8. Trigonométrie

1. $\cos(\frac{5\pi}{6}) = -\frac{\sqrt{3}}{2}$, $\sin(\frac{5\pi}{6}) = \frac{1}{2}$, $\tan(\frac{5\pi}{6}) = -\frac{1}{\sqrt{3}}$.
   $\cos(\frac{7\pi}{6}) = -\frac{\sqrt{3}}{2}$, $\sin(\frac{7\pi}{6}) = -\frac{1}{2}$, $\tan(\frac{7\pi}{6}) = \frac{1}{\sqrt{3}}$.
   $\cos(\frac{9\pi}{6}) = 0$, $\sin(\frac{9\pi}{6}) = -1$, $\tan(\frac{9\pi}{6}) =$ indéfinie.
   $\cos(\frac{4\pi}{3}) = -\frac{1}{2}$, $\sin(\frac{4\pi}{3}) = -\frac{\sqrt{3}}{2}$, $\tan(\frac{4\pi}{3}) = \sqrt{3}$.
   $\cos(\frac{71\pi}{3}) = \frac{1}{2}$, $\sin(\frac{71\pi}{3}) = -\frac{\sqrt{3}}{2}$, $\tan(\frac{71\pi}{3}) = -\sqrt{3}$.
   $\cos(\frac{-512\pi}{3}) = -\frac{1}{2}$, $\sin(\frac{-512\pi}{3}) = -\frac{\sqrt{3}}{2}$, $\tan(\frac{-512\pi}{3}) = \sqrt{3}$.
   $\cos(\frac{-\pi}{2}) = 0$, $\sin(\frac{-\pi}{2}) = -1$, $\tan(\frac{-\pi}{2}) =$ indéfinie.
   $\cos(\frac{7\pi}{2}) = 0$, $\sin(\frac{7\pi}{2}) = -1$, $\tan(\frac{7\pi}{2}) =$ indéfinie.
   $\cos(\frac{5\pi}{4}) = -\frac{\sqrt{2}}{2}$, $\sin(\frac{5\pi}{4}) = -\frac{\sqrt{2}}{2}$, $\tan(\frac{5\pi}{4}) = 1$
2. $\sin(x - \pi) = -\sin(x)$, $\cos(x - \pi) = -\cos(x)$.
   $\sin(x + 4\pi) = \sin(x)$, $\cos(x + 4\pi) = \cos(x)$.
   $\sin(-x + 5\pi) = \sin(x)$, $\cos(-x + 5\pi) = -\cos(x)$.
   $\sin(-x - 12\pi) = -\sin(x)$, $\cos(-x - 12\pi) = \cos(x)$.
   $\sin(\frac{\pi}{2} + x) = \cos(x)$, $\cos(\frac{\pi}{2} + x) = -\sin(x)$.
   $\sin(\frac{3\pi}{2} - x) = -\cos(x)$, $\cos(\frac{3\pi}{2} - x) = -\sin(x)$.
3. $\cos(\frac{\pi}{8}) = \frac{1}{2}\sqrt{(2 + \sqrt{2})}$, $\sin(\frac{\pi}{8}) = \frac{1}{2}\sqrt{(2 - \sqrt{2})}$, $\tan(\frac{\pi}{8}) = \sqrt{2} - 1$.
   $\cos(\frac{3\pi}{8}) = \frac{1}{2}\sqrt{(2 - \sqrt{2})}$, $\sin(\frac{3\pi}{8}) = \frac{1}{2}\sqrt{(2 + \sqrt{2})}$, $\tan(\frac{3\pi}{8}) = \sqrt{2} + 1$.
   $\cos(\frac{\pi}{12}) = \frac{1}{2}\sqrt{(2 + \sqrt{3})}$, $\sin(\frac{\pi}{12}) = \frac{1}{2}\sqrt{(2 - \sqrt{3})}$, $\tan(\frac{\pi}{12}) = 2 - \sqrt{3}$.
   $\cos(\frac{11\pi}{12}) = -\frac{1}{2}\sqrt{(2 + \sqrt{3})}$, $\sin(\frac{11\pi}{12}) = \frac{1}{2}\sqrt{(2 - \sqrt{3})}$, $\tan(\frac{11\pi}{12}) = \sqrt{3} - 2$.
4. a)
   b)
   c)
5. $\cos(\pi - x) = \cos(\pi)\cos(x) + \sin(\pi)\sin(x) = -\cos(x)$.
   $\sin(\pi - x) = \sin(\pi)\cos(x) - \sin(x)\cos(\pi) = \sin(x)$.
6. $\cos(a + b) + \cos(a - b) = \cos(a)\cos(b) - \sin(a)\sin(b) + \cos(a)\cos(b) + \sin(a)\sin(b) = 2\cos(a)\cos(b)$.
   $\cos(a - b) - \cos(a + b) = \cos(a)\cos(b) + \sin(a)\sin(b) - \cos(a)\cos(b) + \sin(a)\sin(b) = 2\sin(a)\sin(b)$.
   $\sin(a + b) + \sin(a - b) = \sin(a)\cos(b) + \sin(b)\cos(a) + \sin(a)\cos(b) - \sin(b)\cos(a) = 2\sin(a)\cos(b)$.

