A SIMPLE ROBUST CONTROLLER TO STABILISE AN ANAEROBIC DIGESTION PROCESS

L. Mailleret, O. Bernard

COMORE, INRIA, BP93, 06 902 Sophia-Antipolis cedex, France
Ludovic.Mailleret, Olivier.Bernard@inria.fr

Abstract: In this paper we propose a simple controller to assign the Chemical Oxygen Demand (COD) of an anaerobic digester. We first present the model of the anaerobic digestion based on a two step description: acidogenesis and methanogenesis. In order to introduce the main ideas, we discuss a simplified model and propose a controller and an observer. Then we show that the principles still apply to the whole system. We derive therefore a controller to regulate the effluent COD. The robustness of the controller is illustrated by a simulation study assuming noisy measurements.

Keywords: anaerobic digestion, mass balance model, controller, observer.

1. INTRODUCTION

Control of bioreactors is a delicate problem since most of the time the available biological models are only rough approximations. Indeed, on the contrary to other domains (physics...) that use laws that have been tested and validated for centuries, the biological systems are known to be highly variable, difficult to measure so that no strong biological law is available. To circumvent this difficulty, Bastin and Dochain (Bastin and Dochain, 1990) have introduced the mass balance based modelling. The main idea of this approach is to design estimators and controllers independently of any modelling of the biological kinetics.

Among the bioreactors, those dedicated to wastewater treatment especially suffer from the modelling uncertainties. A complex ecosystem composed by many different bacterial populations takes place in these processes, and the composition and concentration of the pollutant to degrade is not well known and evolves with respect to time. Moreover, most of the time no measurement of the involved chemical or biological species is available. In these conditions, a control procedure that would guarantee the process stability should be as insensitive as possible to all these parameters.

We will consider an anaerobic wastewater treatment process. Anaerobic digestion is a biological process in which biodegradable organic materials are decomposed in the absence of oxygen to produce methane. The underlying model assumes that two main bacterial populations are present (Bernard *et al.*, 2000). The first one, the acidogenic bacteria X_1, consumes the organic substrate S_1 and produces through an acidogenesis step volatile fatty acids (VFA) S_2. The second population (methanogenic bacteria) X_2, uses the VFA in a methanogenesis step as substrate for growth and produces methane.

Despite its capacity to degrade difficult substrates, this process is known to become unstable under certain circumstances, and requires therefore a monitoring procedure to detect a destabilisation. This must also be associated to a control action that can avoid the risk of acidification of the fermenter.

The paper is organised as follows: we first present a model of the anaerobic digestion in dimension 4. Then we simplify it through a model (in dimension 2) that contains the main features. We develop then a state estimator for this model and we propose a controller that globally stabilises the process. Then we consider the more detailed process model (in dimension 4), and we show how to extend the previous results. We prove that the control law globally stabilises the extended model. Finally we use the control action to estimate some parameters in order to better tune the controller.

2. THE MASS BALANCE MODEL

The dynamical model of the anaerobic digestion process can be deduced from mass balance considerations based on the following reaction scheme (Bernard *et al.*, 1999):

- Acidogenesis:

$$k_1 S_1 \xrightarrow{r_1(.)} X_1 + k_2 S_2$$

  The reaction rate is given by: $r_1(.) = \mu_1(.)X_1$
- Méthanogenesis:

$$k_3 S_2 \xrightarrow{r2(.)} X_2$$

  The reaction rate is given by: $r_2(.) = \mu_2(.)X_2$

The model can be written as follows:

$$\begin{cases} \dot{X}_1 = \mu_1(.)X_1 - \alpha D X_1 \\ \dot{S}_1 = -k_1\mu_1(.)X_1 + D(S_{1in} - S_1) \\ \dot{X}_2 = \mu_2(.)X_2 - \alpha D X_2 \\ \dot{S}_2 = -k_3\mu_2(.)X_2 + k_2\mu_1(.)X_1 + D(S_{2in} - S_2) \end{cases} \tag{1}$$

where D is the dilution rate, the terms S_{1in} and S_{2in} are the influent concentrations of S_1 and S_2 respectively. The k_i represent the yield coefficients associated with bacterial growth, $\mu_1(.)$ and $\mu_2(.)$ are the specific growth rates of X_1 and X_2 (they depend on some variables that are not explicited here and this point will be discussed later on).

