The kinematics of the redundant $N-1$ wire driven parallel robot

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***Abstract*— We address the kinematics of the redundant $N-1$ wire-driven parallel robot, i.e. a robot with $N > 3$ wires connected at the same point on the platform. The redundancy allows one to increase the workspace size. But we show, both theoretically and experimentally that if the wires are not elastic, then the redundancy cannot be used to control the wire tensions. Indeed we show that whatever are the number of wires there will always be only at most 3 wires in tension, while the other $N-3$ wires will be slack. We then show that if the wires are elastic, then the platform positioning will be very sensitive to stiffness identification and wire lengths control. Hence classical redundant control schemes are difficult to use for such robot and alternate use of the geometry of redundant wires have to be considered.**

I. INTRODUCTION

Recently wire-driven parallel robots have gained further interest because of potential new applications such as rescue crane [1], [2], [3], assistance robots, rehabilitation [4] and haptic devices [5], [6]. But their kinematics is much more complex than their rigid legs counterpart, especially as it has appeared recently that their forward (FK) and inverse (IK) kinematics are closely connected. Indeed it may be shown that applying an IK solution for a robot with N wires (i.e. sending as control input the N wire lengths that theoretically will allow to reach a pose with the N wires under tension) is not sufficient to reach the desired pose because the final pose of the platform my be such that less than N wires are under tension [7]. Hence applying an IK solution leads to a pose which maybe any solution of the FK problem with 1 to N wires under tension. Unfortunately solving such FK problem appears to be extremely complex with no known solution at this time [8].

In this paper we focus on the 3 dof redundant $N-1$ wire-driven robot in a crane configuration (i.e. the only force applied on the platform is gravity). Such a robot has $N > 3$ wires connected at the same point on the platform and is used to control the location of the center of mass of the platform. The actuating system of this robot allows one to control the lengths ρ of the wires, i.e the distance between the output point A of the winch system and the center C of the platform (for simplification we will assume that the center of mass of the platform is located at C).

Redundancy may be used for two purposes:

- increasing the workspace size
- optimizing a secondary control criteria. For wire-driven robot such secondary criteria is typically related to the distribution of tension among the wires

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Without lack of exhaustivity we will consider here a specific application of such robot: acting as a lifting crane to provide mobility assistance for elderly and handicapped people. The purpose of the robot is to be able to provide mobility assistance to the end-user in a given room. In that case the A points of the winch system are all located at the same altitude in the ceiling of the room. We define the altitude of the A points as 0 and any pose of the robot in the room will have a negative altitude. A first limit for the pose that can be reached by such a robot is imposed by the effective amount of wire length change that can be coiled by the winch system. We will assume here that these changes are such that potentially any pose in the room can be reached. The second limit on the workspace is imposed by the fact that a pose can be reached by the robot if and only if a minimal number of wires are under tension.

Under that constraint the reachable pose can easily be determined. Indeed consider the plane $\mathcal{P}$ with altitude 0 in which are located all the A points and the convex hull $\mathcal{H}$ of these A points. We will denote by $\mathcal{V}$ the volume that is created by moving $\mathcal{H}$ along a vertical downward direction. It can be shown that if there no limit on the wire tensions, then for any pose in $\mathcal{V}$ there will be a subset of wires with positive tensions (i.e. all the wires are under tension) [9], [10]. As a consequence adding a redundant wire to a $N-1$ robot will indeed increase the workspace size as soon as the added A_{N+1} point lies outside the convex hull of the $A_1, \ldots A_N$ points.

It is then usually assumed that actuation redundancy will also allow to manage wire tensions [11], [12] (e.g. will allow to reduce the maximum of the wire tensions at a given pose). We will examine this assumption in the following sections.

