



COPRIN **project**

Contraintes, OPTimisation et Résolution par INTervalles

Constraints, OPTimization and Resolving through INTervals

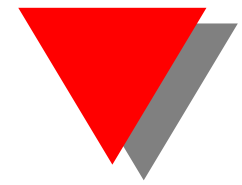


COPRIN project

Contraintes, OPTimisation et Résolution par INTervalles

Constraints, OPTimization and Resolving through INTervals

COPRIN has been created in February 2002



Members of the project

Staff

MERLET Jean-Pierre	(DR 1, scientific head)
DANEY David	(Chargé de Recherche INRIA)
NEVEU Bertrand	(Ingénieur en Chef, P & C, CERTIS)
PAPEGAY Yves	(Chargé de Recherche INRIA)
POURTALLIER Odile	(Chargé de Recherche INRIA, join the team in 2004)
TROMBETTONI Gilles	(Maître de Conférences UNSA)

Students

- 7 PhD students
- 1 post-doc
- 1 engineer

Scientific objectives and Methods



Scientific objectives and Methods

Two main complementary research axis:

Robotics and Interval Analysis

Scientific objectives and Methods



Robotics

- **Robotics Objective 1:** robot modeling and analysis

Scientific objectives and Methods



Robotics

- **Robotics Objective 1: robot modeling and analysis**
 - establishing the performances of a given robot

Scientific objectives and Methods



Robotics

- **Robotics Objective 1: robot modeling and analysis**
 - establishing the performances of a given robot
 - in a **guaranteed manner**

Scientific objectives and Methods



Robotics

- **Robotics Objective 1: robot modeling and analysis**
 - establishing the performances of a given robot
 - in a **guaranteed manner**
 - especially taking into account the **uncertainties** in the modeling and control

Scientific objectives and Methods



Robotics

- **Robotics Objective 1:** robot modeling and analysis
- **Robotics Objective 2:** design methodology

Scientific objectives and Methods



Robotics

- **Robotics Objective 1:** robot modeling and analysis
- **Robotics Objective 2:** design methodology
 - establishing the robot design parameters so that it will fit given requirements

Scientific objectives and Methods



Robotics

- **Robotics Objective 1:** robot modeling and analysis
- **Robotics Objective 2:** design methodology
 - establishing the robot design parameters so that it will fit given requirements
 - methodology provides almost all design solutions

Scientific objectives and Methods



Robotics

- **Robotics Objective 1:** robot modeling and analysis
- **Robotics Objective 2:** design methodology
 - establishing the robot design parameters so that it will fit given requirements
 - methodology provides almost all design solutions
 - methodology is robust with respect to manufacturing tolerances

Scientific objectives and Methods



Robotics

- **Robotics Objective 1:** robot modeling and analysis
- **Robotics Objective 2:** design methodology
- **Robotics Objective 3:** parallel robot, prototypes, applications

Example: new wire-driven parallel robot

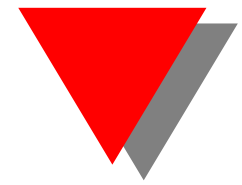


Example: new wire-driven parallel robot



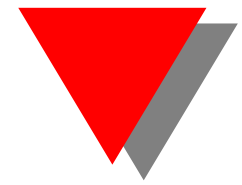
Highly modular

Example: new wire-driven parallel robot



Highly modular
Linear actuators

Example: new wire-driven parallel robot



Highly modular

Linear actuators

Applications:

service robotics

rehabilitation

Scientific objectives and Methods



Scientific objectives and Methods

Interval Analysis/Constraints

- certified solving

Scientific objectives and Methods



Interval Analysis/Constraints

- certified solving
 - of equations and/or inequalities systems

Scientific objectives and Methods



Interval Analysis/Constraints

- certified solving
 - of equations and/or inequalities systems
 - for **real** variables, lying in a **bounded domain**

Scientific objectives and Methods



Interval Analysis/Constraints

- certified solving
 - of equations and/or inequalities systems
 - for **real** variables, lying in a **bounded domain**
 - providing results that are **guaranteed**

Scientific objectives and Methods



Interval Analysis/Constraints

- certified solving
- methods:

Scientific objectives and Methods



Interval Analysis/Constraints

- certified solving
- methods:
 - constraint programming
 - interval analysis
 - symbolic computation

