Singular configurations and direct kinematics of a new parallel manipulator

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Abstract :We present in this paper a new mechanical architecture for a parallel manipulator. We address the problem of the determination of the singular configurations of this architecture. Then we show that the direct kinematic problem has at most 16 solutions and exhibit an algorithm to find all the solutions.

1 Introduction

Many architectures of parallel manipulators have been proposed by various researchers: Fichter [3], Inoue [6], Reboulet [9], Zamanov [11]. Our purpose is to design a parallel manipulator which will be used as a compliant wrist. Therefore this manipulator has to be light and its center of mass must be low. In order to increase the workspace and improve the dynamic behaviour we want to use very simple cylindrical links.

2 The INRIA prototype

The INRIA prototype (figure 1) is composed of a mobile plate and a fixed one, connected by six links whose lengths are identical. These links (A_iB_i) are low-diameter cylindrical beams. The prismatic actuators M_i enables to change the position of the articulation points A_i. By changing the position of these points we are able to control the position and the orientation of the mobile plate. We define a reference frame $(O, \mathbf{x}, \mathbf{y}, \mathbf{z})$ and a relative frame linked to the mobile plate $(C, \mathbf{x_r}, \mathbf{y_r}, \mathbf{z_r})$. We denote by $\mathbf{X} = [x_c, y_c, z_c, \psi, \theta, \phi]$ the generalized coordinates vector whose componants are the coordinates of C in the reference frame and the orientation angles of the mobile plate. We denote by $\rho = [za_1, za_2, za_3, za_4, za_5, za_6]$ the articular coordinates vector where za_i is the z-coordinates of A_i. $\mathbf{n_z}$ denotes the normal of the mobile-plate.

3 Inverse Kinematics

The fundamental relation between the articular componant za_i and the position and orientation of the mobile plate is:

$$(z_c - za_i + zu_i)^2 + (x_c - xu_i)^2 + (y_c - yu_i)^2 = L_i^2 \quad (1)$$

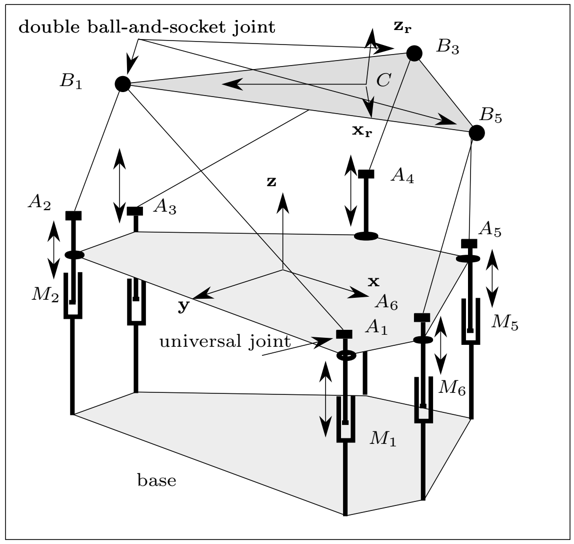

Figure 1: The parallel prototype: the prismatic actuators move the articulation point A_i along a vertical axis.

where xu_i, yu_i and zu_i depend only upon the orientation of the mobile plate and L_i is the length of link i. Therefore for our prototype we have:

$$L_i^2 - L_j^2 = \mathbf{T_{i,j}}(za_i, za_j, \mathbf{X}) = 0 \tag{2}$$

4 Singular configurations

The inverse jacobian matrix $J^{-1}(\mathbf{X})$ relates the articular velocities $\dot{\rho}$ to the cartesian and angular velocities $\dot{\mathbf{X}}$:

$$\dot{\rho} = J^{-1}(\mathbf{X})\ \dot{\mathbf{X}} = (\frac{\partial \rho}{\partial X})\dot{\mathbf{X}} \tag{3}$$

In order to determine the articular velocity vector for a given $\dot{\mathbf{X}}$ the determinant of $J^{-1}(\mathbf{X})$ must be different from zero. If in a given configuration $\mathbf{X_0}$ the determinant is equal to zero, the robot is uncontrollable, and $\mathbf{X_0}$ is a *singular configuration*. To determine these configurations, we may try to find the roots of the determinant of the matrix J^{-1} which is a rather complex non-linear expression. Another approach is based on Grassmann line-geometry and has been explained in [8] and the mathematical background of this geometry can be found in [1],[2],[10]

