A new model of mobile robots with lights and its computational power

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Joint work with

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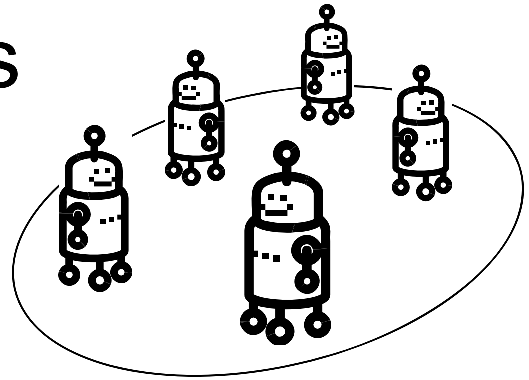
Coordination of Autonomous Mobile Robots

- Autonomous Mobile Robots
  Multiple, Fully decentralized

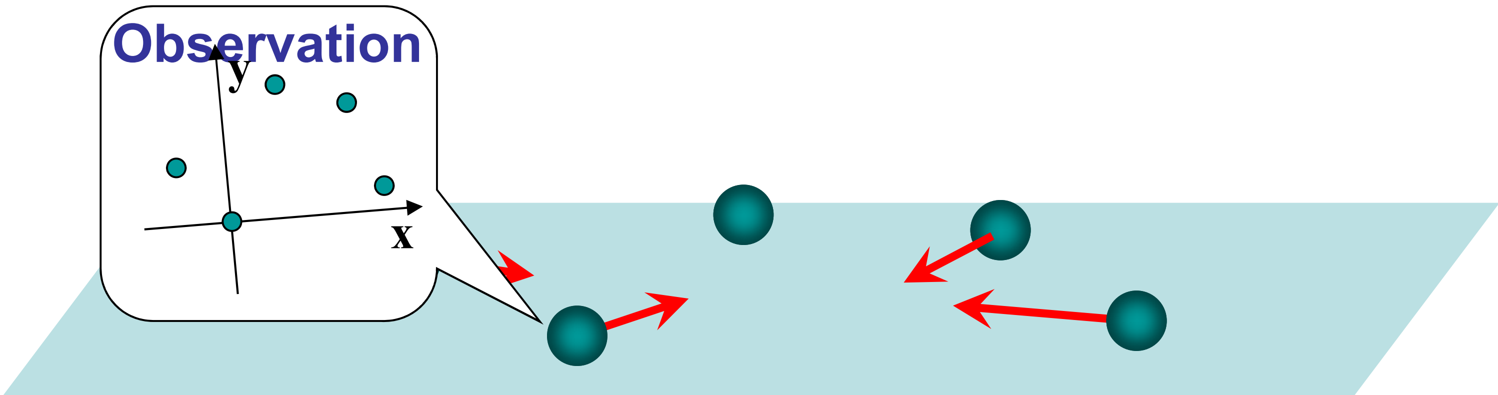


- Coordination task of Mobile Robots
  - Gathering, Convergence, Formation ...

- Challenges from the theoretical aspect
  - Clarifying the “power of lights" to solve gathering problems

