Distributed Computing by Mobile Robots: Solving the Uniform Circle Formation Problem

GRASTA–MAC 2015

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Giovanni Viglietta

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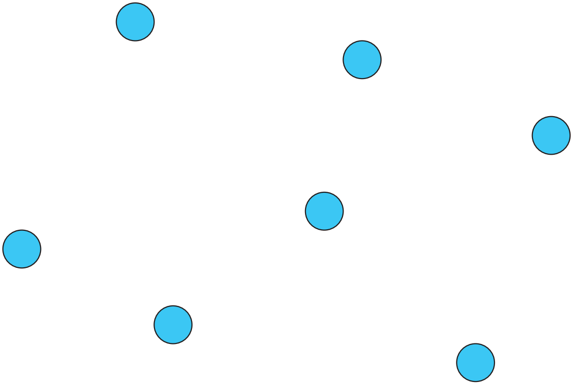
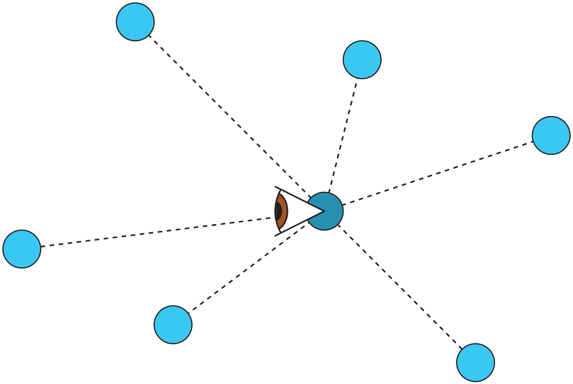
Based on: http://arxiv.org/abs/1407.5917

Anonymous robots sensing and moving in the plane



Swarm of anonymous robots

Anonymous robots sensing and moving in the plane



Sensing the positions of other robots

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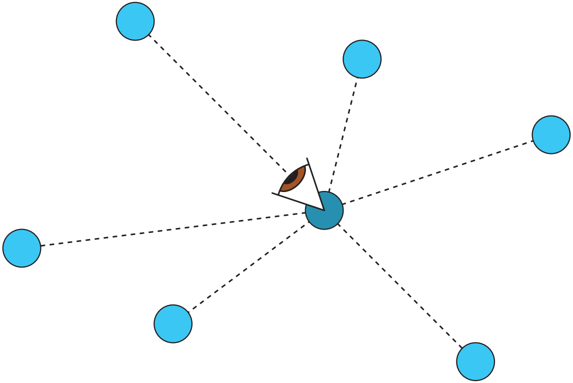

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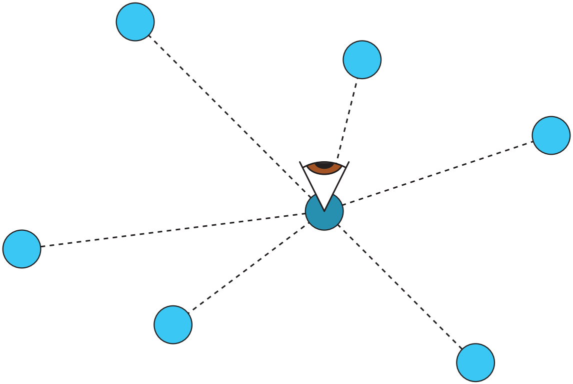

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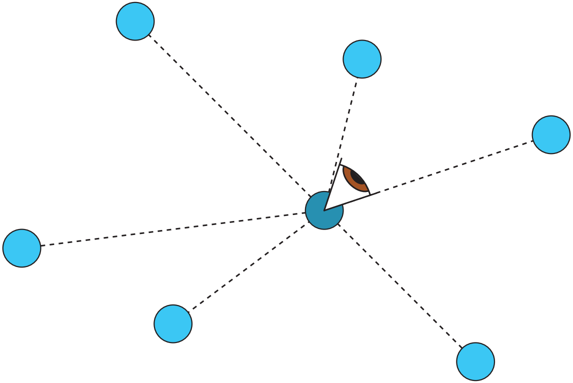

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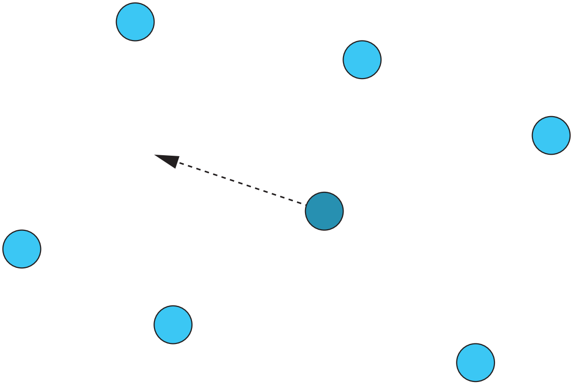
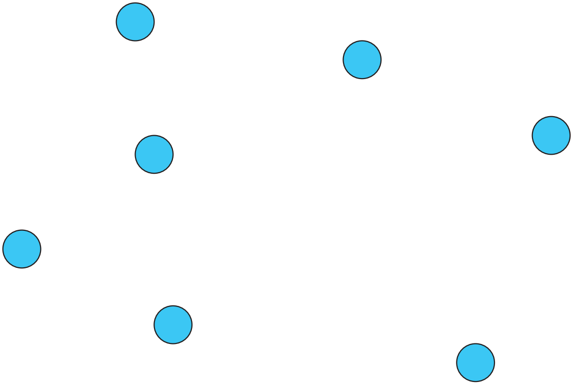


Moving accordingly

Anonymous robots sensing and moving in the plane



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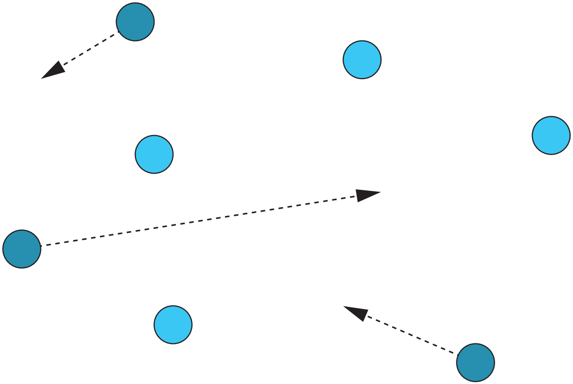

Different robots are activated asynchronously

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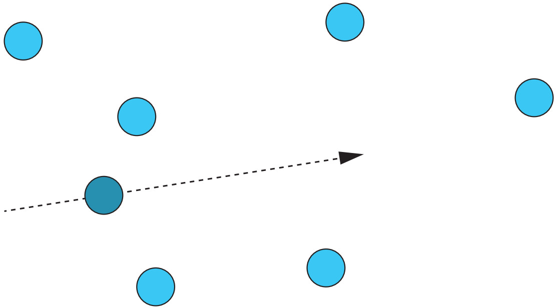

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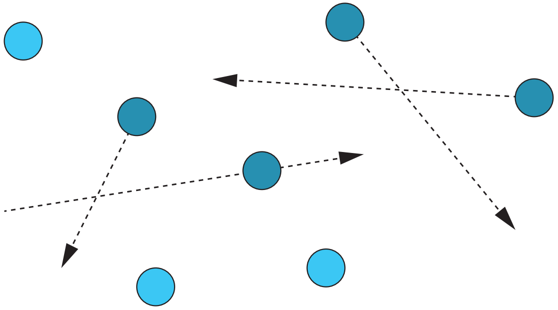

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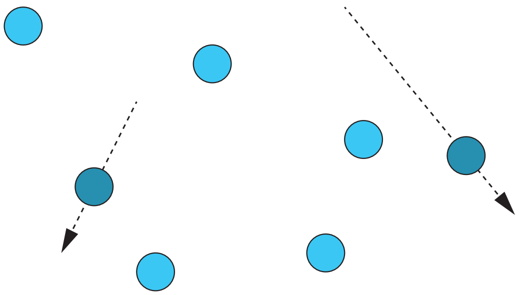


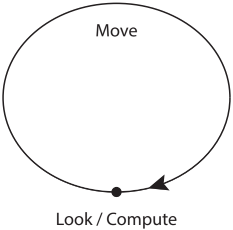
Different robots are activated asynchronously

Anonymous robots sensing and moving in the plane

Robots are:

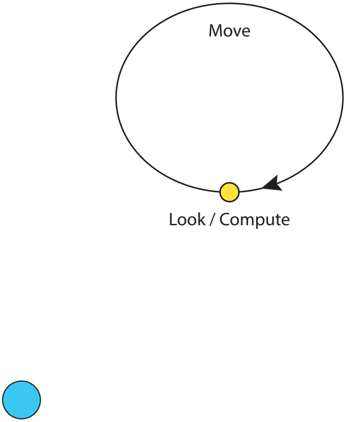
- **Dimensionless** (robots are modeled as geometric points)
- **Anonymous** (no unique identifiers)
- **Homogeneous** (the same algorithm is executed by all robots)
- **Autonomous** (no centralized control)
- **Oblivious** (no memory of past events)
- **Silent** (no explicit way of communicating)
- **Long-sighted** (complete visibility of all other robots)
- **Disoriented** (robots do not share a common reference frame, and a robot's reference frame may change from turn to turn)
  - No common unit distance
  - No common compass
  - No common notion of clockwise direction

Life cycles and asynchronicity



Each robot repeats a Look/Compute/Move cycle

Life cycles and asynchronicity



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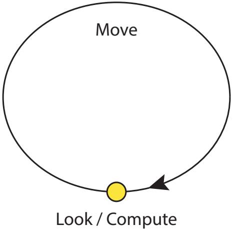
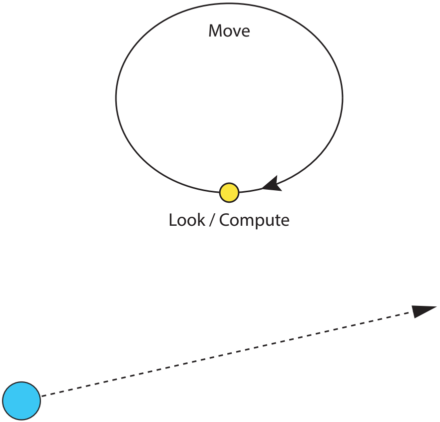


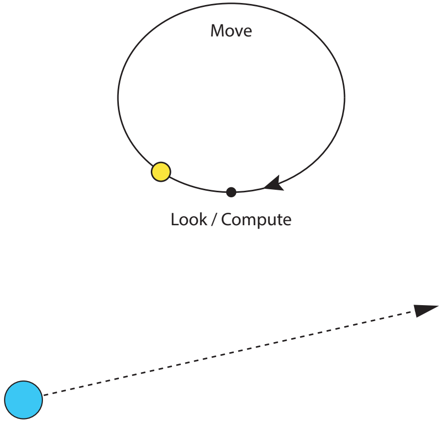
In a Look phase, an instantaneous snapshot is taken of all robots

Life cycles and asynchronicity



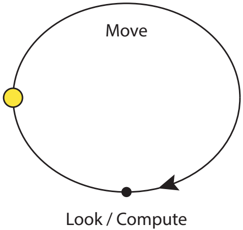
A destination point is computed as a function of the snapshot

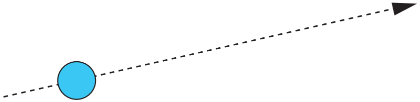
Life cycles and asynchronicity



The destination point is approached with unpredictable speed

Life cycles and asynchronicity





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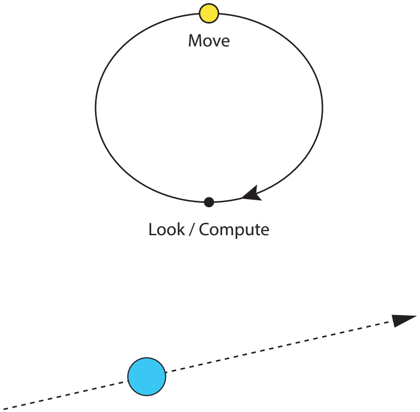

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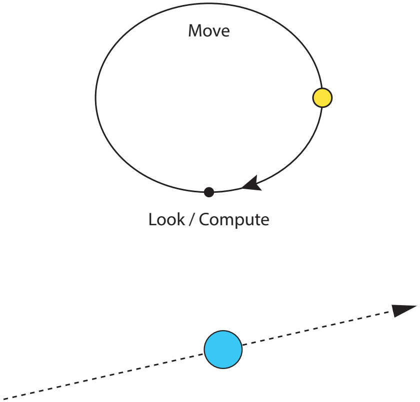

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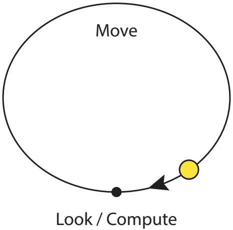

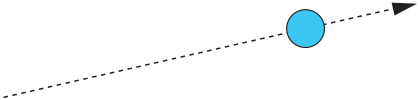

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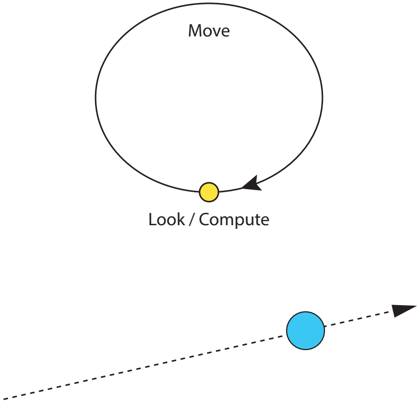
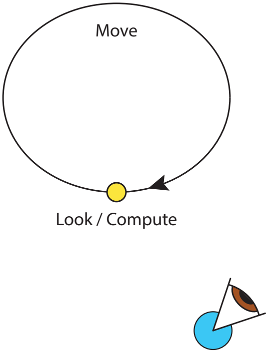
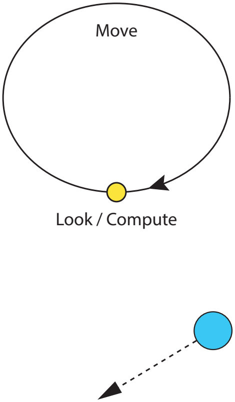


The robot may unpredictably stop before reaching the destination...