5 Determination of the singular configurations

First we have to remind the definition of the Plücker coordinates of lines. A line (Δ) can be defined by its Plücker coordinates. Let us consider two points on the line (Δ), M_1 and M_2 defined in a reference frame R_r with origin R. The Plücker vector of (Δ), denoted $\mathbf{P_\Delta}$ is defined by:

$$\mathbf{P_\Delta} = [\,\mathbf{M_1M_2},\,\mathbf{M}\,] \qquad \text{with} \qquad \mathbf{M} = \mathbf{RM_1} \wedge \mathbf{RM_2} = \mathbf{RM_1} \wedge \mathbf{M_1M_2} = \mathbf{RM_2} \wedge \mathbf{M_1M_2} \tag{4}$$

We may also define the normalized Plücker coordinates as:

$$\mathbf{P'_\Delta} = \frac{\mathbf{P_\Delta}}{\| \mathbf{M_1 M_2} \|} \;\; = \; [\,\mathbf{S'}, \, \mathbf{M'}\,] \tag{5}$$

5.1 Determination of $J^{-1}(\mathbf{X})$

Now let us calculate the matrix $J^{-1}(\mathbf{X})$. Let $\mathbf{F}$ be the force vector applied on the mobile plate, $\mathbf{M}$ the torque vector acting on point C and $\mathbf{f}$ the articular force vector (the stress in the links). It is well known that:

$$\mathbf{\Gamma} = (\frac{\partial \rho}{\partial \mathbf{X}})^{\mathbf{T}} \, \mathbf{f} \;\; = \;\; \mathbf{J^{-T}} \, \mathbf{f} \quad \text{with} \quad \mathbf{\Gamma} = [\,\mathbf{F}, \, \mathbf{M}\,] \tag{6}$$

When, the mechanical system is in equilibrium we have:

$$\sum_{i=0}^{6} f_i \mathbf{n_i} = \mathbf{F} \qquad\qquad \sum_{i=0}^{6} (\mathbf{CB_i} \wedge f_i \mathbf{n_i}) = \mathbf{M} \tag{7}$$

where $\mathbf{n_i}$ is the unit vector of the link. Let $\mathbf{P'_i}$ be the normalized Plücker vector of link i. Equation (7) can be written as:

$$\sum_{i=0}^{6} f_i \mathbf{S'_i} = \mathbf{F} \qquad\qquad \sum_{i=0}^{6} f_i (\mathbf{CB_i} \wedge \mathbf{S'_i}) = \mathbf{M} \tag{8}$$

Therefore:

$$J^{-1}(\mathbf{X}) = \mathbf{P} \qquad \text{with} \qquad \mathbf{P} = [\,\mathbf{P'_1}, \mathbf{P'_2}, \mathbf{P'_3}, \mathbf{P'_4}, \mathbf{P'_5}, \mathbf{P'_6}\,] \tag{9}$$

Therefore the degeneracies of $J^{-1}(\mathbf{X})$ are obtained when one of the Plücker vector of the line associated to a link is linearly dependent of the others Plücker vectors. Grassmann has shown that such a dependency between n Plücker vectors (which therefore span a variety of rank $n-1$) will yield to a geometrical constraint between the n lines. These constraints for a set of n lines (where n lie in the range [3,6]) have been presented in [8]. We will study now how the various sets of n lines of our prototype can span a variety of rank $n-1$.

5.2 Linear dependency of the sets of lines

5.2.1 Set of three lines

One of the Plücker vectors is a linear combination of the others if the two following geometric conditions are satisfied: the three lines belong to a plane P and they intersect all a point M (relation C_2). The set of three links of our prototype can be divided into two families. In the first family, two of the links have a common point (for example 1, 2, 3). The condition C_2 is obtained when:

$$(\mathbf{A_1B_1} \wedge \mathbf{A_2B_2}).\mathbf{A_3B_3} = 0 \qquad \mathbf{A_3B_3} \wedge \mathbf{A_3B_1} = \mathbf{0} \tag{10}$$

We set za_2 to zero and from equations (10) we deduce za_1, za_3. Then by using the $\mathbf{T_{1,3}}$ we get:

$$x_c = \mathbf{F_{21}}(y_c, \psi, \theta, \phi) \qquad\qquad \mathbf{H_{21}}(y_c^2, y_c, z_c, \psi, \theta, \phi) = 0 \tag{11}$$

which define the singular configurations obtained for that case.

