Redundant parallel manipulators

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Received: May 9, 2001

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Abstract

Parallel manipulators are closed-loop mechanisms which feature high nominal load and high positioning ability. Usually they have the same number of actuators than their degree of freedom but in some cases it may interesting to have more actuators than needed i.e. to consider redundant parallel manipulators. We will show that redundancy is interesting for solving the forward kinematics, for singularities and obstacle avoidance, for improvement in force control and for kinematic calibration.

Another aspect of redundant parallel manipulators is their use for building *variable geometry trusses* i.e. highly redundant spatial structures which may be interest for spatial applications or for applications where obstacle avoidance is critical.

1 Parallel manipulators

Parallel manipulators are closed-loop mechanism (i.e. at least one body of the mechanism, usually the end-effector, has a connection with at least two links of the mechanism) in which all the links are connected both at the base and at the gripper of the robot. Manipulators of this type have been designed or studied for a long time. The first one, to the author's knowledge, was designed for testing tyres (see Gough paper [7]). But this mechanical architecture is mainly used for the flight simulator (see for example Stewart [25], Baret [2]). The first design as a manipulator system has been done by Mac Callion in 1979 for an assembly workstation [13].

The most well known parallel manipulator is the general Gough platform described in Figure 1, which will be used as an example throughout this paper. Basically it consists in two plates connected by 6 articulated links. In the following sections the smaller plate will be called the *moving platform* and the larger (which is in general fixed) will be called the *base*. In each articulated link there is one linear actuator and by changing the lengths of the links we are able to control the position [1] of the gripper. The links are connected to the base with universal joints at points A_i and to the moving platform with ball-and-socket joints at the B_i points.

The main features of this type of robot is their high nominal load and very precise positioning ability. This may be illustrated by the robot developed for the European Synchrotron Radiation Facility (ESRF) shown in Figure 2. This robot has a nominal load of 500 kg, and a positioning accuracy better than 1

[1] In this paper position means position and orientation

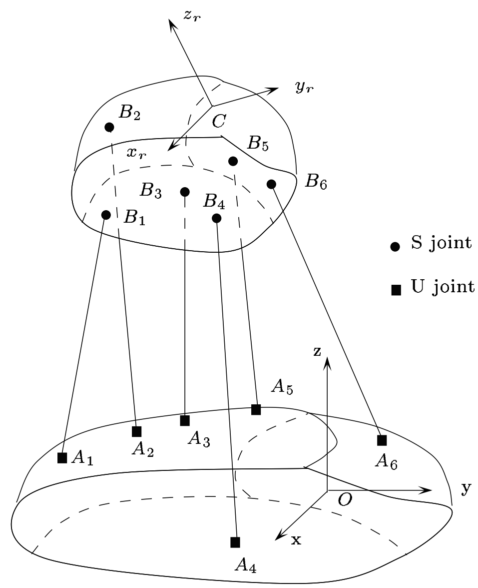

Figure 1: A parallel manipulator: the general Gough platform. Links i is articulated at point A_i, B_i.

μm in a cubic workspace with an edge length of 40 mm. The purpose of this robot is to support special optic devices for controlling the direction of X-rays.

Another example illustrates the positioning ability of parallel manipulators: its application to eye surgery described in [8]. In this application a hypodermic needle is inserted through the wall of the eye to gain access to the interior surface. A rigid glass micropipette is inserted in the hypodermic needle and is guided to various injection sites on the retina by pivoting the needle about its puncture point. This rotation motion around a precise point, which should be accurate for evident reasons, is performed by the parallel robot depicted in Figure 3. Although the structure is basically similar to the structure of a Gough platform the difference lie in the actuation mode. Here we have fixed length legs and the motion of the platform is obtained by changing the vertical position of the A_i points.

2 Redundant parallel manipulators

2.1 Building a redundant parallel robot

In general a parallel manipulator with d degrees of freedom is constituted of d kinematics chains which are connected both to the base and to the moving

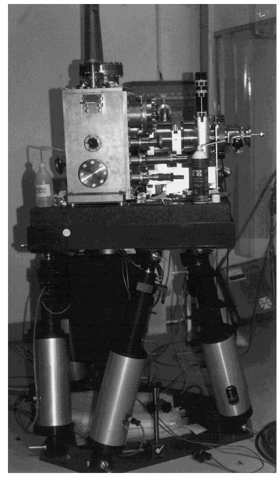

Figure 2: The ESRF parallel robot.

