A Novel Tracking Control Method for a Wheeled Mobile Robots*

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Abstract

In this paper, a novel path tracking control method is proposed for a nonholonomic mobile robot. The proposed controller is based on a bang-bang control technique, the concept of landing curve, and a biologically inspired neural dynamics model that is derived from Hodgkin and Huxley's (1952) membrane equation. The acceleration constraints and the nonholonomic kinematics constraints are full respected in the controller design. The proposed tracking controller is capable of generating bounded real-time acceleration commands that can produce smooth, continuous robot velocities. Stability of the control system and the convergence of tracking errors to zero are rigorously proved using a Lyapunov stability theory. The effectiveness of the proposed algorithm is demonstrated by simulations with a two-wheel driven mobile robot.

1 Introduction

Autonomous mobile robots are increasingly used in well structured environment such as factories, warehouses and offices. The two degrees of freedom mobile robots are able to navigate to track a desired trajectory under the control. In order to have the mobile robot complete its duties well, one of the important issues is the accuracy and reliability of its motion control. Motion control is the strategy by which the robot approaches a desired position and implementation of this strategy. In this sense, the motion controller should generate a series commands which guide the robot to accurately follow a desired path. Any deviation from the desired path, either in the form of an offset or orientation is undesirable, since both lead to lose the path. A good controller, thus, becomes necessary for a stable, efficient tracking performance.

Many previous studies on the problem of path tracking have been conducted and various control strategies have been employed. For example, Jiang [4] proposed a tracking control methodology via time-varying state feedback based on the backstepping technique for both a kinematic and simplified dynamic model of a 2.o.f mobile robot, through using Lyapunov direct method for obtaining semiglobal and global results in the tracking problem for the mobile robot. Lee and Williams [9] proposed a control method for eliminating the track error quickly by controlling two independent driving wheels at same time, where the tracking error system used a wheel Jacobian which is suitable for a robot with two driving wheels. Kanayama [10] presented the decomposition of error between the

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reference posture and the current posture by a PID filtering method. Nelson and Cox [13] adopted the path error vector and used this error vector for the correction of driving velocity and steering velocity. Kimura [12] proposed use of a Lyapunov function for a stable tracking control. Kinayama [14] proposed a method using straight line reference for the robot's locomotion instead of a sequence points. Crawley [3] developed a locomotion control system whose organization has a three layer structure, the top layer is an interpreter, the middle layer is a control loop, the bottom layer is a translator. Some authors also used geometrical or time-optimal approaches. For example, David [7] found time-optimal motions of the robot in Cartesian space and the corresponding control trajectories that move the robot from an initial configuration to a final configuration. Tounsi [11] presented different curves used in the path. In summary, the previous works on the path tracking controller can be classified into the following four categories: linear, nonlinear, geometrical and intelligent approaches. However, they may mainly focus on path error convergence and system stability. As a result, they may neglect the motion smoothness and dynamics constraints. Although the geometric schemes show the smooth tracking motion to guide the mobile robot towards the reference path, but still neglect the dynamics constraints, such as the acceleration bounds which are important factors for avoiding wheel slippage or mechanical shock during the navigation.

Therefore, a good controller should consider the path continuity, smoothness. In addition, the constraints also need to be considered, e.g., the nonholonomic constraint for a mobile robot, the turning radius is bounded, and the dynamics constraints such like acceleration bounds in order to avoid wheel slippage and mechanical damage during the mobile robot navigation. With all above consideration, in this paper, a novel path tracking controller is presented. The proposed controller uses a biologically inspired neural dynamics model, a shunting model, that is derived from Hodgkin and Huxley's membrane equation [1], and it also is based on the bang-bang control technique to produce bounded real-time acceleration commands that can guide the robot to reach the desired trajectory. Hence, the dynamics constraints and nonholonomic kinematic constraints are fully considered in the designed controller. Stability of the control system and the convergence of tracking errors to zero are proved through using a Lyapunov stability theory based on the convergence of bang-bang control. The parameter sensitivity is also addressed for the shunting model that effect the controller's tracking performance.

The paper is organized as follows. Section 2 provides background information about the tracking control problem. Section 3 addresses the philosophy of the proposed algorithm including the stability analysis using a Lyapunov stability theory. In section 4, the simulation results are presented to demonstrate the effective performance of the proposed controller. Section 5 discusses the effect of parameters used in the shunting model. The last part concludes this paper by highlighting the feature properties of the proposed model.