Interval analysis





Interval analysis

Calculating with **intervals** is (almost) as easy than with real numbers



Interval analysis

Calculating with **intervals** is (almost) as easy than with real numbers

Example: $F = x^2 + \cos(x)$, $x \in [0, 1]$

Problem: find $[A, B]$ such that: $A \leq F(x) \leq B \forall x \in [0, 1]$



Interval analysis

Calculating with **intervals** is (almost) as easy than with real numbers

Example: $F = x^2 + \cos(x)$, $x \in [0, 1]$

Problem: find $[A, B]$ such that: $A \leq F(x) \leq B \forall x \in [0, 1]$

$$F = [0, 1]^2 + \cos([0, 1])$$



Interval analysis

Calculating with **intervals** is (almost) as easy than with real numbers

Example: $F = x^2 + \cos(x)$, $x \in [0, 1]$

Problem: find $[A, B]$ such that: $A \leq F(x) \leq B \forall x \in [0, 1]$

$$F = [0, 1]^2 + \cos([0, 1])$$



Interval analysis

Calculating with **intervals** is (almost) as easy than with real numbers

Example: $F = x^2 + \cos(x)$, $x \in [0, 1]$

Problem: find $[A, B]$ such that: $A \leq F(x) \leq B \forall x \in [0, 1]$

$$F = [0, 1]^2 + \cos([0, 1]) = [0, 1] + \cos([0, 1])$$



Interval analysis

Calculating with **intervals** is (almost) as easy than with real numbers

Example: $F = x^2 + \cos(x)$, $x \in [0, 1]$

Problem: find $[A, B]$ such that: $A \leq F(x) \leq B \forall x \in [0, 1]$

$$F = [0, 1]^2 + \cos([0, 1]) = [0, 1] + \cos([0, 1])$$



Interval analysis

Calculating with **intervals** is (almost) as easy than with real numbers

Example: $F = x^2 + \cos(x)$, $x \in [0, 1]$

Problem: find $[A, B]$ such that: $A \leq F(x) \leq B \forall x \in [0, 1]$

$$F = [0, 1]^2 + \cos([0, 1]) = [0, 1] + [0.54, 1]$$



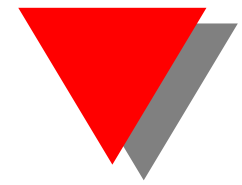
Interval analysis

Calculating with **intervals** is (almost) as easy than with real numbers

Example: $F = x^2 + \cos(x)$, $x \in [0, 1]$

Problem: find $[A, B]$ such that: $A \leq F(x) \leq B \forall x \in [0, 1]$

$$F = [0, 1]^2 + \cos([0, 1]) = [0, 1] + [0.54, 1]$$



Interval analysis

Calculating with **intervals** is (almost) as easy than with real numbers

Example: $F = x^2 + \cos(x)$, $x \in [0, 1]$

Problem: find $[A, B]$ such that: $A \leq F(x) \leq B \forall x \in [0, 1]$

$$F = [0, 1]^2 + \cos([0, 1]) = [0, 1] + [0.54, 1] = [0.54, 2]$$



Interval analysis

Calculating with **intervals** is (almost) as easy than with real numbers

Example: $F = x^2 + \cos(x)$, $x \in [0, 1]$

Problem: find $[A, B]$ such that: $A \leq F(x) \leq B \forall x \in [0, 1]$

$$F = [0, 1]^2 + \cos([0, 1]) = [0, 1] + [0.54, 1] = [0.54, 2]$$

- 0 not included in $[0.54, 2] \Rightarrow F \neq 0 \forall x \in [0, 1]$



Interval analysis

Calculating with **intervals** is (almost) as easy than with real numbers

Example: $F = x^2 + \cos(x)$, $x \in [0, 1]$

Problem: find $[A, B]$ such that: $A \leq F(x) \leq B \forall x \in [0, 1]$

$$F = [0, 1]^2 + \cos([0, 1]) = [0, 1] + [0.54, 1] = [0.54, 2]$$

- 0 not included in $[0.54, 2] \Rightarrow F \neq 0 \forall x \in [0, 1]$
- $F > 0 \forall x \in [0, 1]$



Interval analysis

Calculating with **intervals** is (almost) as easy than with real numbers

Example: $F = x^2 + \cos(x)$, $x \in [0, 1]$

Problem: find $[A, B]$ such that: $A \leq F(x) \leq B \forall x \in [0, 1]$

$$F = [0, 1]^2 + \cos([0, 1]) = [0, 1] + [0.54, 1] = [0.54, 2]$$

- **Advantages:** numerical round-off errors are managed



Interval analysis

Calculating with **intervals** is (almost) as easy than with real numbers

Example: $F = x^2 + \cos(x)$, $x \in [0, 1]$

Problem: find $[A, B]$ such that: $A \leq F(x) \leq B \forall x \in [0, 1]$

$$F = [0, 1]^2 + \cos([0, 1]) = [0, 1] + [0.54, 1] = [0.54, 2]$$

- **Advantages:** numerical round-off errors are managed
- **Drawbacks:** overestimation, calculation sensitive to formulation

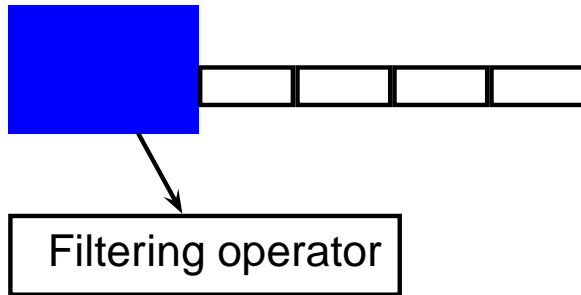
The structure of an IA algorithm



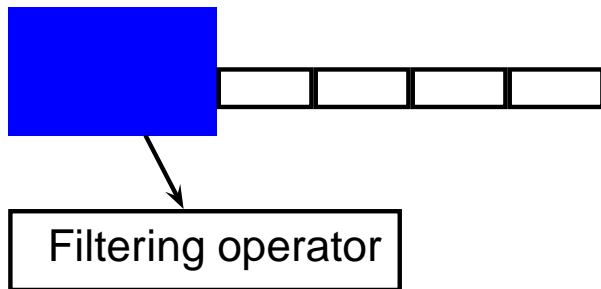
The structure of an IA algorithm



The structure of an IA algorithm

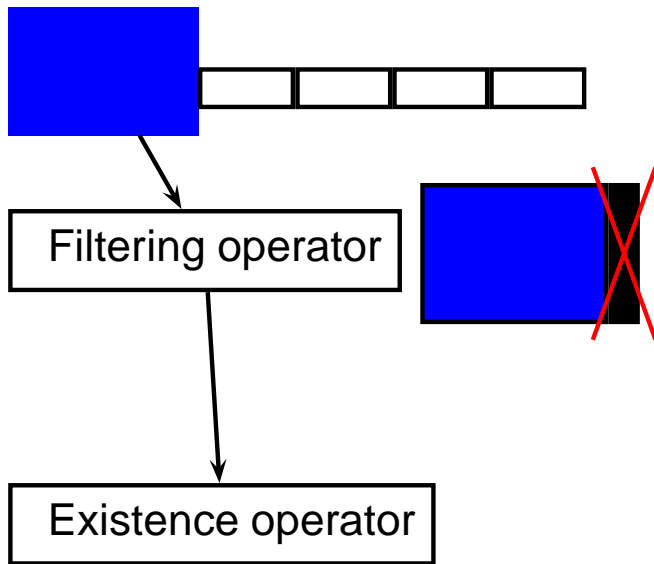


The structure of an IA algorithm

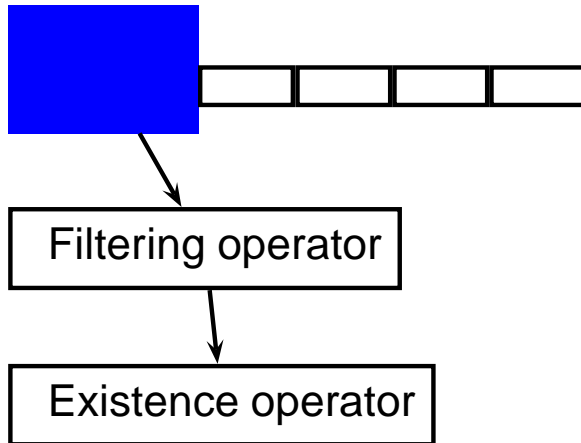


Filtering operator: a set of heuristics that may allow to determine that there is **no solution in the current box** or may **reduce its size**