II. MECHANICAL EQUILIBRIUM

A $N-1$ wire driven parallel robot is in mechanical equilibrium if the wire tension balances the load applied on the platform, which is in our case the action of gravity at C. Let m denotes the mass of the platform and τ be the wire tensions. The force $\mathcal{F}$ applied on the platform at C is (0,0,-mg). Mechanical equilibrium will be obtained if

$$\mathcal{F} = \mathbf{J}^{-\mathbf{T}}\tau \tag{1}$$

where $\mathbf{J}^{-\mathbf{T}}$ is the $3 \times N$ transpose of the inverse jacobian of the robot. Consider all triangles in the $\mathcal{P}$ plane whose vertices are three A points: as soon as the projection of the pose in the $\mathcal{P}$ plane lie inside one of these triangles, then mechanical equilibrium is obtained with positive tension by using only the corresponding wires and any other wire is redundant.

If $N > 3$ the linear system (1) has an infinity of solution and hence it may be thought that one can choose a tension distribution that satisfy (1) and optimize some secondary criteria, such as the sum of the τ. Unfortunately such control scheme cannot be used for two reasons:

- a winch system is basically a length generator. Tension control can be obtained only for elastic wires but only by modifying the wire lengths, which has an effect on the positioning of the platform. This influence will be examined in section IV
- the control scheme assumes that all wires are simultaneously under tension: we will show in the next sections that this assumption does not hold for non-elastic wires

In the next section we will use as example a 4-1 robot

$$A_1(0,0,0), A_2(400,0,0), A_3(0,400,0), A_4(400,400,0) \quad (2)$$

and a load of 80kg. Units are centimeter and Newton.

III. Wire tension for non-elastic wires

In this section we will assume that there is no elasticity in the wires. We will show both theoretically and experimentally the following theorem:

Theorem A: for a redundant N-1 robot with non elastic wires it is not possible to use the redundancy to control the tension in the wires for a given pose and any pose will be reached with only at most 3 wires under tension

A. Theoretical proof

When moving from its current pose to the desired one the platform is in equilibrium. At a given time we may assume that C is in position M with the projection of M in the $\mathcal{P}$ plane lying inside the triangle with vertices A_1, A_2, A_3 while the wires 4 to N have a length that is greater than the distance between M and their A points. Consequently the mechanical equilibrium at M is such that the wires 1, 2, 3 are under tension while the other wire tensions are 0. During the motion toward the desired pose it may happen that the length of one wire j in [4,N] became lower than the distance between M and A_j. We will now prove the following theorem:

Theorem B: consider a redundant N-1 robot with non elastic wires that is in equilibrium with 3 wires in tension while the others N-3 wires have zero tension but have a length that is exactly equal to the distance between C and the corresponding A_i. Then any reduction of the length of any these N-3 wires leads the robot to move to another pose with still only 3 wires in tension

Proof: For the sake of simplicity we will assume that there is no triplet of A_i, A_j, A_k that are aligned. Without lack of exhaustivity we may consider that at M the mechanical equilibrium of the platform is such that wire 1, 2, 3 are under tension while the other wire tensions are equal to 0 and we will suppose that the projection of M onto $\mathcal{P}$ lies strictly inside the triangle A_1, A_2, A_3 (the case where M lies on an edge of this triangle will be treated later on). We will assume that the length of wire 4 is initially exactly the distance $||\mathbf{A_4M}||$ and is then decreased by an infinitesimal amount. We will now determine what will be the new pose M' of the load. The motion from M to M' must be *valid* i.e. it must satisfy the following *validity conditions*:

1) in the new pose the distances $||A_jM'||, j \in [1, N]$ cannot be greater than ρ_j
2) the new pose must satisfy the mechanical equilibrium

Let us assume that wires $i, j, i, j \in [1, 3]$ remains under tension during the motion from M to M'. Let $\mathcal{P}_{ij}$ be the vertical plane that includes A_i, A_j and separates the space in two half-spaces V, V_4, with A_4 belonging to V_4 (figure 1).