2 Background

In this section the general mobile robot model is first introduced, then the tracking control problem with consideration of dynamics constraints will be described.

2.1 A Nonholonomic Mobile Robot Model

A typical nonholonomic mobile robot is shown in Fig. 1, where a mobile robot is located in a two dimensional Cartesian workspace, in which a global coordinate is defined. A local coordinate is attached to the robot with the origin at point c, the robot mass center.

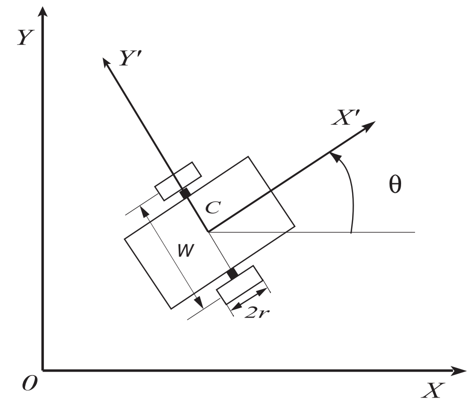

Figure 1: *Model of a wheeled mobile robot*

The robot possesses three degrees of freedom in its positioning which are represented by a posture $p_c = (x_c, y_c, \theta_c)$, where (x_c, y_c) indicate the position of the robot with respect to the global coordinate system and θ_c is the heading angle of the robot. The robot's kinematics is defined as

$$\begin{aligned} \dot{x}_c &= v_c \cos\theta_c, \\ \dot{y}_c &= v_c \sin\theta_c, \\ \dot{\theta}_c &= \omega_c, \end{aligned} \tag{1}$$

where v_c is the linear (tangential) velocity, ω_c is the angular velocity of the robot mass center c. In the control system, two postures are used: the reference posture $p_t = (x_t, y_t, \theta_t)$ and the current posture $p_c = (x_c, y_c, \theta_c)$. The motion of the target robot can be represented by the target posture and the current posture is real posture at this moment, respectively. So the error posture p_e is given in the local coordinate with respect to reference posture as

$$\begin{aligned} e_x &= (x_t - x_c)\cos\theta_t + (y_t - y_c)\sin\theta_t, \\ e_y &= -(x_t - x_c)\sin\theta_t + (y_t - y_c)\cos\theta_t, \\ e_\theta &= \theta_t - \theta_c. \end{aligned} \tag{2}$$

If the reference velocities are given as (v_t, ω_t), and the current velocities are (v_c and ω_c), based on Eq. (2), the error dynamics of the mobile robot is derived as

$$\begin{aligned} \dot{e}_x &= v_t - v_c \cos e_\theta + \omega_t e_y, \\ \dot{e}_y &= -\omega_t e_x + v_c \sin e_\theta, \\ \dot{e}_\theta &= \omega_t - \omega_c. \end{aligned} \tag{3}$$

2.2 Constraints of a Mobile Robot

The nonholonomic constraints states that the robot can only move in the direction normal to the axis of the driving wheels, therefore, if the derivatives $\dot{x}_c$ and $\dot{y}_c$ exist, θ_c is not an independent variable that should satisfy the nonholonomic kinematics constrain as

$$\dot{y}_c \cos\theta_c - \dot{x}_c \sin\theta_c = 0. \tag{4}$$

In addition to the nonholonomic constraint, the acceleration constraint should be considered in order to avoid slippage during the robot navigation. For a mobile robot with two differential wheels move on a planar surface, the linear velocity v_c and the angular velocity ω_c of center c can be derived as

$$\begin{aligned} v_c &= \frac{r}{2}(\omega_r + \omega_l), \\ \omega_c &= \frac{r}{D}(\omega_r - \omega_l), \end{aligned} \quad (5)$$

where r is radium of the wheels, D is the length between wheels, respectively. When considering to avoid slippage or any abrupt change in robot motion, the angular accelerations of the left and right driving wheel must be bounded, i.e., $\omega_{r,l} \leq \omega_{\max}$. Based on the Eq. (5), the linear and angular accelerations are limited by

$$\begin{aligned} |a_c| &= r\dot{\omega}_{\max} \leq a_{\max}, \\ |\beta_c| &= r\frac{\dot{\omega}_{\max}}{D} \leq \beta_{\max}, \end{aligned} \quad (6)$$

where a_c and β_c are the linear acceleration and angular acceleration of center c, respectively.