The structure of an IA algorithm

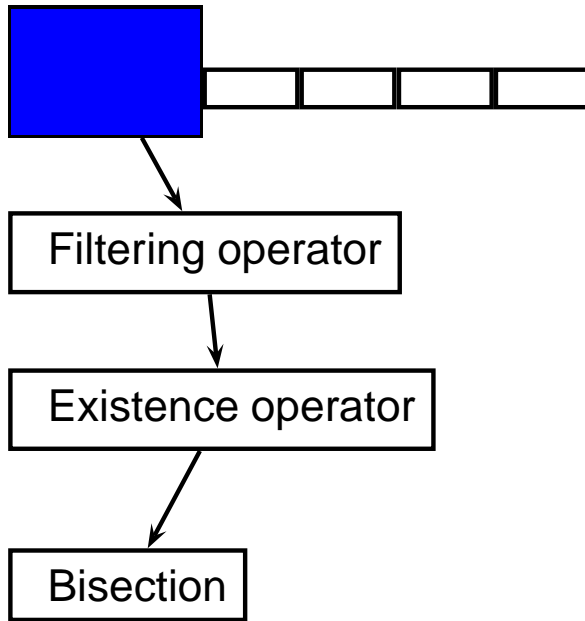


The structure of an IA algorithm

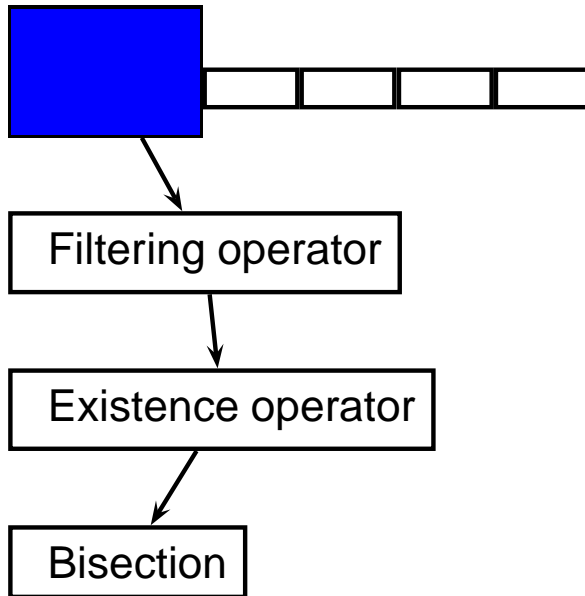


Existence operator: a set of heuristics that may allow to determine that there is **a single solution in the current box** (e.g Kantorovitch theorem)

The structure of an IA algorithm

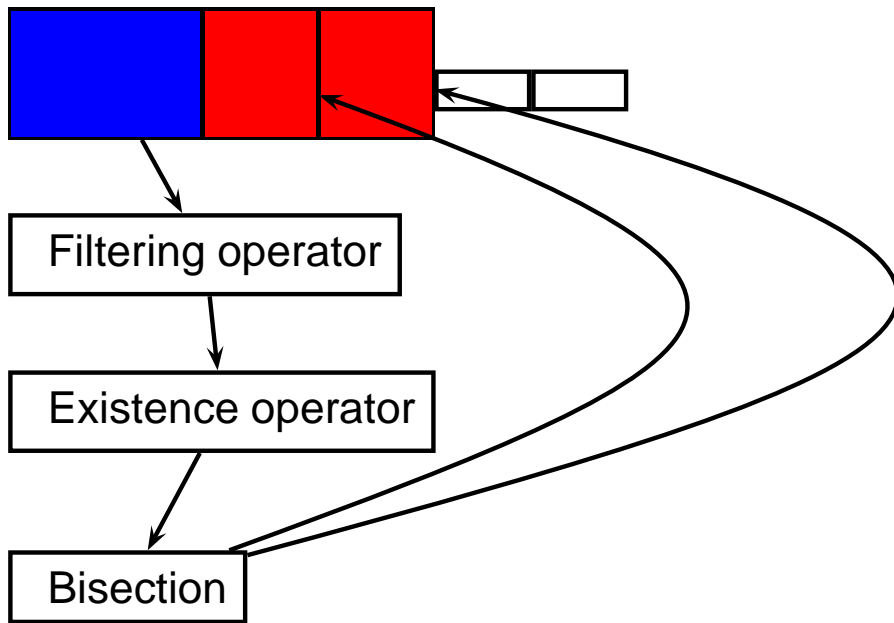


The structure of an IA algorithm

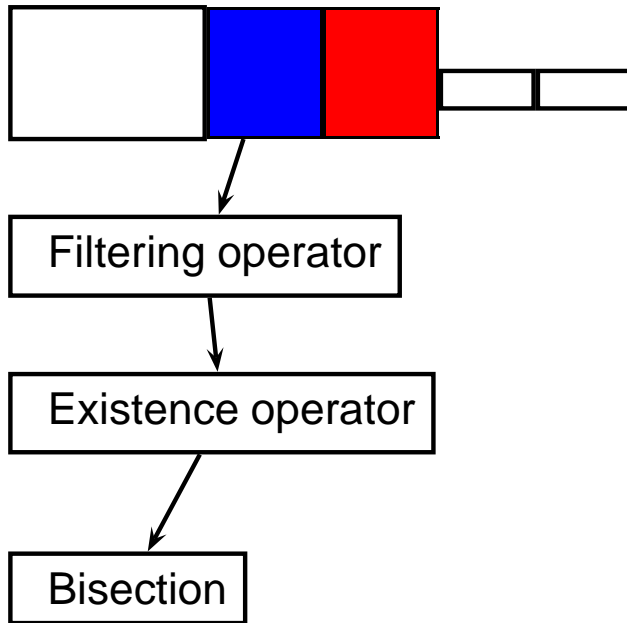


Split the current box, usually in two

The structure of an IA algorithm



The structure of an IA algorithm



An example





An example

Managing a set of inequalities:

