A Family of New Parallel Architectures with Four Degrees of Freedom

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Abstract: This paper describes a novel parallel kinematic chain with 4 DoF. Three of the freedoms of the platform are rotational and only one is translational. The position and velocity kinematics of the mechanism are analyzed with different choices of the actuated joints. Methods for singularity analysis of the proposed mechanisms are also presented.

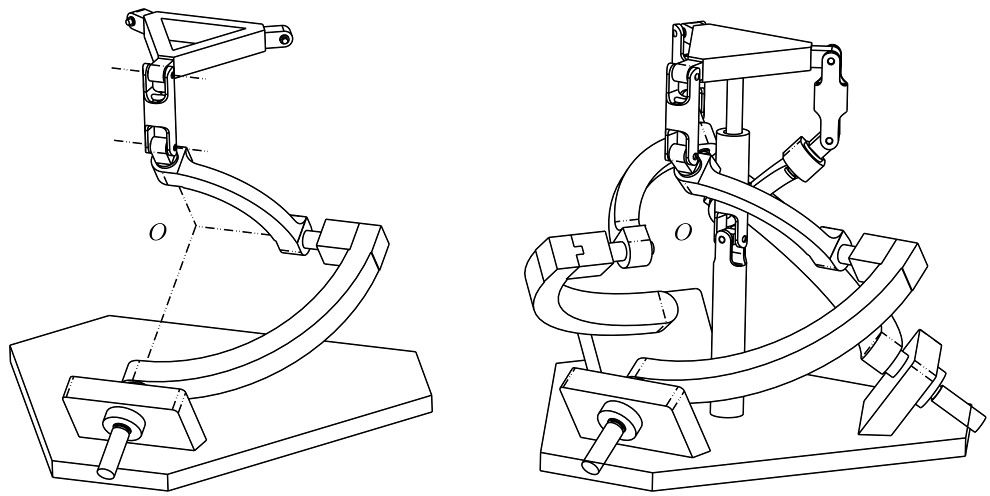
1 Introduction

Most of the work on parallel manipulators, both theoretical and applied, has been directed to platforms with six degrees of freedom as well as 3-DoF planar or spherical mechanisms. However, in recent years there has been increasing interest in mechanisms with fewer than six freedoms that are neither planar nor spherical. It is hoped that such mechanisms can perform successfully many tasks that have so far required 6-DoF platforms and achieve lower device and operation costs, due to simplified designs involving fewer links and actuators.

Many of the existing parallel devices with reduced platform freedoms achieve this reduction by adding one extra serial subchain (i.e., an additional leg) between the base and platform of a Gough-Stewart type parallel manipulator. Then, the mobility of the platform is equal to the degree of freedom of the new subchain. Such a mechanisms can be described as a serial manipulator with non-actuated joints and separate in-parallel actuation.

A major drawback of this approach is that a new mechanism is obtained by a complication (rather than the desired simplification) of the original architecture. Furthermore, the single serial subchain with fewer than six DoF has to bare all the load due to constraining the undesired freedoms of the platform. In practice, this leads to a bulky and expensive design of the extra chain and increased rather than lowered costs of the machine.

A more attractive approach would try to achieve the reduction in the platform freedoms using all serial chains, analogously to planar or spherical architectures. In the classical case of planar/spherical mechanisms a parallel manipulator has three legs and each leg is a 3-DoF serial planar/spherical subchain. Thus, removing all legs but one will leave the freedoms of the platform unchanged since each serial subchain is restricting the platform to the same three DoFs. In a more general parallel mechanism, different legs may block different freedoms, and only the combined effect of several subchains results in the desired space of feasible displacements of the platform. A number of such mechanisms have been proposed, the more successful among them being positioning mechanisms where the platform has three translational DoFs [1, 2, 3, 4].



(a) (b)

Figure 1. (a) A 5R serial subchain (b) a variant of the proposed architecture with three 5R legs and one RRC leg.

There exist a number of applications where the rotational freedoms of the platform are more important than the translational ones, e.g., flight simulation and device orientation [5]. For pure orientation, spherical parallel devices, such as the Agile Eye [6] or parallel wrists such as Argos [7] can be used. However, for tasks that require one translational motion of the platform in addition to the three rotations, there are no known parallel kinematic chains. The present paper fills this gap.

One major area of application of parallel manipulators is flight and motion simulation. For this type of application, rotational freedoms play a major role, while translations are of lesser importance. However, one translational freedom, the heave, is of great significance in flight simulation. (Unfortunately, even the 6-DoF Gough-Stewart platforms most commonly used as flight simulators cannot provide good heave simulation.) Hence, if one were to choose a subset of the platform freedoms for the purposes of flight simulation, a natural choice would be to keep the three rotational freedoms and only one translation. It can be expected, therefore, that the architectures proposed herein can be used in the design of new flight and motion simulators.