2.3 Tracking Problem

The tracking problem in this paper is to design a control law for accelerations a_c and β_c with consideration of constraints. Furthermore, to design the velocities v_c and ω_c, such that the robot follows a reference path with the desired position p_t and desired velocities v_t and ω_t. The appropriate acceleration control law for a_c and β_c is in form of

$$a_c = f(e_x, e_y, e_\theta, v_t, \omega_t, a_{\max}), \quad (7)$$

$$\beta_c = g(e_x, e_y, e_\theta, v_t, \omega_t, \beta_{\max}), \quad (8)$$

such that for arbitrary initial tracking errors, the error posture of the tracking control system will converge to zero.

The control system is working as follows. First using the Eq. (2) to calculate the errors, then the proposed controller generates the accelerations of the robot based on the errors and constraints, after that the velocities of the robot can be derived using

$$\begin{aligned} v_c(k+1) &= v_c(k) + a_c dt, \\ \omega_c(k+1) &= \omega_c(k) + \beta_c dt. \end{aligned} \quad (9)$$

The fourth step is to get the robot kinematics by using the Eq. (1). Finally using a integrator, current posture of the robot is derived as follows: if $\omega_c \neq 0$, the current posture is given as

$$\begin{aligned} x_c(k+1) &= x_c(k) + \frac{v_c}{\omega_c}(\sin\theta_c(k+1) - \sin\theta_c(k)), \\ y_c(k+1) &= y_c(k) - \frac{v_c}{\omega_c}(\cos\theta_c(k+1) - \cos\theta_c(k)), \\ \theta_c(k+1) &= \theta_c(k) + \omega_c dt; \end{aligned} \quad (10)$$

and if $\omega_c = 0$, the current posture is given as

$$\begin{aligned} x_c(k+1) &= x_c(k) + v_c dt \cos\theta_c(k), \\ y_c(k+1) &= y_c(k) + v_c dt \sin\theta_c(k), \\ \theta_c(k+1) &= \theta_c(k). \end{aligned} \quad (11)$$

The system architecture is shown in Fig. 2.

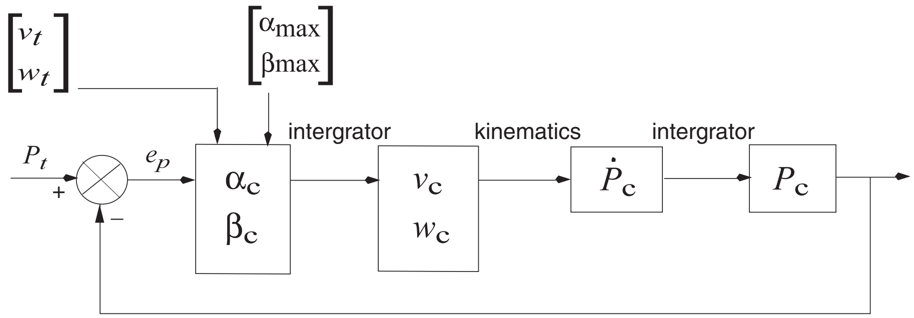

Figure 2: *Structure of the control system*

3 Control Algorithm

In this section, the time-optimal bang-bang control technique is briefly introduced. Then the concept of landing curve that is used for guiding the robot to land smoothly on the target is presented. After that, the biologically inspired shunting model that is used for the generating smooth velocities is introduced. The proposed control algorithm is presented. Finally the stability and convergence of the proposed control systems are proved using a Lyapunov stability analysis.

3.1 Time-optimal Bang-Bang Control

For a bang-bang control, the wheels accelerations can switch from their upper limit to their low limit. A bang-bang control of a double integrator system is represented by

$$\begin{aligned} v_s &= v_t - v_c + (2a_{\max}|x_t - x_c|)^{1/2}, \\ a_c &= a_{\max}\text{sgn}(v_s), \end{aligned} \tag{12}$$

where x_c, v_c and a_c are position, velocity and acceleration of the robot, respectively, and v_s is a switching condition of the acceleration. The bang-bang control is asymptotically stable, the errors converges to the origin if the acceleration is bounded by $a_{\max}$ [15].

