Isogeometric analysis with Axel

Gang Xu, Régis Duvigneau,Bernard Mourrain

INRIA Sophia-Antipolis

gxu@sophia.inria.fr

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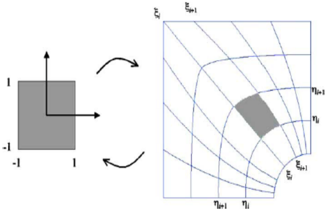
Outline

Main idea

- Use the same standard mathematical representation as in CAD systems (such as NURBS) for both the geometry and the solution field



Task of IGA toolbox

- Libraries for definition and handling of CAD-type geometry
- Isogeometric library offering necessary functionality for building and handling CAD and analysis models
- Isogeometric analysis and optimization interface for typical simulation problem
- Interfaces and tools to support visualization of isogeometric models and the results of isogeometric analysis

Current situation in IGA toolbox

- Software:
  - GoTools: SINTEF library for B-spline Objects
  - Axel: INRIA modeler for parametric and implicit modeling
- Isogeometric plugin:
  - Isogeometric solver and optimizer: heat conduction; incompressible flow simulation(almost done)
  - Visualization tools for IGA: color map and iso-value curves
  - Injectivity test tools of computational domain
  - Parameterization tools of computational domain in IGA
  - Discrete Coons volume generation method
  - Simplex spline plugin for modeling and analysis

Outline

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1. Isogeometric toolbox in EXCITING
2. Isogeometric plugin
   - Isogeometric solver and optimizer
   - Injectivity test tools for computational domain
   - Parameterization tools for computational domain
   - Discrete Coons volume generation method
   - Simplex spline plugin for modeling and analysis
3. Ongoing and future work

Isogeometric solver

Isogeometric C++ framework for the following heat conduction problem

$$\nabla(\kappa(\boldsymbol{x})\nabla \boldsymbol{T}(\boldsymbol{x})) = \boldsymbol{f}(x,y) \quad in \quad \Omega$$
$$\boldsymbol{T}(\boldsymbol{x}) = \boldsymbol{T}_0(\boldsymbol{x}) \quad on \quad \Gamma_D$$
$$\kappa(\boldsymbol{x})\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{n}}(\boldsymbol{x}) = \Phi_0(\boldsymbol{x}) \quad on \quad \Gamma_N$$

where $\boldsymbol{x}$ are the Cartesian coordinates, $\boldsymbol{T}$ represents the temperature field and κ is the thermal conductivity. Dirichlet and Neumann boundary conditions are applied on Γ_D and Γ_N respectively, $\boldsymbol{T}_0$ and Φ_0 being the imposed temperature and thermal flux ($\boldsymbol{n}$ is unit vector normal to the boundary).

Example 1

The heat conduction problem with source term

$$\boldsymbol{f}(x, y) = -\frac{4}{9}\sin(\frac{\pi x}{3})\sin(\frac{\pi y}{3}). \tag{1}$$

For this problem with boundary condition $\boldsymbol{T}_0(\boldsymbol{x}) = 0$ and $\Phi_0(\boldsymbol{x}) = 0$, the exact solution over the computational domain $[0, 3] \times [0, 6]$ is

$$\boldsymbol{T}(x, y) = 2\sin(\frac{\pi x}{3})\sin(\frac{\pi y}{3}). \tag{2}$$

Demo

Visualization of results

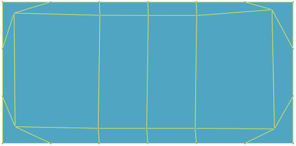


Figure: computational domain

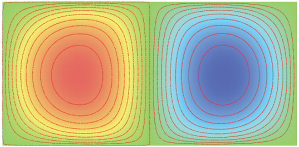


Figure: colormap and isovalue curves

Isogeometric shape optimization

- The shape optimization problem consists in finding the shape which is optimal in that it minimizes a certain cost function while satisfying given constraints.
- For 2D isogeometric shape optimization problem, the design variables are the control points of boundary B-spline curves.

Demo

Aircraft swing optimization

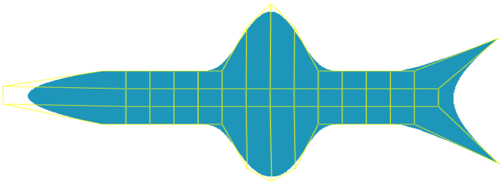


Figure: Before optimization

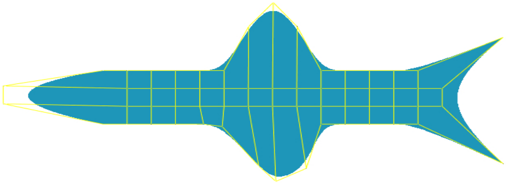
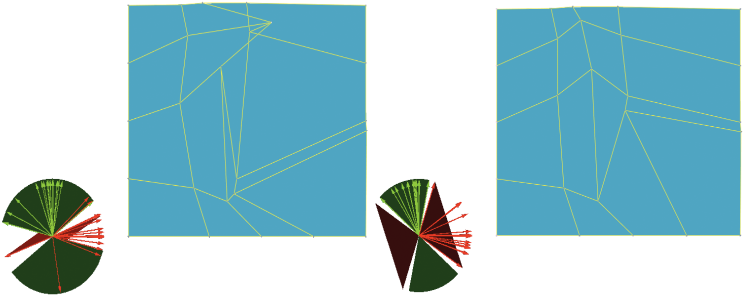