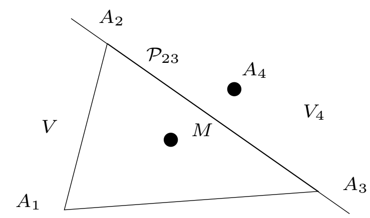

Fig. 1. The vertical plane $\mathcal{P}_{23}$ that contains A_2, A_3 split the space in two half-space V, V_4 with A_4 belonging to V_4

Lemma 1: if there are any wire k, $k \in [5, N]$, $k \neq i, j$ such that A_k belongs to V and such that the initial length of wire k is equal to the distance between A_k and M, then a motion with i, j under tension is not valid

Proof of lemma 1: if i, j remain under tension, then the load rotates around the line A_i, A_j. If A_k lies in V, then a change in ρ_4 leads to an increase in the distance between A_k, M and therefore the motion is not valid. In the example of figure (1) a decrease of ρ_4 leads to an increase in the distance between A_1, M.

Lemma 2: if A_4 lies inside the convex hull of the A_i's, then a decrease of ρ_4 leads the robot in a pose with only 3 wires under tension, one of which is wire 4.

Proof of lemma 2: consider first that at least one of A_i, A_j is included into the convex hull but is not a vertex of the hull. Then by definition of the convex hull the plane $\mathcal{P}_{ij}$ splits the space into 2 half-spaces that both include some of the A_i's. Hence by virtue of lemma 1 a motion from M to M' with i, j under tension is not valid.

Let us not denote by A^j the m A points that are vertices of the convex hull. These points will be numbered sequentially so that A^jA^{j+1} is an edge of the convex hull (the fist point of the list will be numbered A^1 or A^{m+1}). The half-space limited by $\mathcal{P}_{A^jA^{j+1}}$ that contains A_4 also includes all the A_i's and by virtue of lemma 1 any motion will satisfy the first validity condition. On the other hand the vertical plane $\mathcal{P}_{ml}$ associated to any pair of non successive vertices A^m, A^l of the convex hull split the space into two half-spaces that both includes some A_i's and therefore a motion with the associated wires remaining under tension is not valid. Hence only wires whose A points are successive vertices of the convex hull may remain rigid during the motion.

The set of triangles $A^jA^{j+1}A_4$ with $j \in [1, m]$ completely covers the convex hull and the current pose belongs to one of these triangles $A^l A^{l+1} A_4$ (figure 2).

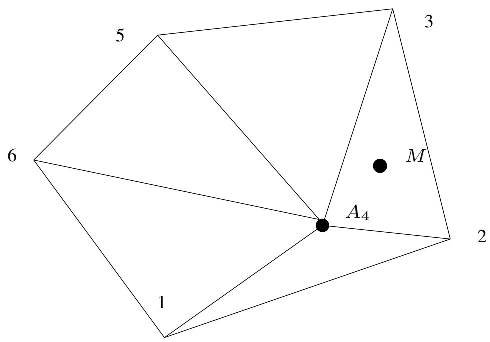

Fig. 2. If A_4 lies inside in the convex hull of the A_i's, then a decrease of ρ_4 leads to a pose where only 3 wires are under tension (here wires 2,3,4)

Any motion with the wires connected to $A^j, A^{j+1}, j \neq l$ will not satisfy the second validity condition. Hence only the motion with the wires connected to A^l, A^{l+1} under tension is valid: the manipulator ends up in the pose where these wires and wire 4 are under tension. The other wires become slack because the distance between their A and M' has decreased.

Lemma 3: if A_4 is a vertex of the convex hull of the A_i's then a decrease of ρ_4 leads the robot in a pose with only 3 wires under tension, one of which is wire 4.

Proof of lemma 3: as for lemma 2 we cannot consider as potential wires leading to a valid motion any wire pair i, j such that A_i or A_j strictly lies inside the convex hull of the A_i's. For the same reason any wire pair i, j whose A_i, A_j are not successive vertices of the convex hull cannot lead to a valid motion. Hence we consider as potential wire pair the one having successive vertices A^j, A^{j+1} of the convex hull. For any triplet A^j, A^{j+1}, A_k that does not include A_4 a decrease of ρ_4 with A^j, A^{j+1} under tension leads to a decrease of the distance between A_k and M: wire k become slack and therefore this motion cannot satisfy the second validity condition. Thereby we consider the triplet A^j, A^{j+1}, A_4. For any of these triplets a decrease of ρ_4 leads to a decrease of the distance between A_k and M for any k different of $j, j+1$ and therefore the first validity condition is satisfied. The second one will be satisfied for any triplet such that the projection of M in $\mathcal{P}$ lies inside the triangle A^j, A^{j+1}, A_4.