Autonomous Mobile Robots

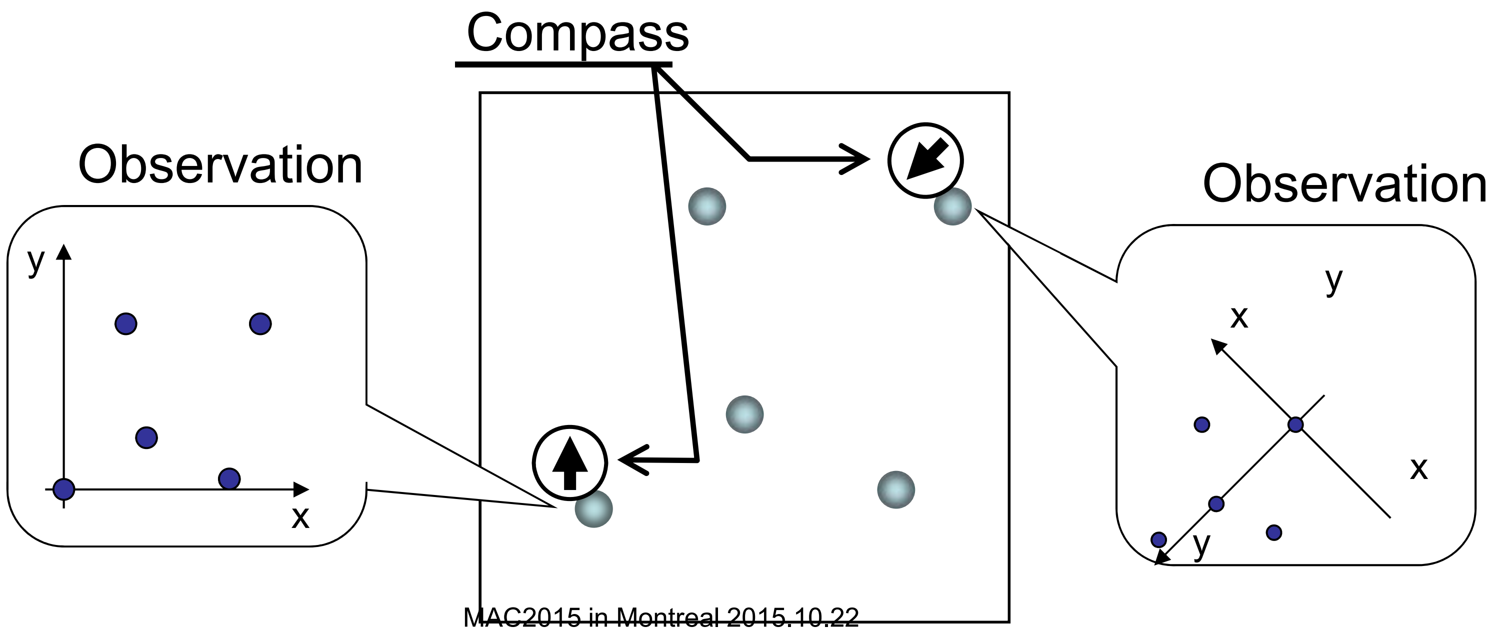
- Robot: Point on an infinite 2D-space
  - Anonymous (No distinguished ID)
  - Oblivious(No persistent memory)
  - Deterministic
  - No communication (Observe the environment and Move)



Observation

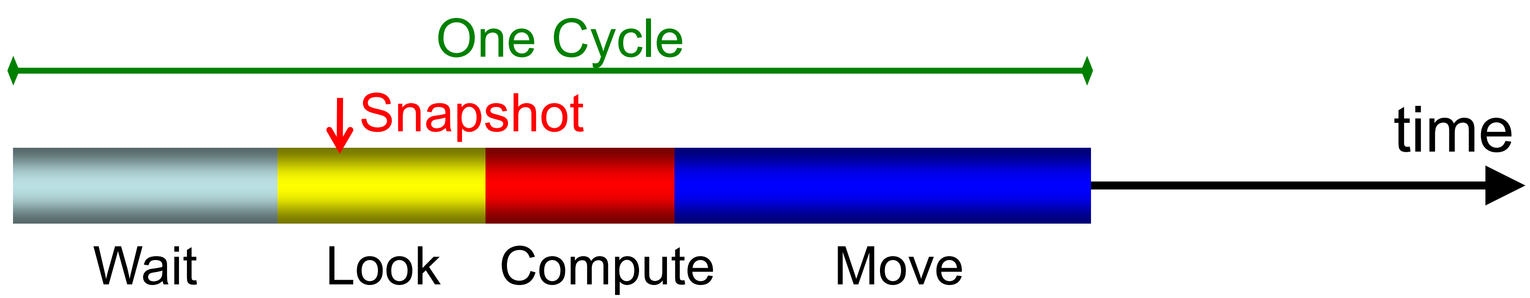
- Each robot has a local x-y coordinate system(LCS)
  - The current position is the origin
- Agreement level of LCSs depends on the model (two axes, one axis, or chirality)

no agreement of axis and chirality



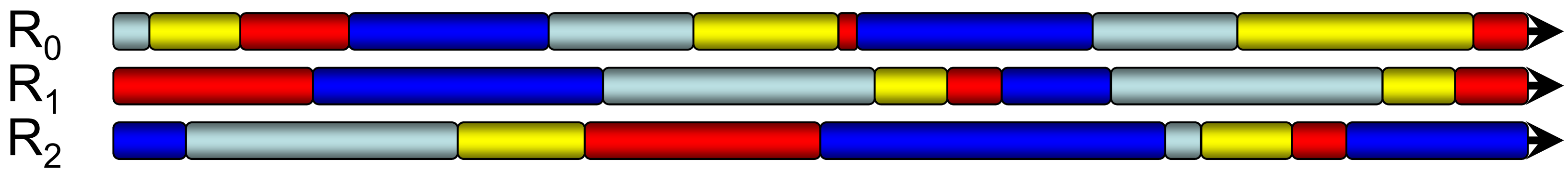
Execution of Robots (Behavior of each robot)