In the second family the three links have no common point (for example 1, 3, 6). The condition C_2 is obtained when:

$$\mathbf{A_1B_1} \wedge \mathbf{A_1M} = \mathbf{0} \qquad \mathbf{A_3B_3} \wedge \mathbf{A_3M} = \mathbf{0} \qquad \mathbf{A_6B_6} \wedge \mathbf{A_6M} = \mathbf{0} \qquad \mathbf{A_1B_1.nz} = \mathbf{0} \tag{12}$$

From these equations we may deduce the coordinates of $\mathbf{M}$ and the values of za_1, za_3, za_6. Then using $\mathbf{T_{1,3}}$ and $\mathbf{T_{1,6}}$, we get the constraint equations:

$$\mathbf{F_{22}}(\ \mathbf{X}\ ) = \mathbf{0} \qquad \mathbf{G_{22}}(\ \mathbf{X}\ ) = \mathbf{0} \qquad \mathbf{H_{22}}(\ \mathbf{X}\ ) = \mathbf{0} \tag{13}$$

5.2.2 Set of four lines

Condition **3a**: The four lines belong to a regulus. A regulus is a family of lines which generates *an hyperboloïd of one sheet*. Since an hyperboloïd of one sheet is doubly ruled, it is generated by two families of lines: the regulus and its complementary regulus. An interesting property is that *if two lines belonging to the same hyperboloïd intersect then one line belongs to the regulus and the other to the complementary regulus.* For any set of four links of our prototype, there are at least two links having a common point (an articulation point of the mobile plate). Therefore, we cannot find four lines which belong to the same regulus.

Condition **3b**:The lines belong to two flat pencils, lying in two distinct planes and having a common line . This case may be divided into two sub-cases. First two pairs of links has each a common point (for example 1,2,3,4). In that case we have:

$$(\mathbf{A_1B_1} \wedge \mathbf{A_2B_2}).\mathbf{B_1B_3} = 0 \qquad (\mathbf{A_3B_3} \wedge \mathbf{A_4B_4}).\mathbf{B_1B_3} = 0 \tag{14}$$

These equations being linear in term of x_c and y_c, we are able to calculate their values. The second sub-case is obtained when there is only one common point between some of the four links (for example 1, 2, 3, 5). In that case we have:

$$(\mathbf{A_1B_1} \wedge \mathbf{A_2B_2}).\mathbf{B_1M} = 0 \tag{15}$$

$$\mathbf{A_3B_3} \wedge \mathbf{A_3M} = \mathbf{0} \qquad \mathbf{A_5B_5} \wedge \mathbf{A_5M} = \mathbf{0} \tag{16}$$

$$\mathbf{A_3B_3.nz} = 0 \qquad \mathbf{A_5B_5.nz} = 0 \tag{17}$$

From equation (17) we deduce za_3, za_5 We use then three of the four equations (16) to determine the coordinates of M. Using the remaining equations and $\mathbf{T_{3,5}}$, $\mathbf{T_{1,2}}$ we get:

$$x_c = \mathbf{F_{3b1}}(y_c, \psi, \theta, \phi) \qquad \mathbf{G_{3b1}}(\ \mathbf{X}\ ) = \mathbf{0} \tag{18}$$

Condition **3c**: All the lines pass through one point. If we choose the quadruplet 1, 2, 4, 6, the condition is fulfilled if:

$$\mathbf{A_4B_4} \wedge \mathbf{A_4B_1} = \mathbf{0} \qquad \mathbf{A_6B_6} \wedge \mathbf{A_6B_1} = \mathbf{0} \tag{19}$$

From these equations we deduce za_4 and za_6 and using $\mathbf{T_{4,6}}$ we get:

$$x_c = \mathbf{F_{3c}}(y_c^2, y_c, \psi, \theta, \phi) \qquad z_c = \mathbf{G_{3c}}(y_c^2, y_c, \psi, \theta, \phi) \qquad \mathbf{H_{3c}}(y_c^2, y_c, \psi, \theta, \phi) = 0 \tag{20}$$