The parameter α ($0 \leq \alpha \leq 1$) represents the percentage of bacteria that are not fixed on the support, and therefore which are affected by the dilution effect: $\alpha = 0$ corresponds to an ideal fixed bed reactor, $\alpha = 1$ corresponds to an ideal continuous stirred tank reactor (CSTR).

The methane solubility is very low, and therefore the methane produced by the methanogenesis step is not stored in the liquid phase. The methane flow rate (denoted q_{CH_4}) can then be written as a function of the state as follows:

$$q_{CH_4} = k_4\mu_2(.)X_2$$

The parameter k_4 corresponds to the yield of methane production and is known. The methane flow rate is on-line measured, let us denote $y = \frac{q_{CH_4}}{k_4}$.

We denote $\xi = (X_1, S_1, X_2, S_2)^T$ the state vector.

The most crucial problem in solving equations (1) is the formulation of reasonable expressions for the corresponding specific growth rates: $\mu_1(.)$ and $\mu_2(.)$. As noted by (Bastin and Dochain, 1990), a large number of models has been proposed to describe bioprocesses kinetics.

In the literature μ_1 has often been represented by a Monod type kinetics and μ_2 was represented by a Haldane kinetics with respect to S_2 that represents the inhibition effect of the substrate (the VFA) on the bacterial growth. Here we will assume that these growth rates are unknown. We only make the following assumptions:

Hypothesis 1. (H1). We assume that:

- μ_1 is a function of S_1, $\mu_1(0) = 0$ and $\mu_1 > 0$ when $S_1 > 0$
- μ_2 is a function of S_2, $\mu_2(0) = 0$ and $\mu_2 > 0$ when $S_2 > 0$

It can be verified that [H1] ensures that the model variables remain positive: indeed the following properties hold for any state variable ξ_i:

$$\text{For } \xi_i = 0, \text{ we have } \dot{\xi}_i \geq 0$$

Before introducing the controller for the whole model, we propose to study a simplified one.

3. STUDY OF A SIMPLIFIED MODEL OF THE ANAEROBIC DIGESTION

3.1 *Simplified model presentation*

In this section we present a simplified model of the anaerobic digestion in order to introduce our controller. This model contains the main features that explain the destabilisation by acid accumulation: the growth rate $\mu_2(S_2)$ is not monotonic and represents both the growth and the inhibition effect of acid accumulation. It can also be seen as a simplification of model (1) if $\mu_1(.)$ is much larger than $\mu_2(.)$ using the singular pertubation theory (Khalil, 1992). This two dimensional model has two locally stable equilibria. Note that the system is unstable because of the inhibition effect of acid accumulation.

$$\begin{cases} \dot{X}_2 = (\mu_2(S_2) - \alpha D)X_2 \\ \dot{S}_2 = D(S_{2in} - S_2) - k\mu_2(S_2)X_2 \end{cases} \tag{2}$$

The yield coefficient k represents the substrate degradation yield.

3.2 *Asymptotic observer design*

In order to monitor the process, we propose here to build a software sensor that will on line estimate, on the basis of the methane flow rate measurements, the VFA and the biomass in the digester.

Let us rewrite the model (2) using the measurement y:

$$\begin{cases} \dot{X}_2 = y - \alpha D X_2 \\ \dot{S}_2 = D(S_{2in} - S_2) - ky \end{cases} \tag{3}$$

Now we propose the following simple estimates $\hat{S}_2$ and $\hat{X}_2$ for S_2 and X_2, respectively:

$$\begin{cases} \dot{\hat{X}}_2 = y - \alpha D \hat{X}_2 \\ \dot{\hat{S}}_2 = D(S_{2in} - \hat{S}_2) - ky \end{cases} \tag{4}$$

Indeed, it is straightforward to see that the equation error

$$e = (e_1,\ e_2)^T = \left(X_2 - \hat{X}_2,\ S_2 - \hat{S}_2\right)^T$$

satisfies:

$$\dot{e} = \left(\alpha D(t)(\hat{X}_2 - X_2),\ D(t)(\hat{S}_2 - S_2)\right)^T$$

and therefore:

$$\dot{e} = -D(t)\begin{pmatrix} \alpha & 0 \\ 0 & 1 \end{pmatrix} e$$

If $D(t)$ satisfies some good properties (*e.g.* $D(t) \geq D_{min} > 0$), then this proves that the error tends toward zero: the estimate converges.