$$x^2 + y^2 \leq 2$$

$$(x - 1)^2 + (y - 1)^2 \leq 2$$

that play a role in the calculation of a parallel robot workspace

VIDEO

Interval Analysis Objectives



Interval Analysis Objectives



IA Objective 1: Improvement of IA methodology

Interval Analysis Objectives

IA Objective 1: Improvement of IA methodology

- new filtering operators



Interval Analysis Objectives



IA Objective 1: Improvement of IA methodology

- new filtering operators
- decomposition and solving of geometric constraints

Interval Analysis Objectives



IA Objective 1: Improvement of IA methodology

- new filtering operators
- decomposition and solving of geometric constraints
- solving of differential equations

Interval Analysis Objectives

IA Objective 1: Improvement of IA methodology

IA Objective 2: Dissemination, software, experimental analysis



Interval Analysis Objectives

IA Objective 1: Improvement of IA methodology

IA Objective 2: Dissemination, software, experimental analysis

- method is not well known



Interval Analysis Objectives

IA Objective 1: Improvement of IA methodology

IA Objective 2: Dissemination, software, experimental analysis

- method is not well known
- lack of available software



Interval Analysis Objectives

IA Objective 1: Improvement of IA methodology

IA Objective 2: Dissemination, software, experimental analysis

- method is not well known
- lack of available software
- interface not convenient for non expert end-user



Interval Analysis Objectives

IA Objective 1: Improvement of IA methodology

IA Objective 2: Dissemination, software, experimental analysis

Tools:

- extensive use of **symbolic computation**
- software (ALIAS library)
- extensive testing

Localization with ultra-sound

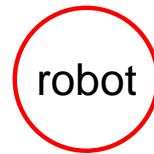


Localization of a robot with ultra-sound (joint work with ETH)

Localization with ultra-sound



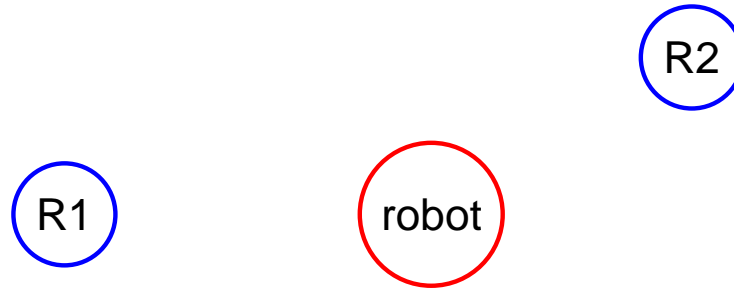
Localization of a robot with ultra-sound (joint work with ETH)



Localization with ultra-sound



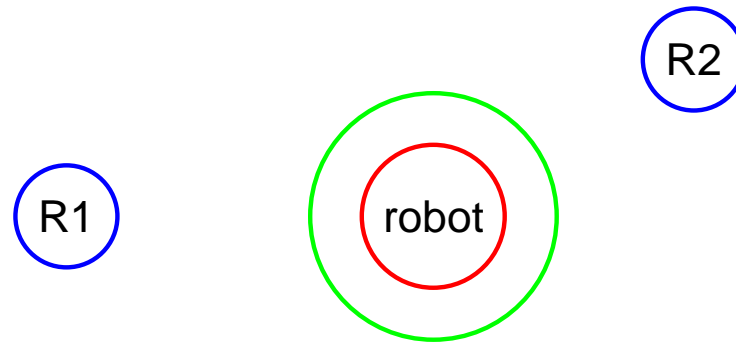
Localization of a robot with ultra-sound (joint work with ETH)



Localization with ultra-sound



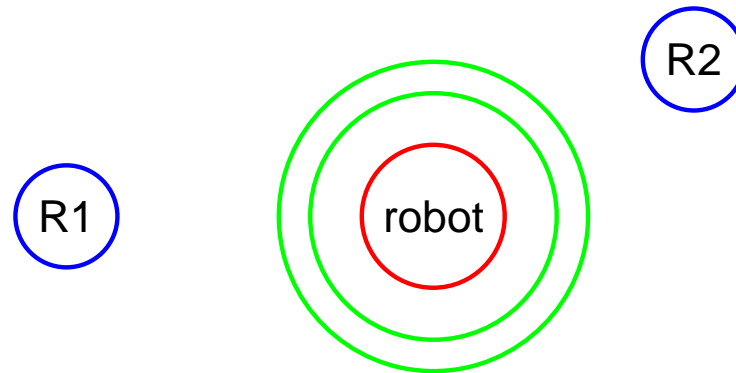
Localization of a robot with ultra-sound (joint work with ETH)



Localization with ultra-sound



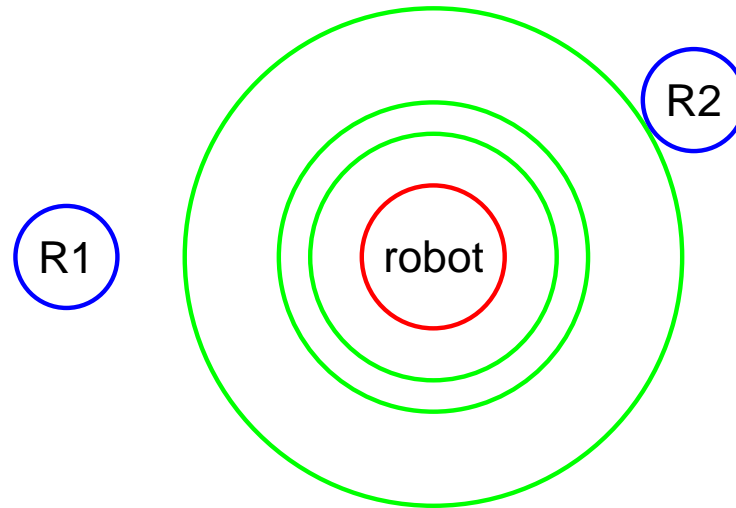
Localization of a robot with ultra-sound (joint work with ETH)