3.2 Landing Curve

In order to satisfy the nonholonomic constraints, the concept of landing curve is employed in the proposed model. This landing curve provides the heading angle and velocity values that guide the robot could softly land on the target path. Here, a cubic spiral is chosen, because it provides an more optimal smooth path than other curves such as clothoids [11]. The curve is defined as

$$y_p = \pm c_e {x_p}^3, \tag{13}$$

where if $x_p = 0$, $y_p' = 0$, and $y_p'' = 0$, and c_e is a positive constant. The sign can be chosen according to e_y, in the following discussion, it is assumed to be positive.

3.3 Shunting Model

A shunting model inspired Hodgkin and Huxley's [1] membrane model of a biological neural system is employed to improve the tracking performance by substituting the sign function in the bang-bang control, since the output of shunting model change smoothly, even with a sudden stimulus. The neural dynamics ζ of a typical shunting model is characterized by [2]

$$\frac{d\zeta}{dt} = -A\zeta + (B - \zeta)S^{+} - (D + \zeta)S^{-}, \tag{14}$$

where A, B and D are positive constants, which represent the passive decay rate, the upper and lower bounds of the neural activity ζ, respectively. Variable S^{+} is total excitatory input to neuron, while S^{-} is the inhibitory input. The neural activity ζ is guaranteed to stay in a finite region $[-D, B]$ for any excitatory and inhibitory inputs [2]. This shunting model was first proposed by Grossberg to understand the real-time adaptive behavior of individuals to complex and dynamic environmental contingencies and has a lot of applications in biological and machine vision, sensory motor control and many other areas [2].

3.4 Proposed Control Algorithm

The proposed control law is developed based on bang-bang control in incorporating with a shunting model and the landing curve with consideration of dynamics constraints. It includes both proportional feedback and derivative term of the offset to guarantee system stability. The control law will be considered into two parts: one is linear velocity control, the other is angular velocity control.

3.4.1 Linear Velocity Control

There is no nonholonomic constraint for the linear velocity, so the dynamics of linear velocity is given by

$$v_s = \dot{e}_x + (2a_{\max}|e_x|)^{1/2} v_{s1}, \tag{15}$$

where v_{s1} is obtained from a shunting model as

$$\frac{dv_{s1}}{dt} = -Av_{s1} + (B - vs_1)f(e_x) - (D + v_{s1})g(e_x).$$

In the shunting equation, the positive input $S^{+} = f(e_x)$ is defined as $\max\{0, e_x\}$, while the negative input $S^{-} = g(e_x)$ is defined as $\max\{0, -e_x\}$. From Eq. (15), consider the acceleration dynamics constraint, the linear acceleration in continuous values is derived as

$$a_c = \begin{cases} a_{\max}, & \text{if } v_s/dt > a_{\max} \\ -a_{\max}, & \text{if } v_s/dt < -a_{\max} \\ v_s/dt, & \text{otherwise} \end{cases} . \tag{16}$$

3.4.2 Steering Control Law

Regarding the steering control law, first it is needed to determine the values of heading angle and angular velocity of the landing curve because of the nonholonomic constraint, then the robot is supposed to follow the landing curve. The heading angle θ_p and angular velocity ω_p of landing curve are given by

$$\begin{aligned}\theta_p &= \theta_t + \arctan[3c_e(\frac{e_y}{c_e})^{2/3}],\\ \omega_p &= \omega_t + \frac{2(\frac{e_y}{c_e})^{-1/3}e_y}{1+\tan(\theta_p-\theta_t)^2}.\end{aligned} \tag{17}$$

Therefore, the dynamics of the robot angular velocity is obtained as

$$\omega_s = (\omega_p - \omega_c) + (2\omega_{\max}|\theta_p - \theta_c|)^{1/2}\omega_{s1}, \tag{18}$$

where ω_{s1} is characterized by a shunting model as

$$\frac{d\omega_{s1}}{dt} = -A\omega_{s1} + (B - \omega_{s1})f(\theta_p - \theta_c) - (D + \omega_{s1})g(\theta_p - \theta_c).$$

Thus the angular acceleration can be derived as

$$\beta_c = \begin{cases} \beta_{\max}, & \text{if } \omega_s/dt > \beta_{\max} \\ -\beta_{\max}, & \text{if } \omega_s/dt < -\beta_{\max} \\ \omega_s/dt, & \text{otherwise} \end{cases}. \tag{19}$$