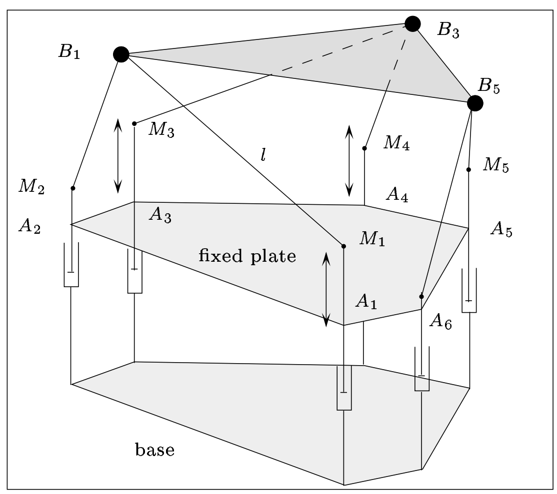

Figure 3: A parallel robot used for eye surgery. The motion of the platform is obtained by changing the height of the M_i points.

platform. For the sake of simplicity we will assume that all the kinematic chains are identical. The mobility m (i.e. the number of degree of freedom) of the platform may be computed with Grübler formula:

$$m = 6(l - n - 1) + \sum_{i=1}^{n} d_i$$

where l is the total number of bodies (including the base), n the total number of joints an d_i the number of degrees of freedom of the joints of the chain i. Assuming that the kinematic chains are identical, let n_1 be the number of bodies in a kinematic chain and let n_2 be the number of 1 d.o.f. joints in the chain. We get:

$$m = 6 + 6\ m\ n_1 - 5\ m\ n_2$$

For building a robot with 6 d.o.f. (m=6) the integer solutions for n_1, n_2 is (5, 6) (considering only the solution with minimal number of joints in the chain). Consequently to get a 6 d.o.f. robot it is sufficient to have one actuated joint in the kinematic chain. Therefore 6 d.o.f. redundant parallel robots will be obtained either by having more than one actuator in the kinematic chain or by having more than 6 kinematic chains. A similar result will be obtained for manipulators with less than 6 d.o.f.

2.2 Redundancy for solving the direct kinematics

For controlling a parallel robot we use the actuators and we have sensors for measuring some of the variable parameters of the kinematic chains. Usually only the parameters of the active joints are measured but we may have redundancy in the sensing with some sensors measuring parameters of passive joints. The output of the sensors is used to compute the location of the moving platform and the errors between the desired position and the actual one is fed to a closed loop algorithm which compute outputs for the actuators. In our example we have linear actuators enabling to modify the leg lengths and we have sensors which measure these leg lengths. But as we are interested in controlling the motion of the moving platform we need to study the kinematics of the robot, which describe the relation between the leg lengths and the position/orientation of the moving platform.

The inverse kinematics enables to compute the leg lengths as a function of the configuration of the moving platform. In general inverse kinematics is straightforward for non redundant parallel manipulators. In our example, if we fix the position and orientation of the moving platform we fix also the location of the points B_i. As the location of the points A_i is known we can compute the leg lengths which are the norm of the vector $\mathbf{A_i B_i}$.

But when controlling a parallel robot we may need also to solve the direct kinematics problem i.e. determining the configuration of the moving platform for a measured set of leg lengths. For example direct kinematics is necessary when the motion of the moving platform is specified in Cartesian coordinates (e.g. the moving platform center has to move along a line).

The direct kinematics problem is much more difficult than the inverse kinematic problem: there is in general more than one solution [16], even for planar manipulators [6]. Furthermore this problem is not yet solved for the manipulator presented in Figure 1. In some special cases all the solutions of the direct kinematics can be found but not in a time compatible with real-time control. Practically this problem is solved by using a numerical iterative procedure which need an initial guess of the solution (usually the guess is provided by the solution computed previously by the algorithm). But this procedure may fail to converge for fast parallel robots (the initial guess is too far away from the solution) and may not comply with real-time requirement.

A solution to this problem is to add information, i.e. sensing redundancy, for example by adding a leg to the robot. For example consider the wrist (with only 3 d.o.f. rotation motion) proposed by Hayward [3],[9],[10] and described in Figure 4, The rotation of the upper cross platform around the ball-and-

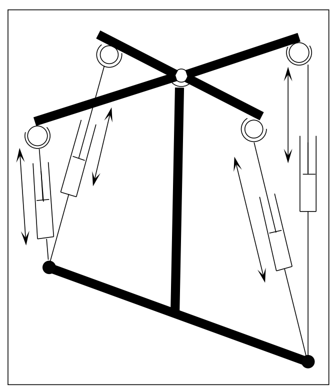

Figure 4: The wrist of Hayward. The rotation around the ball-and-socket joint of the moving platform is obtained by changing the lengths of the four legs.

socket joints of the moving platform is obtained by changing the lengths of the four legs. As we have only three d.o.f. we get a redundant parallel robot whose direct kinematics can be solved in closed-form.