- Wait-look-compute-move cycle
  - **Wait** : Idle state
  - **Look** : Take a snapshot of all robots' current locations (in terms of LCS)
  - **Compute**: Deciding the next position
  - **Move** : Move to the next position
    - Rigid vs Non-Rigid(movement of $\delta>0$)

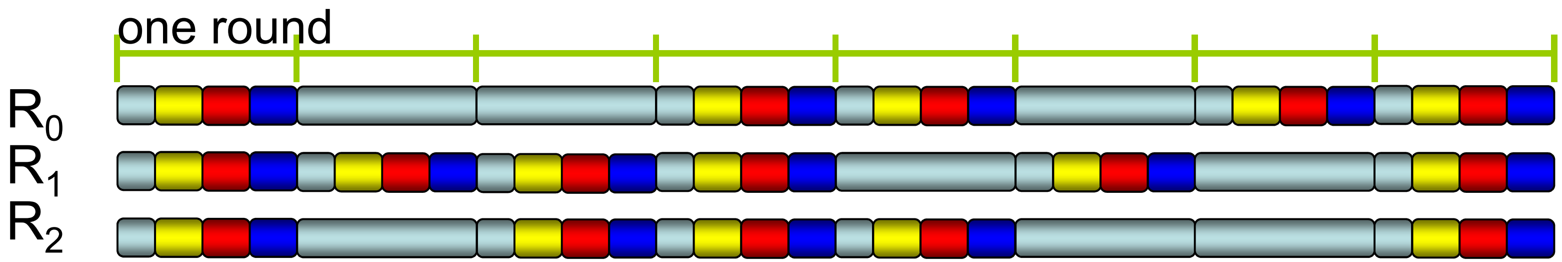


Timing Model(How Cycles are Synchronized)

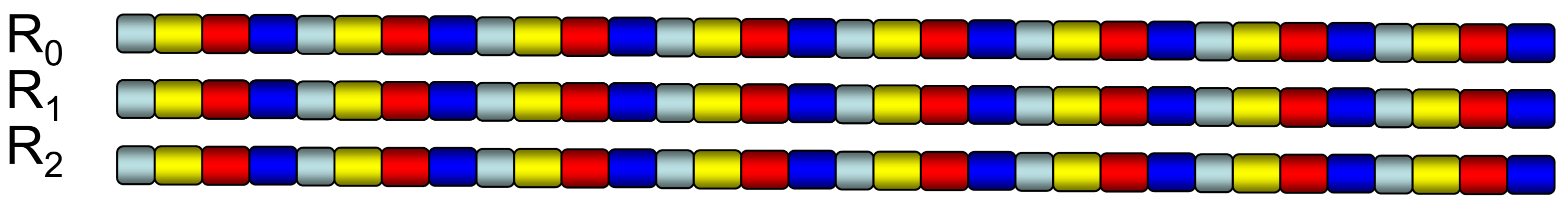
- Async (or CORDA): No bound for length of each step



- Ssync (SYm, ATOM): Synchronized Round
  - Only a subset of all robots becomes active in each round



- Fsync: All robots are completely synchronized



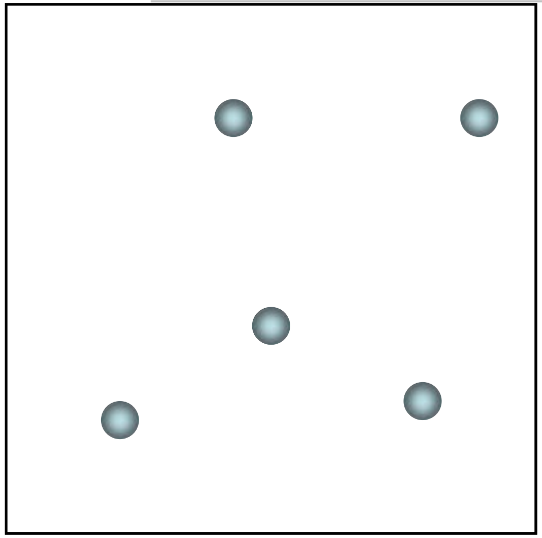
Fairness and Restricted Schedulers in Ssync