Any such triplet that does not include A_2 will not contain any part of the triangle $A_1A_2A_3$ and therefore the equilibrium cannot be satisfied for an infinitesimal motion from M to M'. Hence only the triplets A_4, A_1, A_2 or A_4, A_2, A_3 may satisfy the equilibrium condition. The line going through A_4, A_2 splits the triangle $A_1A_2A_3$ into 2 components, only one of which will include M (figure 3). Hence the robot will move in a configuration where either 1,2,4 or 2,3,4 are under tension while any other wire will become slack.

A special case occurs when M lies on one edge of the triangle A_1, A_2, A_3. This implies that in this pose only 2 wires are under tension (the one involved in the edge) while the third one has 0 tension. A decrease of ρ_4 will lead to a pose M' whose projection onto $\mathcal{P}$ lies in a triangle A_j, A_k, A_l while the distances between M' and the A point of the wires that not involved in the triangle will decrease. Hence these wires will become slack while wires j, k, l will ensure the mechanical equilibrium.

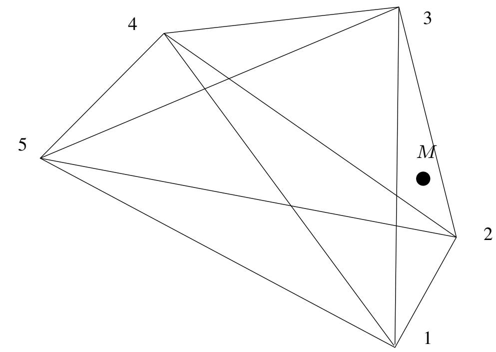

Fig. 3. If A_4 is a vertex of the convex hull of the A_i's, then a decrease of ρ_4 leads to a pose where only 3 wires are under tension (here wires 2,3,4)

The proof of Theorem B is a direct consequence of lemma 2 and 3. During a motion toward the desired pose we will have a sequence of events in which one (or more) wire will become shorter than the distance between its A and the current pose. Hence theorem B applies and the platform reaches a pose in which 3 wires have exactly the length corresponding to the distance between their A and the current pose X, whose projection onto $\mathcal{P}$ lies inside the triangle of the 3 A_i. This proves theorem A.

As an example we consider the 4-1 test robot at the pose (100, 150, -200) with the wire lengths (269.25824, 390.51248, 335.41, 438.74821). If we impose a length value of 438.5 for wire 4, then the robot will move to the pose (100.272188,150,-199.863675) in which only wire 1,3,4 are under tension (at this pose the distance $||\mathbf{A_2C}||$ is 390.2335837301 and wire 2 is no more under tension).

A corollary theorem can be proposed

Theorem C: being given only the coordinates of a pose X we cannot determine which set of 3 wires are under tension at this pose

Proof: the projection of X onto $\mathcal{P}$ belongs to several triangles A_j, A_k, A_l, each one being a candidate to give the wire numbers that are under tension at X. During the proof of theorem B we have seen that the wires that will be under tension at X depends upon the history of the coiling of the wires. Hence we cannot determine the wires under tension if only the coordinates of X are given.

B. Experimental proof

Theorem A may also be illustrated experimentally. For that purpose we have used our assistance lifting crane `MARIONET-ASSIST`, one member of our family of wire-driven parallel robot `MARIONET`, that is used in our apartment as an elderly lifting crane and manipulator. We have attached a load to the 4 wires with low stiffness springs that connect at the same point, leading to a 4-1 robot. Observation of these springs allows one to determine if a given wire is under tension. We start in a pose in which wires (1,2,3) are under tension while the fourth one is slack (figure 4). We then start decreasing the length of the fourth wire. At some point the tension in wire 3 starts decreasing. If we continue decreasing the length of wire 4 then the tension in wire 3 goes to 0 while wire 4 becomes under tension. A direct