For the example of the Gough platform Nair [20] has shown that by adding three legs to the mechanism we can compute a unique solution to the direct kinematic problem, but therefore deeply reducing the available workspace. For spatial mechanism the use of redundancy has to be carefully thought if the purpose is to get a unique solution for the direct kinematics. In our example adding four sensors at two of the universal joints seems to be the best solution to get a unique solution to the problem [15].

2.3 Singular configurations

Usually parallel manipulators have a high stiffness, except in some particular postures, which are called its *singular configurations*. We have explained in a previous issue of this journal [14] how singular configurations are determined. Basically for a non redundant manipulator the forces τ in the joint are related to the forces and torques $\mathcal{F}$ exerted on the moving platform by the relation:

$$\mathcal{F} = J^{-T}\tau \tag{1}$$

where J^{-T} is a 6 by 6 matrix called the transpose of the inverse jacobian matrix, which is configuration dependent. Therefore to compute the vector τ for a given external load we need to solve a linear system. A problem will arise when the determinant of the matrix J^{-T} is equal to 0, as the internal forces may tend to infinity. Therefore the singular configurations are defined as the location of the platform such that $|J^{-T}|$=0.

Another way to look at the singular configurations is to calculate the relation between the linear velocities $\dot{\rho}$ of the actuators as a function of the cartesian and angular velocities $\dot{\mathbf{\Omega}}$ of the platform. We get

$$\dot{\rho} = J^{-1}\dot{\mathbf{\Omega}} \tag{2}$$

If the determinant of J^{-1} is equal to 0, then for a null velocity of the actuator the velocity of the platform may be not equal to 0: one can move the platform without any change in the leg lengths and consequently the manipulator is no more rigid.

For a non redundant parallel manipulator singularities will only occur when the lines going through A_i, B_i are in special geometric configurations. For a redundant manipulator equation (2) is transformed into:

$$\dot{\mathbf{\Omega}} = J\dot{\rho} \tag{3}$$

where the matrix J is now a m by $n > m$ matrix, with m the number of degree of freedom of the moving platform and n the number of actuators. Singularities will occur if the rank of J is lower than m or in other words $det(JJ^T) = 0$. Let us take the example of the wrist presented in Figure 4: here the matrix J^{-1} is 4 by 3 matrix. In a singular configuration its rank will be less than three and this occur only when the determinant of all the four 3 by 3 minor matrices extracted from J^{-1} are all 0. These minor matrices are in fact the inverse jacobian matrices of all the non-redundant manipulators which can be extracted from the redundant robot by suppressing a leg. Consequently if the redundant manipulator is in a singular configuration, then all the extracted robot have to be in a singularity. This means that the lines associated to the legs of these robots have to simultaneously fulfill the special geometric configurations and the probability of such event is very low as shown in [11].

2.4 Minimizing a cost-function using redundancy

2.4.1 Minimizing the articular velocities

For a redundant manipulator the relation between the articular velocities and the cartesian and angular velocities of the platform can be written as:

$$\dot{\mathbf{\Omega}} = J\dot{\rho} \tag{4}$$

Our purpose is now to find the articular velocities $\dot{\rho}$ for a given velocity for the platform. A solution $\dot{\rho_s}$ to this problem can be decomposed in two components $\dot{\rho_x}, \dot{\rho_k}$ such that $J\dot{\rho_x} = \dot{\mathbf{\Omega}}$ and $J\dot{\rho_k} = 0$. In other words $\dot{\rho_k}$ can be any arbitrary vector belonging to the kernel of J. Consequently some latitude is available for the velocities of the actuators.

Assume now that you want to determine the solution $\dot{\rho}$ for this problem which minimize the cost function F with:

$$F = \frac{1}{2}(\dot{\rho} - Z)^T(\dot{\rho} - Z) = \frac{1}{2}||\dot{\rho} - Z||^2$$

where Z is any function of the vector ρ. It can be shown that the solution is:

$$\dot{\rho} = J^+\dot{\mathbf{\Omega}} + (I_n - J^+J)Z$$

where $J^+ = J^T(JJ^T)^{-1}$ is called the pseudo-inverse of the jacobian matrix J. If we choose $Z = 0$, then the solution $\dot{\rho}$ minimize the articular velocities.