Condition **3d**: The four links are coplanar. Two sub-cases are to be considered. First two pairs of line have each a common point (for example 1, 2, 3, 4). In that case, we have:

$$(\mathbf{A_1B_1} \wedge \mathbf{A_2B_2}).\mathbf{A_1B_3} = 0 \qquad (\mathbf{A_3B_3} \wedge \mathbf{A_4B_4}).\mathbf{A_3B_1} = 0 \tag{21}$$

From the linear system (21), we calculate the variables x_c and y_c. In the second case there is only one common point between the links (for example 1, 2, 3, 6). In that case we have:

$$\mathbf{A_1B_1.nz} = \mathbf{0} \qquad \mathbf{A_2B_2.nz} = \mathbf{0} \qquad \mathbf{A_3B_3.nz} = \mathbf{0} \qquad \mathbf{A_6B_6.nz} = \mathbf{0} \tag{22}$$

From these equations we deduce za_1, za_2, za_3, za_6,and then we use $\mathbf{T_{1,2}}$ and $\mathbf{T_{3,6}}$ to get the following equations:

$$\mathbf{x_c} = \mathbf{F_{3d}}(\psi, \theta, \phi) \qquad \mathbf{y_c} = \mathbf{G_{3d}}(\psi, \theta, \phi) \tag{23}$$

5.2.3 Set of five lines

Condition **4b**:The five lines intersect two skew lines (D_1) and (D_2). Without loss of generality we will consider the set of lines 1, 2, 3, 4, 5. First, let us find (D_1) and (D_2) that intersect four lines. The two skew lines (D_1), (D_2) can be defined in two different ways. First $D_1 \in P_{12}$ *and cross the point* B_3, $D_2 \in P_{34}$ *and cross the point* B_1 (P_{ij} is the plane defined by the lines i, j). In that case we have:

$$(\mathbf{A_1B_1} \wedge \mathbf{A_2B_2}).\mathbf{A_1B_3} = 0 \qquad (\mathbf{A_3B_3} \wedge \mathbf{A_4B_4}).\mathbf{A_3B_1} = 0 \tag{24}$$

These equations define a set of lines (D_1, D_2) and in this set a pair of line (D_{a1}, D_{a2}) intersect line 5. These lines will be skew if:

$$\mathbf{A_5B_5}.\mathbf{n_z} \neq 0 \qquad \mathbf{A_5B_5}.\mathbf{n_{12}} \neq 0 \qquad \mathbf{A_5B_5}, \mathbf{n_{34}} \neq 0 \tag{25}$$

where $\mathbf{n_{ij}}$ is the normal to the plane defined by the lines i, j. From equations (24), we calculate the values of x_c and y_c. The second way to define (D_1, D_2) is $D_1 \in (P_{12} \cap P_{34})$, $D_2 = B_1B_3$. The Plücker vector P_{D1} of the intersection line of P_{12}, P_{34} can be calculated as a function of $\mathbf{X}, \rho$. Therefore if line 5 intersect D_1 we have:

$$\mathbf{S'_{D1}}.\mathbf{M'_5} + \mathbf{S'_5}.\mathbf{M'_{D1}} = 0 \tag{26}$$

and line 5 will intersect D_2 if:

$$\mathbf{A_5B_5}.\mathbf{nz} = 0 \tag{27}$$

First, let us set $za_1 = 0$ and calculate za_5 from (27). Then $\mathbf{T_{1,5}}$, and equation (26) yield to:

$$x_c = \mathbf{F_{4b}}(y_c, \psi, \theta, \phi) \qquad \mathbf{G_{4b}}(\mathbf{X}, za_2, za_3, za_4) = 0 \tag{28}$$

Condition **4c**: The lines define three flat pencils having one line in common but lying in distinct planes and with distinct centres. Without loss of generality let us consider links 1, 2, 3, 4, 5. We must have:

$$(\mathbf{A_1B_1} \wedge \mathbf{A_2B_2}).\mathbf{B_1B_3} = 0 \tag{29}$$
$$(\mathbf{A_3B_3} \wedge \mathbf{A_4B_4}).\mathbf{B_1B_3} = 0 \tag{30}$$
$$\mathbf{A_5B_5}.\mathbf{nz} = 0 \tag{31}$$

