TENTH WORLD CONGRESS ON THE THEORY OF
MACHINE AND MECHANISMS
Oulu, Finland, June 20-24, 1999

FINDING THE EXTREMA OF THE LEG LENGTHS OF A GOUGH-TYPE PARALLEL ROBOT WHEN THE PLATFORM IS MOVING IN A GIVEN 6D WORKSPACE

J-P. Merlet
INRIA, BP 93
06902 Sophia-Antipolis Cedex, France

Abstract

The purpose of this paper is to present algorithms for computing either exactly or with a pre-specified accuracy the extremal values of the articular coordinates of a Gough type parallel robot when the platform is constrained to lie within a given 6D workspace. Such algorithms are useful either for the design of the robot or for computing its workspace.

Keywords: *performance analysis, design, parallel robots*

1 Introduction

The Gough platform [Gough 1956],[Gough 1962] (figure 1) is a 6-dof parallel robot which has received a lot of attention in the last ten years, due to its positioning and load performances which outperform more classical robots. Basically a Gough platform consists in a base and a moving platform connected by 6 identical kinematic chains called the legs. Each of these legs is connected to the base with an universal joint and to the moving platform by a ball-and-socket joint. Furthermore a linear actuator enable to change the leg lengths and by an appropriate control of the 6 leg lengths it is possible to control the 6 dof of the moving platform. When designing a Gough platform it is important to determine what will be the extrema of the leg lengths for any posture lying within a given 6D workspace as in most applications the desired workspace is known and the minimal and maximal leg lengths belong to the design parameters. To the best of our knowledge this problem has not been addressed in the literature, the authors focusing on the inverse problem of determining the workspace of a parallel robot being given the extremal values of the leg lengths [Gosselin 1990],[Haugh 1995], [Haugh 1998],[Ji 1996]. A 6D workspace consists in two parts:

- a 3D geometrical object (for example a cube) denoted the *location part* of the workspace, which define the possible locations of a specific point C of the moving platform
- three ranges denoted the *orientation part* which define the possible values of the three orientation angles of the moving platform

The purpose of this paper is to present algorithms for computing the extremal value of the leg lengths for a given 6D workspace either exactly or with an accuracy ϵ which can be fixed arbitrarily.

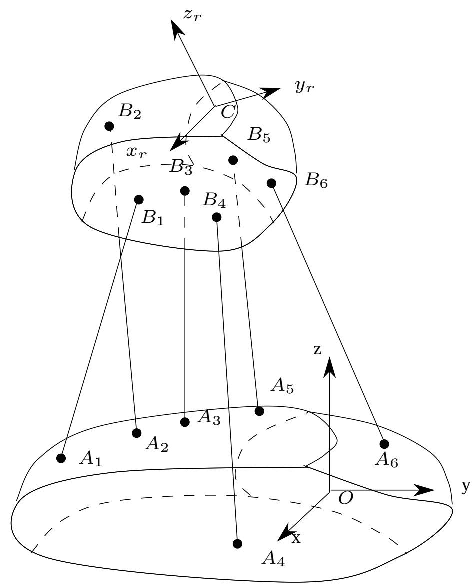

Figure 1: The Gough platform

2 Extended box and general workspace

We define an *extended box* or EB for short as a pair of element: a cartesian box, which represent the possible location of the end-effector, and a set of three ranges, one for each of the rotation angles. An EB is therefore composed of a *location part* (the box) and an *orientation part* and defines a particular type 6D workspace for the robot.

More generally we will assume that any location part can be defined by a set of horizontal polygonal cross-sections,the workspace between two sections being the polyhedra obtained by linking the corresponding vertices of the two polygons (figure 2).

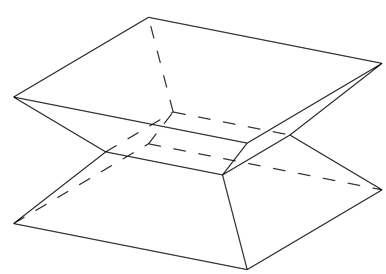

Figure 2: Example of location part

Assume now that we have an algorithm A_1 which enable to compute the extremal leg lengths for an EB (or at least an upper bound and a lower bound of these quantities). We will show now that using this algorithm it is possible to compute the extremal leg lengths for any type of location part with a pre-specified accuracy ϵ.