- All schedulers are assumed to be fair
  - All robots are activated infinitely often
- Restricted Schedulers in Ssync
  - k-bounded
    - Between two cycles of any robot, other robots perform at most k cycles
  - Centralized
    - Robots perform one by one
  - Round-Robin
    - = centralized and 1-bounded

Gathering Problem

- All robots meet at one point on a plane from any initial configuration

  n=2 : rendezvous



- Distinct gathering(D-gathering)

  All robots are located at distinct positions

- Self-Stabilizing gathering (SS-gathering)

  Some robots can be located at a same position

Unsolvability of Rendezvous problem

Schedulers	**Initial Config.**	**Solvability**
Fsync	any	Yes(trivial)
Centralized Ssync	any	Yes(trivial)
k-bounded Ssync (k $\geqq$1)	any	No[1]
Ssync	any	No(↑)
Async	any	No (↑)

[1] X D'efago , M Gradinariu , P Julien , C St'ephane , M Philippe , R Parv'edy , Fault and Byzantine Tolerant Self-stabilizing Mobile Robots Gathering — Feasibility Study — , DISC 2006, LNCS , 4167 , pp 46-60.

Unsolvability of Gathering problem (n ≧3)

Schedulers	Initial Config.	Solvability
Fsync	any	Yes(trivial)
Round-Robin Ssync	Distinct	OPEN
Round-Robin Ssync	SS	No [1]
2-bounded Ssync	Distinct	No [1]
Ssync	any	No (↑)
Async	any	No (↑)[2]

[1] X D’efago , M Gradinariu , P Julien , C St’ephane , M Philippe , R Parv’edy , Fault and Byzantine Tolerant Self-stabilizing Mobile Robots Gathering — Feasibility Study — , , DISC 2006, LNCS , 4167 , pp 46-60, 2006.
[2] G. Prencipe, The effect of synchronicity on the behavior of autonomous mobile robots, Theory of Computing Systems, 38(5),539-558, 2005.

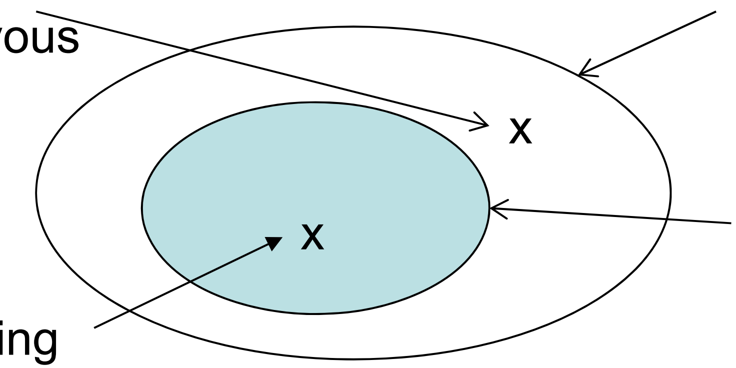
Solvability with other assumptions

- Multiplicity detection
  - Strong multiplicity→gathering (n ≧3）
  - Weak multiplicity→gathering (odd n ≧3）
- Axis agreement
  - Two-axis →gathering on Async (n ≧2)
  - One-axis →gathering on Async (n ≧2)
- Chirality→gathering(n ≧3)

Special feature of rendezvous problem

If Chirality is assumed,

rendezvous problem has a special feature.



[3 I. Suzuki, M. Yamashita, SIAM J. Computing, 28, 4, 1347-1363, 1999.

Robot with lights

- light
  $O(1)$ bits of memory that can store robot's internal state.
  Light is classified by its visibility.

	my light	other's
$\boldsymbol{full-light}$	○	○
$\boldsymbol{internal-light}$ (FSTATE[4])	○	×
$\boldsymbol{external-light}$ (FCOMM[4])	×	○

[4]P.Flocchini , N.Santoro , G.Viglietta , M.Yamashita , Rendezvous of Two robots with Constant Memory ,SIROCCO 2013), LNCS 8179, pp 189-200, 2013.