As Z can be any function of ρ we can assume that Z is a function defined by:

$$Z = \epsilon\nabla\Phi(\rho)$$

where Φ is a function of the vector ρ, ∇ the gradient operator and ϵ a constant which can be positive or negative. In that case it can be shown that the solution for $\dot{\rho}$ described by equation (2.4.1) leads to a maximization of $\Phi(\rho)$ if $\epsilon > 0$ and a minimization of $\Phi(\rho)$ if $\epsilon > 0$.

2.4.2 Minimizing the actuator forces

Assume now that you want to minimize the actuator forces τ for a given external load $\mathcal{F}$. We have seen that:

$$\tau = J^T(\rho)\mathcal{F}$$

consequently we can define:

$$\Phi(\rho) = \frac{1}{2}\tau\ \tau^T \qquad Z = \frac{1}{2}\epsilon\nabla(\tau\ \tau^T) \quad \text{with} \quad \epsilon < 0$$

and this will lead to a motion with minimum articular forces. As mentioned by Tadoroko [27] as the ratio torque/weight of electrical motors is almost the same whatever the size of the motor, the weight of the robot does not increase with the number of actuators. But increasing the number of actuators to gain a redundancy in the actuation enables to minimize, for a given task, the power requirement and therefore the total energy consumption of the robot.

Yi [30] has studied the force transmission factor (i.e. the ratio of actuator torques to the torques exerted on the moving platform) in the redundant spherical shoulder mechanism depicted in Figure 5. All the joints axis of this

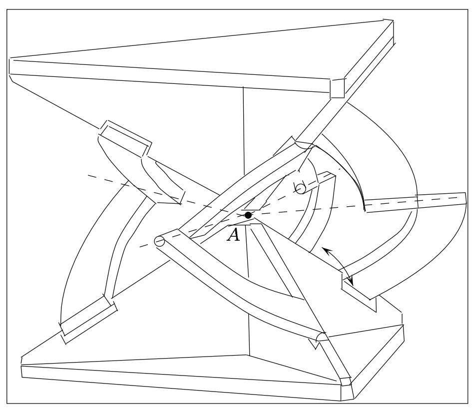

Figure 5: A redundant spherical shoulder mechanism.

mechanism have a common intersection point A and by rotating the four links attached to the base it is possible to control the rotation of the moving platform around A. As we have four legs we have a redundant mechanism. Yi shows that over the workspace the force transmission factor for the three legs non-redundant mechanism ranges from 1 to 10 and only from 1 to 2 for the redundant mechanism.

2.4.3 Dealing with articular limits

If we want to deal with articular limits we can define Z as:

$$Z = \epsilon(\rho - \rho_m) \qquad \epsilon < 0$$

where ρ_m is the middle point for the actuator. In that case the command will insure that during the motion the actuators will remain as close as possible from their central position. Note that in some cases it will be possible to use the redundancy to increase the workspace of the robot despite an increase in the number of links.

2.5 Manipulability

Assume that we have considered for equation (4) the solution which minimize the articular velocities ($Z = 0$) and that the articular velocities are such that:

$$||\dot{\rho}||^2 \leq 1$$

which lead to:

$$\dot{\Omega}^T (JJ^T)^{-1} \dot{\Omega} \leq 1$$

Therefore a sphere in the articular velocities space maps to an ellipsoid in the cartesian velocities space (Figure 6). This ellipsoid is called the *manipulability*

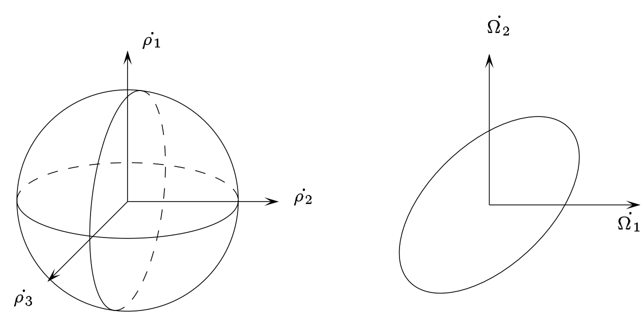

Figure 6: The velocity ellipsoid for a 2 d.o.f. robot with 3 articular coordinates.

ellipsoid. Note that the robot will exhibit the same positioning ability in all directions of the cartesian frame if this ellipsoid is close to a sphere. It can be shown also that the volume v of the ellipsoid is given by $v = \sqrt{det(JJ^T)}$ which means that its volume will be equal to 0 at a singular configuration. Consequently it may be of interest to use the redundancy to maximize the manipulability ellipsoid volume either by modifying the posture when controlling the robot or by designing the robot in such way that the volume is maximum for some tasks [21].