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This observer will be used in the practical implementation of the controller to monitor the VFA concentration and to check that the controller works properly. An important point is that the controller proposed in the sequel will not use the observer estimates.

3.3 *The asymptotic controller design*

The objective of this section is to design a feedback that uses the measurement y to maintain the VFA concentration at a reference value $S_2^\star$. More precisely, we choose the following dynamics for S_2:

$$\dot{S}_2 = D(t)(S_2^\star - S_2) \tag{5}$$

Note that, if $D(t) \geq D_{min} > 0$, this guarantees the convergence of S_2 toward $S_2^\star$.

The chosen dynamics differ from the usual linearising control (Bastin and Dochain, 1990) as it has been for example applied to anaerobic digestion (Perrier and Dochain, 1993; Dochain and Perrier, 1993). Indeed, here the dynamics can not be tuned (it is asymptotic). But, as we will see in the sequel, it leads to a more robust controller.

If we confront (5) with (3), this imposes:

$$D(t) = \frac{k}{S_{2in} - S_2^\star} y \tag{6}$$

Now our result follows:

Lemma 1. For any positive initial condition the control law (6) ensures that S_2 tends asymptotically toward $S_2^\star$

Proof of Lemma 1: First let us take a small $\epsilon > 0$ such that $X_2(0) > \epsilon$ and $S_2(0) > \epsilon$. We first show that the following set:

$$I_\epsilon = \{(S_2, X_2),\ S_2 > \epsilon,\ X_2 > \epsilon\}$$

is positively invariant. This is easily proved by considering the flow on the boundaries:

- For $X_2 = \epsilon$, and $S_2 \geq \epsilon$:

  $$\dot{X}_2(X_2 = \epsilon) = \mu_2(S_2)\epsilon\left(1 - \frac{\alpha k \epsilon}{S_{2in} - S_2^\star}\right)$$

  Let $X_2^\star = \frac{S_{2in} - S_2^\star}{\alpha k}$, then, if $\epsilon < X_2^\star$, we have $\dot{X}_2 > 0$.
- For $S_2 = \epsilon$ and $X_2 > \epsilon$, we show in the same way that if $\epsilon < S_2^\star$ then $\dot{S}_2 > 0$.

  Finally, we choose: $0 < \epsilon < \min(S_2^\star,\ X_2^\star)$.

We have shown that $D(t) = \mu_2(S_2)X_2 > 0$, consequently the observer (4) applied to the system (2) with the control law (6) converges towards the values of X_2 and S_2.

Now let us consider the following Lyapunov function:

$$V_1(\xi) = \tfrac{1}{2}(S_2^\star - S_2)^2 + \tfrac{1}{2}(X_2^\star - X_2)^2$$

$$\dot{V}_1(\xi) = -y\left(\frac{k}{S_{2in} - S_2^\star}(S_2^\star - S_2)^2 + \tfrac{1}{X_2^\star}(X_2^\star - X_2)^2\right)$$

Then $\dot{V}_1 \leq 0$ on the domain I_ϵ, and $\dot{V}_1 = 0$ only for $(S_2^\star,\ X_2^\star)$. It demonstrates the global asymptotic stability of the point $(S_2^\star,\ X_2^\star)$.

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4. STUDY OF THE 4 DIMENSION MODEL

4.1 *Control objectives*

Now we consider the dimension 4 model given by equations (1) and we want to choose and maintain constant the total amount of organic substrate in the fermenter. Let us denote $\tilde{S}_1 = \frac{k_1}{k_2} S_2$. It is the amount of COD needed (according to the yield coefficients) to obtain the quantity of VFA S_2, after the biological degradation of $\tilde{S_1}$ by the acidogenic bacteria. So we introduce the variable S_T that is an equivalent of the total amount of organic substrate in the fermenter:

$$S_T = S_1 + \frac{k_1}{k_2} S_2$$

We want then to assign S_T to a chosen $S_T^\star < S_{Tin}$, where $S_{Tin} = S_{1in} + \frac{k_1}{k_2} S_{2in}$. $S_T^\star$ is the acceptable level of pollution in the outflow of the bioreactor.