The organization of this paper is as follows. In the next section, we describe the new architecture with three different actuation schemes and analyze the global mobility of the mechanisms. Section 3 discusses the position kinematics, while Section 4 examines the velocity kinematics of the family of architectures. Section 5 presents our conclusions.

2 Description and Mobility of the Mechanism

The base and the mobile platform are connected by $m \geq 2$ serial subchains, each with five revolute joints. One such subchain, together with the base and the mobile platform, is shown in Figure 1(a). The first three joints (counting from the base) in each serial subchain form a 3R spherical chain, i.e., their axes intersect in one fixed point, O, the *rotation centre* common to all serial subchains. The remaining fourth and fifth revolute joints in each serial subchain form a planar 2R chain, i.e., the two revolute axes are parallel. Moreover, these last two joint axes in each serial subchain are also parallel to a chosen plane in the moving platform (the *platform plane*). It is also required that the m axes of the fifth joints (the joints on the mobile platform) are not all parallel, i.e., at least two of the m planar 2R subchains have different planes of motion (Figure 1(b), Figure 3).

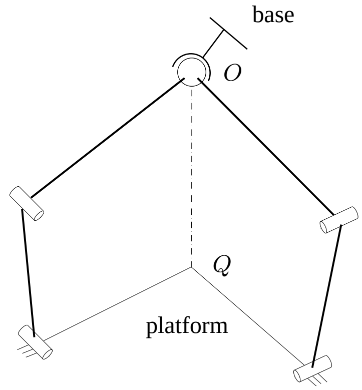


Figure 2. The inverted mechanism.

It may be easier to visualize the four degrees of freedom if we consider the inverted mechanism, i.e., if the mobile platform is assumed fixed and we consider the relative movement of the base of the mechanism. It is clear that the spherical part of the mechanism and, in particular, the common centre of intersection of the spherical revolutes, O, is attached to the mobile platform by m planar 2R chains. Each 2R chain restricts the rotation centre, O, to a plane perpendicular to the mobile platform. Since we have postulated that these m *heave planes* are not all identical and since their intersection is not empty (the mechanism has at least one configuration), they must all intersect in one straight line (*the platform axis*) perpendicular to the platform plane and, therefore, the rotation centre is restricted to move along this line. Hence, the possible motion of the base with respect to the mobile platform is composed of a translation perpendicular to the platform plane followed in series by a spherical wrist at O (Figure 2).

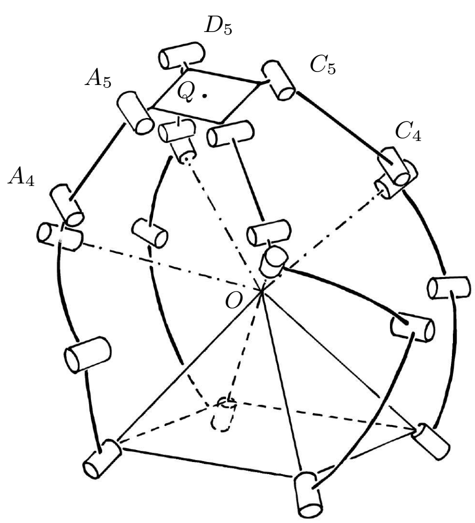


Figure 3. The parallel architecture with four 5R legs.

Returning to the original kinematic chain, it is now clear that the mobile platform can rotate arbitrarily about a point fixed in the base (the rotation centre O) and translate along a direction fixed in the platform (always perpendicular to the platform plane).

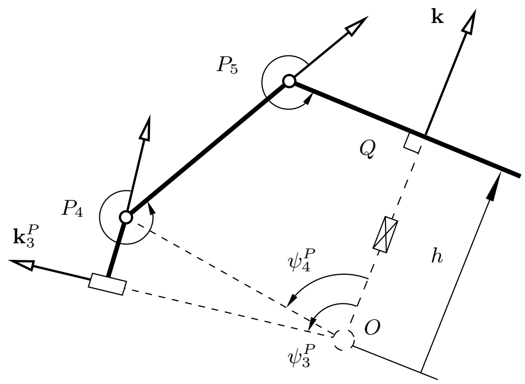


Figure 4. An equivalent mechanism in the heave plane of leg P.

