Assembly modes and direct kinematics of parallel manipulators

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Abstract: Direct kinematics is known to be one of the most difficult problem during the study of parallel manipulators although a solution is needed for most of the practical applications of parallel manipulators.

We consider here a parallel manipulator with an hexagonal base, a triangular mobile plate and time-varying links lengths. We first show that the direct kinematics problem can be reduced to solve a polynomial in one variable which degree will give an upper-bound of maximum number of assembly mode (UBAM for brief) of the manipulator. We establish theoretically an upper-bound of the degree of this polynomial and then we calculate the polynomial. We present an example where the theoretical maximum number of assembly mode is reached.

1 Introduction

The direct kinematics of closed-loop mechanism is known to be one of the most challenging problem when one deal with this kind of mechanism. We consider here specific closed-loop mechanisms called "parallel manipulator" where a mobile plate is connected by articulated links to a fixed plate. Each link has one extremity linked to the base, the other being linked to the mobile plate. The motion of the mobile plate is ensured through one actuator in each link: for example an electric ram enables to change the link's length. The positions of the actuators are called the articular coordinates. It is in general easy to determine the articular coordinates from the position and orientation of the base (i.e. the inverse kinematics problem) but this yield to a set of non-linear equations.

Solving a system of non-linear equations is a difficult task and a numerical resolution is tedious. Furthermore we have no a-priori information about the uniqueness of the solution (except in the case of singular configurations [1]). Nanua and Waldron [2] have initiated a new approach to this problem. They reduce the resolution of the system of non-linear equations to the one of a polynomial in one variable. The number of assembly-modes of the manipulator (i.e. the number of way one can assemble the manipulator with fixed articular coordinates) is clearly related to the degree of this polynomial: it cannot be greater than this degree.

In the case of the manipulator called the TSSM [4], [5], [3] (Triangular Symmetric Simplified Manipulator, see figure 1) these authors show that the direct kinematics problem may be expressed as solution of a polynomial in one variable, which degree is 24. Charentus and Renaud [6] have studied the same manipulator, in the case where the mobile plate is an equilateral triangle. They have shown that the degree of the polynomial can be reduced to 16. Hunt [7] has proposed a conjecture for the same manipulator which states that the number of assembly-modes cannot be greater than 16.

2 The TSSM

2.1 Equivalent mechanism

The TSSM (figure 1) is a 6 d.o.f. parallel manipulator in which a mobile plate is connected to a fixed base through 6 articulated links, each link being connected both at the base and the mobile plate through ball-and-socket and universal joints. By controlling the links lengths we are able to control the position and orientation of the mobile plate [4], [8], [9], [10].

For fixed links lengths the articulation points B_1, B_3, B_5 of the mobile plate are able to describe circles centered in O_{12}, O_{34}, O_{56} whose radius are r_{12}, r_{34}, r_{56}. The characteristics of these circles can be determined using only the knowledge of the links lengths. Thus the TSSM is equivalent to a mechanism constituted of three links articulated on revolute joints and connected to the mobile plate (figure 1) for which the articular coordinates are the angles p_{12}, p_{34}, p_{56}. This mechanism is called the *equivalent mechanism* of the TSSM.

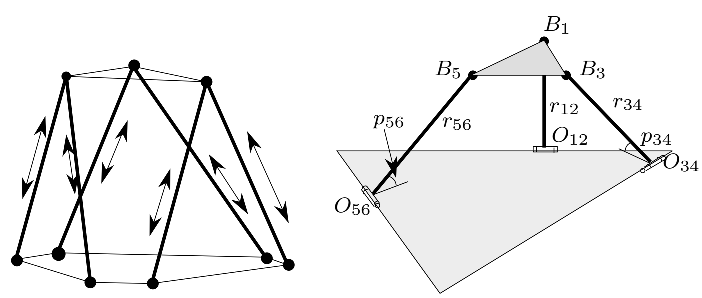

Figure 1: The TSSM and its equivalent mechanism

2.2 Minimal degree of the TSSM polynomial

Hunt [7] has conjectured that an UBAM of the TSSM is 16 by using the following method: if we dismantle one of the link of the equivalent mechanism of the TSSM we get a RSSR mechanism.

