ANALYSIS OF THE INFLUENCE OF WIRES INTERFERENCE ON THE WORKSPACE OF WIRE ROBOTS

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Abstract The workspace of wire robots is limited due to three factors: minimal and maximal lengths of the wires, static equilibrium that imposes to have a positive only force in the wires and wires interference (either intersection between 2 wires or between a wire and the end-effector). The limitation by the two first factors may be reduced by having wires with a large coiling ability and by increasing the number of wires. We propose in this paper algorithms that allow to study the influence of wire/wire and wire/end-effector interference on the workspace, assuming a fixed orientation of the end-effector. These algorithms allow one to determine exactly which region of the workspace is interference free.

Keywords: workspace analysis,self-collision,wire robots,parallel robots

1. Introduction

Wire robots are constituted of an end-effector connected to the ground by a set of wires whose lengths can be modified by coiling devices. Modifying the length of the wires allows one to control the pose of the end-effector. Recently wires robots have attracted the attention of many researchers as they have some interesting features:

- *high speed*: the moving parts of wire robots have a very low mass and fast coiling devices are available. Hence high speed wire robots have been developed such as the Falcon robot (Kawamura, 2000)
- *large workspace*: compared to their rigid link equivalent the actuators of wire robots have a larger possible change in lengths which allows to obtain a larger workspace
- *force sensing*: like all parallel structures a carefully instrumented wire robots may be used both as a positioning device and also as a force sensor

Wire robots have also drawbacks:

- *lower accuracy*: the errors in the positioning of the end-effector is clearly related to the accuracy in the measurement of the wire lengths. Here the larger change in length of the actuators will lead to a larger error in the lenght measurements for a given accuracy of the sensors. Furthermore the deformation of the wires (which are due to the mass of the wires) are difficult to model: hence their straight-line model is only an approximation
- *control problem*: wires can only pull and are not rigid. Hence control should address the problems of vibrations in the wire together with force control

Wire robots have been used in multiple applications: robotics crane such as the Robocrane (Albus, 1993; Bostelman, 1996) or the device proposed in (Arai, 1992; Higuchi, 1988; Viscomi, 1994), haptic devices (Baumann, 1997), pose measurements (Geng, 1994; Jeong, 1999), medical assistance devices (Homma, 1994; Wendlandt, 1994), sport training (Morizono, 1997), telescope supporting system (Su, 2001).

We have started recently the development of a modular wire robot that will be used in virtual reality application in conjunction with a *workbench* which allows a 3D rendering of large virtual scene. This system uses a magnetic sensing device that measure the position/orientation of the head of the user, an information that is used to update the scene presented on the screen. Although this system is effective it has two main drawbacks: no metallic object should be added in the surrounding of the workbench otherwise a tedious calibration has to be performed and no tactile feedback is provided The use of a wire robot in place of the current sensing system will allow to get a similar pose measurement ability while providing a minimal interference with the screen view and may also provide some force-feedback ability that will be useful in many applications such as medical training. Another specificity of our system is its modularity: we aim to provide modular coiling devices whose location on the ground can be easily adjusted to adapt the wires robot to the task (optimal location of the coiling devices will be determined using algorithms such as the one described in (Merlet, 2003)).

Previous works on wires robot have focused on kinematics (Maier, 1998; Yamamoto, 2000), planar robots (Ming, 1994-1; Ming, 1994-2; Osumi, 2000) and force controllable workspace (Verhoeven, 1998; Verhoeven, 2000). Clearly modularity should address first the workspace problem. Three elements play a role on the limitation of the workspace of wires robots:

- *limitation due to the wire lengths*: here there is no differences between a wires robot and a classical parallel robot and this subject is now mastered using for example the algorithms described in (Gosselin, 1990) when the orientation is constant or in (Merlet, 1999) in the general case

- *limitation due to the force in the wires*: forces in the wires should always be positive
- *limitation due to the interference between wires and between the wires and the end-effector*

The purpose of this paper is to address the last point which, to the best of the author knowledge, has never been addressed for wires robot.