From a purely geometric point of view the mechanism may have any number of legs not smaller than two. Since it is desirable to have the actuators at the base it is natural to have four serial subchains (Figure 3). To avoid excessive link interference and increase the rotational workspace, one of the four serial subchains can be designed with only 4 DoF, namely the three rotational and one translational freedoms of the platform, and placed in the middle between the other three legs. Such an architecture can be seen in Figure 1(b). The variant shown has a universal joint at the rotation centre, O, and a cylindrical joint along the platform axis. The fourth chain can, alternatively, have a spherical joint at the rotations centre O, and a prismatic actuator controlling the distance between O and the platform plane. This chain can also have the standard design of a 6-DoF (or 7-DoF) leg of a Gough-Stewart platform, the two extra revolutes at the platform remaining inactive since the leg will always be perpendicular to the platform plane. The mechanism can also be made with only three 5R legs as long as, in addition to the three base joints, any one other joint is actuated. We emphasize that the degree of freedom of the mechanism is determined only by (any) two 5R legs (with different heave planes), the addition of extra 5R chains or 4-DoF chains like the one in Figure 1(b) does not affect the mobility. Thus, the mechanism in Figure 1(b) will have the same 4-DoF with or without the RRC chain, with or without the third or an extra fourth 5R chain. With only two 5R legs we have a single-loop chain with 10 joints and 4 DoF and the Grübler-Kutzbach mobility criterion is satisfied. With additional legs the mechanism is overconstrained and the formula is violated.

We assume that the base reference frame has its origin at the rotation centre O. The reference frame of the mobile platform is positioned at the intersection, Q, of the platform plane and the platform axis. The mobile Qz axis is along the platform axis (and perpendicular to the platform plane) and the mobile coordinate unit vectors are $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$. We denote each of the 5R legs with one of the letters A, B, C, D and we label all notations associated with the serial subchain with the corresponding letter. At each joint, we define a reference frame (the link frame of the adjacent link closer to the platform) in a way consistent with the Denavit-Hartenberg formalism. If the frame's origin is not O we denote it by P_i, the coordinate unit vectors by $\mathbf{i}_i^P$, $\mathbf{j}_i^P$, $\mathbf{k}_i^P$ and the joint angles by θ_i^P, where $P = A, \ldots, D$, $i = 1, \ldots, 5$. We remind that the $\mathbf{k}_i^P$ vectors (the z axes of the link frames) are all along the axes of the joints. We choose the positive directions on the z axes from point O to the physical joints, for the first three frames, and counterclockwise in the platform plane (as seen from the end of $\mathbf{k}$), for the fourth and fifth frames.

3 Position Kinematics

3.1 Inverse kinematics

For the inverse kinematics problem, we assume that the pose $(\mathbf{R}, \mathbf{t})$ of the mobile platform is given, where $\mathbf{R}$ is the orthogonal rotation matrix with columns $\mathbf{i}, \mathbf{j}, \mathbf{k}$ and $\mathbf{t}$ is the vector $\overrightarrow{OQ}$. Since the Qz axis always passes through O, $\mathbf{t} = h\mathbf{k}$ and the pose of the platform is defined by $(\mathbf{R}, h)$. We need to find the configuration of the mechanism and, in particular, the joint angles of the actuated joints, θ_1^P, $P = A, \ldots, D$.

The distance, h, between O and Q determines the location with respect to the platform of the last three axes of each serial subchain. This relationship can be derived by analyzing the planar mechanism in Figure 4. This is an equivalent mechanism which describes the motion of the planar part of each subchain, P. The planar chain in the figure is a 1-DoF closed loop with four joints: the revolutes at P_4 and P_5, a prismatic along OQ and a spherical joint at O. The prismatic joint, the spherical joint and the links drawn with dashed lines are not a physical part of the spatial mechanism but represent the restrictions imposed by the other legs on the serial subchain P.

We note that in Figure 3 (as well as all other figures) and in the discussion of position kinematics we assume that all four points P_5 are in the platform plane. This is not a necessary requirement, and if the fifth joint axes are in different parallel planes, we will have different points Q_P and parameters h_P each differing from h by a constant.

For each value of h there are, in general, two possible positions of point P_4 in the platform, each described by the value of the angle $\psi_4^P = \angle QOP_4$:

$$\psi_4^P = \arccos \frac{h}{\sqrt{h^2 + r_P^2}} \pm \arccos \frac{R_P^2 + h^2 + r_P^2 - \ell_{45}^2}{2R_P\sqrt{h^2 + r_P^2}}\,, \quad P = A, \ldots, D\,, \tag{1}$$

where $R_P = |OP_4|$, $\ell_{45} = |P_4P_5|$ and r_P is the distance between P_5 and the Qz axis, $|QP_5|$ in Figure 3 (although, in general, the points P_5 need not be co-planar). There are two corresponding possible solutions for the vector $\mathbf{k}_3^P$ in the platform frame. Each is determined by the angle $\psi_3^P = \angle QOP$, which can be obtained from ψ_4^P by the addition of the constant angle α_{43}^P between $\overrightarrow{OP}_4$ and $\mathbf{k}_3^P$.