Now we rewrite system (1) using the new variable S_T, and injecting the output y:

$$\begin{cases} \dot{X_1} = (\mu_1(S_1) - \alpha D) X_1 \\ \dot{S_1} = D(S_{1in} - S_1) - k_1 \mu_1(S_1) X_1 \\ \dot{X_2} = y - \alpha D X_2 \\ \dot{S_T} = D(S_{Tin} - S_T) - \dfrac{k_3 k_1}{k_2} y \end{cases} \tag{7}$$

4.2 *Observer design*

In the model (7), the equations for S_T and X_2 are the same as equations (3). Therefore the observer designed for the simplified model can be used without any modification to estimate X_2 and S_T. Remark that $S_1 \le S_T$ and $S_2 \le \frac{k_2}{k_1} S_T$. Thus, the estimate of the total COD in the digester provides us an upper bound for S_1 and S_2.

4.3 *Controller design*

Let us look for $D(t) > 0$ ensuring, with a chosen $S_T^\star$:

$$\dot{S_T} = D(t)(S_T^\star - S_T)$$

it leads to

$$D(t) = \frac{k_3 k_1}{k_2 (S_{Tin} - S_T^\star)} y(t) \tag{8}$$

Notations: Now $X_2^\star = \frac{k_2(S_{Tin} - S_T^\star)}{\alpha k_3 k_1}$, so with the control law (8), $\dot{X_2} = \alpha D(t)(X_2^\star - X_2)$.

Let us define the following function:

$$\tilde{D}(S_1) = \frac{k_3 k_1}{k_2 (S_{Tin} - S_T^\star)} \mu_2 \big(\frac{k_2}{k_1} (S_T^\star - S_1) \big) X_2^\star$$

for the considered value of $S_{Tin}, S_T^\star$ and $X_2^\star$. Note that $\tilde{D}(S_1)$ is $D(t)$ computed for $X_2 = X_2^\star$ and $S_T = S_T^\star$.

We denote also: $\tilde{\mu}_1(S_1) = \frac{\mu_1(S_1)}{\tilde{D}(S_1)}$.

Lemma 2. For any positive initial condition the control law (8) ensures that S_T tends asymptotically toward $S_T^\star$.

Proof of Lemma 2: First we show that the model trajectories are bounded. For this we define the two following variables:

$$z_1 = k_1 X_1 + S_1$$

$$z_2 = S_T + \frac{k_3 k_1}{k_2} X_2$$

Now it is straightforward that:

$$-D(z_1 - S_{1in}) \le \dot{z}_1 \le -\alpha D(z_1 - S_{1in})$$

$$-D(z_2 - S_{Tin}) \le \dot{z}_2 \le -\alpha D(z_2 - S_{Tin})$$

proving that z_1 and z_2 are bounded; thus the state variables are bounded too.

Now we will use the boundedness of the state variables to show that X_2 and S_2 remain positive for any positive initial condition and therefore show that $D(t)$ remains positive. We will assume for the proof that the acidogenic growth rate is bigger than the methanogenic one, and therefore that a dilution rate that would not wash out the methanogenic bacteria would not wash out the acidogenic ones. Consequently, X_1 remains lower bounded by a positive number.

- We have easily $D(t) \ge 0$

  $$\dot{X_2}(X_2 = \epsilon) = \alpha D(t)(X_2^\star - \epsilon)$$

  Then $X_2 \ge min(X_2^\star, X_2(t_0)) > 0$. And:

  $$D(t) = 0 \Leftrightarrow S_2 = 0$$

- Now we have to show that S_2 also remains positive for any positive initial condition.