Life cycles and asynchronicity



...and execute a new Look/Compute phase



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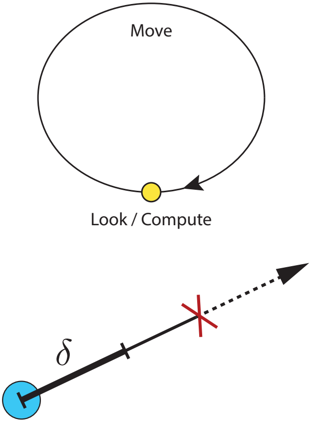
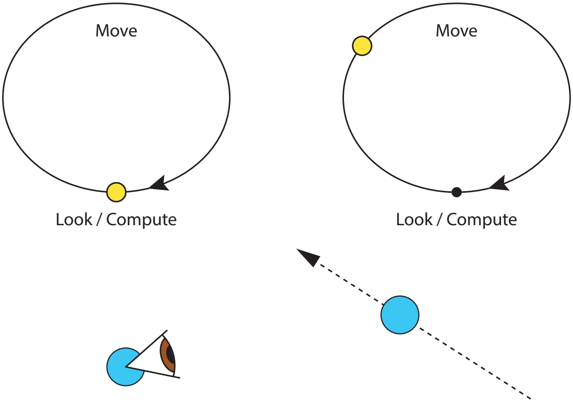


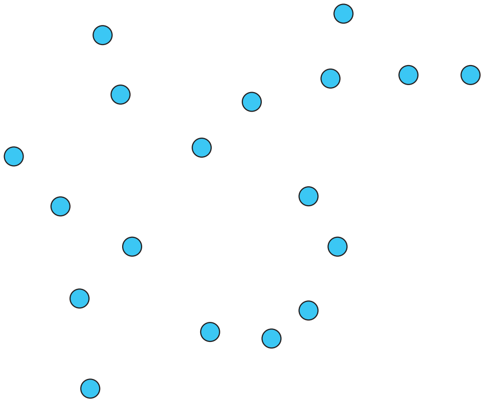
At each cycle, a robot is guaranteed to move by at least δ

Life cycles and asynchronicity



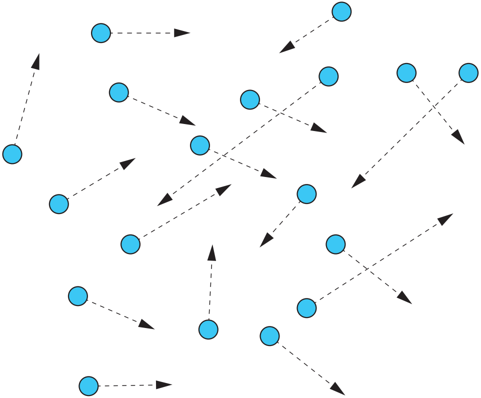
Different robots execute independent cycles, asynchronously

Pattern Formation problem



Problem: form a given pattern from any initial configuration

Pattern Formation problem



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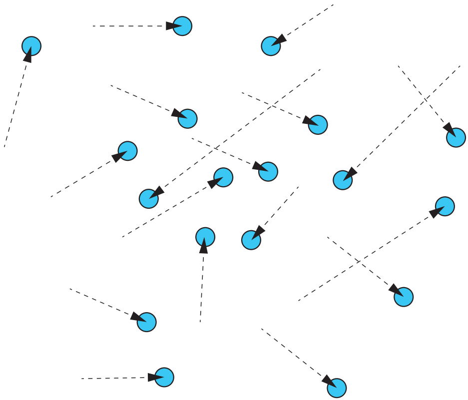

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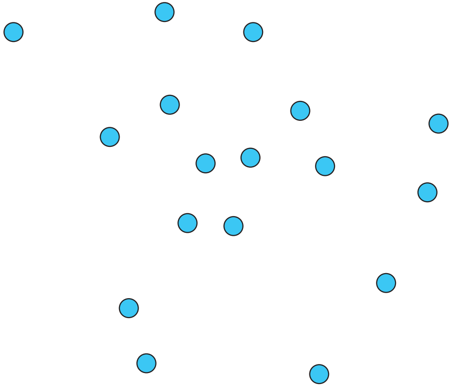

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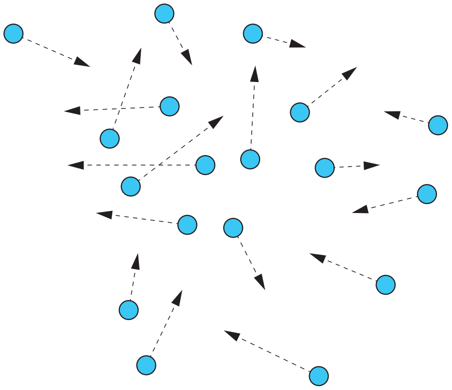

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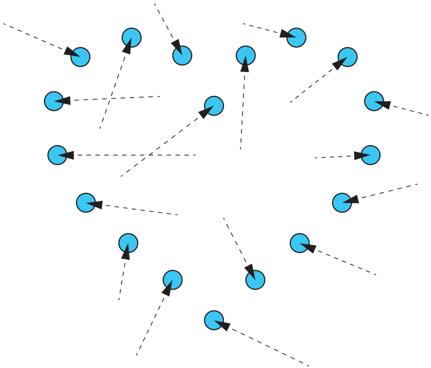

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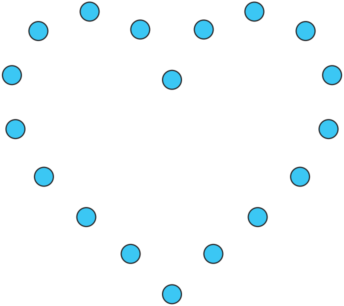

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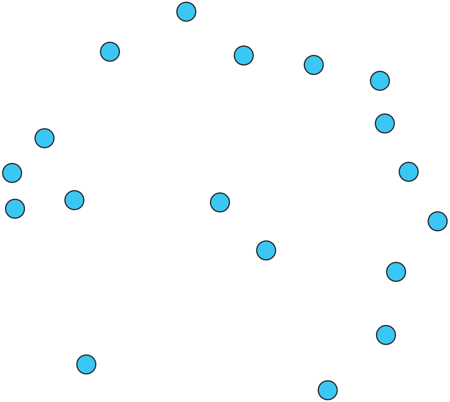
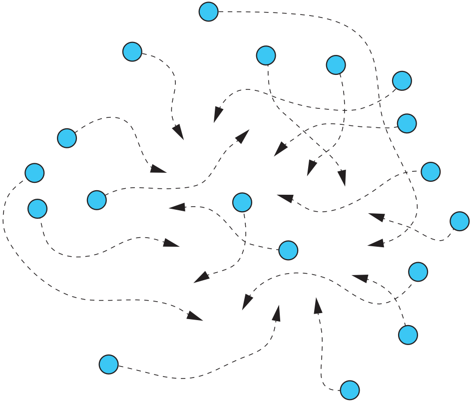


The pattern may be rotated, reflected, and scaled



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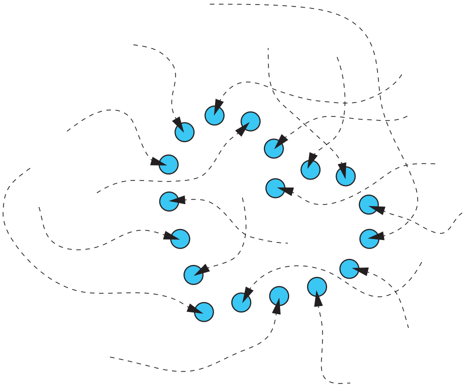

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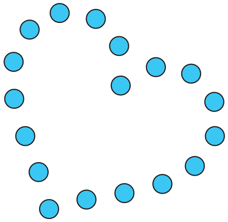

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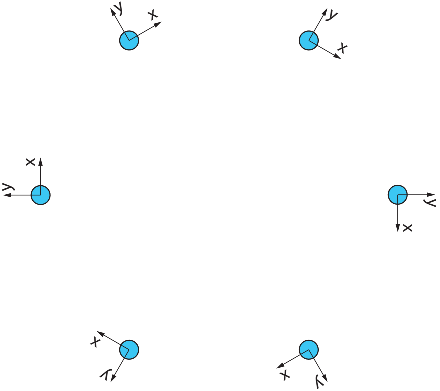
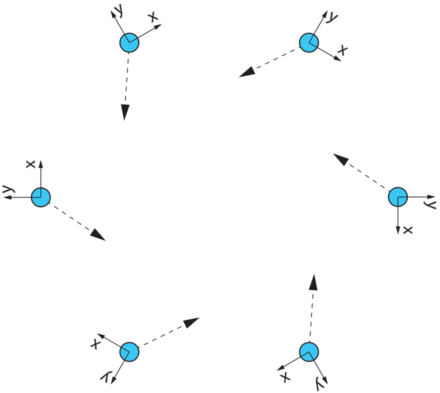


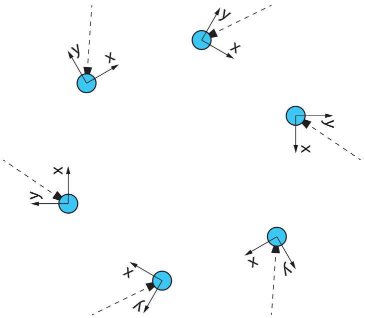
Let the initial configuration be rotationally symmetric

Pattern Formation problem

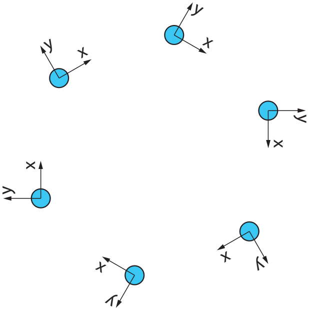


All robots have the same view and compute symmetric destinations

Pattern Formation problem



If they are all activated synchronously, they remain symmetric

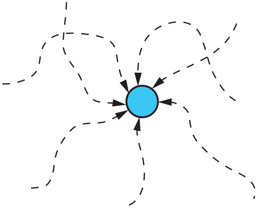


Hence Pattern Formation is unsolvable if the pattern is asymmetric

Pattern Formation problem

No pattern is formable from every possible initial configuration, except:

- **Single point** (aka Gathering problem)
  $\Longrightarrow$ Solved! (Cieliebak–Flocchini–Prencipe–Santoro, 2012)



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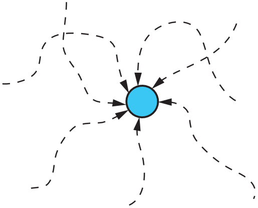
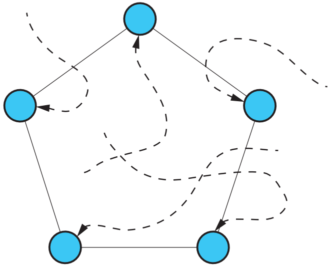
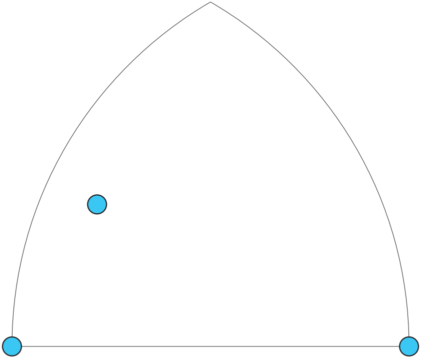




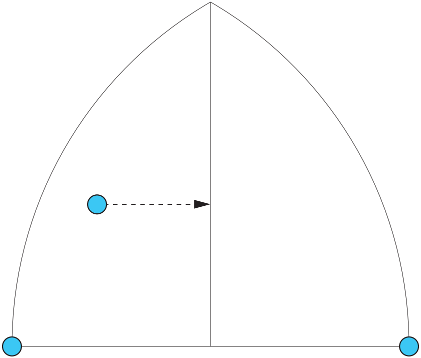
- **Regular polygon** (aka Uniform Circle Formation problem)
  $\Longrightarrow$ This talk

Uniform Circle Formation for $n = 3$

Suppose the triangle formed by the robots is scalene

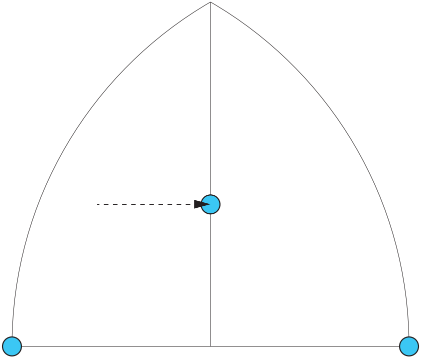


Identify the longest edge

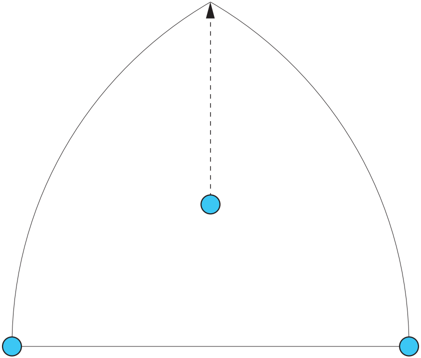


Move the third vertex parallel to the longest edge...