Let us set $za_2 = 0$ and deduce za_1, za_5 from equation (29) and (31). Then by using $\mathbf{T_{1,5}}$ we get:

$$\mathbf{F_{4c}}(\ \mathbf{X}\ ) = \mathbf{0} \qquad \mathbf{G_{4c}}(\ \mathbf{X}, za_3, za_4\ ) = 0 \tag{32}$$

Condition **4d**: The lines either belong to a same plane P or pass through a unique point M, $M \in P$. Let us examine the different possible cases:

- Links 1,2,3 $\in P$, and 4,5 pass through the same point M. Therefore we have:

$$(\mathbf{A_1B_1} \wedge \mathbf{A_2B_2}).\mathbf{A_3B_3} = \mathbf{0} \qquad \mathbf{A_5B_5} \wedge \mathbf{A_5B_3} = \mathbf{0} \tag{33}$$

  From equation (33) we calculate the values of x_c, y_c and z_c.

- The links 1,2,5 $\in P$, and 3,4 pass through M. Thus we have:

$$\mathbf{A_1B_1}.\mathbf{nz} = \mathbf{0} \tag{34}$$
$$\mathbf{A_2B_2}.\mathbf{nz} = \mathbf{0} \tag{35}$$
$$\mathbf{A_5B_5}.\mathbf{nz} = \mathbf{0} \tag{36}$$

  We set $za_5 = 0$ and calculate za_1, za_2 from the equations (34), (35). Using $\mathbf{T_{1,2}}$ and equation (36) we deduce the following relations:

$$x_c = \mathbf{F_{4d1}}(y_c, \psi, \theta, \phi) \qquad \mathbf{G_{4d1}}(y_c^2, y_c, \psi, \theta, \phi) = 0 \tag{37}$$

- links 1,2 $\in P$, and 3,4,5 pass through the same point M. We have:

$$(\mathbf{A_1B_1} \wedge \mathbf{A_2B_2}).\mathbf{B_1B_3} = 0 \qquad \mathbf{A_5B_5} \wedge \mathbf{A_5B_3} = \mathbf{0} \tag{38}$$

We set za_2 to zero and find the following relations:

$$\mathbf{F_{4d2}}(y_c^3, y_c^2, y_c, z_c^2, z_c, \psi, \theta, \phi) = 0 \qquad \mathbf{G_{4d2}}(y_c^2, y_c, z_c^2, z_c, \psi, \theta, \phi) = 0 \tag{39}$$

5.2.4 Set of six lines

Condition **5a**: The three lines belonging respectively to the three flat pencils spanned by the links (1,2), (3,4) and (5,6) and lying in the mobile-plate plane intersect at a unique point M. Therefore we have:

$$(\mathbf{A_1B_1} \wedge \mathbf{A_2B_2}).\mathbf{A_1M} = 0 \qquad (\mathbf{A_3B_3} \wedge \mathbf{A_4B_4}).\mathbf{A_3M} = 0 \tag{40}$$

$$(\mathbf{A_5B_5} \wedge \mathbf{A_6B_6}).\mathbf{A_5M} = 0 \qquad \mathbf{B_1M}.\mathbf{nz} = 0 \tag{41}$$

Unfortunately once we have reported the M coordinates in the last equation, we obtain a relation:

$$\mathbf{F_{5a}}(\ \mathbf{X},\ \rho\ ) = \mathbf{0} \tag{42}$$

Condition **5b**: All the lines meet one line Δ. It can be show that (Δ) is the line passing through two articulation points B_i, B_j. If we consider B_3 and B_5 we have:

$$(\mathbf{A_1B_1} \wedge \mathbf{A_2B_2}) \wedge \mathbf{nz} = \mathbf{0} \tag{43}$$

We set za_1 to 0 and calculate za_2 from one of the equation (43). Then using $\mathbf{T_{1,2}}$ and the remaining equation we get:

$$x_c = \mathbf{F_{5b}}(\psi, \theta, \phi) \qquad y_c = \mathbf{G_{5b}}(\psi, \theta, \phi) \tag{44}$$