2. Wires interference

In this paper we consider a wires robot with n wires whose coiling devices are located at point A_i on the ground and at point B_i on the end-effector. A reference frame is attached to the ground so that the coordinates of A_i are xa_i, ya_i, za_i and a mobile frame is attached to the end-effector so that the coordinates of B_i are xb_i, yb_i, zb_i. The location of the end-effector is defined by the coordinates X, Y, Z in the reference frame of a specific point C of the end-effector while its orientation is defined by the rotation matrix R that allows to express in the reference frame the components of a vector whose components in the mobile frame are known.

Two wires i, j will interfere if they have a common point M such that $\mathbf{A_iM} = l_i\mathbf{A_iB_i}$ and $\mathbf{A_jM} = l_j\mathbf{A_jB_j}$ with l_i, l_j in [0,1]. We get

$$\mathbf{A_iA_j} = \mathbf{A_iM} - \mathbf{A_jM} = l_i\mathbf{A_iB_i} - l_j\mathbf{A_jB_j} \quad (1)$$

Equation (1) is a system of 3 equations which are linear in the unknowns l_i, l_j. Two equations of this system are chosen to determine l_i, l_j and using these values in the third equation allows one to get an interference condition that is free of l_i, l_j. Geometrically speaking this interference condition is equivalent to state that the points A_i, A_j, B_i, B_j *lie in the same plane.*

3. Interference for a fixed orientation of the end-effector

3.1 Interference condition and constraints planes

When the orientation is fixed the interference condition implies that C lie in a plane $\mathcal{P}$. An interesting property of the interference condition is that when two wires i, j have the same z coordinates and the orientation of the end-effector is such that the z components Bz_i, Bz_j of the vectors $\mathbf{CB_i}, \mathbf{CB_j}$ are identical, then the interference condition is written as:

$$U\ (Z - Bz_i - za_i) = 0 \tag{2}$$

where U is not a function of X, Y, Z and cancel only if B_iB_j is parallel to A_iA_j. Hence interference may occur only if B_iB_j is parallel to A_iA_j or if the points B_i, B_j lie in the same horizontal plane than A_i, A_j.

The values of l_i, l_j may be written as:

$$l_i = \frac{N(l_i)}{D(l_i)} \quad l_j = \frac{N(l_j)}{D(l_j)}$$

The region of the workspace in which interference will occur is bounded by geometrical elements for which l_i or l_j are exactly 0 or 1. Hence it is necessary to investigate the constraint conditions $N(l_i) = 0, D(l_i) = 0, N(l_i) = D(l_i)$ and $N(l_j) = 0, D(l_j) = 0, N(l_j) = D(l_j)$. All these constraint conditions are also equations of planes for C that will be called the *constraints planes*. Note that we have $D(l_i) = D(l_j)$ and hence only 5 constraints planes that will be denoted $\mathcal{C}^{N(l_i)=0}$, $\mathcal{C}^{N(l_j)=0}$, $\mathcal{C}^{D(l_i)=0}$, $\mathcal{C}^{N(l_i)=D(l_i)}$ and $\mathcal{C}^{N(l_j)=D(l_j)}$. Interference between the wires will thus happen when C belong to a region of $\mathcal{P}$ that is bounded by the intersection of $\mathcal{P}$ with the constraints planes.

The constraints planes have interesting properties: the planes obtained for $N(l_i) = 0$ and $N(l_j) = 0$ are parallel as the coefficients in X, Y, Z of these equations are identical and similarly the planes obtained for $N(l_i) = D(l_i)$ and $N(l_j) = D(l_j)$ are parallel.