Solvability of Rendezvous problem

[1]

schedule	solvability
centralized	○
k-bounded($k \geq 1$)	×

S.Das, P. Flocchini, G.Prencipe, N. Santoro, M.Yamashita, 2012 ICDCS (2012)

Rigid

[4]

schedule	*full*	*int.*	*ext.*
ASYNC	≤ 4	?	12
SSYNC	2	6	≤ 3
FSYNC	1	1	1

Non-Rigid

schedule	*full*	*int.*	*ext.*
ASYNC	4	?	3*
SSYNC	2	3 *	3
FSYNC	1	1	1

* with knowledge of δ

[1] X D’efago , M Gradinariu , P Julien , C St’ephane , M Philippe , R Parv’edy , Fault and Byzantine Tolerant Self-stabilizing Mobile Robots Gathering — Feasibility Study — , DISC 2006, LNCS , 4167 , pp 46-60, 2006.

[4] P.Flocchini , N.Santoro , G.Viglietta , M.Yamashita , Rendezvous of Two robots with Constant Memory , SIROCCO 2013 , LNCS 8179, pp 189-200, 2013.

External-light vs. Internal-light

- External > Internal for Rendezvous

lights	schedulers	Rigidness	# of lights
Internal	Ssync	Rigid	6
External	Sysnc	Non-rigid	3

lights	Schedulers	Rigidness	# of lights
Internal	Ssync	Non-rigid(δ)	3
External	Sysnc	Non-rigid	3

lights	schedulers	Rigidness	# of lights
Internal	Ssync	Non-rigid(δ)	3
External	Aysnc	Non-rigid(δ)	3

Rigidness vs. Non-rigidness (δ)

- Rigid > Non-Rigid

- Non-rigid(δ) > Rigid

Rigidness	Schedulers	light	# of lights
Rigid	Ssync	internal	12
Non-Rigid(δ)	Async	internal	3

Rigidness	Schedulers	light	# of lights
Rigid	Ssync	internal	6
Non-Rigid(δ)	Ssync	internal	3

Gathering problem for robots with lights

- To solve gathering problem by robots with lights
  - Chirality can not be assumed

    If chirality is assumed then

    Gathering ∈(The set of patterns formable

    by non-oblivious robots on Ssync)

    =(The set of patterns formable

    by oblivious robots on Ssync)
  - How to look at lights of robots at the same location

How to look at lights of robots at the same location

$r_1 \sim r_4$: robots
$l_1 \sim l_4$: lights of $r_1 \sim r_4$

α

r_1, r_2

$l_1 = A$
$l_2 = B$

β

r_3, r_4

$l_3 = C$
$l_4 = C$

full-light

r_1
Point
α:A,B
Point
β:C,C

multiset
=strong multiplicity detection

variation

multiset	*set*	*arbitrary*
r_1 looks	r_1 looks	r_1 looks
Point α={A,B}	Point α={A,B}	Point α={B}
Point β={C,C}	Point β={C}	Point β={C}

Solvability of gathering problem(our result)

[1]

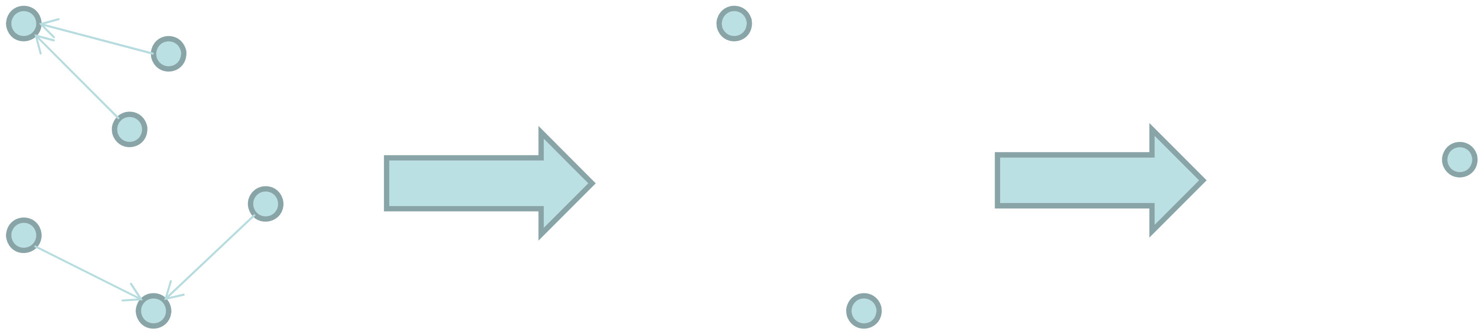
schedule	Initial config.	solvability
2-bouded centralized	Distinct	×
round-robin	SS	×