Uniform Circle Formation for $n = 3$



...until the triangle is isosceles



Then move the apex to the final position

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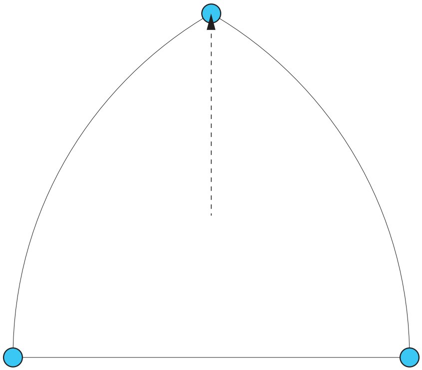

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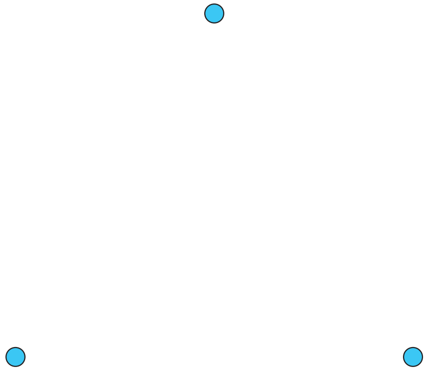
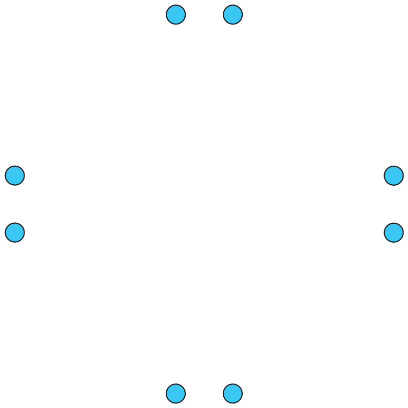


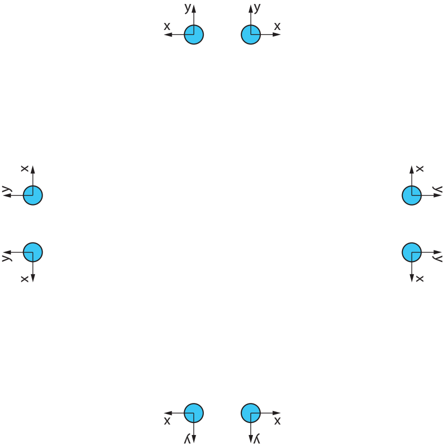
Correctness: only one robot is ever allowed to move

Resolving Biangular configurations



In a **Biangular** configuration all robots may have the same view

Resolving Biangular configurations



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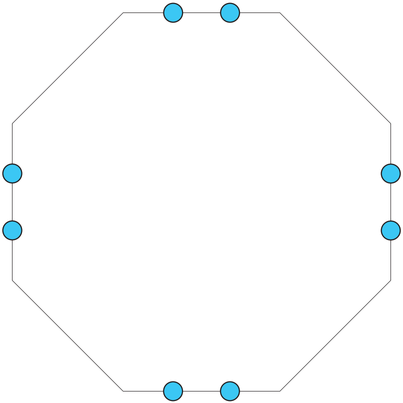
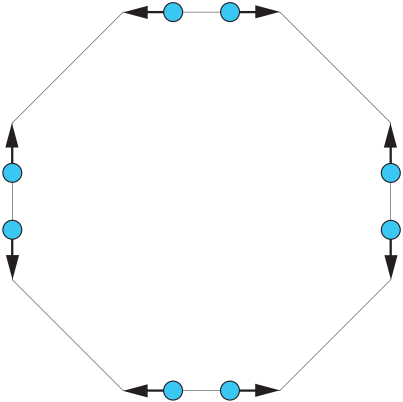
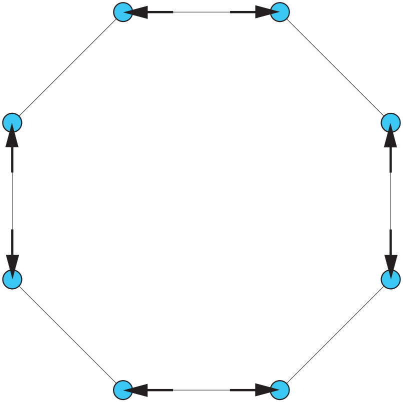


Solution: identify a "supporting polygon" (Dieudonné–Petit, 2009)



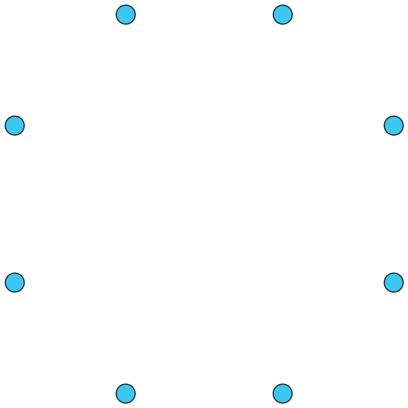
Each robot moves to the closest vertex

Resolving Biangular configurations



During the motion, the supporting polygon remains fixed (if $n > 4$)

Resolving Biangular configurations



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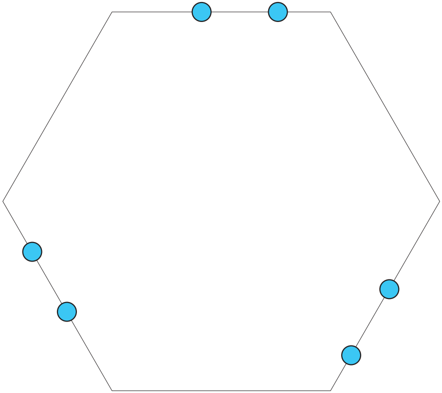
Pre-regular configurations

The possible (asynchronous) evolutions of a Biangular configuration are called **Pre-regular** configurations



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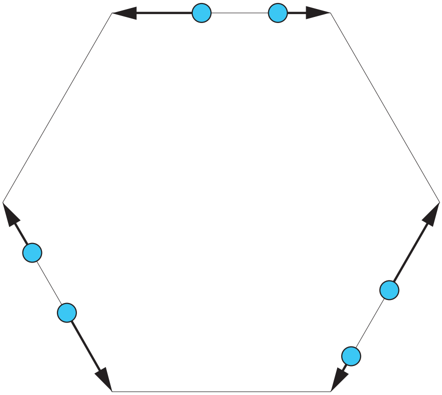
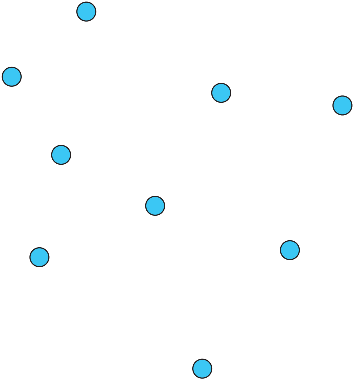


For consistency, they must be resolved in the same fashion

Tool: Smallest Enclosing Circle (SEC)



There always exists a unique **SEC**, which is easily computable

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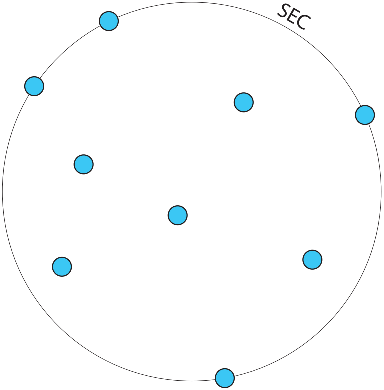

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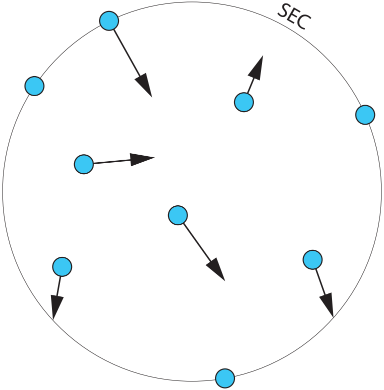

Strategy: always preserve the SEC, until a Pre-regular is formed

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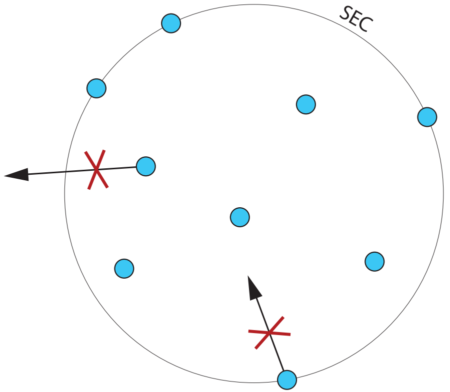
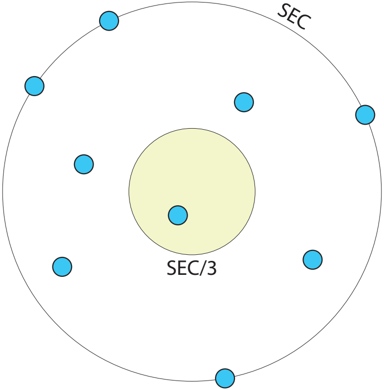


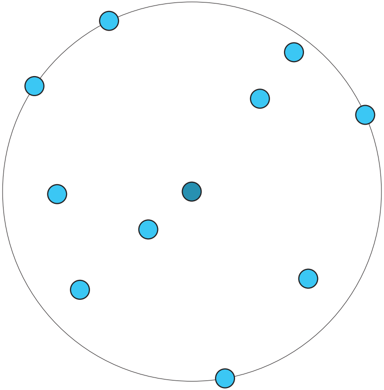
Strategy: always preserve the SEC, until a Pre-regular is formed

Tool: Smallest Enclosing Circle (SEC)



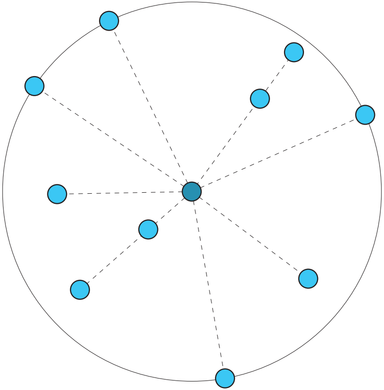
Another circle plays an important role: **SEC/3**

Central configurations



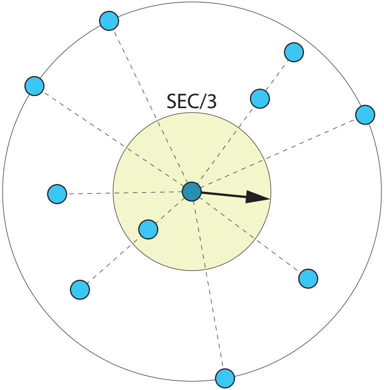
If there is a robot at the center of the SEC

Central configurations



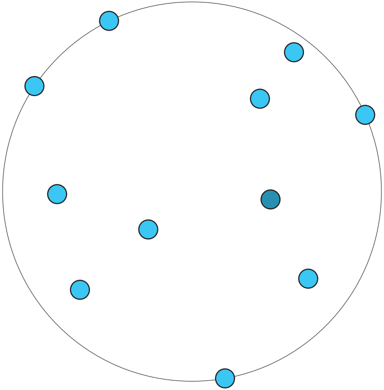
Identify the occupied radii

Central configurations



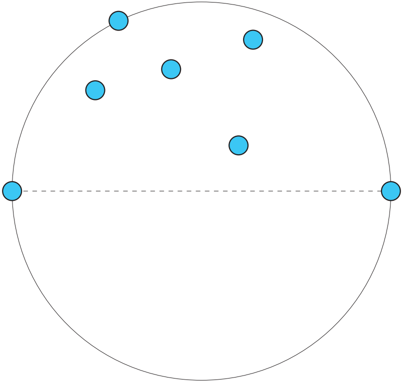
Move to an unoccupied radius all the way to SEC/3

Central configurations



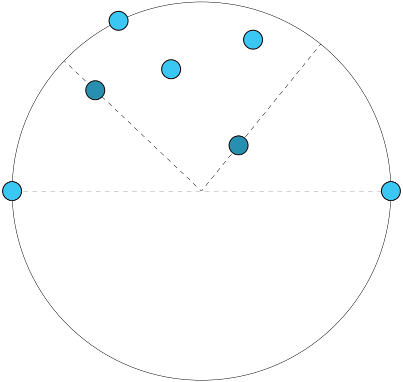
No Central configuration will ever be formed again

Half-disk configurations



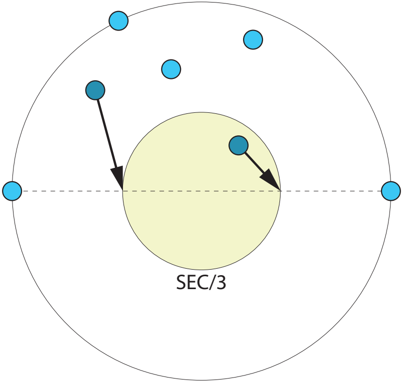
If all the robots lie in one half of the SEC

Half-disk configurations



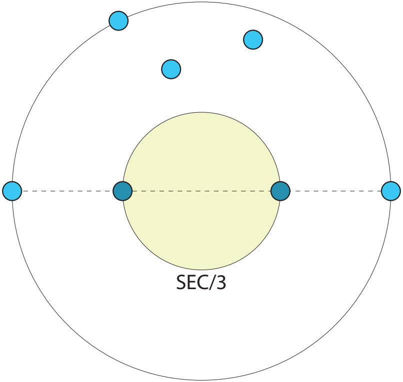
Identify the robots “angularly closest” to the diameter

Half-disk configurations



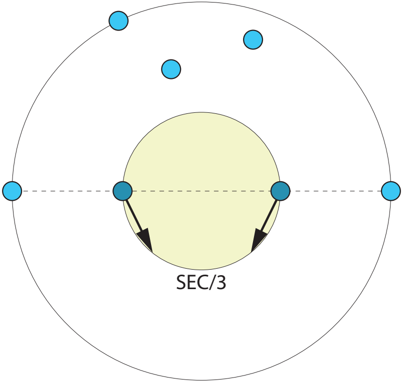
Move them to the diameter and make sure they are in SEC/3

Half-disk configurations



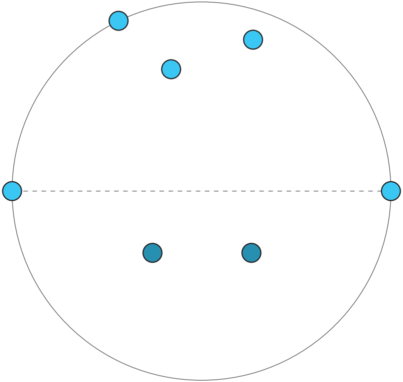
Move them to the diameter and make sure they are in SEC/3

Half-disk configurations



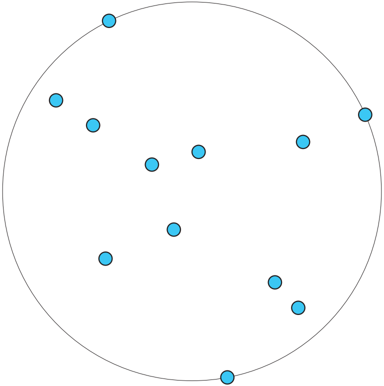
Make them cross the diameter but remain in SEC/3

Half-disk configurations



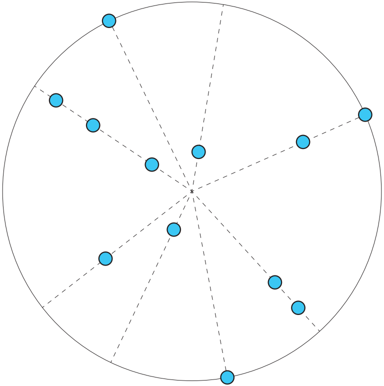
No half-disk configuration will ever be formed again