For the sake of simplicity we will illustrate our algorithm for the case where we want to determine the maximal length ρ_{max} of one leg (the determination of the minimal leg length can be derived directly from the algorithm). This value is initialized by taking some particular postures within the 6D workspace $\mathcal{W}$ (for example the vertices of the polygonal cross-sections with a few orientation among the orientation ranges) and retaining the maximal lengths among all the postures. In our algorithm we will use a list $\mathcal{S}$ of EB with n elements in it. Any EB $\mathcal{B}_i$ in this list will have the same orientation part than the 6D workspace. This list is initialized with an EB whose location part is the cartesian bounding box of the volume and therefore $n = 1$. A trivial geometrical algorithm enable to determine if an EB fully lie inside $\mathcal{W}$, is totally outside $\mathcal{W}$ or is partially inside $\mathcal{W}$. The algorithm uses an index i initialized to 1 and uses the following step:

1. if $i > n$ return ρ_{max}
2. compute the maximal leg length ρ for the EB $\mathcal{B}_i$ using the algorithm A_1
3. if $|\rho - \rho_{max}| < \epsilon$, then $i = i + 1$ and go to step 1
4. if $|\rho - \rho_{max}| > \epsilon$:
   (a) if $\mathcal{B}_i$ fully lie inside $\mathcal{W}$, then $\rho_{max} = \rho$, $i = i + 1$, go to step 1
   (b) if $\mathcal{B}_i$ is totally outside $\mathcal{W}$, then $i = i + 1$, go to step 1
   (c) if $\mathcal{B}_i$ is partially inside $\mathcal{W}$ we bisect $\mathcal{B}_i$. The bisection process create 8 new EB which are put at the end of $\mathcal{S}$. Therefore $n = n + 8$ and we go to step 1

Hence this algorithm enable to compute ρ_{max} with an accuracy ϵ for any type of location part. A direct consequence is that as soon as an algorithm A_1 for the EB has been determined we may deal with any type of 6D workspace. The purpose of the following sections is to present such an algorithm.

3 Extremal leg lengths for an extended box

In the sequel we will assume that the location part of the 6D workspace is a cartesian box.

3.1 Translation workspace

We assume here that all the orientation ranges are reduced to a point or, in other words, that the orientation of the platform is fixed. Let B_i denote the attachment points of the legs on the platform and A_i the attachment points of the legs on the platform. As the orientation is fixed when C moves within its cartesian box $\mathcal{C}$ all the B_i moves within a cartesian box $\mathcal{C}_{B_i}$ of same size than $\mathcal{C}$, whose center is obtained by translating the center of $\mathcal{C}$ by the vector $\mathbf{CB_i}$. For physical reason we may assume that the cartesian box of the B_i does not include the attachment point A_i of the leg on the base. It is then trivial that the following result hold:

- the maximal leg length is obtained when B_i lie at one of the vertices of $\mathcal{C}_{B_i}$
- the minimal leg length is obtained when B_i lie at one of the vertices of $\mathcal{C}_{B_i}$ except if the projection A_i^p of A_i on the planes including the faces of $\mathcal{C}_{B_i}$ lie within a face. In that case the minimal distance may be $||A_i A_i^p||$.

For computing the extremal leg lengths the procedure is therefore

1. compute the coordinates of the 8 vertices V_j of $\mathcal{C}_{B_i}$ by translating the vertices of $\mathcal{C}$ by the vector $\mathbf{CB_i}$
2. the maximal leg length is $\mathrm{Max}(||V_j A_i||)$ for j in [1,8]
3. compute the projection $A_{i_k}^p$ of A_i on the 6 planes containing the 6 faces F_m of $\mathcal{C}_{B_i}$

4. the minimal leg length is $\text{Min}(||V_j A_i||, ||A^p_{i_k} A_i||)$ for j in [1,8], k in [1,6] and for $A^p_{i_k}$ belonging to a face F_m with m in [1,6].