Now, the vector along the third joint axis can be obtained in the fixed frame by

$$\mathbf{k}_3^P = \mathbf{R}_z(\theta^P)\mathbf{R}_y(\psi_3^P)\,, \quad P = A, \ldots, D\,, \tag{2}$$

where θ^P is the angle at which we need to rotate the platform about its $\mathbf{k}$ axis in order for $\mathbf{i}$ to coincide with $\overrightarrow{QP}_5$. Since we need the sine and cosine functions of ψ_3^P rather than the angle itself we can use the following equation instead of Equation 1.

$$2\lambda(h)\cos\psi_3^P + 2\mu(h)\sin\psi_3^P = \nu(h)\,, \quad P = A, \ldots, D\,, \tag{3}$$

where

$$\begin{aligned} \lambda(h) &= h\cos\alpha_{43}^P - r_P\sin\alpha_{43}^P, \\ \mu(h) &= h\sin\alpha_{43}^P + r_P\cos\alpha_{43}^P, \\ \nu(h) &= h^2 + R_P^2 + r_P^2 - l_{45}^P. \end{aligned} \tag{4}$$

Once the vectors $\mathbf{k}_3^P$, $P = A, \ldots, D$, are known, the problem reduces to the inverse kinematics of a general spherical parallel manipulator. The vector, $\mathbf{k}_2^P$, along the second joint axis of the subchain P can be obtained from the condition that it forms constant angles with $\mathbf{k}_1^P$ and $\mathbf{k}_3^P$. Geometrically, $\mathbf{k}_2^P$ lies along the intersection of two cones with axes along $\mathbf{k}_1^P$ and $\mathbf{k}_3^P$. Algebraically, finding θ_1^P amounts to solving a quadratic equation and, in general, there are two solutions for $\mathbf{k}_2^P$ for each given $\mathbf{k}_3^P$ [6]. Therefore, the inverse kinematics for each 5R subchain has a maximum of four solutions and the inverse kinematics of the mechanism as a whole has 64 solutions at most. The number is reduced to only 8 if the 2R planar parts allow only one solution for $\mathbf{k}_3^P$.

3.2 Direct kinematics

The direct kinematics problem consists in finding the pose $(\mathbf{R}, h)$ of the platform when the values of the joint variables of the actuated joints are given. The direct kinematics of the mechanism is dependent on the method of actuation, i.e., on which of the versions of the mechanism (discussed in Section 2) is chosen.

3.2.1 One prismatic actuator

This is the easiest case. The joint variable of the prismatic actuator provides the value of h. This in turn determines the configuration of the equivalent mechanism in Figure 4, up to a sign. Hence the location of the third joint axes, $\mathbf{k}_3^P$, $P = A, B, C$, are known in the frame of the platform. Then, the problem reduces to the direct kinematics problem of a spherical parallel manipulator. As shown in [6], this problem has a maximum of 8 solutions. Therefore the direct kinematics of the 4-DoF mechanism has at most 64 solutions, just as the inverse kinematics.

3.2.2 Four 5R chains

The values of θ_1^P, $P = A, \ldots, D$, are given. This defines the second joint axes, $\mathbf{k}_2^P$. Then, the vectors of the third joint axes must satisfy

$$\mathbf{k}_2^P \cdot \mathbf{k}_3^P = \cos \alpha_{23}^P \,, \;\; P = A, \ldots, D \,, \tag{5}$$

where α_{23}^P are the the constant angles between the second and third joints. Moreover, the angles between $\mathbf{k}$ and each of the vectors $\mathbf{k}_3^P$ depend only on h and not on the platform's orientation:

$$\mathbf{k} \cdot \mathbf{k}_3^P = \cos \psi_3^P \,, \;\; P = A, \ldots, D \,, \tag{6}$$

the relationship between h and $\cos \psi_3^P$ being given by Equation (3).

Furthermore, the angles between the heave planes of each leg are constant (since these planes are fixed in the platform) and this gives $m - 1 = 3$ independent equations for the angles between the vectors $\mathbf{k}_3^P$ for different P:

$$\mathbf{k}_3^A \cdot \mathbf{k}_3^P = \cos \psi_3^A \cos \psi_3^P + \sin \psi_3^A \sin \psi_3^P c_{AP} \,, \;\; P = B, C, D \,, \tag{7}$$

where c_{AP} is the cosine of the constant angle between the heave planes of legs A and P. Equation (7) was obtained by decomposing each vector $\mathbf{k}_3^P$ into a component in the platform plane and another along the platform axis.