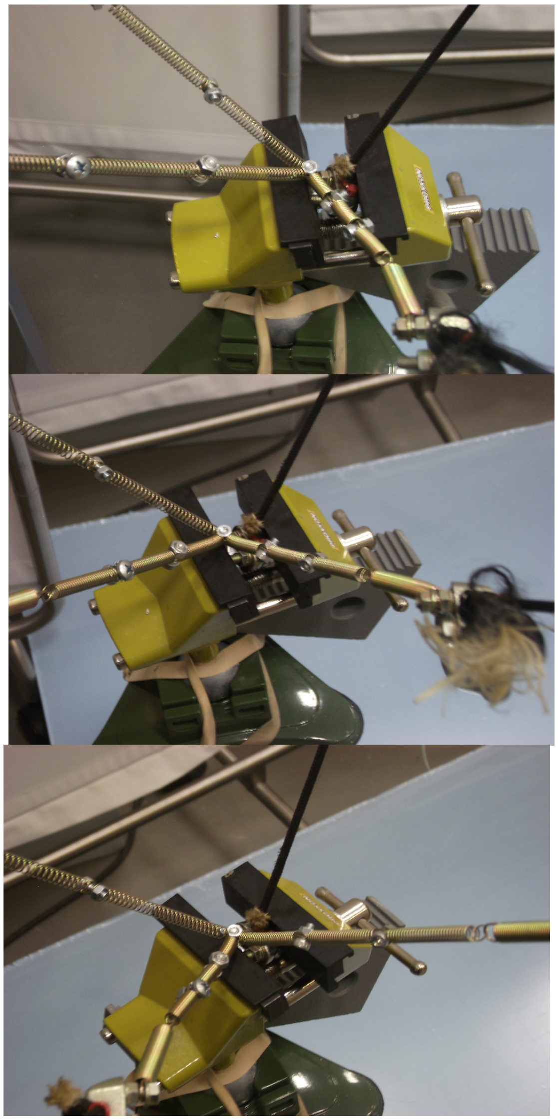

Fig. 4. An experiment with a 4-1 robot: we start in a configuration in which 3 wires are under tension (1,2,3), the fourth one being slack (top picture). Then we start decreasing the length of wire 4. At some point the tension in wire 3 is almost 0 (middle picture). Continuing decreasing the length of wire 4 leads to a pose in which the tension in wire 3 is 0 while wire 4 is becomes under tension (bottom picture).

consequence of the theoretical and experimental proofs is that the use of the pseudo-inverse to calculate the tension in the wires is not appropriate [13].

IV. Elastic wires

In this section we consider for the sake of simplicity a 4-1 robot but the result presented in this section can be extended to any $N-1$ robot.. We assume that we have elastic wires that are supposed to be perfect linear springs. If l_i is the length at rest of wire i and ρ_i its current length we have

$$\tau_i = k(\rho_i - l_i) \tag{3}$$

where k is the stiffness constant of the wire. We address the IK problem: the pose is given and we have to determine the wire lengths l_i that have to be fed to the winch controller. For a given pose the length ρ of the wires should be equal to the distance between C and A and the equilibrium conditions (1) has to be satisfied. These conditions constitute a linear system of 3 equations in the 4 unknowns τ. We assume that this system is not singular and we calculate $\tau_2, \tau_3\tau_4$ as a function of τ_1. We may choose τ_1 to optimize some criteria, for example for minimizing

$$H = \sum_{j=1}^{j=4} \tau_j^2$$

H is a quadratic function in τ_1 and it is therefore trivial to determine τ_1 that leads to the minimum of H. Then equation (3 allows to determine the four l_is. However finding the solution of the IK does not guarantee that the platform will move at the desired pose as the FK may have several solutions. Furthermore we have uncertainties both on k and on the l_i that will be provided by the control system. Hence a study of the FK is necessary.