On a SUN Ultra 1 workstation the computation time of this algorithm is 0.26 ms. If we use this algorithm to compute the extremal leg lengths of the volume of figure 2 with an accuracy of at least 0.02% the computation time is 767 ms.

4 General workspace

4.1 Special case of Gough platform

In this section we assume that all the B_i are coplanar and similarly that all the A_i are coplanar. We define a reference frame whose z axis if perpendicular to the plane of the A_i and such that the z coordinates of the A_i is equal to 0. Similarly we define a mobile frame, whose origin is C and whose z axis is perpendicular to the plane of the B_i and such that the z coordinate of the B_i is equal to 0. Let x_c, y_c, z_c be the coordinates of C and ψ, θ, ϕ be the orientation angles. To determine the extremal leg lengths we have to solve the constrained optimization problem which is to find the extremal values of ρ with:

$$\rho^2 = F(x_c, y_c, z_c, \psi, \theta, \phi)$$

under the constraints:

$$\begin{array}{lll} x_c \in [x_1, x_2] & x_c \in [y_1, y_2] & z_c \in [z_1, z_2] \\ \psi \in [\psi_1, \psi_2] & \theta \in [\theta_1, \theta_2] & \phi \in [\phi_1, \phi_2] \end{array}$$

The F functions are polynomials of degree 2 in x_c, y_c, z_c (the degree 2 term being $x_c^2 + y_c^2 + z_c^2$, the other terms being linear) while the ψ, θ, ϕ parameters appear through their sine and cosine. A preliminary remark is that the extremal leg lengths will be obtained when at least one of the parameter x_c, y_c, z_c is extremal. Indeed assume that the extremal leg length will be obtained for a posture which does not satisfy this property. The leg has a fixed direction $\mathbf{u}$ in that posture and it will be possible to move C without changing the platform orientation so that the attachment point B of the leg remains on the line defined by $A, \mathbf{u}$ and either come closer to A (therefore reducing the leg length) or moving away from A (and therefore increasing the leg length). Consequently this posture cannot define an extremal of the leg length. For all variable p_i in the set $\{x_c, y_c, z_c\}$ which vary in the range $[p_1, p_2]$ we define a new variable α_i such that:

$$p_i = p_1 + \frac{1 + \sin(\alpha_i)}{2}(p_2 - p_1) \tag{1}$$

Thus we insure that p_i lie within its range. For the orientations parameters we define a new parameter λ_i such that:

$$p_i = p_1 + \lambda_i (p_2 - p_1) \tag{2}$$

If we impose that λ_i lie in the range [0,1], then we insure that p_i lie within its range. According to the preliminary remark The optimization problem is solved by equating to 0 n of the partial derivatives:

$$\frac{\partial F}{\partial \alpha_j} \qquad \frac{\partial F}{\partial \lambda_j}$$

where n is at most 5 (this case being obtained when only one of the parameters in $\{x_c, y_c, z_c\}$ has a fixed value). We have therefore to solve:

- 3 systems of 5 equations in 5 unknowns obtained when only one of the parameters in $\{x_c, y_c, z_c\}$ has a fixed value
- 3 systems of 4 equations in 4 unknowns obtained when two of the parameters in $\{x_c, y_c, z_c\}$ has a fixed value
- 1 system of 3 equations in 3 unknowns obtained when all the parameters in $\{x_c, y_c, z_c\}$ has a fixed value

After solving each of the systems we obtain a potential extremal leg length value and we retain the minimal and maximal values among all the potential values to get the extremal leg lengths.

For lack of space we cannot describe further the procedure, which require a lot of algebraic geometry manipulation but is still tractable as all the equations are factorizable (the algebraic manipulation are performed using the library ALP developed in our laboratory).

4.2 General case of Gough platform

In the general case the previous optimization problem cannot be solved easily as the previous systems have a largely higher complexity. For efficiency reason we have decided to use another approach based on *interval analysis* [Hansen 1992], [Moore 1979].