Equations (5), (6), (7) provide 11 algebraic equations for the 16 unknowns h, $\mathbf{k}$, $\mathbf{k}_3^P$. The remaining 5 equations are the conditions for unit length of the unknown vectors. All these equations have to be solved together with (3) which relate $\cos \psi_3^P$ and $\sin \psi_3^P$ to h. At this time, it is not known what is the maximum number of solutions of this problem.

3.2.3 Three 5R chains

Let the fourth actuator control θ_2^A. This means that the vector $\mathbf{k}_3^A$ is known. Then, Equations (5) and (7) for $P = A, B$, as well as (6) for $P = A, B, C$, give us 7 different equations for the 10 unknowns h, $\mathbf{k}$, $\mathbf{k}_3^P$, $P = A, B$.

4 Velocity Kinematics

In this section we assume that the mechanism is in a known configuration. For the purposes of velocity analysis, we choose the instantaneous reference frame $O\mathbf{ijk}$. The origin is fixed in the base while the coordinate vectors are constant in the mobile platform. For a chosen Cartesian frame in space we associate a standard basis, $\{\boldsymbol{\varrho}_x, \boldsymbol{\varrho}_y, \boldsymbol{\varrho}_z, \boldsymbol{\tau}_x, \boldsymbol{\tau}_y, \boldsymbol{\tau}_z\}$, in the six-dimensional space of twists, $\mathcal{S}$. The elements of this basis are the three rotations and three translations about the coordinate axes. The coordinates of a twist, $\boldsymbol{\xi} = (\omega_x, \omega_y, \omega_z, v_x, v_y, v_z)$, in the standard basis are given by

$$\omega_\sigma = \boldsymbol{\tau}_\sigma \circ \boldsymbol{\xi}\,; \;\; v_\sigma = \boldsymbol{\varrho}_\sigma \circ \boldsymbol{\xi}\,, \quad \sigma = x, y, z\,, \tag{8}$$

where "$\circ$" is the reciprocal screw product. Note that in (8) the rotational coordinates are generated by (the wrenches along) the translational basis screws and vice versa.

Below, we perform our analysis assuming four 5R chains. At the end of the section we make specific comments on the differences occurring with the other versions of the architecture.

4.1 The screw system of the platform twists

The relationship between the instantaneous motion of the platform, the *output twist* $\boldsymbol{\xi} = (\boldsymbol{\omega}, \mathbf{v})$, and the joint velocities $\dot{\theta}_i^P$ is given by the twist equations of the serial subchains:

$$\boldsymbol{\xi} = \sum_{i=1}^{5} \dot{\theta}_i^P \boldsymbol{\xi}_i^P\,, \quad P = A, \ldots, D\,, \tag{9}$$

where the joint screws are denoted by $\boldsymbol{\xi}_i^P$.

Equation (9) is a necessary and sufficient condition for the output twist, $\boldsymbol{\xi}$, and the joint velocities, $\dot{\theta}_i^P$, to be feasible. Therefore, the space of all the possible platform twists, $\mathcal{T}$, is given by

$$\mathcal{T} = \bigcap_{P=A}^{D} \mathcal{T}_P\,, \tag{10}$$

where $\mathcal{T}_P$ is the output twist space of the subchain P,

$$\mathcal{T}_P = \text{Span}\,(\boldsymbol{\xi}_1^P, \ldots, \boldsymbol{\xi}_5^P)\,, \quad P = A, \ldots, D\,. \tag{11}$$

For any subspace, $\mathcal{L}$, of vector space $\mathcal{S}$, let $\mathcal{L}^\perp$ be the reciprocal companion of $\mathcal{L}$,

$$\mathcal{L}^\perp = \{\boldsymbol{\rho} \mid \boldsymbol{\rho} \circ \boldsymbol{\xi} = 0 \;\; \forall \boldsymbol{\xi} \in \mathcal{L}\}\,, \tag{12}$$

Since $(\mathcal{L}^\perp)^\perp = \mathcal{L}$, Equation (10) implies

$$\mathcal{T}^\perp = \sum_{P=A}^{D} \mathcal{T}_P^\perp\,. \tag{13}$$

It is easy to see that, when $\boldsymbol{\xi}_1^P, \ldots, \boldsymbol{\xi}_5^P$ are linearly independent, the 1-dimensional space $\mathcal{T}_P^\perp$ is spanned by the screw $\boldsymbol{\varrho}^P = (\mathbf{k}_4^P, \mathbf{0})$, i.e., a rotation with an axis through O and perpendicular to the heave plane of the subchain. If the joint screws of the subchain are linearly dependent, $\mathcal{T}_P^\perp$ will be of dimension at least two and will contain screws outside Span $(\boldsymbol{\varrho}^P)$. Physically, the reciprocal screws in $\mathcal{T}_P^\perp$ represent wrenches (a pure force in the case of $\boldsymbol{\varrho}^P$) which, if applied to the platform, can be resisted by the subchain P with zero torque from the actuator.