A. *Forward kinematics*

In the FK problem the l_i's are given and we have to determine the pose of the load. We first solve the equation (1) to obtain τ_2, τ_3, τ_4 as functions of τ_1. The first equation of (3) allows to determine τ_1 as a function of ρ_1. The three remaining equations are linear in x, y, z. After solving this system, which gives x, y, z as functions of $\rho_1, \rho_2, \rho_3, \rho_4$, we report the result in the IK equations

$$||\mathbf{A_k C}||^2 - \rho_k^2 = 0 \tag{4}$$

We will denote by 4_j this equation applied for wire j. The above equations constitute a system of 4 equations in the unknowns $\rho_1, \rho_2, \rho_3, \rho_4$. Equation $(4_2 - 4_1)$ is linear in ρ_4 and is solved for this variable. The 3 remaining equations, denoted a_1, a_2, a_3, are of degree (6,6,2), (3,3,3), (9,9,3) in ρ_1, ρ_2, ρ_3. The four equations $a_1, \rho_3 a_1, a_2, a_3$ are linear in the four monomial $1, \rho_3, \rho_3^2, \rho_3^3$ and hence the determinant of the matrix of this linear system should be 0, which leads to a polynomial P_1 of degree 15 in ρ_1, ρ_2. Taking the resultant of a_1, a_2 in ρ_3 leads to a polynomials P_2 in ρ_1, ρ_2 of degree 18. The resultant of P_1, P_2 factors out in 2 polynomials of degree 76 and 96 in ρ_1. Theoretically solving this polynomial will lead to possible values for ρ_1, then the common roots of P_1, P_2 will give ρ_2. Solving a_1 will lead to ρ_3, which together with ρ_1, ρ_2 will allow to determine ρ_4. Having computed the four ρ enable then to obtain x, y, z.

Although this complete the theoretical solution, the degree of the involved polynomials are too high to be used in practice and consequently we have to resort to a numerical procedure. Equations a_1, a_2, a_3 together with $(4_2-4_1), (4_3-4_1), (4_4-4_1), (4_1)$ constitutes a system of 7 equations in the 7 unknowns $x, y, z, \rho_1, \rho_2, \rho_3, \rho_4$. It may be shown that all these unknowns can be bounded and consequently interval analysis is an appropriate tool for solving these equations, the solving time being less than one second.

B. Positioning sensitivity

We have investigated the sensitivity of the calculation of x, y, z with respect to possible uncertainties on k, l_i. For that purpose we have considered the 4-1 robot with the A coordinates (2) and we have used the IK to determine what should be the l_i to reach the pose $x = 100, y = 200, z = -200$ while minimizing $\sum_{j=1}^{j=4} \tau_i^2$ for $k = 1000$. The nominal values are $l_1 = l_2 = 299.558$, $l_3 = l_4 = 412.1083$ which leads to $\tau_1 = \tau_2 = 441.45$, $\tau_3 = \tau_4 = 202.2383$. If we set k to 100 the nominal values are $l_1 = l_2 = 295.5855$, $l_3 = l_4 = 410.288$ with wire tensions $441.45, 441.45, 202.23833, 202.23833$.

We have then considered 1000 values for the l_i's and k that were randomly perturbed around their nominal values l_{nom}, k_{nom} (by ± 3 for the l_i's and $\pm 0.1k$ for k) and we have used the FK to determine x, y, z. The average values of x, y, z and their maximal variation around these averages are presented in table I while table II presents the average values of the τ and their maximal variation around these averages. The following results were also obtained:

Average pose (x, y, z)			
k_{nom}=100	99.633266	200.58488	-198.945539
k_{nom}=1000	100.032878	200.22398	-199.629886
Pose error $(\Delta x, \Delta y, \Delta z)$			
k_{nom}=100	[-7.53, 4.57]	[-6.86, 5.75]	[-7.60, 2.87]
k_{nom}=1000	[-5.48, 4.79]	[-4.79, 4.26]	[-5.14, 4.24]