How to look of lights: set

schedule	$full$	$int.$	$ext.$
SSYNC	3 (non-rigid)	?	2(with δ)
centralized	≤ 2	?	2(non-rigid)
round-robin	≤ 2	2(rigid,SS)	≤ 2

[1] X D'efago , M Gradinariu , P Julien , C St'ephane , M Philippe , R Parv'edy , Fault and Byzantine Tolerant Self-stabilizing Mobile Robots Gathering — Feasibility Study — , DISC 2006, LNCS , 4167 , pp 46-60.

Overview of algorithms



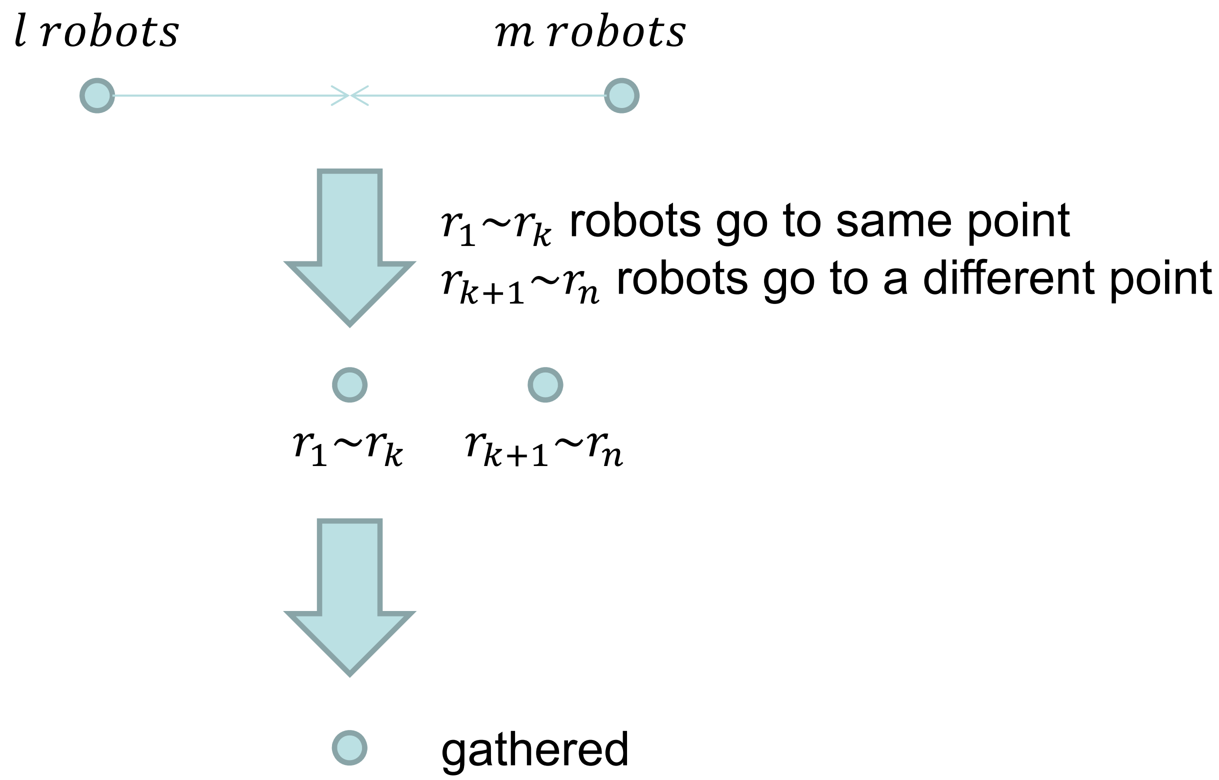
Algorithm 1[4]：from initial configuration to 1 or 2 points
Algorithm 2[5]：extension of two-robot algorithms

[4] P.Flocchini , N.Santoro , G.Viglietta , M.Yamashita , Rendezvous of Two robots with Constant Memory , SIROCCO 2013 , LNCS 8179, pp 189-200, 2013.5

[5] T Izumi, Y Katayama, N Inuzuka, and K Wada, Gathering Autonomous Mobile Robots with Dynamic Compasses: An Optimal Result, DISC 2007, LNCS 4731, pp 298-312, 2007,