  We have assumed that $\exists \epsilon_1 > 0,\ X_1 > \epsilon_1$. Since $\dot{S_T} = D(t)(S_T^\star - S_T)$ and $D(t) \ge 0$, $S_T > 0$ for a positive initial condition. Since $S_T = S_1 + \frac{k_1}{k_2} S_2$, S_1 and S_2 can not cancel at the same time. For $S_2 = \epsilon_2 > 0$ small enough, $\exists \epsilon_3 > 0, S_1 > \epsilon_3$. And consequently, $\exists A > 0, k_2 \mu_1(S_1) X_1 > A$. Now we compute $\dot{S_2}(S_2 = \epsilon_2)$:

  $$\dot{S_2}(S_2 = \epsilon_2) > A + B \mu_2(\epsilon_2) X_2$$

  With $A > 0$. $B < 0$ if $S_T^\star < S_{1in}$. If we choose a good value for ϵ_2 and since X_2 is positive and bounded, we prove $\dot{S_2}(S_2 = \epsilon_2) > 0$ for any positive initial condition and so S_2 can not cancel.

We have shown that for any positive initial condition the state variables are bounded and remain positive, so $D(t)$ is bounded and remains positive too.

Note that $D(t)$ is also a function of the state variables and as $D(t) > 0$ we can make a time change $t_2 = \int_0^t D(\tau)d\tau$(see *e.g.* (Hofbauer and Sigmund, 1988)). The system (7) becomes:

$$\begin{cases} \dfrac{dX_1}{dt_2} = (\dfrac{\mu_1(S_1)}{D(t)} - \alpha)X_1 \\ \\ \dfrac{dS_1}{dt_2} = (S_{1in} - S_1) - \dfrac{k_1\mu_1(S_1)}{D(t)}X_1 \\ \\ \dfrac{dX_2}{dt_2} = \alpha(X_2^\star - X_2) \\ \\ \dfrac{dS_T}{dt_2} = S_T^\star - S_T \end{cases} \tag{9}$$

The two last equations of (9) are decoupled from the two others and linear. It guarantees that for any positive initial condition X_2 and S_2 tends asymptotically to $X_2^\star$ and $S_2^\star$.

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Hypothesis 2. (H2). We assume that:

- for $S_1 \in [0 \quad S_T^\star]$ the function $\mu_1(S_1)$ is increasing, and that
- for $S_2 \in [0 \quad \frac{k_2}{k_1}S_T^\star]$ the function $\mu_2(S_2)$ is increasing

This hypothesis is very reasonable: in practice we choose a low value for $S_T^\star$ that will be under the inhibition threshold of μ_2.

Lemma 3. Under hypothesis [H2] the point $\xi^\star$ of (1) with the controller (8) is globally asymptotically stable.

Proof of Lemma 3: Now let us consider the system (1) when X_2 and S_T have reached their equilibria. We have:

$$\begin{cases} \dot{X_1} = (\mu_1(S_1) - \alpha\tilde{D}(S_1))X_1 \\ \\ \dot{S_1} = \tilde{D}(S_1)(S_{1in} - S_1) - k_1\mu_1(S_1)X_1 \end{cases} \tag{10}$$

Since $\tilde{D}(S_1) > 0$, we make the following change of time; we set $t_2 = \int_0^t \tilde{D}(S_1(\tau))d\tau > 0$, the system becomes then:

$$\begin{cases} \dfrac{dX_1}{dt_2} = (\tilde{\mu}_1(S_1) - \alpha)X_1 \\ \\ \dfrac{dS_1}{dt_2} = (S_{1in} - S_1) - k_1\tilde{\mu}_1(S_1)X_1 \end{cases}$$

First, remember that we are on the surface given by $S_1 + \frac{k_1}{k_2}S_2 = S_T^\star$ and therefore $S_1 < S_T^\star$. Under hypothesis [H2], the application μ_1 is increasing on the interval $S_1 \in [0, S_T^\star[$ and $\mu_2\big(\frac{k_2}{k_1}(S_T^\star - S_1)\big)$ is decreasing. Finally, on this interval, $\tilde{\mu}_1$ is an increasing application such that $\tilde{\mu}_1(0) = 0$. This corresponds to the classical chemostat equation, whose behaviour is simple (Smith and Waltman, 1995): it has a single globally stable equilibrium.

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Note that the demonstration is not based on any analytical formulation of the growth rates μ_1 and μ_2, that is particularly important for the robustness of the controller.