From (13) it follows that $\mathcal{T}^\perp$ includes at least $\mathcal{T}_{max}^\perp = \text{Span}\,(\boldsymbol{\varrho}^A, \ldots, \boldsymbol{\varrho}^D)$. The vectors $\mathbf{k}_4^P$ are all coplanar and not all parallel (Section 2), hence $\mathcal{T}_{max}^\perp$ is a planar pencil of rotations through O in a plane parallel to the platform (first special 2-system with zero pitch, [8].

$$\mathcal{T}_{max}^\perp = \text{Span}\,(\boldsymbol{\varrho}_x, \boldsymbol{\varrho}_y)\,. \tag{14}$$

Since $\mathcal{T}_{max}$ is a subspace reciprocal to $\mathcal{T}_{max}^\perp$, it must be a twist subspace of dimension four spanned by three pure rotations through O and a pure translation perpendicular to the plane of the planar pencil, i.e.,

$$\mathcal{T}_{max} = \text{Span}\,(\boldsymbol{\varrho}_x, \boldsymbol{\varrho}_y, \boldsymbol{\varrho}_z, \boldsymbol{\tau}_z)\,. \tag{15}$$

When all serial subchains are nonsingular, i.e., their five screws are linearly independent, the output twist space of the manipulator is $\mathcal{T} = \mathcal{T}_{max}$. Otherwise, $\mathcal{T}$ can be a smaller subspace of $\mathcal{T}_{max}$.

This confirms our conclusions in Section 2 that the feasible motions of the platform are three rotations and one translation.

4.2 The velocity equations

The 24 scalar equations in (9) are not all independent. (If they were, the instantaneous mobility of the mechanisms would be zero). The output twist needs to be in $\mathcal{T}_{max}$ and hence reciprocal to $\boldsymbol{\varrho}_x$ and $\boldsymbol{\varrho}_y$. This means that, in the chosen coordinate system, the v_x and v_y coordinates of $\boldsymbol{\xi}$ are both zero. The same is true for the first three joint screw, $\boldsymbol{\xi}_i^P$, $i = 1, 2, 3$ (all their translational coordinates are zeroes since their axes are through O). Thus, in the scalar equations for the v_x and v_y coordinates in (9) there will be only terms from the 4th and 5th joint twists:

$$0 = \boldsymbol{\varrho}_\sigma \circ (\boldsymbol{\xi}_4^P \dot{\theta}_4^P + \boldsymbol{\xi}_5^P \dot{\theta}_5^P)\,,\ \sigma = x, y\,. \tag{16}$$

In other words, the translation of the origin caused by these two twists must be perpendicular to the platform plane. The two equations in (16) are linearly dependent and equivalent to the single equation for each leg P:

$$0 = (\mathbf{k} \cdot \mathbf{r}_4^P)\,\dot{\theta}_4^P + (\mathbf{k} \cdot \mathbf{r}_5^P)\,\dot{\theta}_5^P\,,\ \ P = A, \ldots, D\,, \tag{17}$$

where $\mathbf{r}_i^P = \overrightarrow{OP_i},\ i = 4, 5$.

Now we can specify a minimal system of equations, which determines the velocities in every configuration. It consists of the $4 \times 4 = 16$ equations for the ω_x, ω_y, ω_z and v_z coordinates in (9) together with the 4 equations (17). We have 20 equations for the 20 joint velocities and 4 components of the output twist. The 4 equations for every P in (9) can be written as

$$\mathbf{s} = \sum_{i=1}^{5} \dot{\theta}_i^P \mathbf{s}_i^P\,,\quad P = A, \ldots, D\,, \tag{18}$$

where the 4-vectors $\mathbf{s}$, $\mathbf{s}_i^P$ are formed by the ω_x, ω_y, ω_z and v_z coordinates of the corresponding twists $\boldsymbol{\xi}$, $\boldsymbol{\xi}_i^P$. The 20 equations (17) and (18) are a necessary and sufficient condition for the 24 joint and output velocities to be feasible.

Equation (17) implies that the joint velocities of the 4th and 5th joint velocities are not independent unless both points P_4 and P_5 are in the Oxy plane. Note that such a configuration is possible only if the links are specially proportioned. Also, this would be a singularity of increased instantaneous mobility [9] and the mechanism as a whole will instantaneously have more than 4 DoF. If we assume that in the given configuration this is not the case, then we can further simplify the system of velocity equations.