TABLE I

VARIATION OF THE POSITIONING WITH A ± 3 UNCERTAINTY ON THE l_i'S AND $\pm 0.1k$ UNCERTAINTY ON THE k

k_{nom}=100	
mean τ_1, $\Delta\tau_1$ 470.68, [-184.1, 123.7]	mean τ_2,$\Delta\tau_2$ 423.2, [-137, 170.7]
mean τ_3, $\Delta\tau_3$ 181.42, [-177.5, 230.7]	mean τ_4,$\Delta\tau_4$ 277.13, [-274.8, 135]
k_{nom}=1000	
mean τ_1, $\Delta\tau_1$ 467.6, [-192, 133.8]	mean τ_2,$\Delta\tau_2$ 463.06, [-187.45, 138.1]
mean τ_3, $\Delta\tau_3$ 278.35, [-274.2, 146.1]	mean τ_4,$\Delta\tau_4$ 277.13, [-241.7, 168.7]

TABLE II

TENSION AVERAGE AND VARIATIONS AROUND THESE AVERAGES WITH A ± 3 UNCERTAINTY ON THE l_i'S AND $\pm 0.1k$ UNCERTAINTY ON k

- in all cases there was only at most a **single** valid 4-1 solution for the FK
- there was a 4-1 solution (all 4 wires under tension) for 960 cases (k=100) and 160 cases (k=1000) while there was always a solution in a 3-1 configuration

Using k=1000 we have performed another test for the nominal pose $x = 130$, $y = 180$, $z = -200$. The mean value of x, y, z were 128.55, 181.558, -200.789 with a variation around these values of [-7.833, 5.56], [-8.61, 6.028], [-6.955, 4.708]. The mean value for $\tau_1, tau_2, tau_3, \tau_4$ were 434.832, 366.715, 316.01, 332.344 with a variation of [-176.741, 213.368], [-211.942, 210.86], [-297.098, 175.482], [-327.109, 183.161].

Another source of error is the uncertainties on the location of the A_i's that may be due to a poor calibration or to deformation of the frame supporting the robot when loaded. To evaluate this influence we have used the nominal wire lengths to reach the pose $x = 100, y = 200, z = -200$ and we have assumed a perfect knowledge of k and a perfect wire lengths control, while we have randomly perturbed the x, y coordinates of the A_i's with a value in the range [-2,2]. Tables III and IV present the average pose, tensions and their maximal variations around the averages.

Average pose (x, y, z)			
k_{nom}=100	97.971473	199.911506	-200.850671
k_{nom}=1000	99.678064	200.047538	-199.993619
Pose error $(\Delta x, \Delta y, \Delta z)$			
k_{nom}=100	[-4.1195, 4.4631]	[-4.55, 4.578]	[-3.044, 3.055].
k_{nom}=1000	[-3.4, 3.446]	[-2.923, 2.684]	[-2.735, 2.519].

TABLE III

VARIATION OF THE POSITIONING WITH A ± 2 UNCERTAINTY ON THE A

k_{nom}=100	
mean τ_1, $\Delta\tau_1$ 452.238063, [-158.657, 136.74]	mean τ_2,$\Delta\tau_2$ 442.731039, [-147.916, 147.59]
mean τ_3, $\Delta\tau_3$ 265.256553, [-147.916, 147.59]	mean τ_4,$\Delta\tau_4$ 280.263655, [-272.85, 126.315].
k_{nom}=1000	
mean τ_1, $\Delta\tau_1$ 439.025901,[-156.45, 157.649]	mean τ_2,$\Delta\tau_2$ 441.118404, [-157.12, 156.826]
mean τ_3, $\Delta\tau_3$ 272.519396, [-269.671, 147.625]	mean τ_4,$\Delta\tau_4$ 237.373109,[-231.759, 182.077]

TABLE IV

TENSION AVERAGE AND VARIATIONS AROUND THESE AVERAGES WITH A ± 2 UNCERTAINTY ON THE A

We have then combined the errors on the A_i's (± 2 on the x, y coordinates), on the l_i's (± 3) and on k (± 10 %) applied in the configuration corresponding to the pose $x = 100, y = 200, z = -200$. A random sampling of 3000 measures has been performed.