Localization with ultra-sound



Localization of a robot with ultra-sound (joint work with ETH)

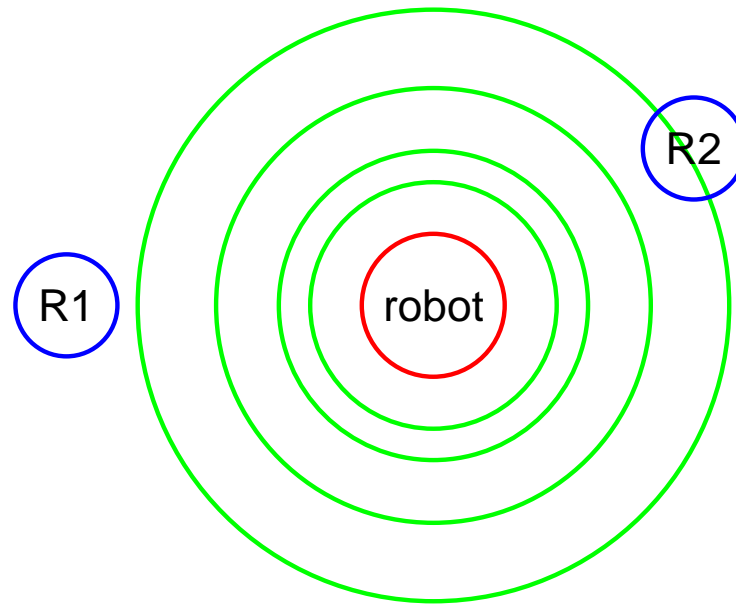


R2 receives the ping at time t_2

Localization with ultra-sound



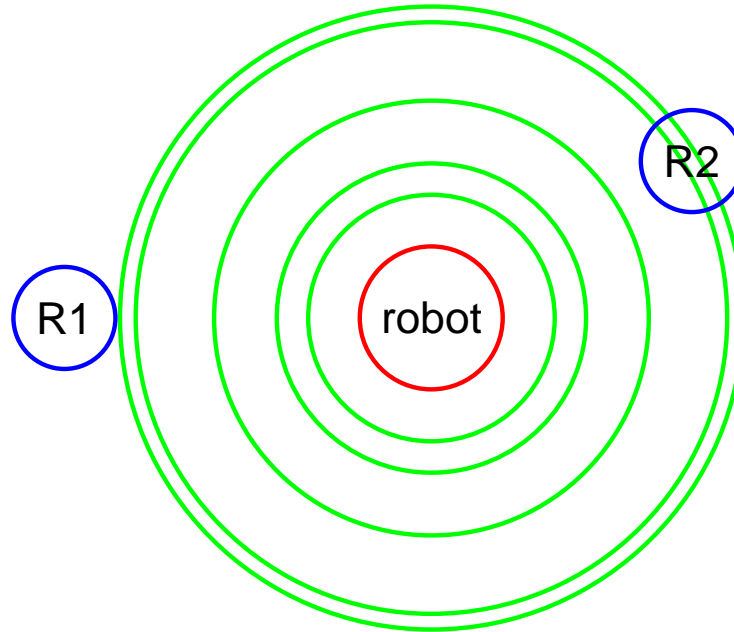
Localization of a robot with ultra-sound (joint work with ETH)



Localization with ultra-sound



Localization of a robot with ultra-sound (joint work with ETH)



R1 receives the ping at time t_1

Localization with ultra-sound



Localization of a robot with ultra-sound (joint work with ETH)

Localization is based on the measurement of $t_2 - t_1$

With 2 receivers: assuming perfect Dirac ping

- $||RR_2|| - ||RR_1|| = c(t_2 - t_1) \Rightarrow$ robot lie on a hyperbola

Localization with ultra-sound



Localization of a robot with ultra-sound (joint work with ETH)

Localization is based on the measurement of $t_2 - t_1$

With 2 receivers: assuming perfect Dirac ping:

- $||RR_2|| - ||RR_1|| = c(t_2 - t_1) \Rightarrow$ robot lie on a hyperbola

In practice we have sinusoidal ping:

- measured time is an interval
- robot lie on a "thick" hyperbola

Localization with ultra-sound



Localization of a robot with ultra-sound (joint work with ETH)

With three receivers

- measurement of $t_2 - t_1, t_3 - t_1$
- robot located at the intersection of 2 "thick" hyperbola

Analysis



Analysis



Usually f, c are assumed to be perfectly known

Analysis



but in practice f, c are uncertain

- c in $[1465, 1496]$ m/s (± 5 degrees temperature variation)
- f in $[295, 305]$ kHz

Influence of these uncertainties on the robot localization ?