Co-radial configurations



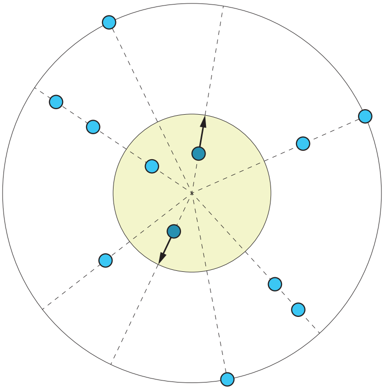
If there are co-radial robots

Co-radial configurations



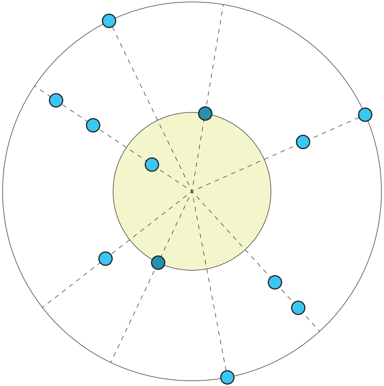
Identify the occupied radii

Co-radial configurations



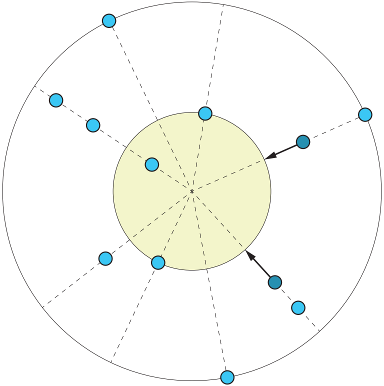
Move radially to SEC/3 the non-co-radial robots that lie inside it

Co-radial configurations



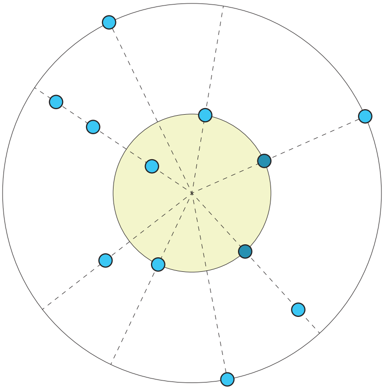
Move radially to SEC/3 the non-co-radial robots that lie inside it

Co-radial configurations



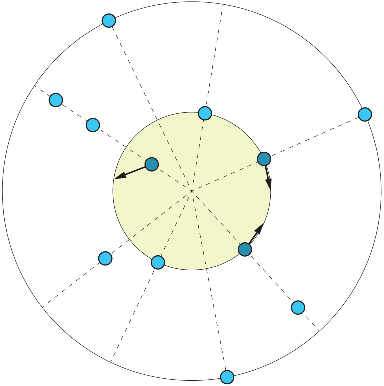
Move to SEC/3 the innermost co-radial robots that lie outside

Co-radial configurations



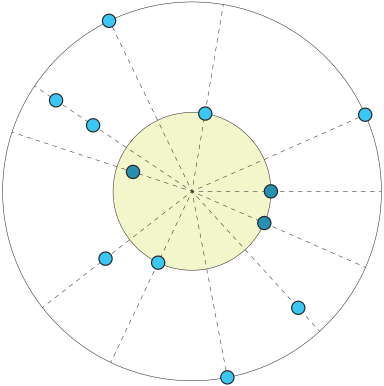
Move to SEC/3 the innermost co-radial robots that lie outside

Co-radial configurations



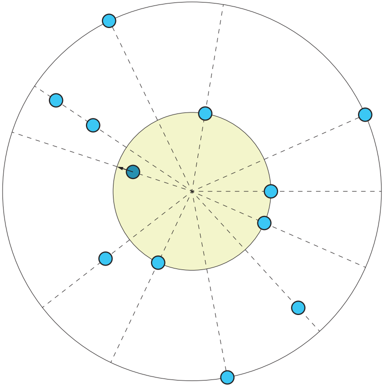
Move the innermost co-radial robots laterally by a small angle

Co-radial configurations



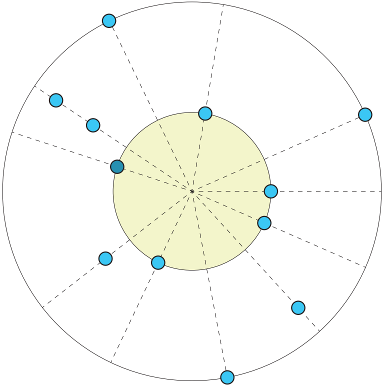
If moves are small enough, no new co-radialities are formed

Co-radial configurations



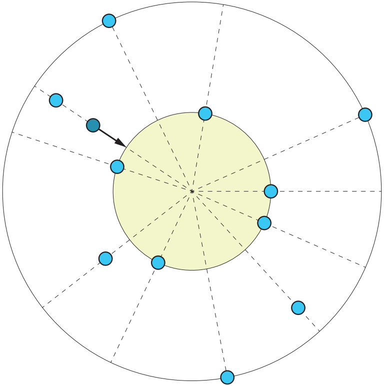
Move radially to SEC/3 the non-co-radial robots that lie inside it

Co-radial configurations



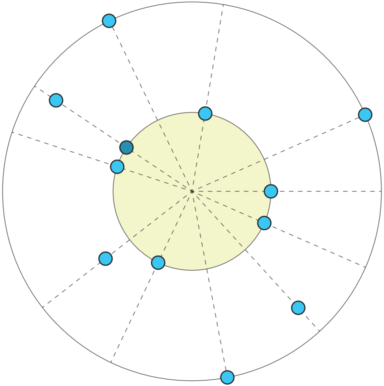
Move radially to SEC/3 the non-co-radial robots that lie inside it

Co-radial configurations



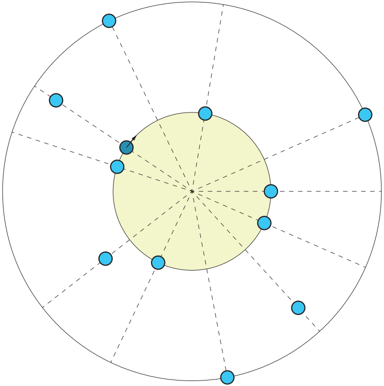
Move to SEC/3 the innermost co-radial robots that lie outside

Co-radial configurations



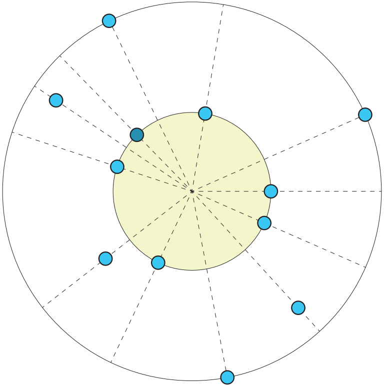
Move to SEC/3 the innermost co-radial robots that lie outside

Co-radial configurations



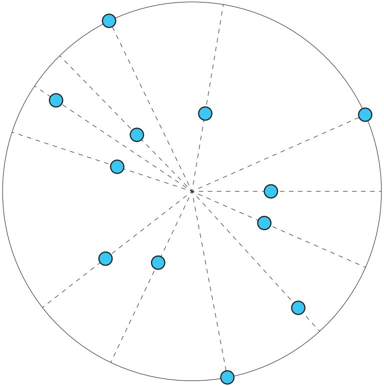
Move the innermost co-radial robots laterally by a small angle

Co-radial configurations



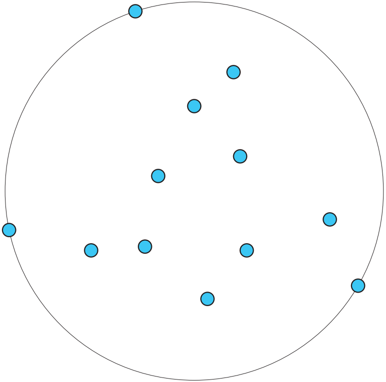
Move the innermost co-radial robots laterally by a small angle

Co-radial configurations



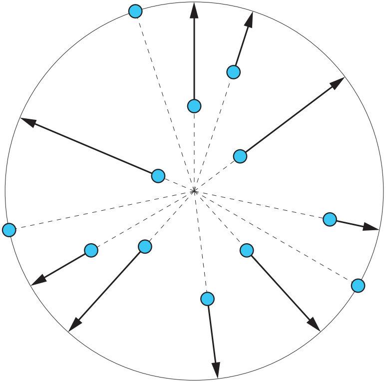
No Co-radial configuration will ever be formed again

General case: overview



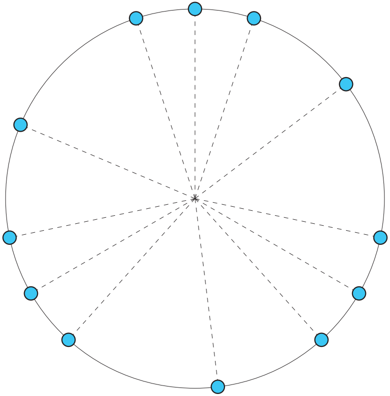
If we are not in one of the previous special cases

General case: overview



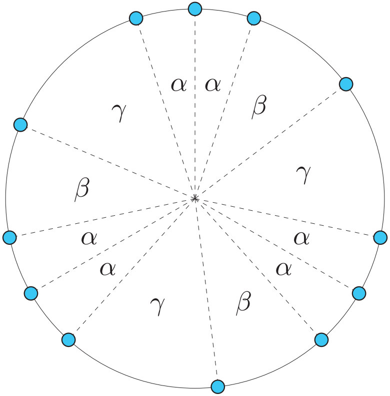
All robots move radially to SEC

General case: overview



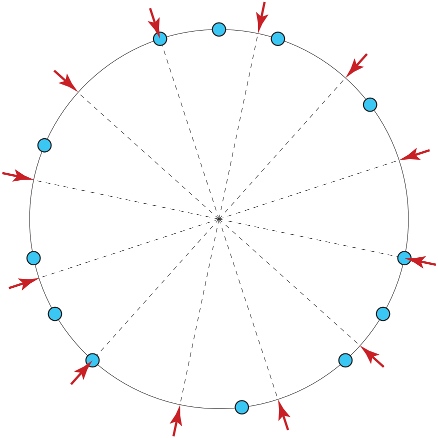
All robots move radially to SEC

General case: overview



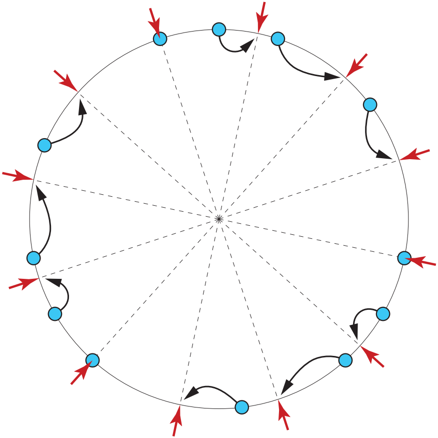
Consider the angle sequence

General case: overview

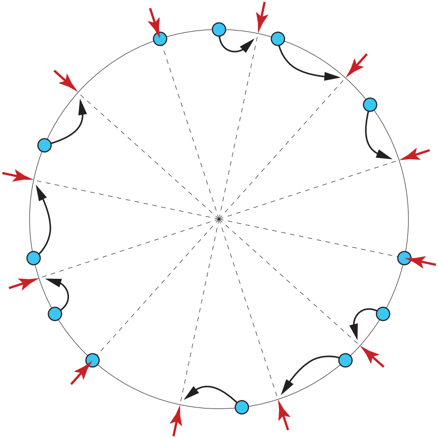


To solve Uniform Circle Formation, all angles must become equal

General case: overview

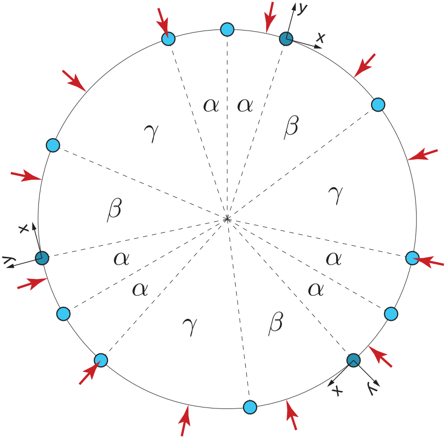


Identify a **target** for each robot

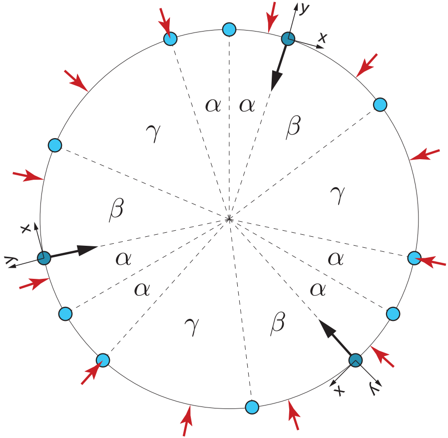


If all robots move together, they may forget their targets!

General case: overview

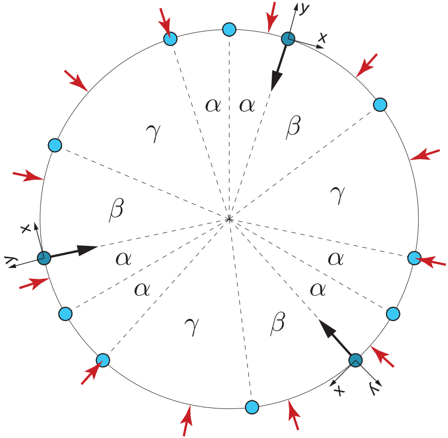


Robots that "see" the same angle sequence are **analogous**



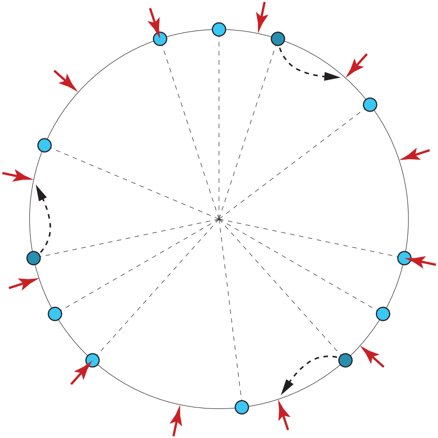
The scheduler can force analogous robots to move together

General case: overview



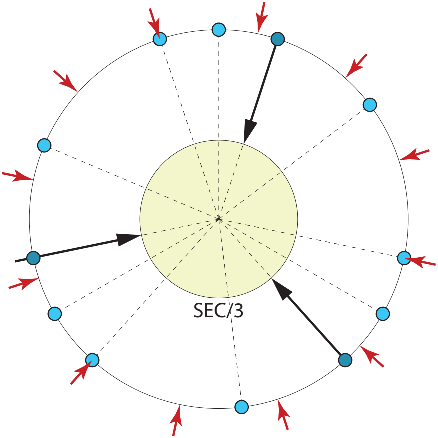
Hence the algorithm will move one **analogy class** at a time