3.5 Stability Analysis

For the driving velocity control, since the constants B and D are chosen equal to 1, i.e., the variable v_{s1} will stay $\in [-1, 1]$ under any e_x. Thus the control law is same as bang-bang control, the stability of the bang-bang control guaranteed that the tangential path error e_x converges to zero. Meanwhile in order to show that e_y and e_θ converge to zero too, a Lyapunov function is defined as

$$v = \frac{1}{2}{e_y}^2 + \frac{1}{2}{e_\theta}^2. \tag{20}$$

So the derivative of v is given as

$$\dot{v} = e_y\dot{e}_y + e_\theta\dot{e}_\theta. \tag{21}$$

From Eq. (3), have

$$\dot{v} = e_y(v_c \sin e_\theta - \omega_t e_x) + e_\theta(\omega_t - \omega_c). \tag{22}$$

Because it is assumed that $\theta_c \to \theta_p$ and $e_x \to 0$, eventually, have

$$e_y\dot{e}_y = \frac{-e_y \tan(\theta_p - \theta_t)}{(1+\tan(\theta_p-\theta_t)^2)^{1/2}}. \tag{23}$$

Since

$$\tan(\theta_p - \theta_t) = 3c_e(\frac{e_y}{c_e})^{2/3}\mathrm{sgn}(e_y),$$

then Eq. (23) becomes

$$e_y\dot{e}_y = \frac{-3e_y v_c c_e(\frac{e_y}{c_e})^{2/3}\mathrm{sgn}(e_y)}{(1+\tan(\theta_p-\theta_c)^2)^{1/2}} < 0, \tag{24}$$

because $c_e > 0$ and $e_y \mathrm{sgn}(e_y) > 0$. Similarly for the second term of Eq. (21), $\omega_c \to \omega_c$, the result becomes

$$e_\theta \dot{e}_\theta = \frac{-2e_\theta v_c \sin e_\theta (\frac{e_y}{c_e})^{-1/3} \mathrm{sgn}(e_y)}{(1 + \tan(\theta_p - \theta_c)^2)^{1/2}} < 0, \tag{25}$$

because $(e_y/c_e)^{-1/3}\mathrm{sgn}(e_y) > 0$ and $e_\theta \sin e_\theta > 0$, if $0 < |e_\theta| < \pi/2$. From the results of Eq.s (24) and (25), it can be concluded that $\dot{v} < 0$, and $\dot{v} = 0$ if and only if $e_y = 0$ and $e_\theta = 0$. Therefore, the proposed control system is stable. The tracking errors are guaranteed to converge to zero.

4 Simulation Results

In this section, to demonstrate the effectiveness, the proposed control algorithm will be applied to track a straight line and a circular path with a large initial tracking error.

4.1 Tracking a Straight Line

First the proposed model is to track a straight line shown in Fig. 3A by dotted line. In the simulation, the dynamics constraints are chosen as $a_{\max} = 0.3m/sec^2$ and $\beta_{\max} = 1.2rad/sec^2$. The target velocities are chosen as $v_t(0) = 1m/s$ and $\omega_t(0) = 0rad/s$. The initial error posture is $e_x(0) = 0m$, $e_y(0) = 1m$, and $e_\theta = 0.523rad$. The initial linear and angular velocities of the robot are $v_c(0) = 1m/s$ and $\omega_c(0) = 0rad$. The real time tracking performance is shown in Fig. 3A, there the actual robot traveling path is shown by solid line. The tracking errors in the tangential (driving) direction, the lateral direction and the orientation are shown in Fig. 3B by solid line, dashed line and dashdot line, respectively. The linear and angular velocities are shown in Fig. 3C, and Fig. 3D shows the linear and angular accelerations. From the results, it shows that the robot starts from a position with a large initial tracking error, then is trying to track the desired line. It takes a short time for the robot to catch up the desired line and land on the desired path smoothly. The position errors also converge to zero quickly, i.e., the proposed controller has very smooth and quick tracking response. Moreover, although the robot starts with a large posture error, its tracking performance still does a good tracking. Thus unlike the linearization based tracking controllers, the proposed controller can deal with large initial errors. From Fig.s 3C and 3D, it shows that at the starting phase the robot has large accelerations which reach its dynamics constraints. In this initial stage, if there are no acceleration constraints, The accelerations may go very large that will cause damage to the robot.