Figure: After optimization

Outline

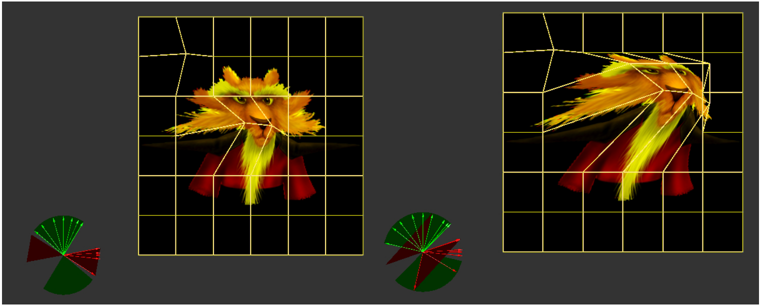
Easy-to-check method for injectivity

- Used for parameterization of computational domain
- **Demo**



Application in image warping

- Self-intersection should be avoided in image warping.
- **Demo**



Outline

Parameterization of computational domain

- Mesh generation in FEA
- Impact on analysis results
- Given boundary curves(surfaces), there will be various different parameterization.

Demo

Parameterization tools in Axel

- 2D problem: given four boundary B-spline curves, find the inner control points such that the resulted planar B-spline surface is a good computational domain for IGA.
- 3D problem: given six boundary B-spline surfaces, find the inner control points such that the resulted B-spline volume is a good computational domain for IGA.
- Contribution:
  - For problems with exact solutions, shape optimization based method is developed
  - For general problems, constraint optimization method is proposed

Problems with exact solution

- Inspired from shape optimization
- Shape optimization: optimize the boundary model to minimize the error function (cost function) of simulation
- Our idea for optimal optimization: optimization variables in shape optimization are the inner control points of computational domain rather than the boundary control points
- Only for problems with exact solutions

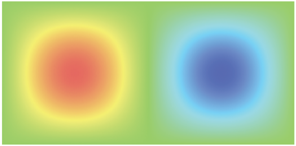


Figure: colormap of exact solution

Demo

Example

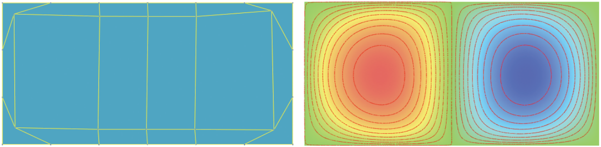


Figure: Initial computational domain and result

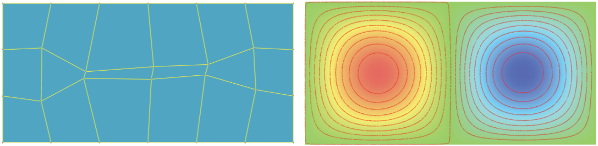
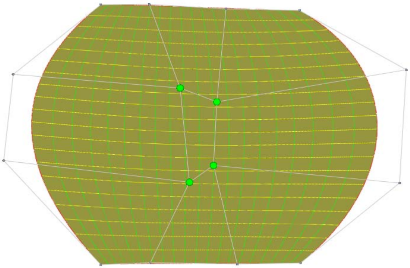


Figure: Final computational domain and result

General problems

- Desired computational domain
  - injective (no self-intersections)
  - as uniform as possible
  - orthogonal isoparametric curves
- Constraint optimization problem



Overview

Input: four coplanar boundary B-spline curves
Output: inner control points and the corresponding planar B-spline surfaces $\boldsymbol{F}(u, v)$

- construct the initial inner control points by discrete Coons method
- compute G_{ij} from the Jacobian determinant
- solve the constraint optimization problem

$$
\begin{aligned}
min \quad & \iint \| \boldsymbol{F}_{uu} \|^2 + \| \boldsymbol{F}_{vv} \|^2 + 2 \| \boldsymbol{F}_{uv} \|^2 \, dudv \\
+\omega & \iint \| \boldsymbol{F}_u \|^2 + \| \boldsymbol{F}_v \|^2 \, dudv \\
s.t. \quad & G_{ij} \geq 0
\end{aligned}
$$

- generate corresponding planar B-spline surface $\boldsymbol{F}(u, v)$ as computational domain **Demo**

Example

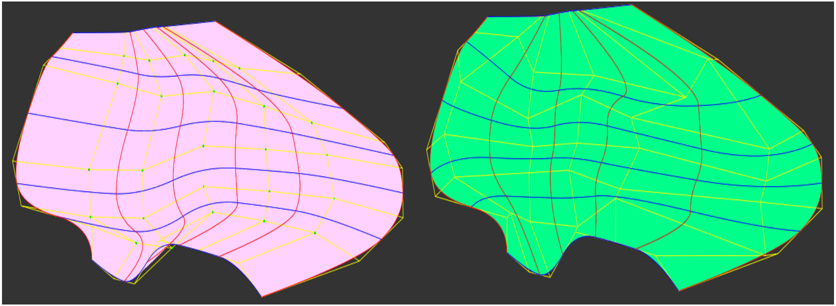


Figure: Initial and final computational domain, FEMesh.