General case: overview



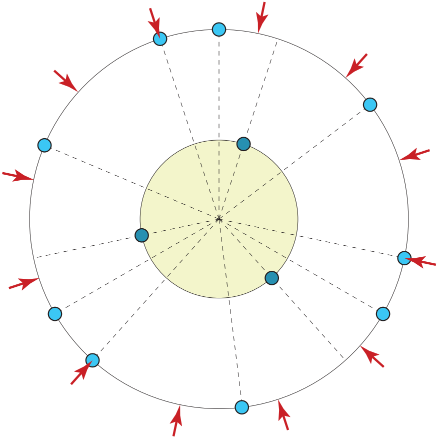
Strategy: choose analogous robots that can “see” their targets

General case: overview

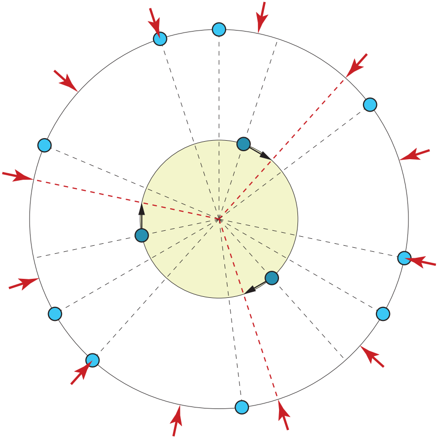


Move these **walkers** radially to SEC/3

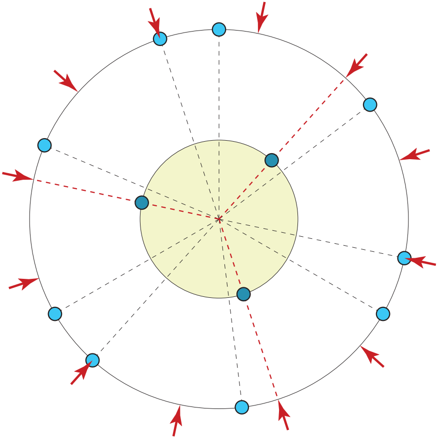
General case: overview



Move these **walkers** radially to SEC/3

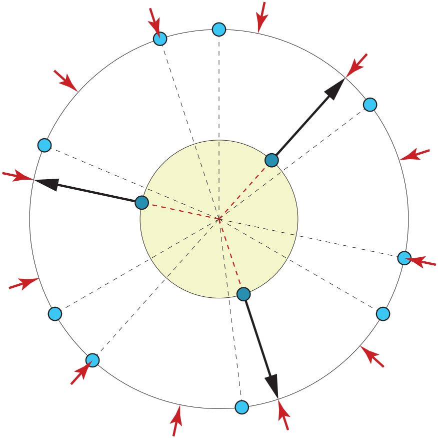


Move them laterally to their targets



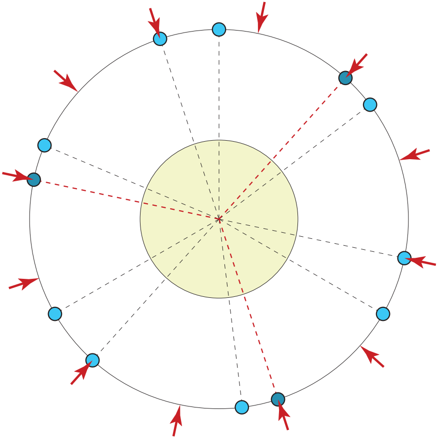
Move them laterally to their targets

General case: overview



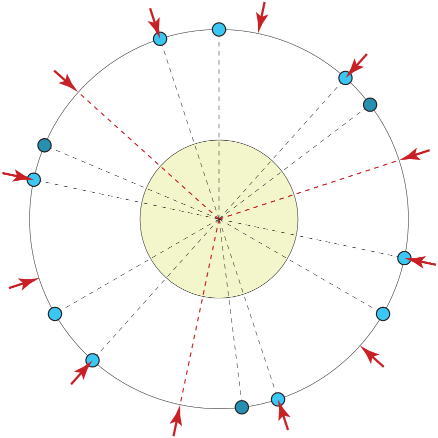
Move them back to SEC

General case: overview



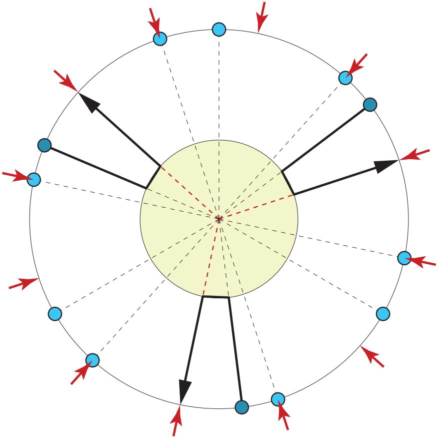
Move them back to SEC

General case: overview



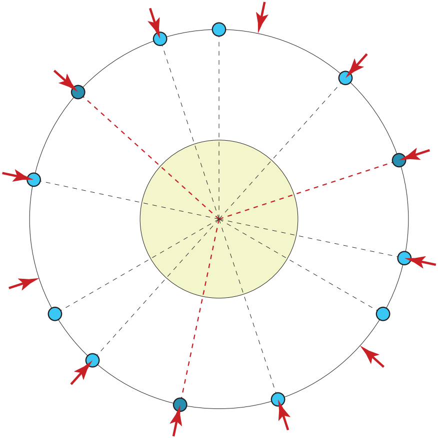
Repeat with another analogy class, until all targets are reached

General case: overview



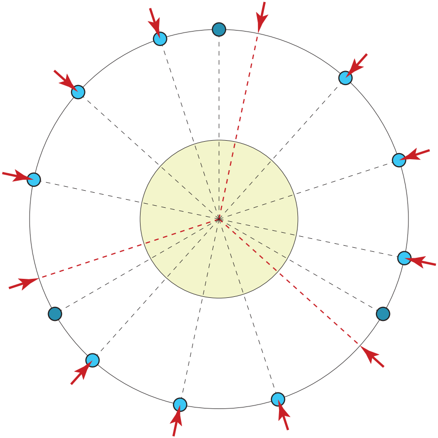
Repeat with another analogy class, until all targets are reached

General case: overview



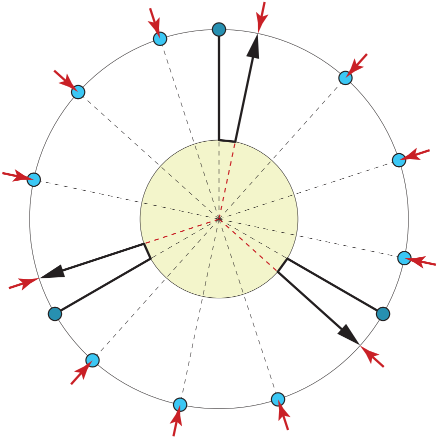
Repeat with another analogy class, until all targets are reached

General case: overview



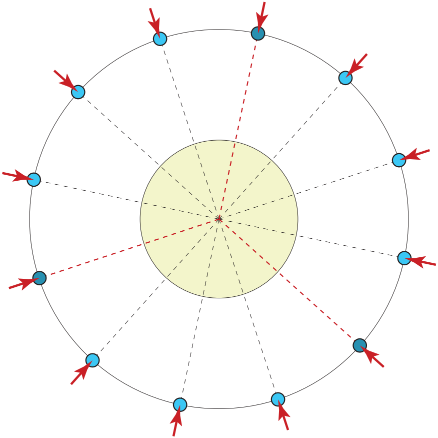
Repeat with another analogy class, until all targets are reached

General case: overview



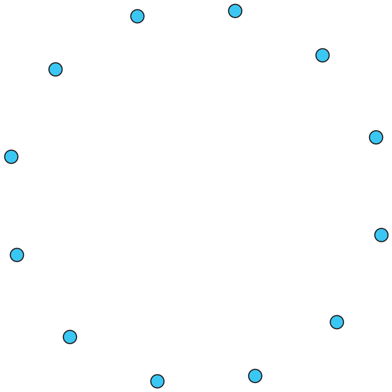
Repeat with another analogy class, until all targets are reached

General case: overview



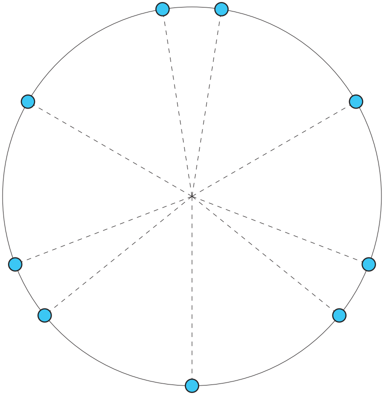
Repeat with another analogy class, until all targets are reached

General case: overview



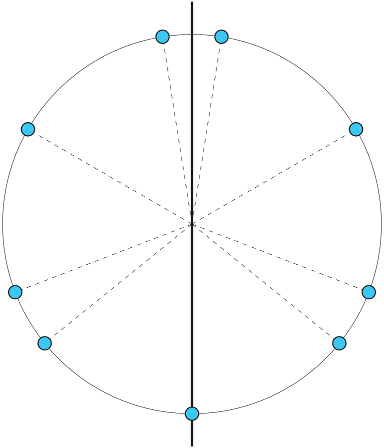
Repeat with another analogy class, until all targets are reached

How to identify the target set

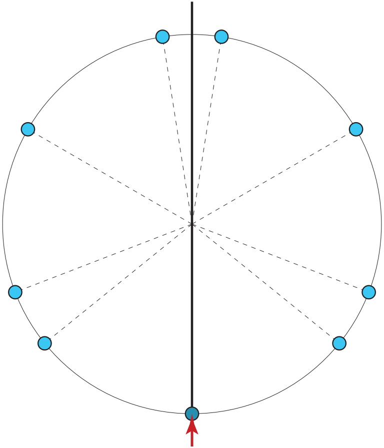


Suppose the configuration has an axis of symmetry

How to identify the target set

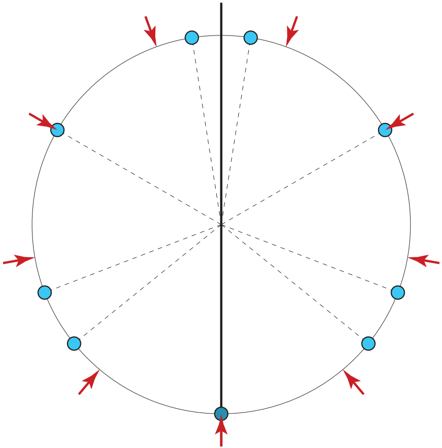


Suppose the configuration has an axis of symmetry



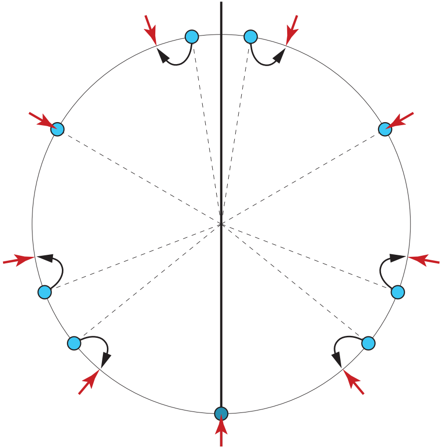
If a robot lies on the axis, it is on its target by definition

How to identify the target set



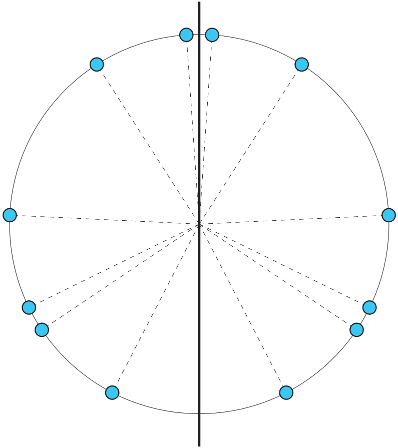
The other targets are determined accordingly

How to identify the target set



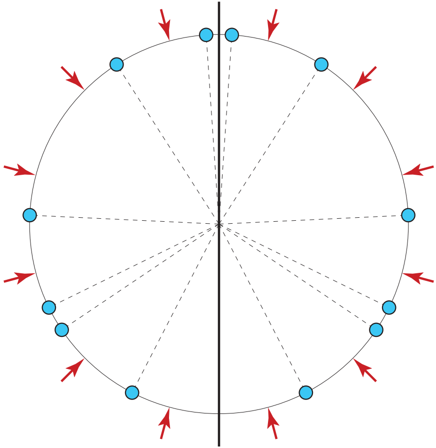
The other targets are determined accordingly

How to identify the target set



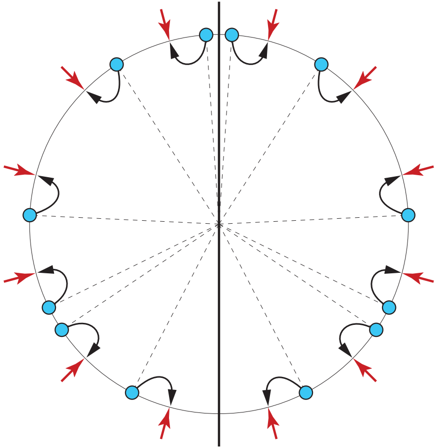
Suppose that no robot lies on the axis of symmetry

How to identify the target set



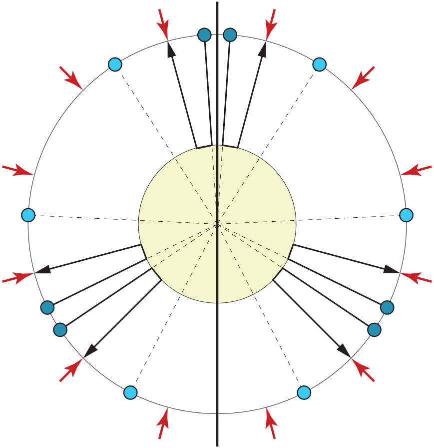
Then no target lies on the axis of symmetry, either

How to identify the target set



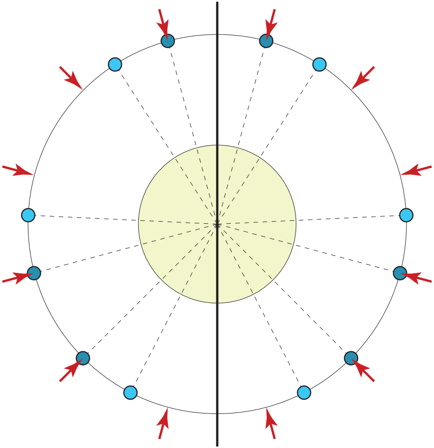
Then no target lies on the axis of symmetry, either