Example

- round-robin schedule
- internal-light



Order of cycle:
$r_1 \to r_2 \to \cdots \to r_n$

Initial state: A

Solvability of gathering problem(our result)

[1]

schedule	Initial config.	solvability
2-bouded centralized	Distinct	×
round-robin	SS	×

How to look at lights: set

3(with δ, Distinct)

schedule	$full$		$ext.$
SSYNC	3 (non-rigid)	?	2(with δ)
centralized	≤ 2	?	2(non-rigid)
round-robin	≤ 2	2(rigid,SS)	≤ 2

2(non-rigid, arbitrary)

[1] X D'efago , M Gradinariu , P Julien , C St'ephane , M Philippe , R Parv'edy , Fault and Byzantine Tolerant Self-stabilizing Mobile Robots Gathering — Feasibility Study — , DISC 2006, LNCS , 4167 , pp 46-60.

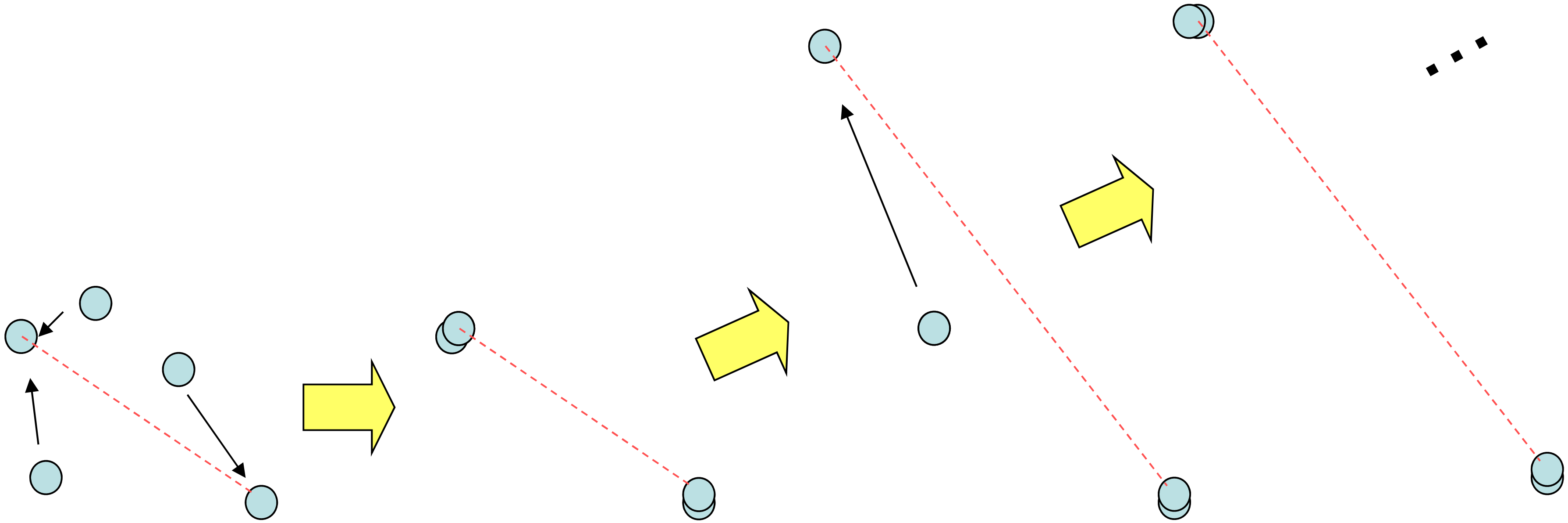
Concluding Remarks

- We have revealed some solvability in assumptions that are not solvable without light.
- We have to investigate relationship between internal and external lights.
  - 2 robots: external >internal
  - n(≧3) robots: external >>internal?

Thank you !

n-robot algorithm under unique LDS(1/2)

- Robots are located at two points
  →All robots execute the two-robot algorithm
- Robots are located at more than two points
  →All robots move to one of two endpoints of LDS



Correctness of Conditional n-robot Alg.