Equations (16) show that the combined twist due to the fourth and fifth joints, $\boldsymbol{\xi}_4^P \dot{\theta}_4^P + \boldsymbol{\xi}_5^P \dot{\theta}_5^P$, is in $\mathcal{T}_{max}$. However, when P_4 and P_5 are not both in Oxy, there is only one screw, $\boldsymbol{\xi}_{45}^P$, which is both in Span$\,(\boldsymbol{\xi}_4^P, \boldsymbol{\xi}_5^P)$ and $\mathcal{T}_{max}$. This is the rotation about the intersection line of the Oxy plane and the plane defined by $\boldsymbol{\xi}_4^P$ and $\boldsymbol{\xi}_5^P$ (Figure 5). When the two planes are parallel, $\boldsymbol{\xi}_{45}^P$ is a translation perpendicular to both planes, i.e., $\boldsymbol{\tau}_z$.

Defining the 4-vector $\mathbf{s}_{45}$ from $\boldsymbol{\xi}_{45}^P$ by suppressing two of the coordinates as above, we can write the velocity equations of the mechanism as

$$\mathbf{s} = \sum_{i=1}^{3} \dot{\theta}_i^P \mathbf{s}_i^P + \dot{\theta}_{45}^P \mathbf{s}_{45}^P\,,\quad P = A, \ldots, D\,, \tag{19}$$

where $\mathbf{s}_{45}^P = (\mathbf{k}_4^P, r_{45}^P)$ and $\dot{\theta}_{45}^P = \dot{\theta}_4^P + \dot{\theta}_5^P$ when $\overrightarrow{P_4P_5}$ intersects Oxy, while $\mathbf{s}_{45}^P = (\mathbf{0}, 1)$ and $\dot{\theta}_{45}^P = l_{45}\dot{\theta}_4^P = -l_{45}\dot{\theta}_5^P$ when $\overrightarrow{P_4P_5}$ is parallel to Oxy. Here r_{45}^P is the distance from O to the line of intersection, with the positive direction chosen along the projection of $\overrightarrow{OP_5}$ on Oxy.

Equation (19) shows that the velocity kinematics of the studied architecture is described by an instantaneously equivalent parallel mechanism. The equivalent mechanism has four legs with four joints each, with joint screws $\boldsymbol{\xi}_1^P, \boldsymbol{\xi}_2^P, \boldsymbol{\xi}_3^P, \boldsymbol{\xi}_{45}^P$. All joint screws as well as the platform twist are in the 4-dimensional space $\mathcal{T}_{max} = $ Span$\,(\boldsymbol{\varrho}_x, \boldsymbol{\varrho}_y, \boldsymbol{\varrho}_z, \boldsymbol{\tau}_z)$. In other words we have an instantaneous 4-dimensional analogue of planar and spherical mechanisms, where all twists stay always within a 3-dimensional twist space. The twists and wrenches involved in the velocity and singularity analysis of such mechanisms with $n < 6$ DoF can be

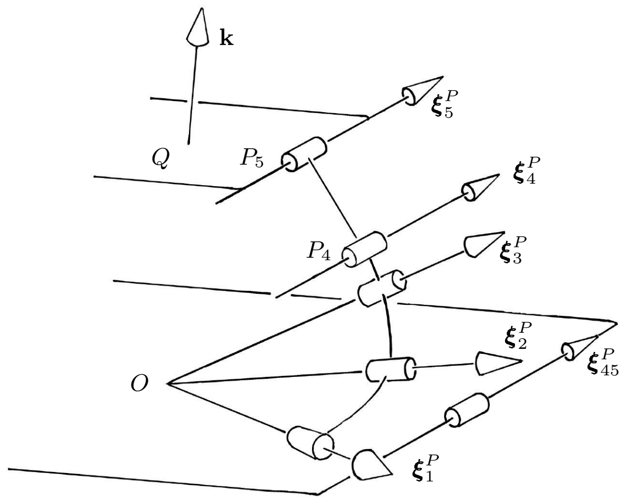


Figure 5. The joint screws of leg P.

treated as n-dimensional and the Jacobian matrices involved are $n \times n$ [9, 10]. Velocity analysis in such cases amounts to an n-dimensional version of screw calculus. However, one needs to remember that when using reciprocal screws (i.e., wrenches) one needs to consider a different set of n coordinates.