It is known [11] that a point of the coupler of this mechanism describes a sixteenth order surface, the *RSSR spin surface*. In order to find the possible configurations of mobile plate we have to intersect this surface with the circle described by the extremity of the dismantled link. A sixteenth order surface is intersected by a circle in no more than 32 points. Working on the conjecture that the RSSR spin-surface contains the imaginary spherical circle eight times Hunt deduces that at least 16 points are imaginary, and therefore there is at most 16 assembly-modes for the TSSM.

Thus to demonstrate this conjecture we have to determine the circularity of the RSSR spin-surface. In the following part we will use the notation defined in figure 2.

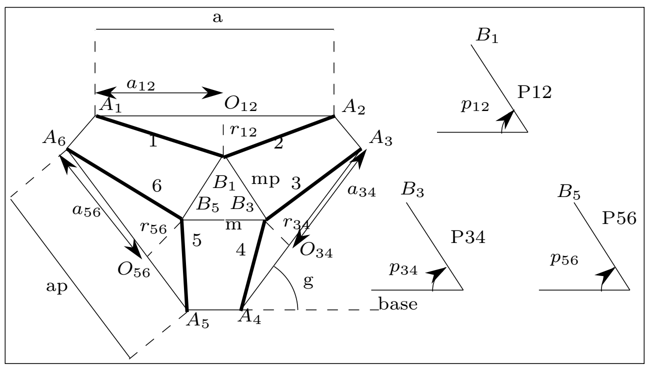

Figure 2: Notation (the TSSM is represented in top view)

Let X, Y, Z denote the coordinates of B, a point on the coupler of the RSSR. The coordinates of the articulation points B_1, B_3 can be expressed as a function of the unknown angles p_{12}, p_{34}. Thus we are able to write three equations relating the known distance between the articulation points to the unknowns p_{12}, p_{34}, X, Y, Z. We have:

$$a_{12} = \frac{\rho_1^2 + a^2 - \rho_2^2}{2a} \qquad a = 2xa_2 \tag{1}$$

$$x_{O_{12}} = -xa_2 + a_{12} = \frac{\rho_1^2 - \rho_2^2}{4xa_2} \quad y_{O_{12}} = ya_2 \quad r_{12}^2 = \rho_1^2 - a_{12}^2 \tag{2}$$

If n_{ij} denote the unit vector between O_{ij} and the corresponding articulation points we get :

$$n_{12} = -\cos(p_{12})\mathbf{j} + \sin(p_{12})\mathbf{k} \tag{3}$$

In the same way:

$$a_{34} = \frac{\rho_3^2 + ap^2 - \rho_4^2}{2ap} \tag{4}$$

$$x_{O_{34}} = xa_3 - a_{34}\cos(g) \quad y_{O_{34}} = ya_3 - a_{34}\sin(g) \quad r_{34}^2 = \rho_3^2 - a_{34}^2 \tag{5}$$

$$n_{34} = -\cos(p_{34})\sin(g)\mathbf{i} + \cos(p_{34})\cos(g)\mathbf{j} + \sin(p_{34})\mathbf{k} \tag{6}$$

We may write then

$$\mathbf{OB_1} = \mathbf{OO_{12}} + r_{12}\mathbf{n_{12}} \tag{7}$$

$$\mathbf{OB_3} = \mathbf{OO_{34}} + r_{34}\mathbf{n_{34}} \tag{8}$$

We are able to express the norm of the vectors $\mathbf{B_1B_3}$, $\mathbf{B_1B}$, $\mathbf{B_3B}$ i.e. the distances between the articulation points of the mobile whose values are mp and m, m. This yields to the following three equations :

$$||B_1B_3||^2 - mp^2 = 0 \quad ||B_1B||^2 - m^2 = 0 \quad ||B_3B||^2 - m^2 = 0 \tag{9}$$