6 Direct Kinematics

6.1 Equivalent mechanism

Let us consider a manipulator with a fixed set of z_a. Clearly the point B_i is able to describe only a circle whose center is located on the line joining the two articulation centers of the links associated to this point (for example B_1 lie on a circle whose center is on the line A_1, A_2). We have shown in [7] that the position of the centers of these circles and their radii can be calculated from the links lengths and the z_a. Therefore we may consider that our prototype is now equivalent to the mechanism described in Figure 2. This mechanism is constituted of three links articulated on revolute joints and connected to the mobile plate. The links lengths are l_{12}, l_{34}, l_{56} and the orientation of the links are defined by the angles p_{12}, p_{34}, p_{56}. For a fixed geometry of this mechanism we will investigate what are the different assembly-modes i.e. we will find what are the various sets of angle p_{12}, p_{34}, p_{56} such that the geometry is respected. These angles define the position of the point B_i and therefore the different configurations of the mobile plate of our prototype i.e. the different solutions of the direct kinematics problem.

6.2 Maximum number of assembly-modes for the equivalent mechanism

If we dismantle one of the link of the equivalent mechanism we get a RSSR mechanism. It is known [4] that a point of the coupler of this mechanism describes a sixteenth order surface, the *RSSR spin surface*.

In order to find the possible configurations of mobile plate we have to intersect this surface with the circle described by the extremity of the dismantled link: indeed every point on the surface which match the extremity of the dismantled link will correspond to an assembly mode of the equivalent mechanism.

A sixteenth order surface is intersected by a circle in no more than 32 points. But we have demonstrated in [7] that the RSSR spin-surface contains the imaginary spherical circle eight times and therefore we deduce that at least 16 points are imaginary, and therefore there is at most 16 assembly-modes for our prototype.

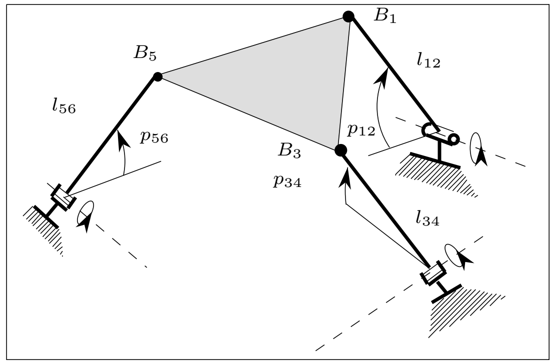

Figure 2: The equivalent mechanism of the INRIA prototype

6.3 Polynomial formulation of the direct kinematics problem

Let us consider the equivalent mechanism. The position in the reference frame of the point B_1, B_3, B_5 are fully defined by the geometry of the mechanism and the three angles p_{12}, p_{34}, p_{56}. As the distances between the B_i are known constant we may write three equations :

$$||\mathbf{B_1B_3}|| - d_{13} = 0 \qquad ||\mathbf{B_1B_5}|| - d_{15} = 0 \qquad ||\mathbf{B_3B_5}|| - d_{35} = 0 \tag{45}$$

where d_{ij} is the distance between point B_i, B_j. In these equations appear the sine and cosine of the unknown angles. Let us denote :

$$t_{12} = \tan(\frac{p_{12}}{2}) \qquad t_{34} = \tan(\frac{p_{34}}{2}) \qquad t_{56} = \tan(\frac{p_{56}}{2}). \tag{46}$$

The sine and cosine appearing in equations (45) can be expressed as polynomial function of the t_{ij} and therefore these equations are now polynomials in t_{12}, t_{34}, t_{56}. Innocenti [5] has shown that by combining these equations we can get a polynomial $\mathcal{P}$ in t_{12} only, whose order is 16. Therefore to solve the direct kinematics problem of our manipulator we use its geometry and the values of its articular coordinates to construct the polynomial $\mathcal{P}$. Then we solve this polynomial in t_{12}, find the corresponding values of t_{34}, t_{56} (which are unique for a given t_{12}). From these values we get the three unknowns angles p_{12}, p_{34}, p_{56} which define the position of the three points B_1, B_3, B_5 and therefore the position and orientation of the mobile plate of our manipulator. From the order of the polynomial $\mathcal{P}$ we have another confirmation of the fact that there will be at most 16 solutions to the direct kinematics problem.

A numerical procedure has been implemented and an extensive research has shown that effectively in some cases the polynomial $\mathcal{P}$ may have 16 real roots which means that there will be 16 solutions. Table 1 gives an example of these cases and the corresponding configurations are shown in Figures 3, 4, 5, 6. It must be noticed that this method involves a heavy computational burden. Therefore it cannot be used in a real-time context. For real-time application an iterative procedure has been shown to be very efficient.