- for $k_{nom} = 100$ the mean value of obtained pose is 98.726651, 200.361376, -201.308687 with a variation of [-9.449, 7.102], [-8.158, 7.217], [-6.337, 6.5]. The average values of the τ are 440.999788 476.440767 225.695517 188.101464 with a variation of [-164.39,75 158.733], [-197.12,34 123.558], [-225.53,40 199.768], [-187.458,61 233.732].
- for $k_{nom} = 1000$ the mean value of obtained pose is 100.149153, 199.46656, -200.443448 with a variation of [-8.575, 6.676], [-5.935, 6.607], [-5.716,6.453]. The average values of the τ are 439.317306, 496.090587, 188.53118, 276.871677 with a variation of [-173.728,

168.846], [-228.579, 110.658], [-188.125, 248.888], [-276.619, 160.55].

C. Using tension measurements

We have also investigated the use of wire tension measurements to determine the pose of the platform. For given τ_i's the equations (1) are linear in x, y, z. After solving this system we report the result in $(4_2 - 4_1), (4_3 - 4_1), (4_4 - 4_1)$. Together with (4_2) these equations constitutes a system of 4 equations in the unknowns $\rho_1, \rho_2, \rho_3, \rho_4$. The second equation is linear in ρ_4 so we may obtain 3 equations in ρ_1, ρ_2, ρ_3. Successive elimination allows to obtain an univariate polynomial which factors into 2 polynomials of degree 144 and 30 in ρ_1^2.

Clearly solving such high degree polynomial may be difficult but as the unknowns can be easily bounded this system can be solved with interval analysis. Wire tension measurement are typically noisy and we may wonder about the sensitivity of the pose determination in the presence of uncertainties in the measurements. To test this sensitivity we have chosen a nominal value for the τ_i's (441.45,441.45,202.238331,202.238331) that corresponds to the pose $x = 100, y = 100, z = -200$ and we have added a random perturbation in the range [-20,20] for 1000 samplings. The variation on the x, y, z around their nominal values were obtained as [-12.2518,12.3812],[-17.258,14.728],[-19.163,15.1576]. Even with a random error on the τ_i's in the range [-1,1] we get a positioning error in the range [-1,1]. Note that when the perturbation is small we may assume the ρ_i's to be constant and the equation (1) define 3 planes in the x, y, z space that are parallel to the x, y, z axis. When the τ_i's are changing the planes move along their axis. Hence the pose of the robot will be obtained as the intersection of 6 half-planes leading to cube whose edges have a length $2 \sum \Delta\tau_i / \rho_i$.

Hence even with very accurate force measurement the error on the pose calculation may be quite large.

D. General summary

All these data show that, although in theory having elastic wires allows one to manage the wire tensions distribution, the practical application of such control scheme will be very difficult. Indeed even if we assume that the wires are perfect linear springs, a small uncertainties on the wire stiffness combined with unavoidable uncertainties on the wire lengths will lead to large positioning errors and very large variations on the wire tensions. Furthermore positioning based on tension measurements seems to be also quite difficult. It remains to determine if alternate sensing methods (such as the use of vision as proposed in [14]) may improve redundancy management.

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

This paper has shown that the actuation redundancy of a $N - 1$ wire-driven parallel robot can be used to increase the workspace size but cannot be used to optimize tensions in the wires: for non-elastic wires we have shown theoretically and experimentally that at all time at most 3 wires will be simultaneously under tension while for elastic wires a numerical analysis has shown that the positioning and tensions in the wires are very sensitive to uncertainties on the wire lengths and stiffness of the wires.

B. Future Works

One advantage of wire-driven parallel robot is their modularity: a proper mechanical design allows one to connect and disconnect the wires at will. We may benefit from this modularity to use the actuation redundancy for managing tensions in the wires. For that purpose we have considered two options:

- connecting counterweights to the platform with redundant wires which is a mean for transforming the wire from a length-only actuation into a force supplier
- temporary connecting redundant wires not on the platform but a specific point on non redundant wires

Both options will be presented in a companion paper.

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