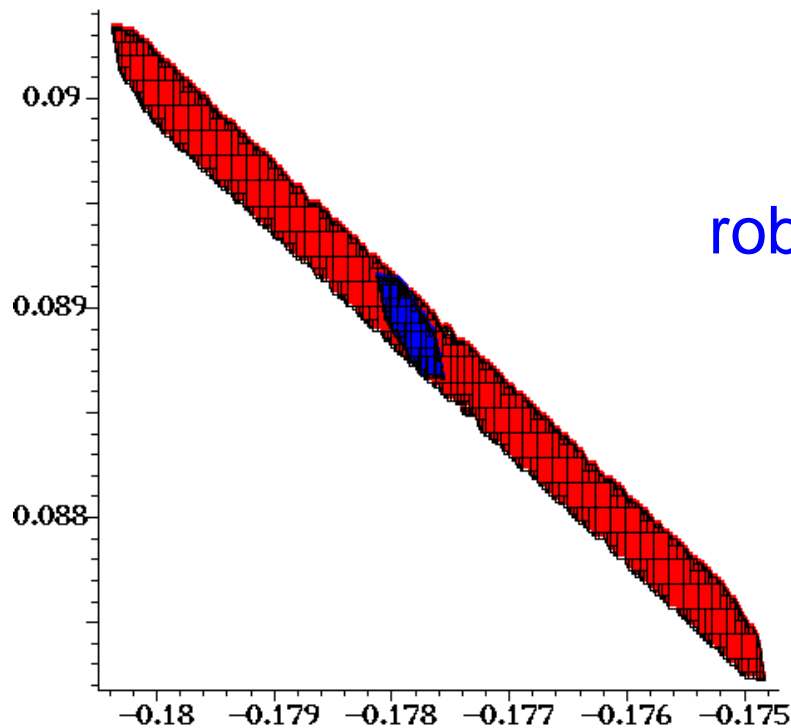
Analysis



but in practice f, c are uncertain

- c in $[1465, 1496]$ m/s (± 5 degrees temperature variation)
- f in $[295, 305]$ kHz

Influence of these uncertainties on the robot localization ?



robot location discarding uncertainties

possible real location

Synthesis



Synthesis



not satisfied with the localization accuracy ?



find the location of the 3rd receiver so that the localization accuracy is lower than a given threshold

Synthesis



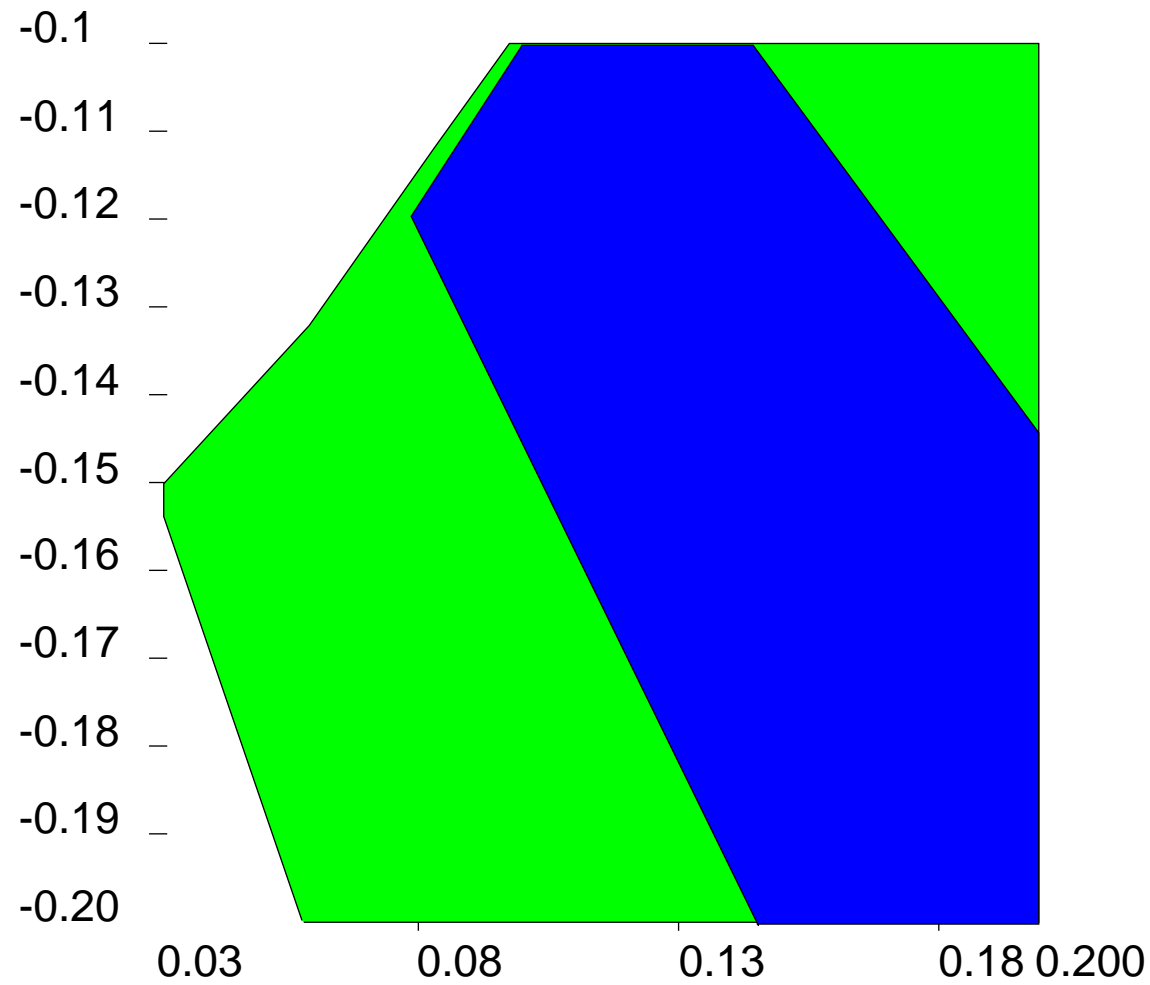
not satisfied with the localization accuracy ?



find the location of the 3rd receiver so that the localization accuracy is lower than a given threshold

IA methods allows to find **all** 3rd receiver location that allow to respect this requirement