7 Conclusion

A light parallel manipulator currently under development at INRIA has been presented. We have addressed the problem of its singular configurations and its direct kinematics. A geometrical approach enables to find the constraint equations on the generalized coordinate vector such that the resulting configuration of the manipulator is singular. This approach

solution	x_c	y_c	z_c	ψ	θ	ϕ
1	-0,0	0,000001	10,0	0,0	10,0	0,0
2	2,473130	0,632074	8,176453	-51,849020	105,730039	52,066359
3	-2,473130	0,632074	8,176453	51,849020	105,730039	-52,066359
4	0,0	-2,601499	7,755148	-180,0	108,417756	180,0
5	-0,496780	-0,294736	3,618337	-60,879349	44,743072	-119,045570
6	0,496780	-0,294736	3,618337	60,879349	44,743072	119,045570
7	-0,034355	-0,008744	2,006402	-54,971335	10,790311	-125,012525
8	0,034355	-0,008744	2,006402	54,971335	10,790311	125,012525
9	0,0	0,003463	1,819111	0,0	2,092780	180,0
10	0,0	0,125864	1,199592	-0,0	20,259768	-180,0
11	-0,0	0,010573	0,920016	180,0	6,487758	0,0
12	0,0	1,558260	-1,257076	-0,0	77,305505	180,0
13	-1,833759	1,900667	-4,541377	-111,759977	109,237706	112,032327
14	1,833759	1,900667	-4,541377	111,759977	109,237706	-112,032327
15	0,0	-2,435690	-5,078887	0,0	101,691828	0,0
16	-0,0	-0,061267	-6,687341	0,0	10,080083	0,0

Table 1: 16 configurations with identical articular coordinates for the INRIA prototype (Euler's angles in degree).

will enable to determine it some of the singular configurations lie in the workspace of the manipulator. We have shown that the direct kinematic problem has up to 16 solutions and we have exhibited a set of articular coordinates for which the mobile plate may effectively be in 16 different positions.

