a homomorphism between two finitely presented left modules to define an isomorphism, and we give an explicit formula for the inverse of an isomorphism. Then, we constructively study Fitting's major theorem, which shows how to enlarge matrices presenting isomorphic modules by blocks of 0 and I to get equivalent matrices. The consequences of this result on the Auslander transposes and adjoints of the finitely presented left modules are given. Finally, we show how to deduce simple proofs of Schanuel's lemma for finitely presented modules and of the fact that Fitting ideals associated with a finitely presented D-module M do not depend on any presentation of M, when D is a commutative ring. The different results developed are implemented in the OreMorphisms package.