$2\cos(\frac{p+q}{2})\sin(\frac{p-q}{2}) = \cos(\frac{p+q+p-q}{2}) + \cos(\frac{p+q-p+q}{2}) = \cos(p) + \cos(q)$.
$2\sin(\frac{p+q}{2})\sin(\frac{p-q}{2}) = \cos(\frac{p+q-p+q}{2}) - \cos(\frac{p+q+p-q}{2}) = \cos(q) - \cos(p)$.
$2\sin(\frac{p+q}{2})\cos(\frac{p-q}{2}) = \sin(\frac{p+q+p-q}{2}) + \sin(\frac{p+q-p+q}{2}) = \sin(p) + \sin(q)$.
$2\cos(\frac{p+q}{2})\sin(\frac{p-q}{2}) = \sin(\frac{p-q+p+q}{2}) + \sin(\frac{p-q-p-q}{2}) = \sin(p) - \sin(q)$.

7. $\cos(x)+\sin(x) = \cos(x)-\cos(x+\frac{\pi}{2}) = -2\sin(x+\frac{\pi}{4})\sin(-\frac{\pi}{4}) = \sqrt{2}\sin(x+\frac{\pi}{4})$.
$\cos(x)-\sin(x) = \cos(x)+\cos(x+\frac{\pi}{2}) = 2\cos(\frac{\pi}{4})\ cos(x+\frac{\pi}{4}) = \sqrt{(}2)\cos(x+\frac{\pi}{4})$.
$\cos(x-\frac{\pi}{3}) = \cos(x)\cos(\frac{\pi}{3}) + \sin(x)\sin(\frac{\pi}{3}) = \frac{1}{2}(\cos(x) + \sqrt{3}\sin(x))$.
$\cos(x+\frac{\pi}{3}) = \cos(x)\cos(\frac{\pi}{3}) - \sin(x)\sin(\frac{\pi}{3}) = \frac{1}{2}(\cos(x) - \sqrt{3}\sin(x))$.

8. $\cos(x) = -\frac{\sqrt{3}}{2}$, $x \equiv \frac{\pi}{6}[2\pi]$, $x \equiv -\frac{\pi}{6}[2\pi]$. Dans $]-\pi;\pi]$: $\frac{\pi}{6}$ et $-\frac{\pi}{6}$.
$\sin(x) = \frac{1}{2}$, $x \equiv \frac{\pi}{6}[2\pi]$, $x \equiv \frac{5\pi}{6}[2\pi]$. Dans $]-\pi;\pi]$: $\frac{\pi}{6}$ et $\frac{5\pi}{6}$.
$\tan(x) = -1$, $x \equiv \frac{\pi}{4}[\pi]$. Dans $]-\pi;\pi]$: $\frac{\pi}{4}$ et $-\frac{3\pi}{4}$.
$\sin(2x) = \sqrt{3}$, ensemble vide.
$\cos(2x+\frac{\pi}{4}) = \frac{1}{2}$, $x \equiv \frac{\pi}{24}[\pi]$, $x \equiv -\frac{7\pi}{24}[\pi]$. Dans $]-\pi;\pi]$: $\frac{\pi}{24}$, $-\frac{23\pi}{24}$, $-\frac{7\pi}{24}$ et $\frac{17\pi}{24}$
$\cos(\frac{x}{2}+\frac{\pi}{4}) = \cos(\frac{x}{3}-\frac{\pi}{6})$, $x \equiv -\frac{5\pi}{2}[12\pi]$ ou $x \equiv -\frac{\pi}{10}[\frac{2\pi}{5}]$.Dans $]-\pi;\pi]$: $-\frac{5\pi}{2}$ et $-\frac{\pi}{10}$.
$2\cos^2(x)+\cos(x)-1 = 0$, $\cos(x) = -1$ ou $\cos(x) = \frac{1}{2}$, soit $x \equiv -\pi[2\pi]$ ou $x \equiv \frac{\pi}{3}[2\pi]$ ou $x \equiv -\frac{\pi}{3}[2\pi]$.
$\cos(x) = \sin(\frac{2x}{3})$, $\cos(x) = \cos(\frac{\pi}{2}-\frac{2x}{3})$, $x \equiv \frac{3\pi}{10}[\frac{6\pi}{5}]$ ou $x \equiv -\frac{3\pi}{2}[6\pi]$
$\cos(4x)+\sin(2x) = 0$, ou encore $\cos(4x) = \cos(\frac{\pi}{2}+2x)$, ce qui donne $x \equiv \frac{\pi}{4}[\pi]$ ou $x \equiv -\frac{\pi}{12}[\frac{\pi}{3}]$.
$\cos(x)+\sin(x) = \frac{\sqrt{3}}{2}$, soit $\sin(x+\frac{\pi}{4}) = \frac{\sqrt{3}}{2\sqrt{2}}$, pas de valeur remarquable.
$\sqrt{3}\cos(x)+\sin(x) = 1$, donne $\cos(\frac{\pi}{6}-x) = \frac{1}{2}$, ce qui donne $x \equiv -\frac{\pi}{6}[2\pi]$ ou $x \equiv \frac{\pi}{2}[2\pi]$.