A direct consequence is that one of the half space bounded by $N(l_i) = 0$ or $N(l_j) = 0$ will be such that $N(l_i) \geq (\leq)0$ **and** $N(l_j) \geq (\leq)0$. Similarly one of the half space bounded by $N(l_i) = D(l_i)$ or $N(l_j) = D(l_j)$ will be such that $N(l_i) \leq D(l_i)$ **and** $N(l_j) \leq D(l_j)$. Combined with the constraints on $D(l_i)$ this will allow us to determine elements on which either l_i or l_j is exactly 0 (1) while on one side of the element we will have $l_i(l_j) > 0(1)$ and $l_i(l_j) < 0(1)$ on the other side.

Hence only three constraints planes have to be calculated: the planes separating the half-space where $N(l_i) \geq (\leq)0$ **and** $N(l_j) \geq (\leq)0$ and $N(l_i) \leq D(l_i)$ **and** $N(l_j) \leq D(l_j)$ together with the plane $D(l_i) = 0$. These three constraints plane will be called the *separating planes* $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$. Determining the two first separating planes among the five constraints planes is easy. We select an arbitrary point on $\mathcal{C}^{N(l_i)=0}$ and we calculate $N(l_j)$ for this point: if $N(l_j) \geq 0$ then we get $\mathcal{S}_1 = \mathcal{C}^{N(l_i)=0}$ otherwise $\mathcal{S}_1 = \mathcal{C}^{N(l_j)=0}$ and a similar procedure is used for determining $\mathcal{S}_2$ among $\mathcal{C}^{N(l_i)=D(l_i)}$ and $\mathcal{C}^{N(l_j)=D(l_j)}$.

3.2 Algorithm for calculating the interference region

The *interference region* for the wires i, j is the region of $\mathcal{P}$ for which each point satisfies $l_i, l_j \in [0, 1]$. This region can be calculated as follows:

1 calculate the intersecting lines D_1, D_2, D_3 between $\mathcal{P}$ and $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{S}_3$

2 compute the intersection points between D_1, D_2, D_3

3 for each D_i:

   (a) order the set $\{S_0^i, \ldots, S_k^i\}$ of $k+1$ intersection points that belong to D_i along this line

   (b) for each segment $S_l^i S_{l+1}^i$, $l \in [0, k-1]$ takes the mid-point M of the segment and then 2 points on each side of the segment close to M. Calculate the value of l_i, l_j for these two points. If for one point we have $0 \leq l_i \leq 1, 0 \leq l_j \leq 1$ while for the other point one of these constraints is violated, then the segment is part of the border of the interference region

4 connect all the parts of the segments that has been determined as part of the border of the interference region

A similar algorithm may be derived for computing the intersection of the interference region with a given plane (for example for a given Z) that are called the *interference lines*. Both algorithms have been implemented in Maple and their computation time is less than 10 s. Consider for example the 6-wired robot whose A_i, B_i are:

A_1	0 200 150	A_2	173.205081 -100 150
A_3	-173.205081 -100 150	A_4	-173.205081 100 -150
A_5	173.205081 100 -150	A_6	0 -200 -150
B_1	-25.980762 15 0	B_2	25.980762 15 0
B_3	0 -30 0	B_4	-25.980762 -15 0
B_5	0 30 0	B_6	25.980762 -15 0

with a minimal wire length of 100 and a maximal wire length of 500. Figure 1 present a cross-section of the workspace for $Z = 0$ and an orientation of the end-effector defined by the Euler angles $\psi = 70°$, $\theta = 30°$, $\phi = 0°$ as derived from the limitation of the wires lengths together with the interference lines. It may be seen that the wires interference will play a role for trajectory planning as some interferences lines are inside the potential workspace of the wires robot.