Example

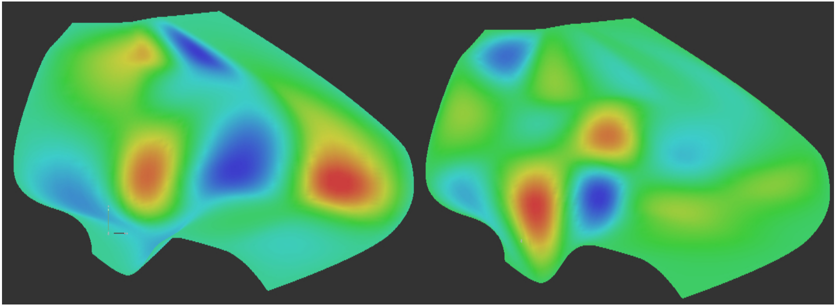


Figure: Initial and final simulation result

Outline

Discrete Coons patches (Farin, 1999)

Given the boundary control points $\boldsymbol{P}_{0,j}, \boldsymbol{P}_{n,j}, \boldsymbol{P}_{i,0}, \boldsymbol{P}_{i,m}$, $i = 0, \ldots, n, j = 0, \ldots, m$, the inner control points $\boldsymbol{P}_{i,j}$ $(i = 1, \ldots, n-1, j = 1, \ldots, m-1)$ can be constructed by the discrete Coons method as follows:

$$\begin{aligned}\boldsymbol{P}_{i,j} &= (1-\frac{i}{n})\boldsymbol{P}_{0,j} + \frac{i}{n}\boldsymbol{P}_{n,j} + (1-\frac{j}{m})\boldsymbol{P}_{i,0} + \frac{j}{m}\boldsymbol{P}_{i,m} \\ &\quad -[1-\frac{i}{n} \quad \frac{i}{n}]\begin{pmatrix} \boldsymbol{P}_{0,0} & \boldsymbol{P}_{0,m} \\ \boldsymbol{P}_{n,0} & \boldsymbol{P}_{n,m} \end{pmatrix}\begin{pmatrix} 1-\frac{j}{m} \\ \frac{j}{m} \end{pmatrix}\end{aligned}$$

Discrete Coons volume

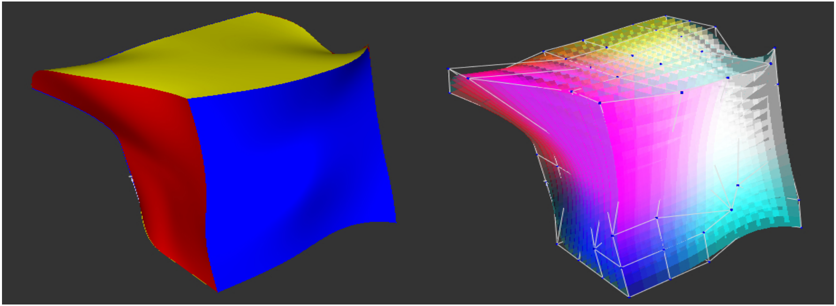


Figure: Boundary surfaces and resulted volume

Outline

Simplex spline

- Spline object defined on triangulation(tetrahedronization)
- **Demo**

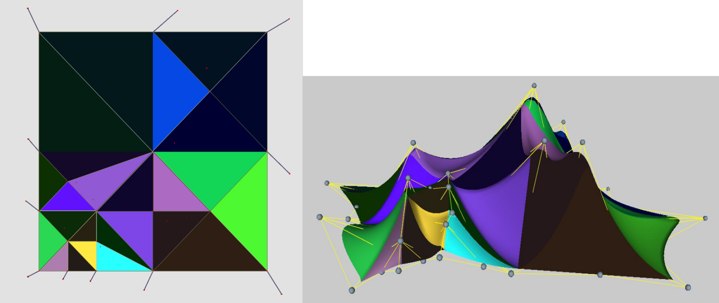


Figure: An example of simplex spline

Outline

Ongoing work

- IGA with triangular B-spline
- PHT spline plugin for IGA

Future work— 3D problem

- 3D solver and optimizer using B-splines
- Visualization tools for 3D IGA: color map and iso-surfaces
- Injectivity test tools for 3D computational domain
- Parameterization tools for 3D computational domain
- More complex geometry in CAD industry

Thanks for your attention!

Question?