An interval number is a real, closed interval $(\underline{x}, \overline{x})$. Arithmetic rules exist for interval numbers. For example let two interval numbers $X = (\underline{x}, \overline{x})$, $Y = (\underline{y}, \overline{y})$, then:

$$X + Y = [\underline{x} + \underline{y}, \overline{x} + \overline{y}]$$
$$X - Y = [\underline{x} - \overline{y}, \overline{x} - \underline{y}]$$

An *interval function* is an interval-valued function of one or more interval arguments. An interval function F is said to be *inclusion monotonic* if $X_i \subset Y_i$ for i in $[1, n]$ implies:

$$F(X_1, \ldots, X_n) \subset F(Y_1, \ldots, Y_n)$$

A fundamental theorem is that any rational interval function evaluated with a fixed sequence of operations involving only addition, subtraction, multiplication and division is inclusion monotonic. The leg length of a Gough platform is obtained from the coordinates of C and the orientation angles as an inclusion monotonic function as the operators used in this function are only addition, subtraction, multiplication and the trigonometric operator $\sin, \cos$. Consequently using the arithmetic rules we may obtain for an EB a lower and an upper bound of the leg lengths and any EB resulting from the bisection of the initial EB will have a lower and an upper bound of the leg lengths included in the range found for the initial EB. Interval analysis involves only basic operations and is therefore quite fast. Note that some tricks (especially using the monotonicity of the leg lengths and Taylor expansion at the first and second order) enable to get rather sharp bounds without adding to much computation time.

As mentioned in the previous sections the algorithm A_1 does not need to give exact values for the extremal leg lengths: consequently interval analysis is perfectly suitable. We use the special interval analysis package BIAS/Profil which provide the basic operations of interval analysis. This package has been included in the algebraic geometry library ALP developed in our laboratory which contain some optimization routine enabling to determine efficiently sharp bounds on the extremal leg lengths.

5 Conclusion

The first purpose of the presented algorithm is for design: it is clearly quite important to be able to determine the necessary lengths and stroke of the linear actuator of a Gough platform during the design process. As this step has to be repeated for various geometries during the design process an efficient and fast algorithm is needed.

But this algorithm may also be used for other purposes. For example it has been used to determine all the locations of C that can be reached with a fixed set of orientations or all the locations of C that can be reached with at least one orientation (the so-called *reachable workspace*) [Merlet 1998].

6 Acknowledgment

Part of this work was supported by the European ESPRIT project FRISCO (LTR 21.024).

References

[] Gosselin C. Determination of the workspace of 6-dof parallel manipulators. *ASME J. of Mechanical Design*, 112(3):331–336, September 1990.

[] Gough V.E. Contribution to discussion of papers on research in automobile stability, control and tyre performance, 1956-1957. Proc. Auto Div. Inst. Mech. Eng.

[] Gough V.E. and Whitehall S.G. Universal tire test machine. In *Proceedings 9th Int. Technical Congress F.I.S.I.T.A.*, volume 117, pages 117–135, May 1962.

[] Hansen E. *Global optimization using interval analysis*. Marcel Dekker, 1992.

[] Haugh E.J., Adkins F.A., and Luh C.M. Domain of operation and interference for bodies in mechanisms and manipulators. In J-P. Merlet B. Ravani, editor, *Computational Kinematics*, pages 193–202. Kluwer, 1995.

[] Haugh E.J., Adkins F.A., and Luh C.M. Operational envelopes for working bodies of mechanisms and manipulators. *ASME J. of Mechanical Design*, 120(1):84–91, March 1998.

[] Ji Z. Analysis of design parameters in platform manipulators. *ASME J. of Mechanical Design*, 118:526–531, December 1996.

[] Merlet J-P. Determination of 6D workspaces of a Gough-type 6 d.o.f. parallel manipulator. In *RoManSy*, Paris, July, 6-9, 1998.

[] Moore R.E. *Methods and Applications of Interval Analysis*. SIAM Studies in Applied Mathematics, 1979.