4.2 Tracking a Circular Path

The proposed algorithm is then applied to track a circle path shown in Fig. 4A. All the conditions are same as in previous case except $\omega_t = 0.1rad/s$. The radium of desired circle path is 5 m. The tracking performance is shown in Fig. 4A. The tracking errors are shown in Fig. 4B. It shows that the robot also starts from a position with large initial error from the desired path. The robot can track the desired path smoothly and the tracking errors are also converge to zero quickly. Meanwhile all the dynamics constraints are satisfied.

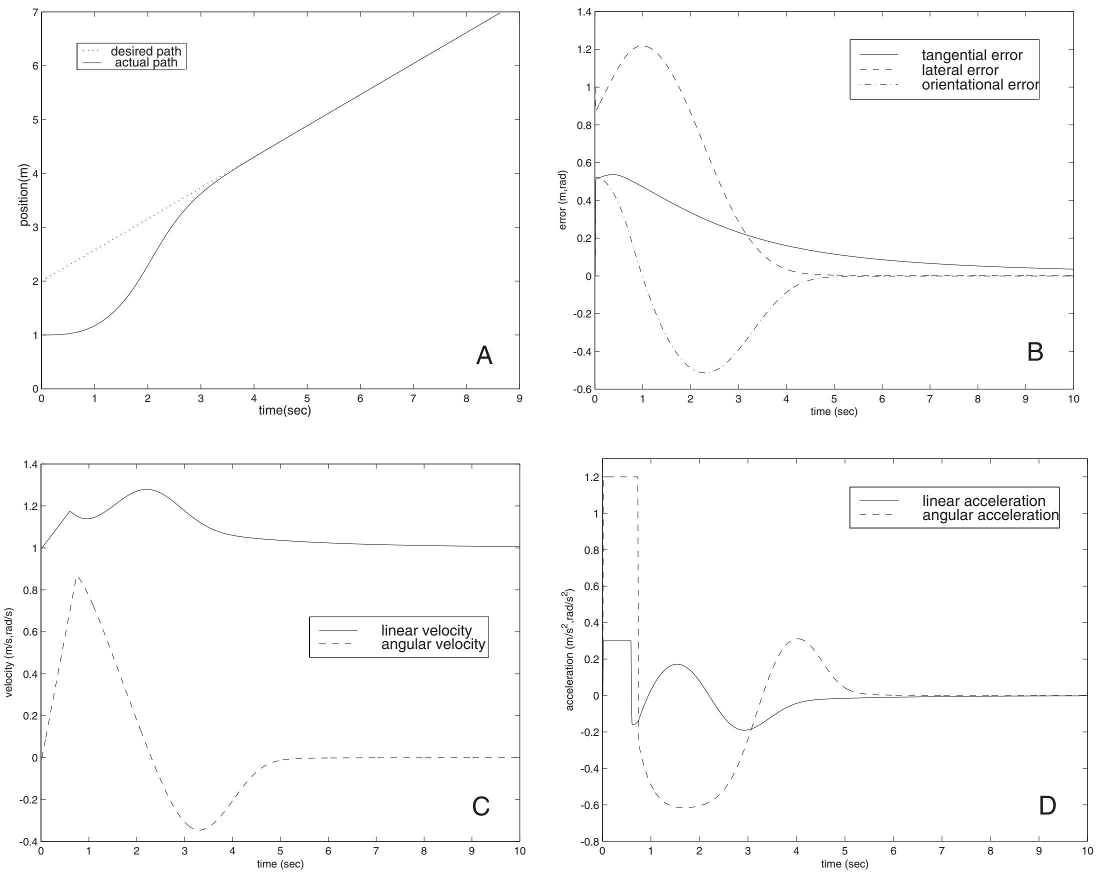

Figure 3: *Tracking a straight line.* A: *Tracking performance;* B: *Tracking errors;* C: *Velocities of the robot;* D: *Accelerations of the robot;*

5 Discussion

The parameter sensitive of a model is a very important factor to be considered when propose or evaluate a model. An acceptable model should be not very sensitive to changes in its parameters. A shunting model is employed in this proposed algorithm. In this section the parameter sensitivity of the shunting model is discussed and the simulation results are provided. There are three parameters A, B and D as introduced in the shunting model. The B and D are the upper and lower bounds of the output. In the proposed algorithm they are chosen to be equal to 1 based on bang-bang control technique. The parameter A represents the passive decay rate that is one of important parameters that effect the model response. A larger value of A results in a smaller value of the steady state caused by the excitatory input, and a shorter duration to reach its steady state. Therefore, the tracking performance of the proposed algorithm is effected by values of A. Based on a series of simulation results, the best suitable value of $A = 1$ was chosen in the above simulation. If A is chosen to be 5, the tracking performances of the proposed controller for the same desired straight line and circle are shown in the Fig.s 5A and 5B, respectively.