How to identify the target set



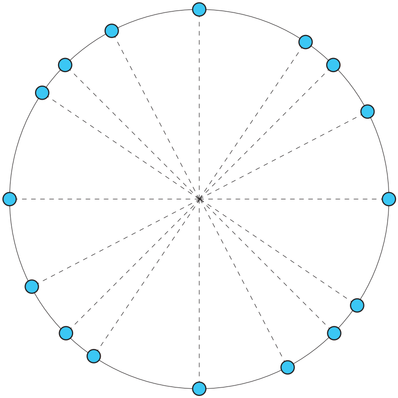
When an analogy class moves, the axis of symmetry is preserved

How to identify the target set



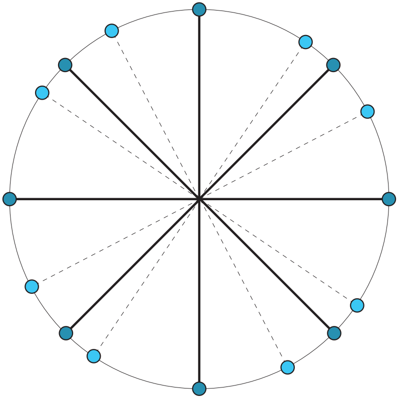
Hence also the targets are preserved

How to identify the target set



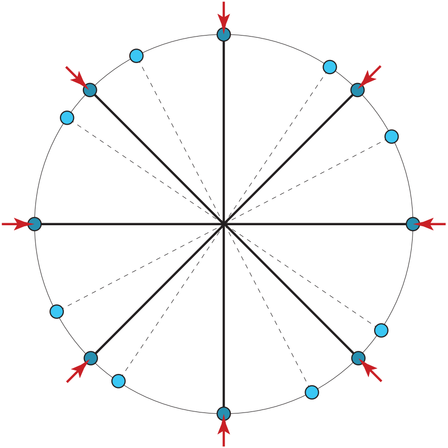
Suppose the configuration has no axis of symmetry

How to identify the target set



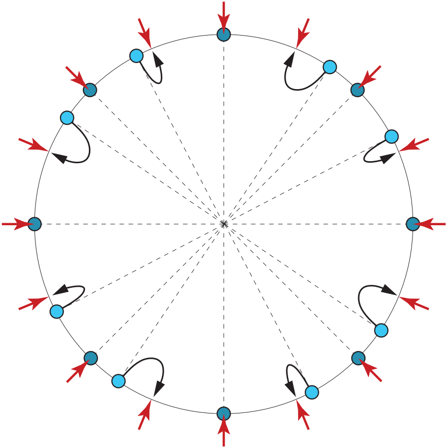
Robots having the “correct” angular distance are **concordant**

How to identify the target set



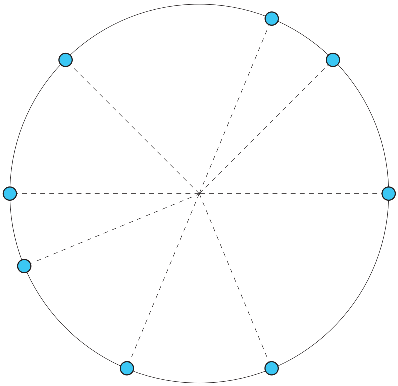
The targets are determined by the largest **concordance class**

How to identify the target set



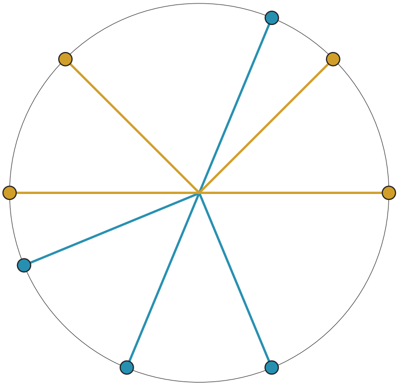
The targets are determined by the largest **concordance class**

How to identify the target set



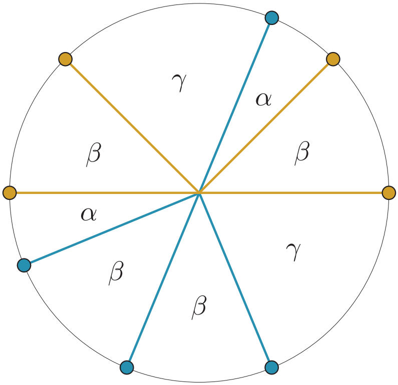
If there is more than one largest concordance class...

How to identify the target set



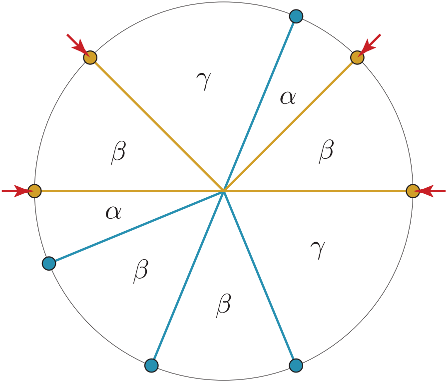
If there is more than one largest concordance class...

How to identify the target set



…they are all "non-equivalent", so one can always be chosen

How to identify the target set



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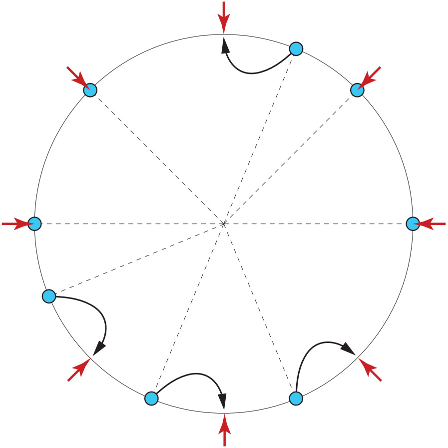
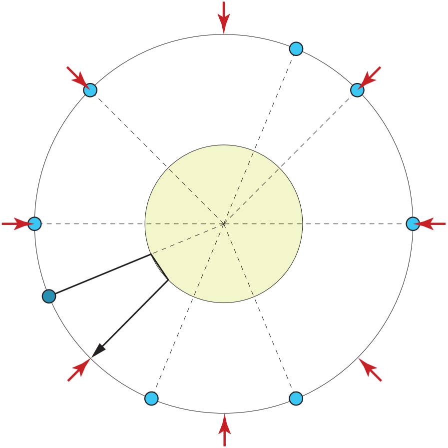
How to identify the target set



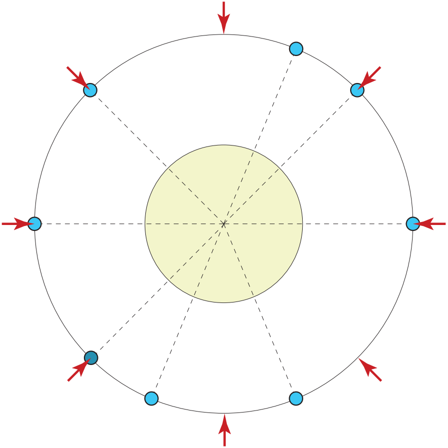
...they are all "non-equivalent", so one can always be chosen

How to identify the target set



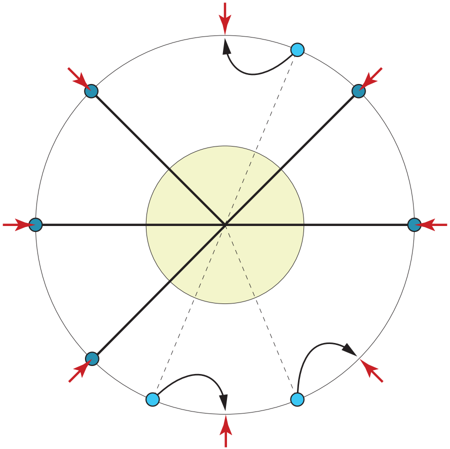
After a move, the chosen concordance class becomes the largest

How to identify the target set



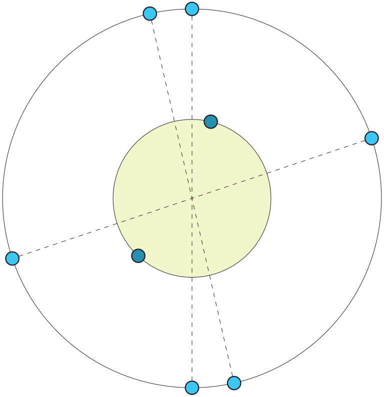
After a move, the chosen concordance class becomes the largest

How to identify the target set



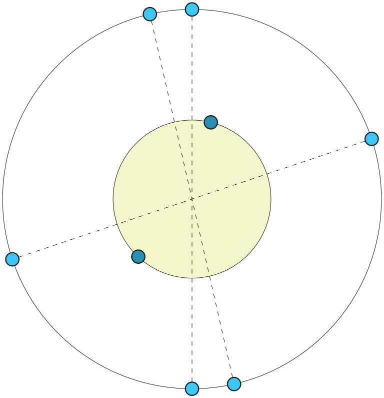
So the targets are preserved, until an axis of symmetry is created

How the walkers “remember” their targets



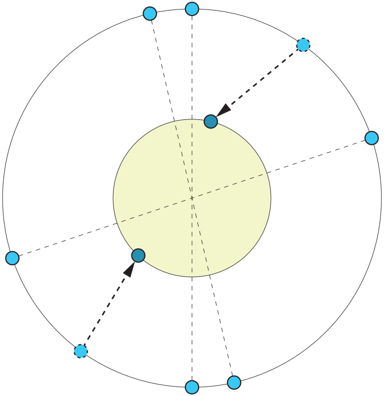
Target locations depend on the walkers’ initial positions on SEC

How the walkers “remember” their targets



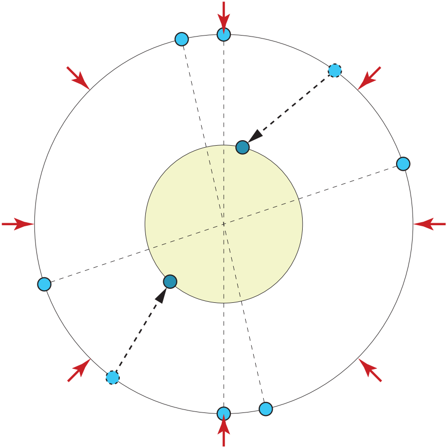
Once in SEC/3, they can only “guess” the initial positions

How the walkers "remember" their targets



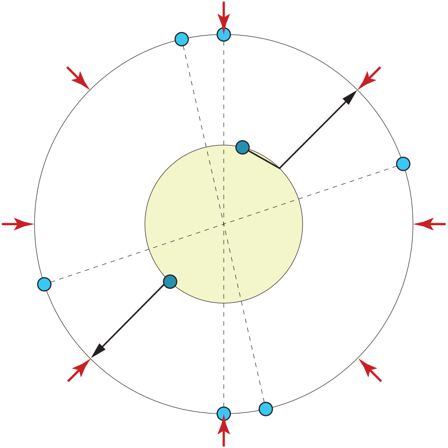
Reasonable guess: we were equidistant from our adjacent robots

How the walkers "remember" their targets



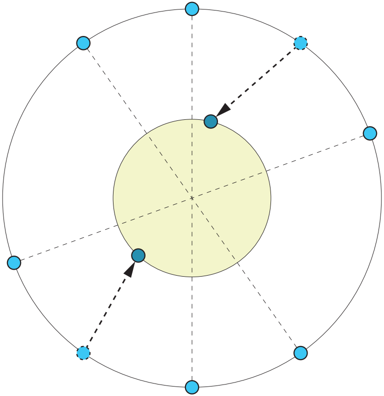
Now they can reconstruct a consistent set of targets...

How the walkers “remember” their targets



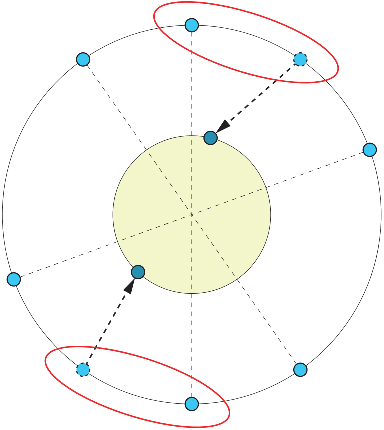
…and move to the appropriate ones

How the walkers “remember” their targets



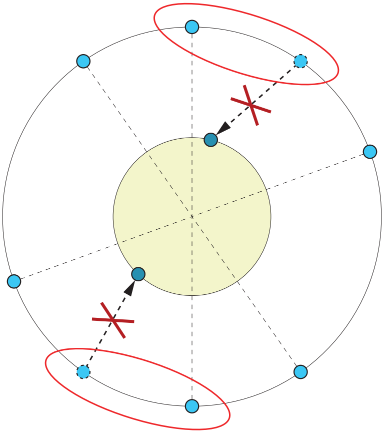
Problem: the guessed locations may not form an analogy class...

How the walkers “remember” their targets



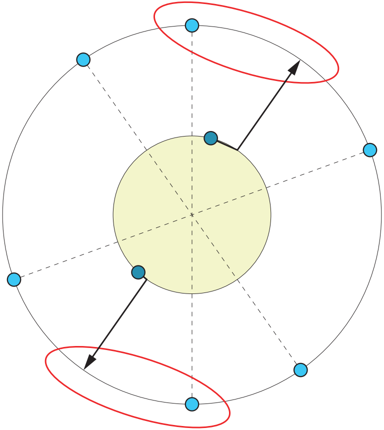
…they may be a proper subset of one!

How the walkers "remember" their targets



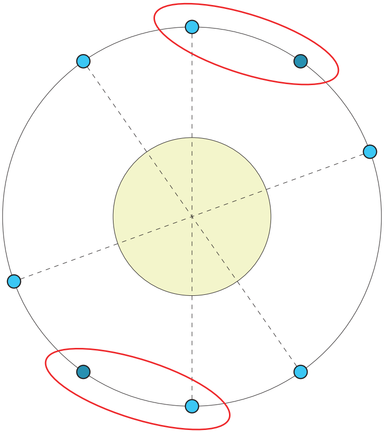
Wrong guess: the walkers are supposed to form an analogy class

How the walkers "remember" their targets



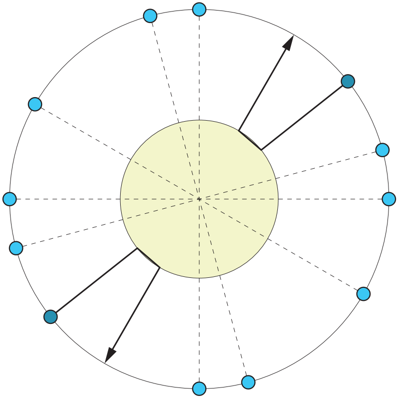
Solution: move to the guessed positions!