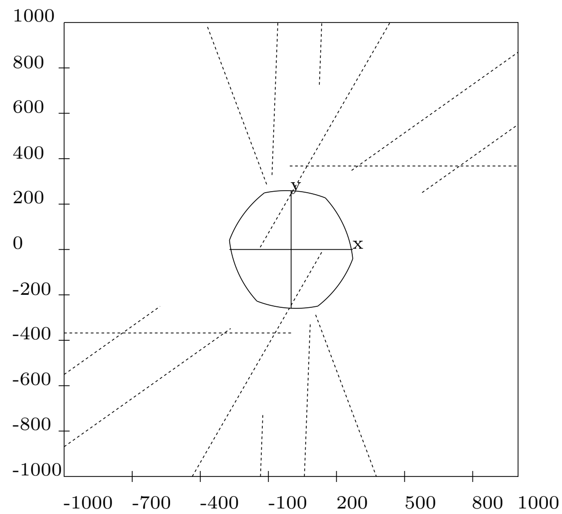

Figure 1. Cross-section of the workspace at $Z = 0$ for $\psi = 70$ degree, $\theta = 30$ degree and $\phi = 0$. The interference lines for all pair of wires are represented in dashed lines

4. Interference between the end-effector and the wires

Another factor that may limit the workspace is the interference between the wires and the end-effector. We will assume here that the end-effector is a polyhedral object with N planar facets $\mathcal{F}_i$. Each facet i is defined in the mobile frame by a set of m^i ordered vertices $\{V_1^i, \ldots, V_m^i\}$ and the exterior of the end-effector is defined by a unit vector $\mathbf{n}^i$ that is perpendicular to the facet plane. Interference will occur when a wire cross the interior of the end-effector. The purpose of this section is to determine the region of the workspace, called the *end-effector interference region*, for which such interference will occur, assuming that the orientation of the end-effector is fixed.

The border of the end-effector interference region will be obtained when one wire lie exactly in a facet plane and has an intersection with a facet edge. A necessary condition for a wire j to be in the i-th facet plane is:

$$\mathbf{n^i}.\mathbf{A_j B_j} = 0 \quad (3)$$

Wire j will have an intersection with a facet edge $V_k^i V_{k+1}^i$ if there is a point M that is common to $A_j B_j$ and $V_k^i V_{k+1}^i$. We have

$$\mathbf{A_j V_k^i} = \mathbf{A_j M} - \mathbf{V_k^i M} = l_l \mathbf{A_j B_j} - l_m \mathbf{V_k^i V_{k+1}^i} \quad (4)$$

in which it is necessary to have l_l, l_m in the range [0,1]. A point of the border of the end-effector interference region will be such that either l_l or l_m is exactly 0 or 1. Equation (4) is a system of 3 linear equations in l_l, l_m: two equations are used to determine l_l, l_m and we get a constraint equation with the third one. This equation is linear in X, Y, Z as are the constraint equation (3), the numerators and the denominator of l_l, l_m.

The border of the end-effector interference region is hence obtained by dealing with constraint planes as in the previous section: the same algorithm may thus be used. For each facet and each edge we have 4 constraints planes (the 2 numerators and the denominator of l_l, l_m and the constraints plane issued from equation (4) and for each facet we have another constraints plane issued from equation (3). Hence we have to deal with a total of $n \sum_{k=1}^{k=N}(4m^k + 1)$ constraints planes.

5. Conclusion

Limitation to the workspace due to interference is a problem that has rarely been addressed. In this paper we have proposed algorithms that allows one to determine exactly the location of the end-effector (its orientation being fixed) where interference between 2 wires or one wire and a polyhedral end-effector will occur. We have shown on an example that indeed interference is a factor that may limit the workspace of wires robots.

References

Albus J., Bostelman R., and Dagalakis N. The NIST ROBOCRANE. *J. of Robotic Systems*, 10(5):709–724, July 1993.

Arai T. and others . A hybrid drive parallel robot for heavy material handling. *IEEE Robotics and Automation Magazine*, 9(1):45–54, March 2002.

Baumann R., Maeder W., Glauser D., and Clavel R. The Pantoscope: a spherical remote-center-of-motion parallel manipulators for force reflection. In *IEEE Int. Conf. on Robotics and Automation*, pages 718–723, Albuquerque, April, 21-28, 1997.

Bostelman R., Albus J., Dagalakis N., and Jacoff A. RoboCrane project: an advanced concept for large scale manufacturing. In *Proceedings Association for Unmanned vehicle systems International*, pages 509–521, Orlando, July, 15-19, 1996.