Compare to above simulation results to the results in Section 4, it shows that although the

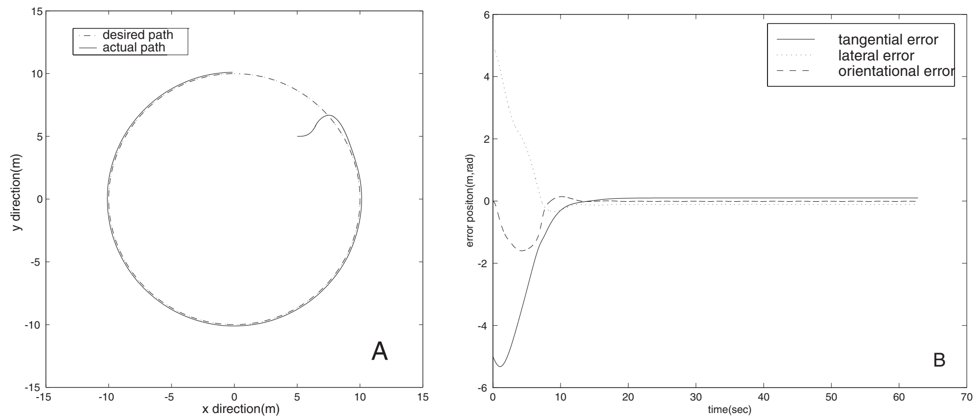

Figure 4: *Performance of the proposed algorithm.* A: *Control performance for tracking a circle path;* B: *Tracking errors.*

robot can still track the desired paths, the tracking performance is not as good as $A = 1$, and the convergence of tracking errors takes longer time.

6 Conclusion

A novel path tracking algorithm for a nonholonomic mobile robot is proposed based on bang-bang control technique, the concept of landing curve, and a biologically inspired shunting model. A smooth tracking method is developed with consideration of acceleration constraints and the nonholonomic constraints. The system stability and the convergence of tracking errors to zeros are rigorously proved using a Lyapunov stability theory. The effectiveness of the proposed tracking controller is demonstrated by simulation studies. This proposed method is mainly hardware-independent, hence it can be applied easily to various kinds of mobile robots with two driving wheels.

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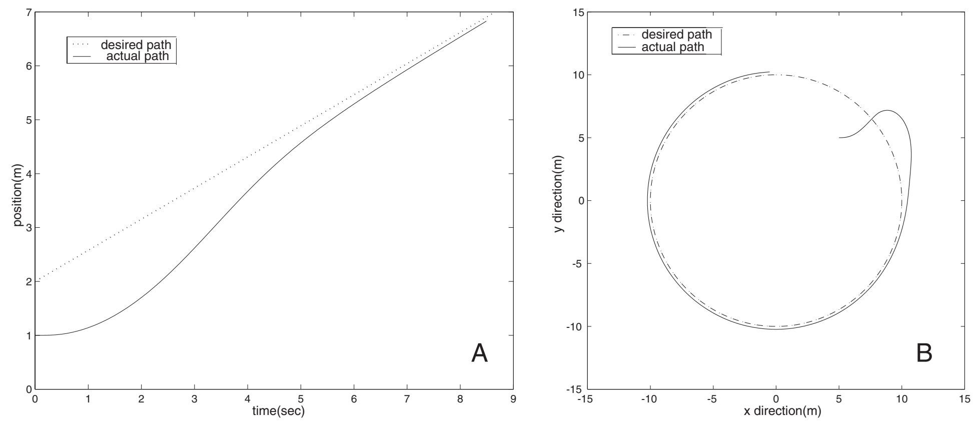

Figure 5: *Tracking performance of the proposed algorithm with large A value, A=5 instead of 1 in Fig. 5 and 6.* A: *Tracking a straight line;* B: *Tracking a circular path.*

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