How the walkers “remember” their targets



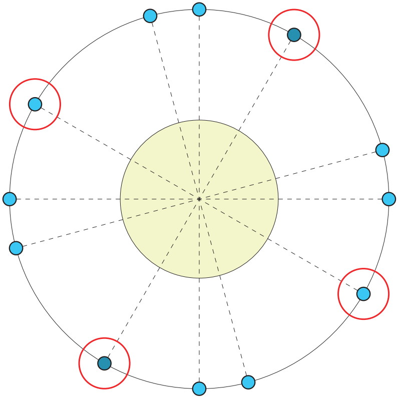
This causes two analogy classes to merge

Making steady progress



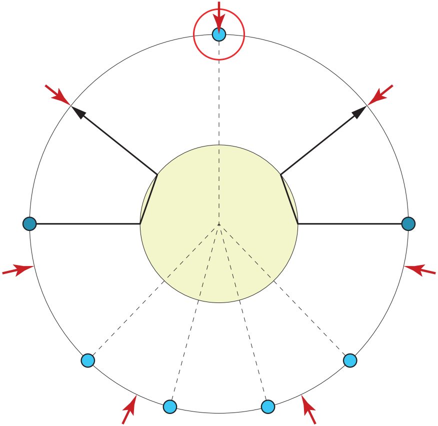
When the walkers complete their journey, two things can happen

Making steady progress



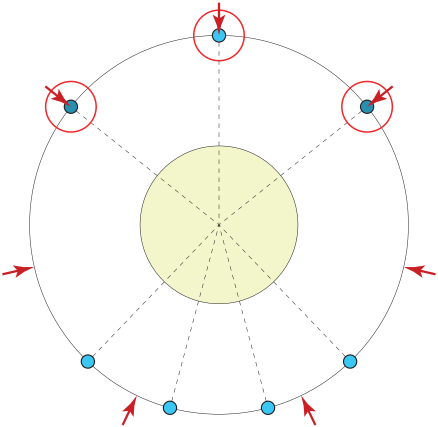
Either two analogy classes merge (and the targets may change)...

Making steady progress



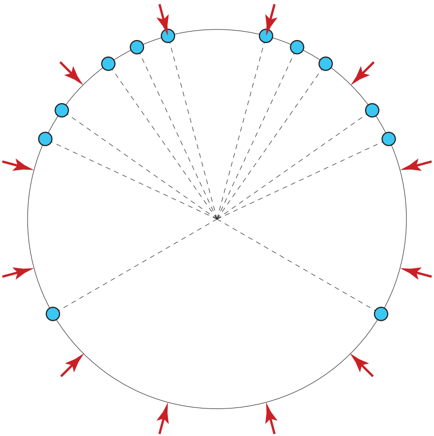
...or more robots reach their targets

Making steady progress



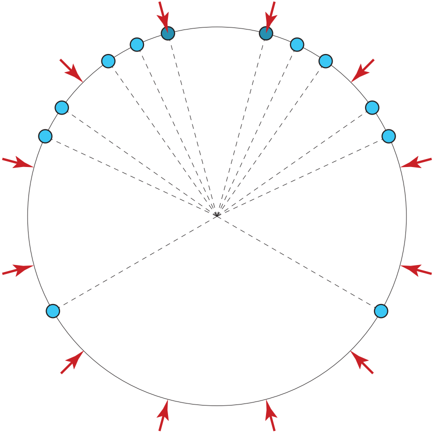
Eventually all robots will reach their targets

Locked configurations



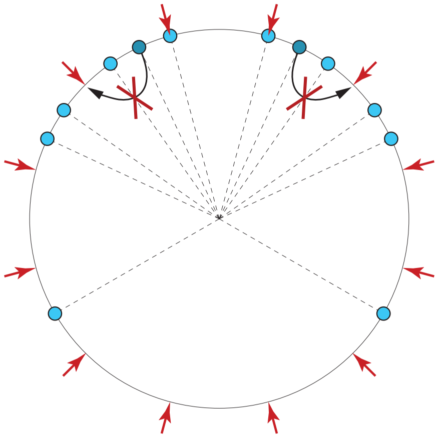
Sometimes no analogy class is able to move to reach its targets

Locked configurations



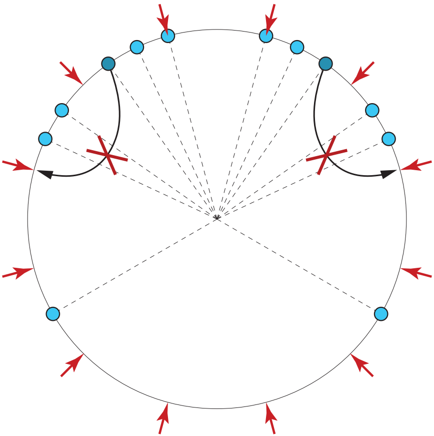
Either because it is already there...

Locked configurations



...or because it would form a Co-radial configuration...

Locked configurations



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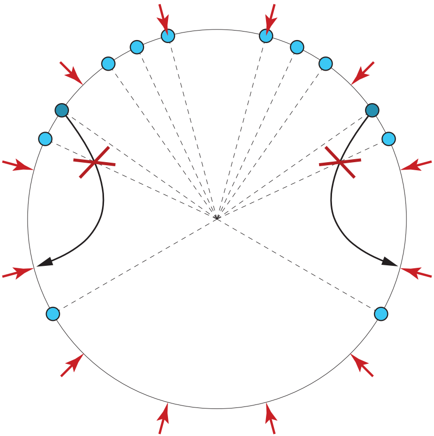
Locked configurations



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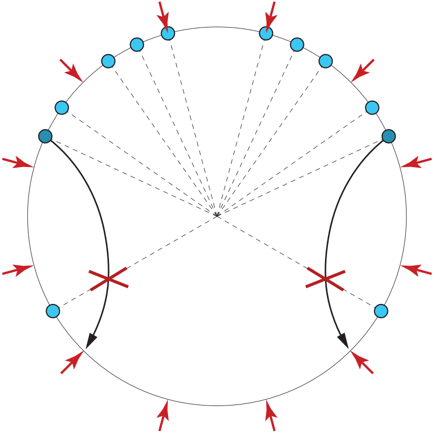
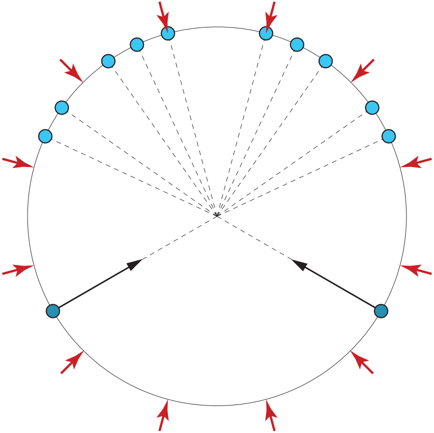
Locked configurations



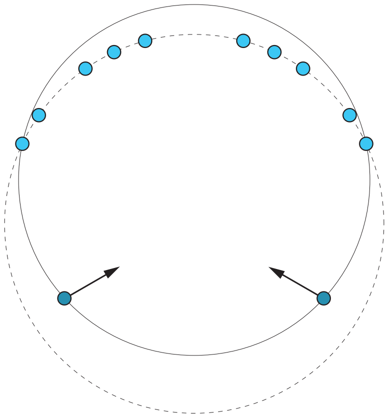
...or because it would form a Co-radial configuration...

Locked configurations



…or because it would alter the SEC

Locked configurations



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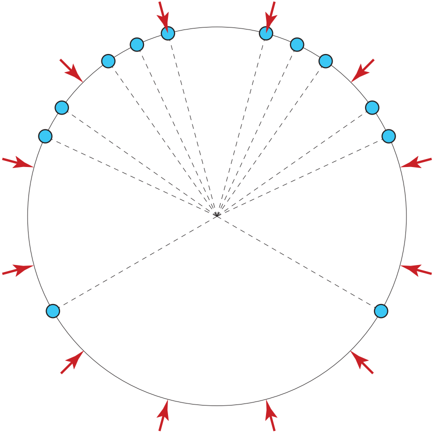
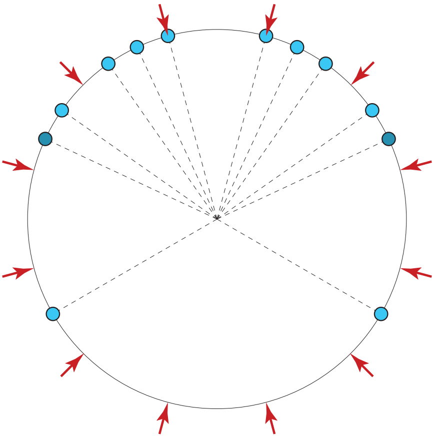
Locked configurations



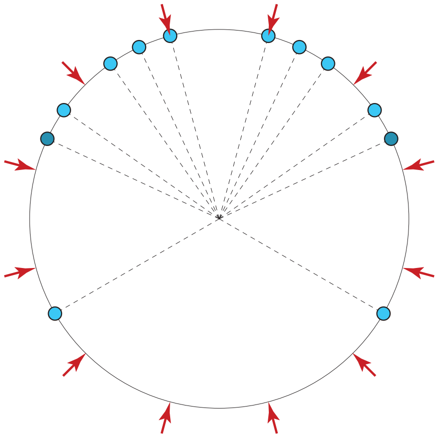
In this case the configuration is **locked**

Locked configurations



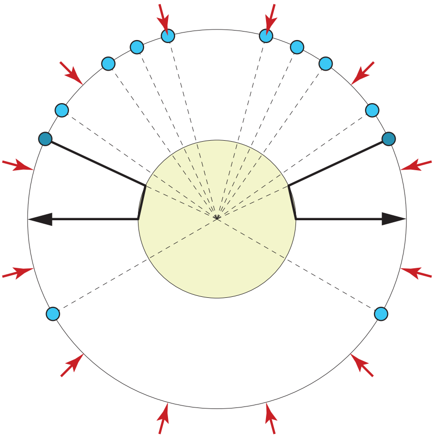
Strategy: identify an **unlocking** analogy class

Locked configurations



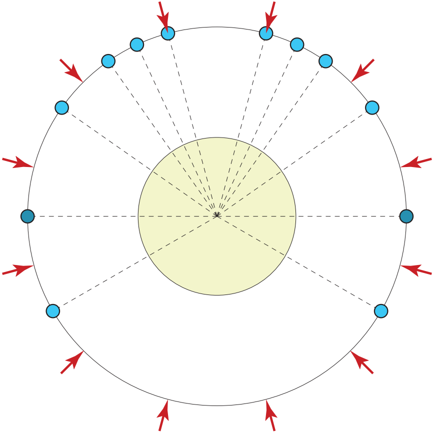
It exists because the configuration is not Half-disk!

Locked configurations



The unlocking class makes a preliminary move...

Locked configurations



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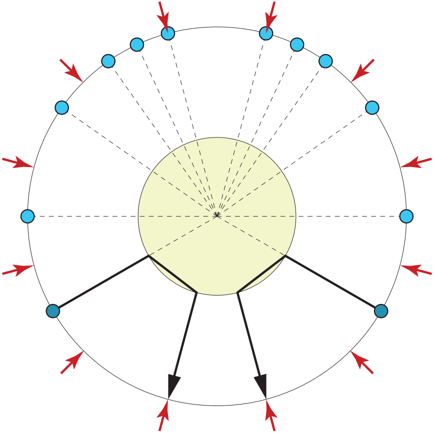
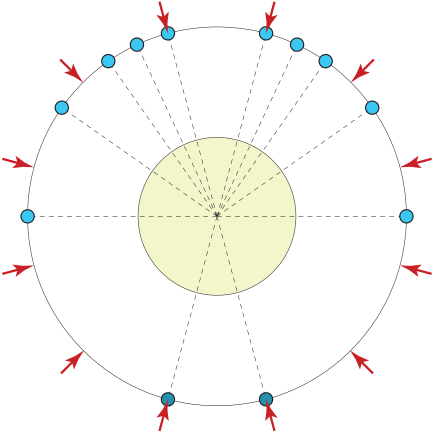


...and the previously unmovable class becomes free to move

Locked configurations



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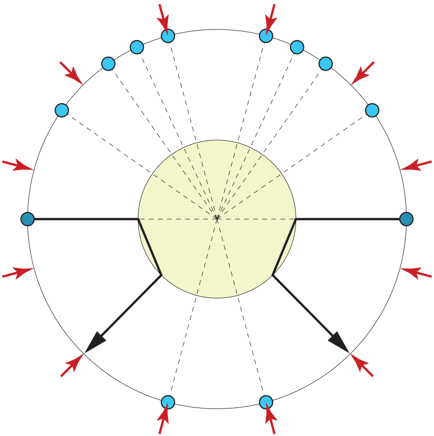
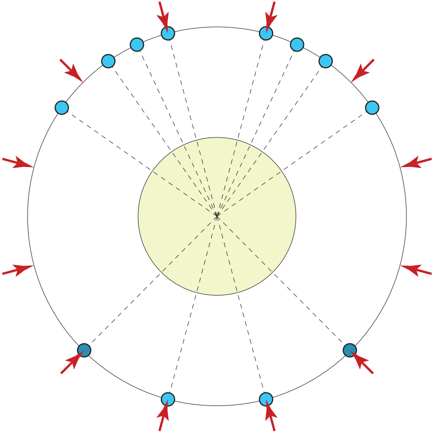


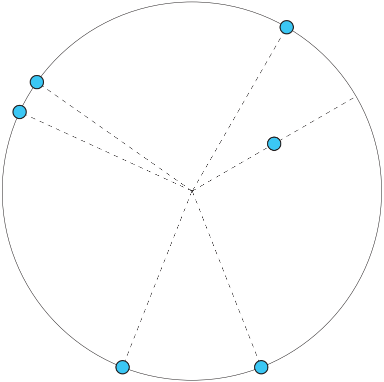
The unlocking step does not make robots lose their progress

Locked configurations



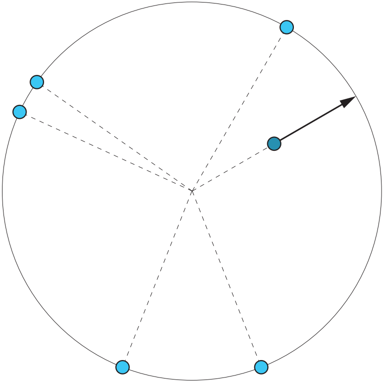
And after the unlocking step, steady progress is made again

Accidental formation of Pre-regular configurations



What if a Pre-regular configuration is formed “accidentally”?