Synthesis



Synthesis



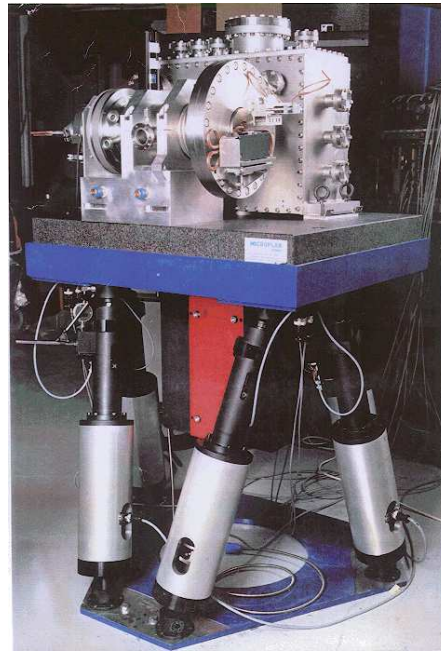
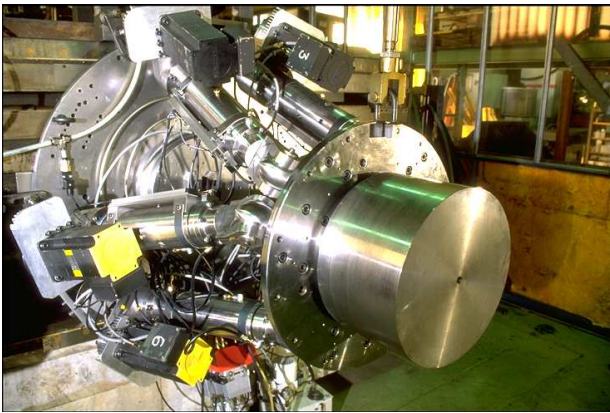
- this methodology allows to design robots that fit a list of requirements
- it has been used for designing industrial robots and our own prototypes



machine-tool (CMW)

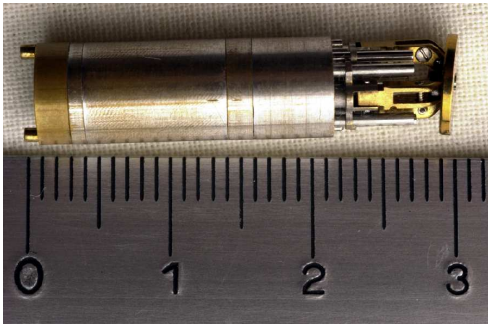
Fine positioning (ESRF)

Space telescope (Alcatel)





MIPS micro robot



ARES micro-robot



wire robot

this methodology is used to manage the **modularity** of our wire-driven parallel robot

- find the geometry that allows to lift an elderly people whatever his/her location in a given room

Wire-driven parallel robot



Wire-driven parallel robot



All purpose device with 1 to 6 d.o.f., redundant or not

- highly **modular**: geometry, amplification of actuator motion
- **powerful**: high ratio load/mass
- **fast**: potentially faster than the speed of sound

Examples:

4 dof crane motion video , fast planar motion (3.5m/s)

Wire-driven parallel robot



Potential applications:

- **domestic robotics**: windows washing

Wire-driven parallel robot



Potential applications:

- **domestic robotics**: windows washing
- **entertainment**: actor motion in theater, fast change in scenes

Wire-driven parallel robot



Potential applications:

- **domestic robotics**: windows washing
- **entertainment**: actor motion in theater, fast change in scenes
- **catastrophe**: portable multi-dof crane (ADT)

Wire-driven parallel robot



Potential applications:

- **domestic robotics**: windows washing
- **entertainment**: actor motion in theater, fast change in scenes
- **catastrophe**: portable multi-dof crane (ADT)
- **haptic interface**: virtual reality, training with force-feedback

Wire-driven parallel robot



Potential applications:

- **domestic robotics**: windows washing
- **entertainment**: actor motion in theater, fast change in scenes
- **catastrophe**: portable multi-dof crane (ADT)
- **haptic interface**: virtual reality, training with force-feedback
- **assistance robotics**: lifting of elderly people (**lifting video**)

Wire-driven parallel robot



Potential applications:

- **domestic robotics**: windows washing
- **entertainment**: actor motion in theater, fast change in scenes
- **catastrophe**: portable multi-dof crane (ADT)
- **haptic interface**: virtual reality, training with force-feedback
- **assistance robotics**: lifting of elderly people (**lifting video**)
- **rehabilitation**

Example: rehabilitation



Example: rehabilitation



Patient suffering from loss of arm coordination after a cardiovascular stroke

Example: rehabilitation



Patient suffering from loss of arm coordination after a cardiovascular stroke

Classical rehabilitation **training**: arm pointing to colored marks moving randomly on a computer screen

Example: rehabilitation



Patient suffering from loss of arm coordination after a cardiovascular stroke

Classical rehabilitation **training**: arm pointing to colored marks moving randomly on a computer screen

Drawbacks:

- no **monitoring** of the arm motion

Example: rehabilitation



Patient suffering from loss of arm coordination after a cardiovascular stroke

Classical rehabilitation **training**: arm pointing to colored marks moving randomly on a computer screen

Drawbacks:

- no **monitoring** of the arm motion
- no objective mean to qualify the motion **quality**

Example: rehabilitation



Patient suffering from loss of arm coordination after a cardiovascular stroke

Classical rehabilitation **training**: arm pointing to colored marks moving randomly on a computer screen

Drawbacks:

- no **monitoring** of the arm motion
- no objective mean to qualify the motion **quality**
- fatigue induced by pointing the arm

Example: rehabilitation



Patient suffering from loss of arm coordination after a cardiovascular stroke

Robot assisted rehabilitation

Example: rehabilitation



Patient suffering from loss of arm coordination after a cardiovascular stroke

Robot assisted rehabilitation

- use **trajectory tracking** to monitor and qualify motions

Example: rehabilitation



Patient suffering from loss of arm coordination after a cardiovascular stroke

Robot assisted rehabilitation

- use **trajectory tracking** to monitor and qualify motions
- relieve partly **arm gravity** for focusing on **coordination**

Example: rehabilitation



Patient suffering from loss of arm coordination after a cardiovascular stroke

Robot assisted rehabilitation: (rehabilitation video)

Example: rehabilitation



Patient suffering from loss of arm coordination after a cardiovascular stroke

Robot assisted rehabilitation: (rehabilitation video)