Geng Z. and Haynes L.S. A 3-2-1 kinematic configuration of a Stewart platform and its application to six degree of freedom pose measurements. *Robotics & Computer-Integrated Manufacturing*, 11(1):23–34, March 1994.

Gosselin C. Determination of the workspace of 6-dof parallel manipulators. *ASME J. of Mechanical Design*, 112(3):331–336, September 1990.

Higuchi T., Ming A., and Jiang-Yu J. Application of multi-dimensional wire crane in construction. In *5th Int. Symp. on Robotics in Construction*, pages 661–668, Tokyo, June, 6-8, 1988.

Homma K. and Arai T. Upper limb motion assist system with parallel mechanisms. In *2nd Japan-France Congress on Mechatronics*, pages 388–391, Takamatsu, November, 1-3, 1994.

Jeong J.W., Kim S.H., and Kwak Y.K. Kinematics and workspace analysis of a parallel wire mechanism for measuring a robot pose. *Mechanism and Machine Theory*, 34(6):825–841, August 1999.

Kawamura S. and others . High-speed manipulation by using parallel wire-driven robots. *Robotica*, 18(1):13–21, 2000.

Maier T. and Woernle C. Inverse kinematics for an underconstrained cable suspension manipulator. In *ARK*, pages 97–104, Strobl, June 29- July 4, 1998.

Merlet J-P. Determination of the optimal geometry of modular parallel robots. In *IEEE Int. Conf. on Robotics and Automation*, Taipei, September, 14-19, 2003.

Merlet J-P. Determination of 6d workspaces of Gough-type parallel manipulator and comparison between different geometries. *Int. J. of Robotics Research*, 18(9):902–916, October 1999.

Ming A. and Higuchi T. Study on multiple degree of freedom positioning mechanisms using wires, Part 1, Concept, Design and Control. *Int. J. Japan Soc. Prec. Eng.*, 28(2):131–138, June 1994.

Ming A. and Higuchi T. Study on multiple degree of freedom positioning mechanisms using wires, Part 2, Development of a planar completely restrained positioning mechanism. *Int. J. Japan Soc. Prec. Eng.*, 28(3):235–242, September 1994.

Morizono T., Kurahashi K., and Kawamura S. Realization of a virtual sports training system with parallel wire mechanism. In *IEEE Int. Conf. on Robotics and Automation*, pages 3025–3030, Albuquerque, April, 21-28, 1997.

Osumi H. and others . Development of a manipulator suspended to parallel wire structure. In *IEEE Int. Workshop on Intelligent Robots and Systems (IROS)*, Takamatsu, Japan, October 30- November 5, 2000.

Su Y.X. and others . Development of a large parallel-cable manipulator for the feed-supporting system of a next-generation large radio telescope. *J. of Robotic Systems*, 18(11):633–643, 2001.

Verhoeven R., Hiller M., and Tadoroko S. Workspace, stiffness, singularities and classification of tendon driven Stewart platforms. In *ARK*, pages 105–114, Strobl, June 29- July 4, 1998.

Verhoeven R. and Miller M. Estimating the controllable workspace of tendon-based Stewart platforms. In *ARK*, pages 277–284, Piran, June, 25-29, 2000.

Viscomi B.V., Michalerya W.D., and Lu L-W. Automated construction in the ATLSS integrated building systems. *Automation in Construction*, 3(1):35–43, May 1994.

Wendlandt J.M. and Sastry S.S. Design and control of a simplified Stewart platform for endoscopy. In *33nd Conf. on Decision and Control*, pages 357–362, Lake Buena Vista, December, 14-16, 1994.

Yamamoto M. and Mohri A. Inverse kinematic analysis for imcompletely restrained parallel wire mechanisms. In *IEEE Int. Workshop on Intelligent Robots and Systems (IROS)*, Takamatsu, October 30- November 5, 2000.