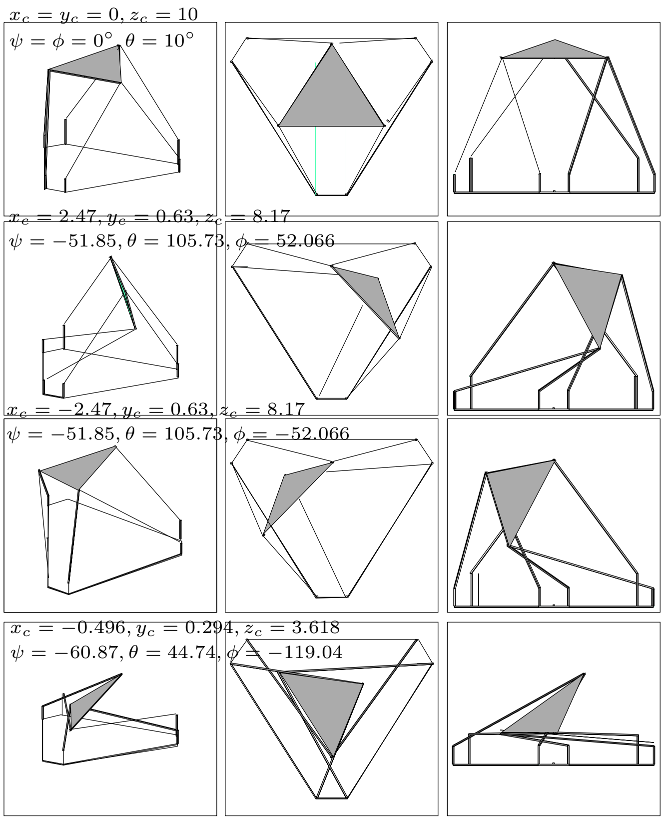

Figure 3: Solution 1-4

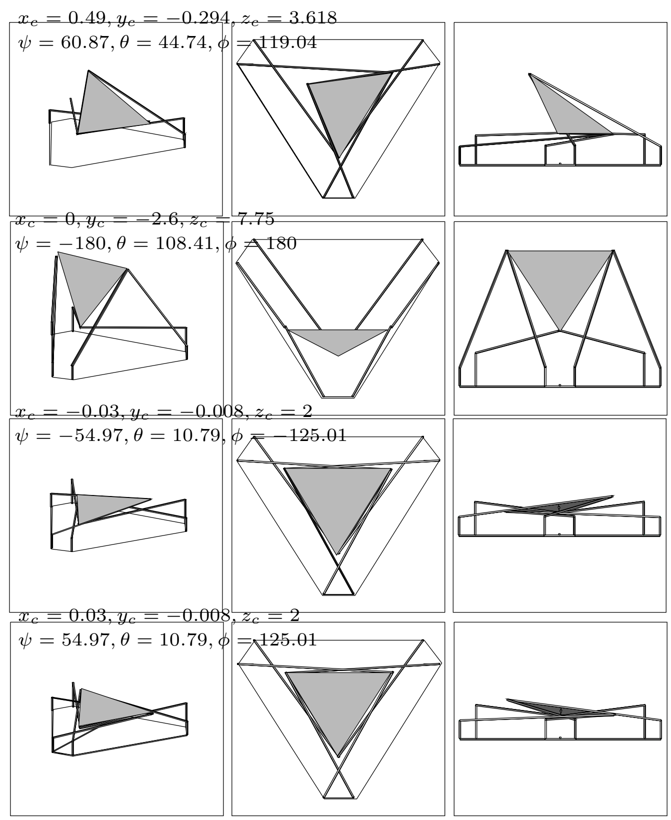

Figure 4: Solution 5-8

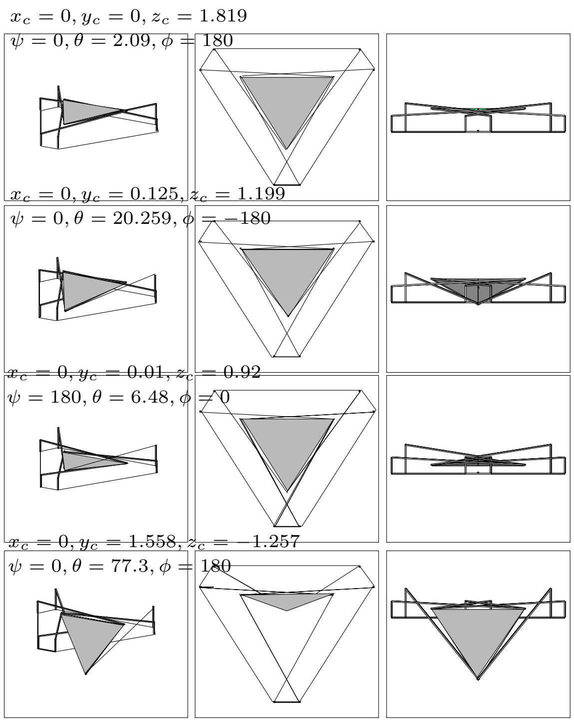

Figure 5: Solution 9-12

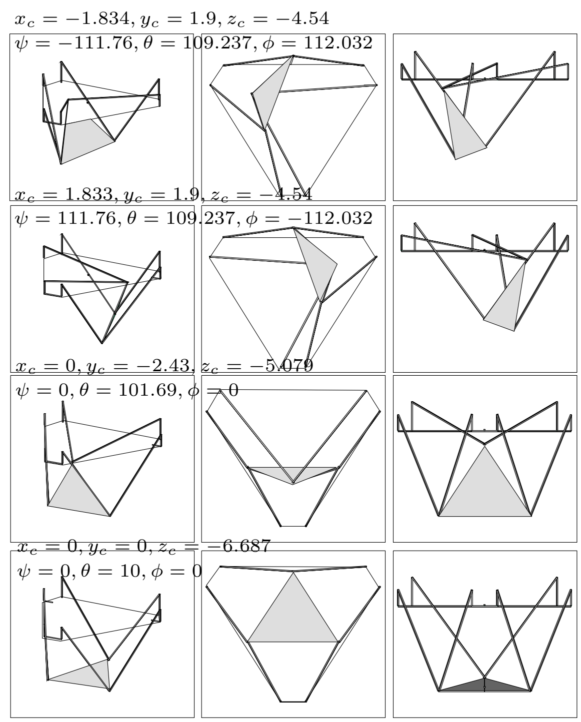

Figure 6: Solution 13-16

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