- gravity effects decreased by 50%

Example: rehabilitation



Patient suffering from loss of arm coordination after a cardiovascular stroke

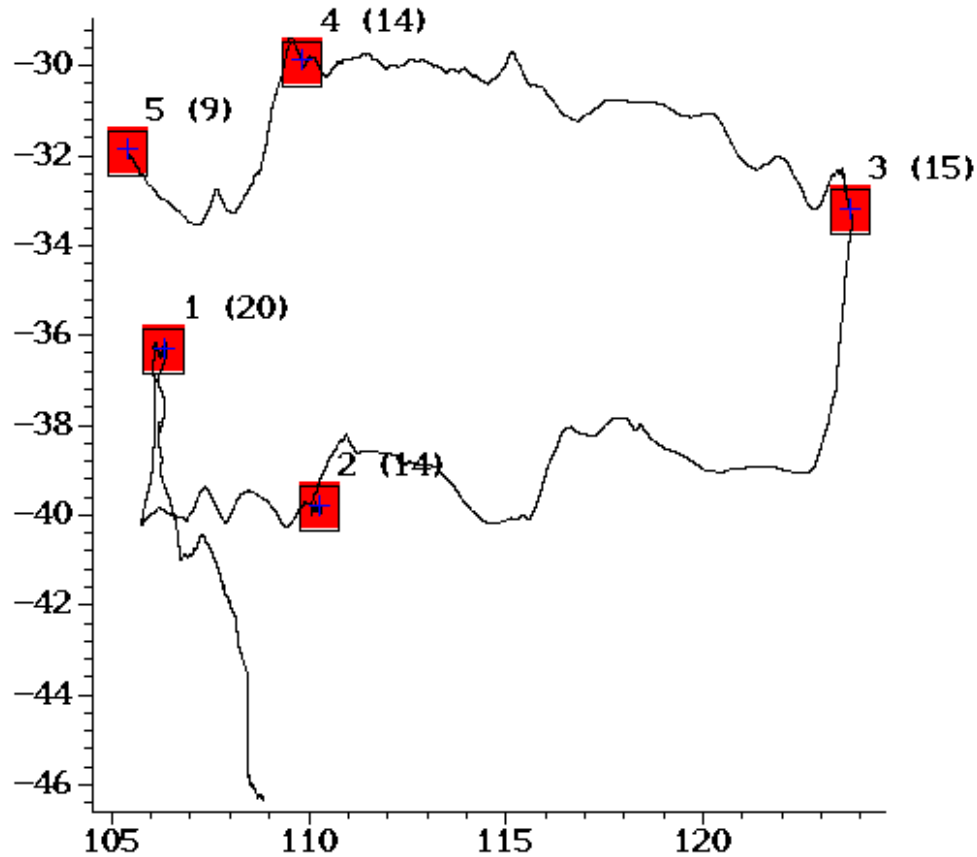
Robot assisted rehabilitation: (rehabilitation video)

- gravity effects decreased by 50%
- trajectory tracking: straightness of the trajectory allows to qualify motion quality

Example: rehabilitation



Trajectory tracking



Recent objectives



Recent objectives



Focus on service robotics

Recent objectives



Focus on service robotics

- developing various **low-cost assistance robotized devices**

Recent objectives



Focus on service robotics

- developing various **low-cost assistance robotized devices**
- **user-centered** (systematic collaboration with end-users)

Recent objectives



Focus on service robotics

- developing various **low-cost assistance robotized devices**
- **user-centered** (systematic collaboration with end-users)
- developing **methodologies** to adapt the device to the end-user and its surrounding

Recent objectives



Focus on service robotics

- developing various **low-cost assistance robotized devices**
- **user-centered** (systematic collaboration with end-users)
- developing **methodologies** to adapt the device to the end-user and its surrounding
- developing various **interfaces** to manage the end-user abilities

Recent objectives



Focus on service robotics

- developing various **low-cost assistance robotized devices**
- **user-centered** (systematic collaboration with end-users)
- developing **methodologies** to adapt the device to the end-user and its surrounding
- developing various **interfaces** to manage the end-user abilities
- **low intrusivity**: use already accepted objects, systems are invisible if not in use

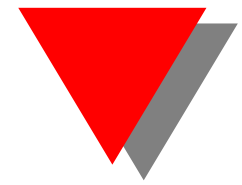
Recent objectives



Focus on service robotics

- developing various **low-cost assistance robotized devices**
- **user-centered** (systematic collaboration with end-users)
- developing **methodologies** to adapt the device to the end-user and its surrounding
- developing various **interfaces** to manage the end-user abilities
- **low intrusivity**: use already accepted objects, systems are invisible if not in use
- **provide information for doctors**: early detection of emerging pathologies

Recent objectives



Focus on service robotics

- developing various **low-cost assistance robotized devices**
- **user-centered** (systematic collaboration with end-users)
- developing **methodologies** to adapt the device to the end-user and its surrounding
- developing various **interfaces** to manage the end-user abilities
- **low intrusivity**: use already accepted objects, systems are invisible if not in use
- **provide information for doctors**: early detection of emerging pathologies
- **safety**: improve emergency detection, prevent fall

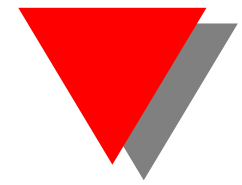
Recent objectives



Focus on service robotics

- developing various **low-cost assistance robotized devices**
- **user-centered** (systematic collaboration with end-users)
- developing **methodologies** to adapt the device to the end-user and its surrounding
- developing various **interfaces** to manage the end-user abilities
- **low intrusivity**: use already accepted objects, systems are invisible if not in use
- **provide information for doctors**: early detection of emerging pathologies
- **safety**: improve emergency detection, prevent fall
- **smart device**: communicating devices, using all types of media (hertzian, IR, optical, . . .)

Recent objectives



Focus on service robotics

- developing various **low-cost assistance robotized devices**
- **user-centered** (systematic collaboration with end-users)
- developing **methodologies** to adapt the device to the end-user and its surrounding
- developing various **interfaces** to manage the end-user abilities
- **low intrusivity**: use already accepted objects, systems are invisible if not in use
- **provide information for doctors**: early detection of emerging pathologies
- **safety**: improve emergency detection, prevent fall
- **smart device**: communicating devices, using all types of media (hertzian, IR, optical, . . .)
- active participation to the large scale initiative PAL (Personally Assisted Living)

Example: assistance for elderly people (video)