The same concept can be introduced for the acceleration. If the actuator acceleration $\ddot{\rho}$ are subject to:

$$||\ddot{\rho}||^2 \leq 1$$

the Cartesian acceleration of the end-effector is mapped into an ellipsoid defined by:

$$\ddot{\Omega} M^T J J^T M \ddot{\Omega} \leq 1$$

where M is the inertia matrix in Cartesian space. This ellipsoid is called the *dynamic manipulability* ellipsoid. For controlling a redundant robot it may be of interest to use the redundancy to modify the form and orientation of the ellipsoid to maximize the acceleration in a direction specified by the task to perform. Reboulet has used this concept for controlling its fast parallel robot Speed-R-Man [23] which can perform only translation motions. (Figure 7).

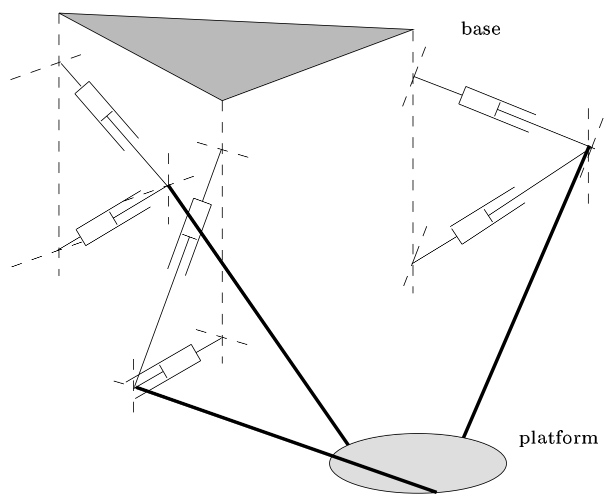

Figure 7: The 3 d.o.f. Speed-r-man redundant robot: its structure is similar to the Delta robot, enabling only translation motion, except that two linear actuators are used for each arm. A mechanism in each arm (in thick lines) insure that the platform remains always parallel to the base.

2.6 Redundancy and calibration

Calibration is the process of estimating the real values of some kinematic parameters of a robot, using sensors measurements and an a-priori model of the robot. For non-redundant parallel manipulators the calibration process consists in measuring the real posture of the moving platform when the articular coordinates have been computed with the a-priori model for a given posture. The errors between the specified posture and the real one is used to estimate the kinematic parameters of the robot: for example for the Gough platform the purpose of calibration is to estimate the location of the A_i, B_i points together with the dead lengths of the links.

Although some papers have addressed this problem [5],[12],[31] it seems that the calibration of non-redundant manipulators is a difficult task: the accuracy of the end-effector posture measurements has to be extremely high for getting an improved geometric model of the robot.

At the opposite, autonomous calibration can be performed with redundant parallel robots: the sensing redundancy is used to calibrate the robot without any additional sensing system. For example Nahvi and Hollerbach [19] describes the calibration process of the Hayward's wrist: using the four leg lengths measurements in about 50 different postures they were able to estimate the sensors offsets and some coordinates of the A_i, B_i points.

2.7 Redundancy and wire parallel robots

Instead of using rigid links some authors have suggested to use wires as links: for example Albus [1] describes the SPIDER crane robot which has the structure of Gough platform with the base over the platform and the rigid links substituted by wires whose lengths can be modified using winches. Although this concept leads to very light manipulators the main drawback is that the workspace is reduced compared to a more classical robot as the wire should always be in a traction state. Furthermore the elasticity of the wires deeply reduce the stiffness of the robot and its bandwidth.

This drawback can be partially solved by using redundancy. For example Ming [17],[18] suggested that a planar robot with 3 d.o.f. should use four wires and propose the robot described in Figure 8. The use of redundant wires essentially enables to increase the stiffness of the robot.