5. SIMULATION STUDY

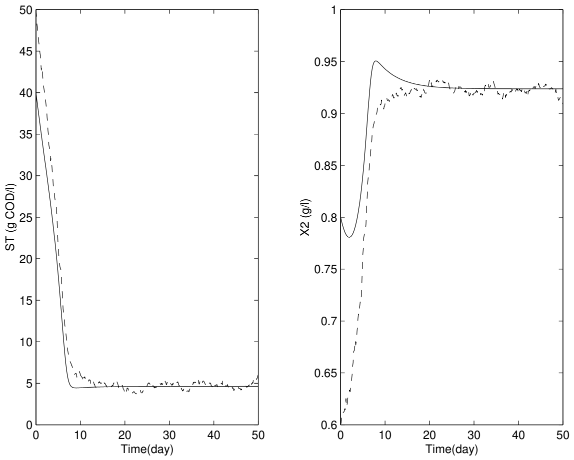

Fig. 1. Simulation of the observer for S_T and X_2 with a 40% noise on the output y.

In this section, the parameter values given by (Bernard *et al.*, 2000) have been chosen to run the model. A multiplicative white noise η of variance 0.4 has been added to the measurement of the methane flow rate generating a 40% proportional noise:

$$y = \frac{q_{CH_4}}{k_4}(1+\eta)$$

With the kinetics expression of (Bernard *et al.*, 2000), hypothesis [H2] is fullfiled, and therefore the hypotheses of Lemma 3 are satisfied.

The result of the observer are plotted on Figure 1. Note that the measurement noise is filtered, and the quality of the estimates is good.

The controller results can be seen on Figure 2. Despite the high level of noise the controller action is efficient and the total COD remains in a reasonable interval. The converging rate is fast and the controller is robust towards variations of the influent COD concentration.

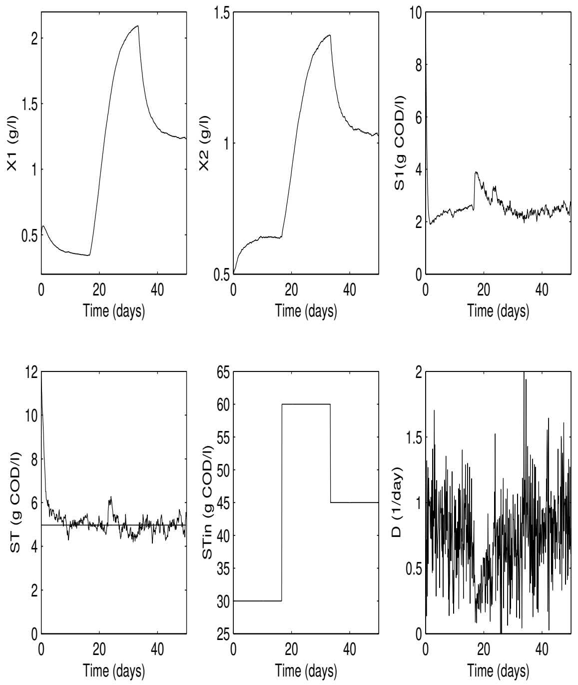

Fig. 2. Simulation of the controller with variations of the influent COD charge and with a 40% noise on the output y.

6. CONCLUSION

The proposed controller is very simple in its design: it is only an output feedback law. The only required parameter is the feedback gain that depends on some yield parameters, so the controller can be tuned online just by changing the gain of the feedback in order to obtain the desired equilibrium value for S_T. Moreover if these yield coefficients are unknown, they can be very simply identified by measuring the equilibrium state obtained by a given value of this gain. Indeed, from equation (8) it can be seen that:

$$\frac{k_3 k_1}{k_2} = \frac{D}{y}(S_{Tin} - S_T^\star)$$

This very simple control law turns out to be very robust to the noise and to variations of the COD influent concentration.

Note finally that the control can also be applied to the model in dimension 6 presented in (Bernard *et al.*, 2000): this model is in fact an extension of the model (1) with two equations in cascade: an equation for the alkalinity and an equation for the total dissolved inorganic carbon. Since these two new variables do not affect X_1, S_1, X_2 and S_2, the convergence of the system (1) will drive the two other variables to an equilibrium value. Therefore the control law will also globally stabilise the model in dimension 6.

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