We get three equations which can be written as :

$$E_1\cos(p_{12}) + E_2\sin(p_{12}) + E_3 = 0 \tag{10}$$

$$F_1\cos(p_{34}) + F_2\sin(p_{34}) + F_3 = 0 \tag{11}$$

$$\begin{aligned} &K_{11}\sin(p_{34})\sin(p_{12}) + (K_{21}\cos(p_{34}) \\ &+K_{22})\cos(p_{12}) + K_{32}\cos(p_{34}) + K_{33} = 0 \end{aligned} \tag{12}$$

where the E_i, F_j coefficients does not depend upon the angles but only upon the three coordinates of B. Equations 10,12 are linear in term of $\sin(p_{12}), \cos(p_{12})$. We solve this linear system and write the equation $\cos(p_{12})^2 + \sin(p_{12})^2 = 1$ which yields:

$$\begin{aligned} &(N_1 - N_2)\cos(p_{34})^2 + N_3\sin(p_{34})\cos(p_{34}) \\ &+N_4\sin(p_{34}) + N_5\cos(p_{34}) + N_6 + N_2 = 0 \end{aligned} \tag{13}$$

Then $\sin(p_{34})$ is determined using equation 11. If we put this value in equation 13 and write $\sin(p_{34})^2 + \cos(p_{34})^2 = 1$ we get two equations:

$$I_1\cos(p_{34})^2 + I_2\cos(p_{34}) + I_3 = 0 \tag{14}$$

$$H_1\cos(p_{34})^2 + H_2\cos(p_{34}) + H_3 = 0 \tag{15}$$

where the coefficients of I_i, H_j are function only of the coordinates of B. The orders of these coefficients are 4, 4, 4, 2, 3, 4. The equations 14, 15 yield to :

$$\begin{vmatrix} |I_1H_2| & |I_1H_3| \\ |I_1H_3| & |I_2H_3| \end{vmatrix} = 0 \tag{16}$$

where

$$|I_iH_j| = I_iH_j - I_jH_i \tag{17}$$

Using this method we get a sixteenth order polynomial. Its higher degree term is:

$$F_{21}^4(Y^2 + X^2 + Z^2)^8(N_{13} - N_{21})^2 \tag{18}$$

If $(N_{13} - N_{21})$ is equal to zero the mobile plate is reduced to a line. As for F_{21} it cannot be equal to zero. Therefore the circularity of the RSSR spin-surface is 8 and the conjecture of Hunt is verified. Thus there is at most 16 assembly-modes for the TSSM.

By expressing the coordinates of point B as a function of the angle p_{56} we are able to determine a sixteenth order polynomial for the direct kinematics problem of the TSSM. This polynomial has only even power (this mean that for a given configuration of the mobile the symetrical configuration with respect to the base is also a solution of the polynomial). A numerical resolution has shown that there may be effectively up to sixteen solutions to this polynomial : figure 3 shows one of the eight over-the-base assembly mode of a given TSSM.

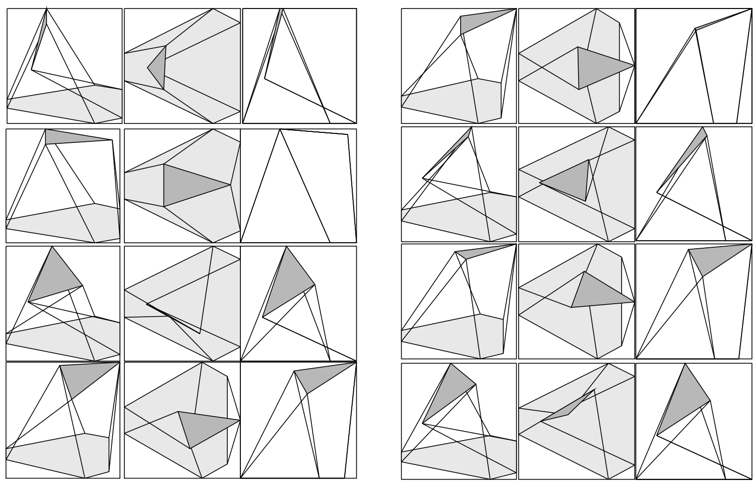

Figure 3: 8 over-the-base assembly-modes of the TSSM (perspective, top and side view)

3 Conclusion

We have shown that for the TSSM there will not be possible to find a closed-form for the solution of the direct kinematics problem. An algorithm has been proposed for finding all the assembly modes of a TSSM (although all may not be reachable without dismantling the manipulator) for a given set of links lengths. A generalization of this algorithm has been proposed in [12] for various kind of parallel manipulators.

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