4.3 The input-output velocity equation

We can now proceed to eliminate the passive joint velocities from Equation (19) and obtain a linear relationship between the input and output velocities. For each leg, we find a screw (interpreted as a wrench), $\mathbf{w}_1^P$, reciprocal to all joint screws except the active-joint screw $\mathbf{s}_1^P$. The wrench can be considered 4-dimensional because we are interested only in screws with ω_x and ω_y coordinates equal to zero. (These two coordinates are of no interest since the screws in Span $(\boldsymbol{\varrho}_x, \boldsymbol{\varrho}_y)$ are reciprocal to *all* twists in $\mathcal{T}_{max}$.). The coordinates of such a 4-dimensional wrench are

$$\mathbf{w} = (f_z, \mathbf{m}) = (f_z, m_x, m_y, m_z), \tag{20}$$

Given a twist in $\mathcal{T}_{max}$, $\mathbf{s}_1^P = (\omega_x, \omega_y, \omega_z, v_z) = (\boldsymbol{\omega}, v_z)$, the reciprocal product is given by

$$\mathbf{w} \circ \mathbf{s} = f_z v_z + \mathbf{m} \cdot \boldsymbol{\omega}\,, \tag{21}$$

When the passive joint screws in the serial subchain are linearly independent, the underlying screw of the wrench $\mathbf{w}_1^P$ is determined in a unique way. When $r_{45}^P < \infty$ and $\mathbf{s}_{45}^P$ is a rotation we have:

$$\mathbf{w}_1^P = (-\frac{1}{r_{45}^P}\mathbf{k}_2^P\mathbf{k}_3^P\mathbf{k}_4^P \;\;,\;\; \mathbf{k}_2^P \times \mathbf{k}_3^P)\,, \quad P = A, \ldots, D\,, \tag{22}$$

When $r_{45}^P = \infty$, i.e. $\mathbf{s}_{45}^P$ is a translation, $\mathbf{w}_1^P$ becomes:

$$\mathbf{w}_1^P = (0, \mathbf{k}_2^P \times \mathbf{k}_3^P)\,, \quad P = A, \ldots, D\,, \tag{23}$$

We take the reciprocal product of $\mathbf{w}_1^P$ with Equation (19) and obtain the input output velocity equation

$$\begin{bmatrix} (\mathbf{k}_2^A \times \mathbf{k}_3^A)^T & -\frac{1}{r_{45}^A}\mathbf{k}_2^A\mathbf{k}_3^A\mathbf{k}_4^A \\ (\mathbf{k}_2^B \times \mathbf{k}_3^B)^T & -\frac{1}{r_{45}^B}\mathbf{k}_2^B\mathbf{k}_3^B\mathbf{k}_4^B \\ (\mathbf{k}_2^C \times \mathbf{k}_3^C)^T & -\frac{1}{r_{45}^C}\mathbf{k}_2^C\mathbf{k}_3^C\mathbf{k}_4^C \\ (\mathbf{k}_2^D \times \mathbf{k}_3^D)^T & -\frac{1}{r_{45}^D}\mathbf{k}_2^D\mathbf{k}_3^D\mathbf{k}_4^D \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ v_z \end{bmatrix} = \begin{bmatrix} \mathbf{k}_1^A\mathbf{k}_2^A\mathbf{k}_3^A & 0 & 0 & 0 \\ 0 & \mathbf{k}_1^B\mathbf{k}_2^B\mathbf{k}_3^B & 0 & 0 \\ 0 & 0 & \mathbf{k}_1^C\mathbf{k}_2^C\mathbf{k}_3^C & 0 \\ 0 & 0 & 0 & \mathbf{k}_1^D\mathbf{k}_2^D\mathbf{k}_3^D \end{bmatrix} \begin{bmatrix} \dot\theta_1^A \\ \dot\theta_1^B \\ \dot\theta_1^C \\ \dot\theta_1^D \end{bmatrix} \tag{24}$$

where the terms with r_{45}^P become 0 when $r_{45}^P = \infty$. This equation is equivalent to (19) when the passive joint screws in a subchain, $\mathbf{s}_2^P$, $\mathbf{s}_3^P$ and $\mathbf{s}_{45}^P$, are linearly independent [9]. Then, (24) can be used for the singularity analysis of the mechanism [11, 9].

4.4 Alternative versions of the architecture

When the fourth chain has 4 DoF, the fourth equation in (19) is the twist equation of the subchain and the fourth equation in (24) is simply $v_z = \dot{h}$.

When the fourth actuator is on $\boldsymbol{\xi}_2^A$, (19) has only 3 screw equalities. The input-output equation (24) has a different fourth equation given by

$$[(\mathbf{k}_1^A \times \mathbf{k}_3^A)^T, -\frac{1}{r_{45}^P}\mathbf{k}_2^A\mathbf{k}_3^A\mathbf{k}_4^A]^T \mathbf{s} = \mathbf{k}_1^A\mathbf{k}_2^A\mathbf{k}_3^A\dot{\theta}_2^A \,. \qquad (25)$$

5 Conclusions

In this paper, we presented a family of new 4-DoF parallel architectures. The equations describing the position and velocity kinematics of the mechanisms were derived. The proposed architectures make it possible to supplement an orientational parallel structure with a translational motion fixed in the mobile frame, which is ideal for motion simulation.

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