- Lemma 3
  - ∠LDSy = ∠ formed by LDS and the global y-axis < ε

  → Wait-Approach Relation is guaranteed

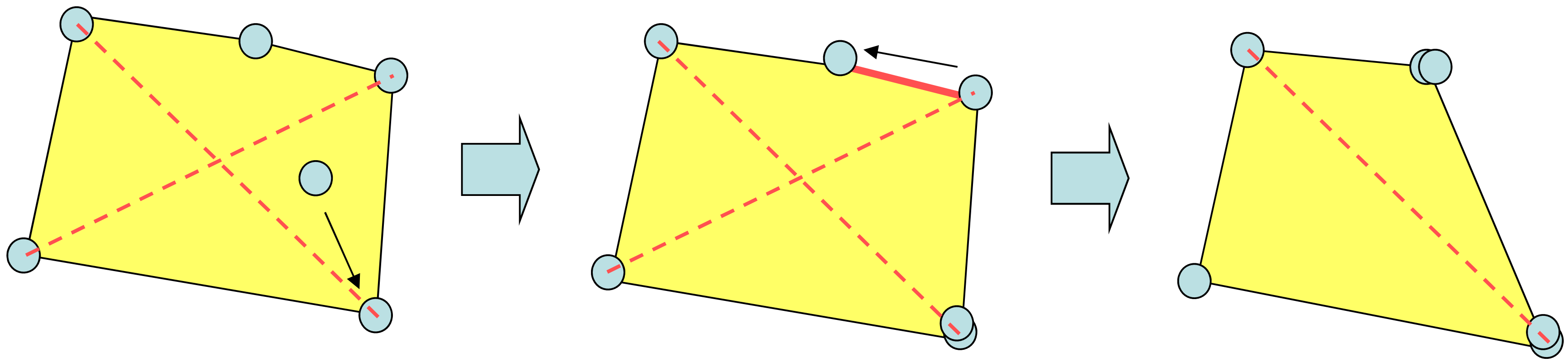
  (regardless of the title angle of each robots)

- Lemma 4
  - At any round, ∠LDSy decreases by ε〜2ε

  unless gathering is achieved

Unique LDS Election (1/2)

- If two or more LDSs exist, each robot calculates the convex hull(CH)
  - Robots on the boundary : Wait
  - Inner robots : Moves to one of vertices
- Contracting the shortest edge of the CH



#edges of the CH decreases

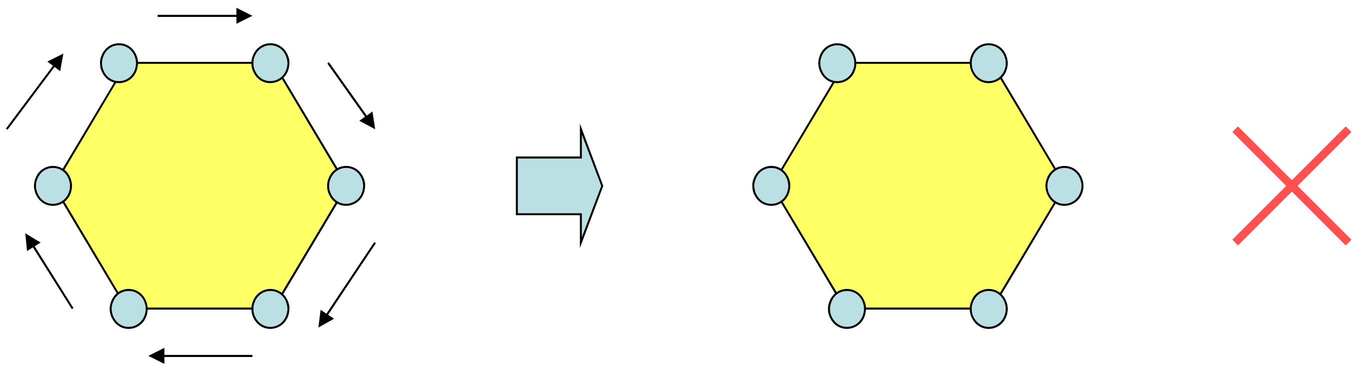
→Eventually unique LDS is elected (or gathered)

Unique LDS Election(2/2)

- If all edges have a same length

→ Robots moves to the center-of-gravity of the CH

  - All robots simultaneously move → gathered
  - A part of robots move → Symmetry is broken



Conclusion

- Gathering mobile robots with dynamic compasses
  - Tilt angle $\leqq \pi/2-\varepsilon$ (Optimal)
  - Semi-synchronous model
  - Arbitrary #robots
- Open problem
  - Asynchronous model
    - $\pi/2$ < Maximum Tilt angle < $\pi/4$
    - Recently, two robots are solved for <π/3
    - #robots = 2, dynamic compass