3 Variable geometry trusses

The development of orbital stations leads to the need of active structures, essentially used as space cranes [4],[29],[26], with severe constraints on their weights. Miura has investigated very early the use of redundant parallel mechanisms for building such structures. The idea is to combine elementary blocks like the module described in Figure 9. This module is an hexagon similar in

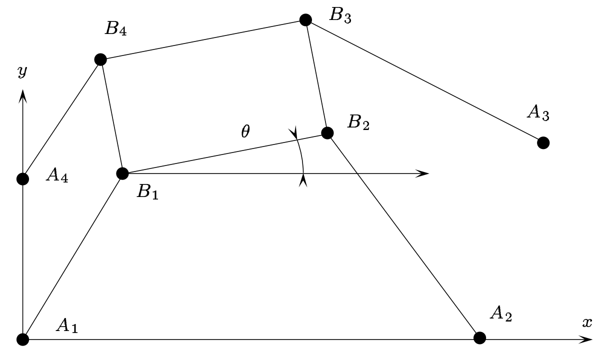

Figure 8: The planar wires parallel robots of Ming

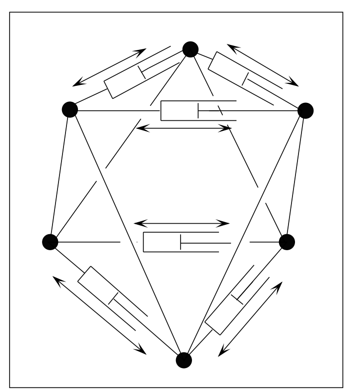

Figure 9: The basic tetrahedra of Miura

design to a classical parallel robot but whose base and platform have a variable geometry: linear actuators are used to modify the distances between the A_i, B_i points. Then these modules are set up as a pile to build a variable geometry truss like the one depicted in Figure 10. These trusses are very light, easy to

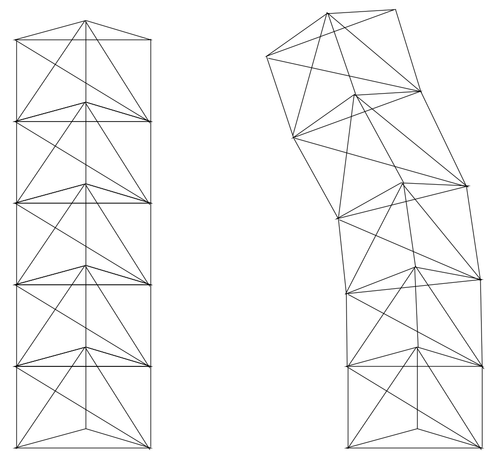

Figure 10: A variable geometry truss

assemble and highly redundant. Their kinematics is however rather complex but many authors have addressed this problem [22],[24],[28].

Another interest of the variable geometry trusses is their use for tasks where obstacle avoidance is very important. An example is the Logabex robot presented in Figure 11 whose weight is 120 kg for a nominal load of 75 kg and whose height may vary from 2m to 2.7m. This robot was intended to be used for repairs in a nuclear plant in a place with a restricted access and various obstacles.

4 Conclusion

Redundant parallel robots present many interests in various applications: increase of reliability, simpler direct kinematics, better load distribution in the actuators. The price to pay for such advantages are a higher complexity in the design, a more complex inverse kinematics and some difficulties with the control.

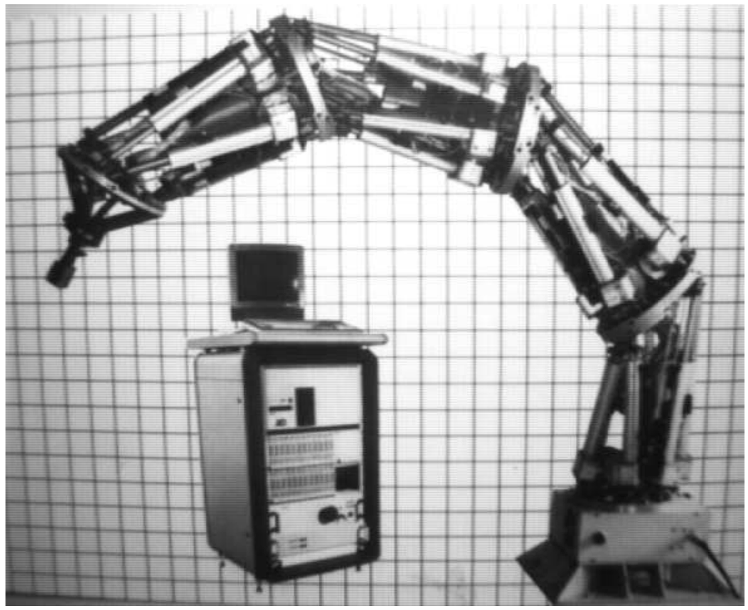

Figure 11: The Logabex L4 robot

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