Accidental formation of Pre-regular configurations



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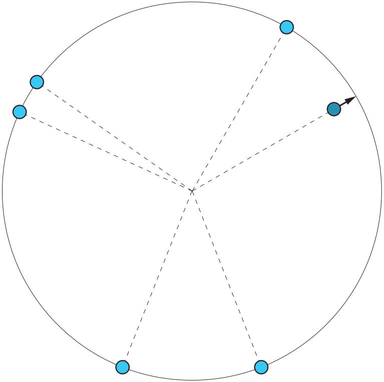

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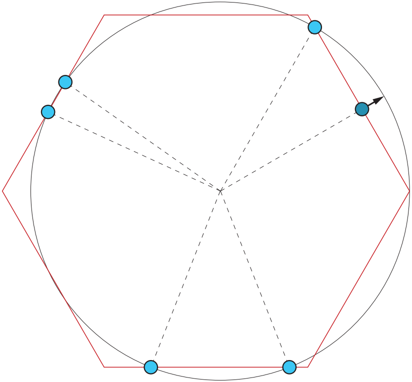

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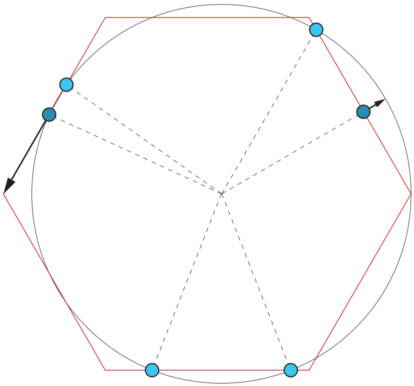
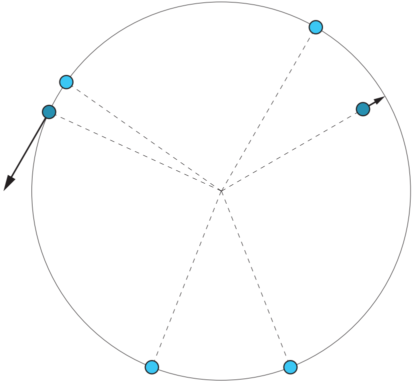


Due to asynchronicity, the behavior may be inconsistent

Accidental formation of Pre-regular configurations



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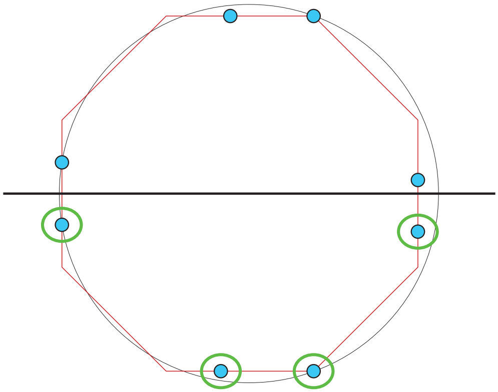
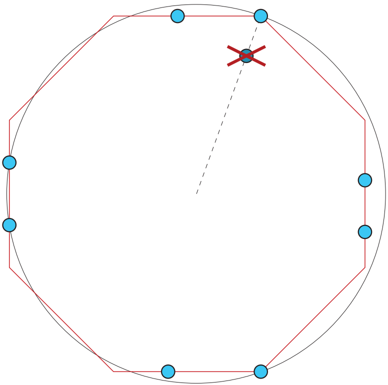
Accidental formation of Pre-regular configurations



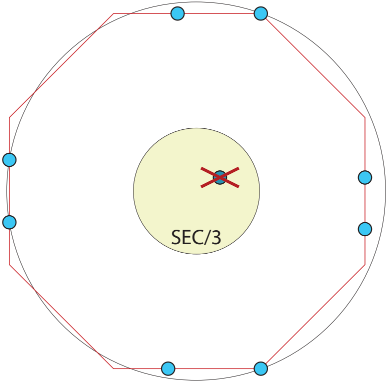
Lemma: no Pre-regular configuration is Half-disk

Accidental formation of Pre-regular configurations



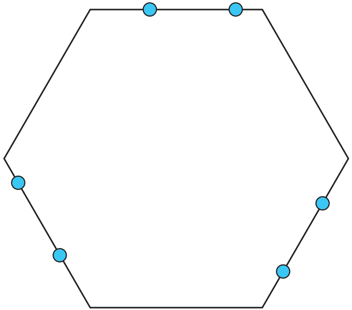
Lemma: no Pre-regular configuration is Co-radial

Accidental formation of Pre-regular configurations

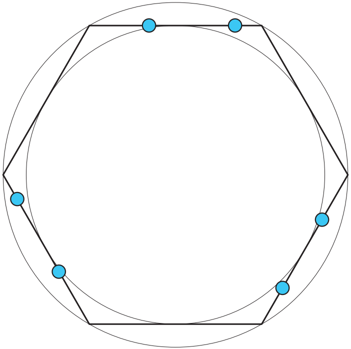


Lemma: no Pre-regular configuration has robots in SEC/3

Accidental formation of Pre-regular configurations

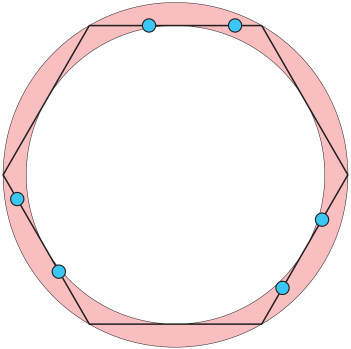


Proof: suppose the configuration is Pre-regular



All robots lie in a thin-enough annulus...

Accidental formation of Pre-regular configurations



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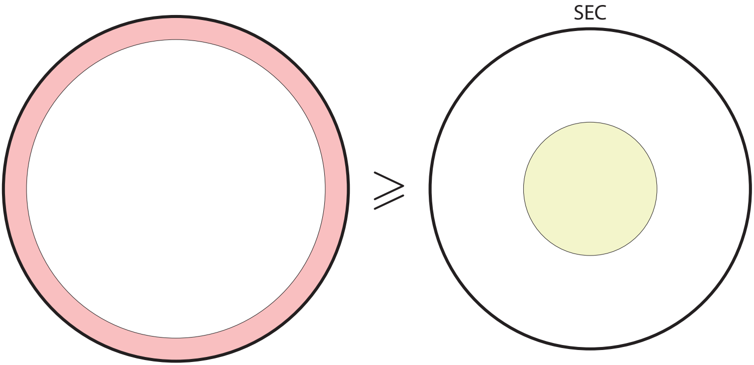
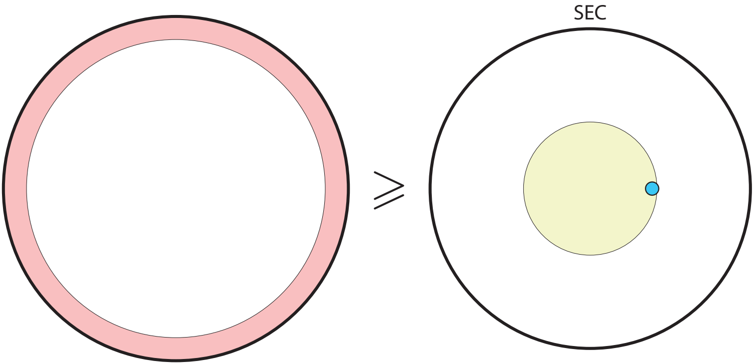
Accidental formation of Pre-regular configurations



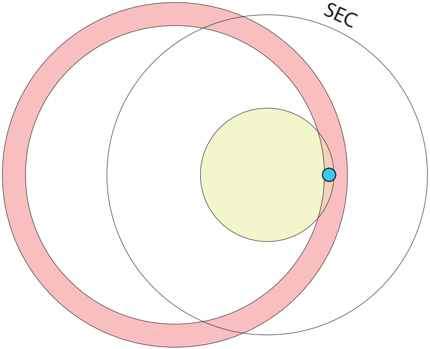
...whose outer circle is at least as large as the SEC

Accidental formation of Pre-regular configurations



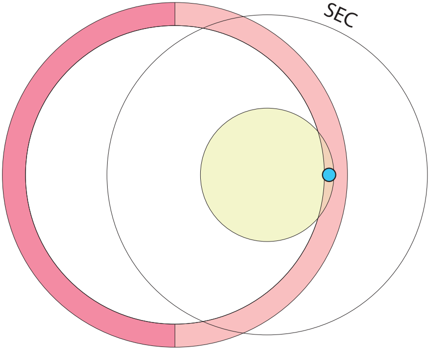
Suppose there is a robot in SEC/3

Accidental formation of Pre-regular configurations



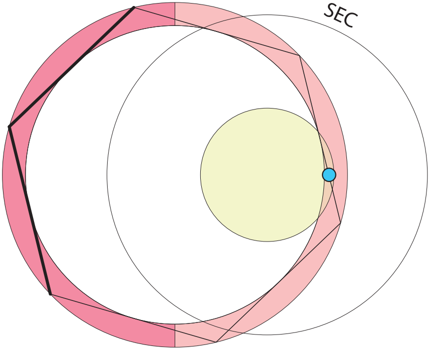
Then the annulus and SEC/3 must overlap

Accidental formation of Pre-regular configurations



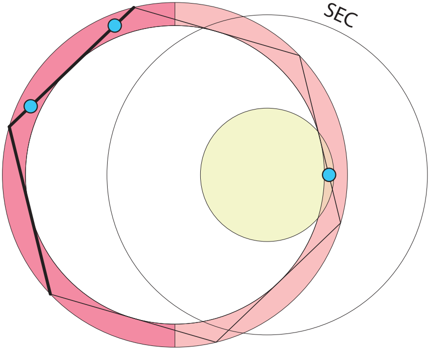
Hence a half-annulus lies outside SEC...

Accidental formation of Pre-regular configurations



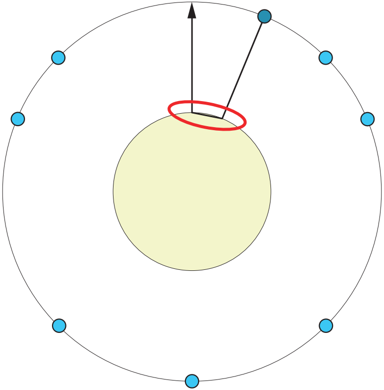
...but it contains two consecutive edges of the polygon...

Accidental formation of Pre-regular configurations



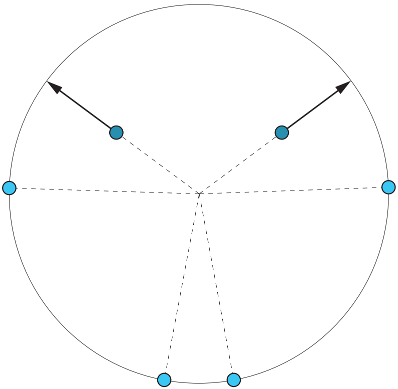
...and one of them must contain robots: contradiction!

Accidental formation of Pre-regular configurations



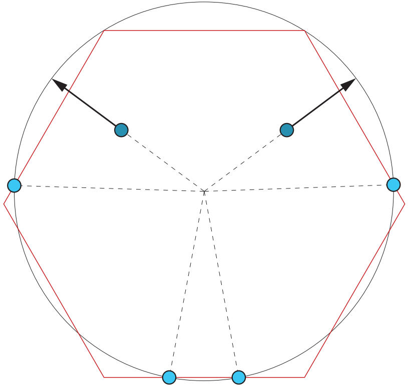
Hence all non-radial moves are safe, as they happen in SEC/3

Accidental formation of Pre-regular configurations



Radial moves are not safe, even if only one analogy class moves

Accidental formation of Pre-regular configurations



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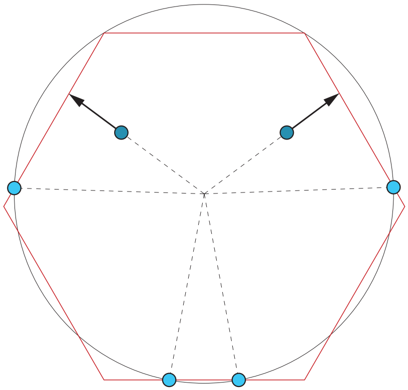
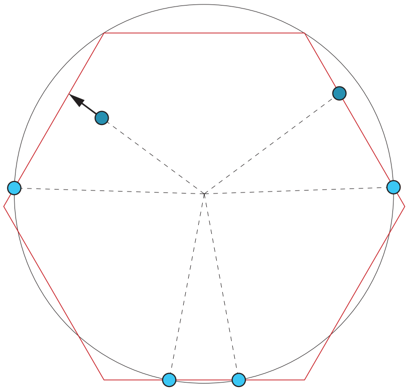
Accidental formation of Pre-regular configurations



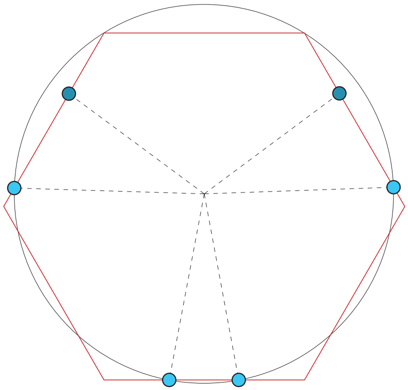
Strategy: add **critical points** corresponding to Pre-regulars...

Accidental formation of Pre-regular configurations



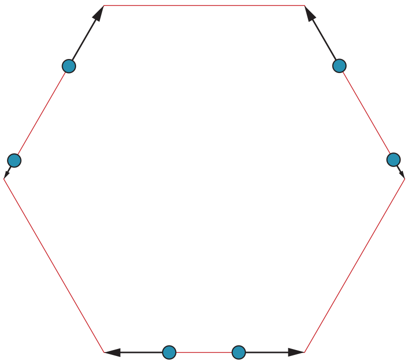
...stop at the next critical point and wait for each other

Accidental formation of Pre-regular configurations



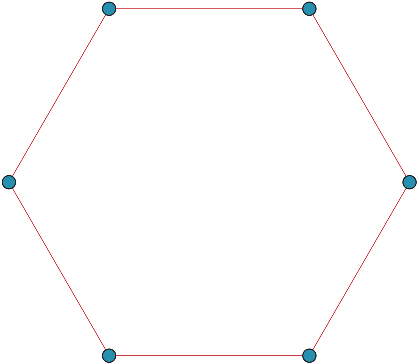
Whenever a Pre-regular is formed, all robots are stopped

Accidental formation of Pre-regular configurations



So they correctly transition

Accidental formation of Pre-regular configurations



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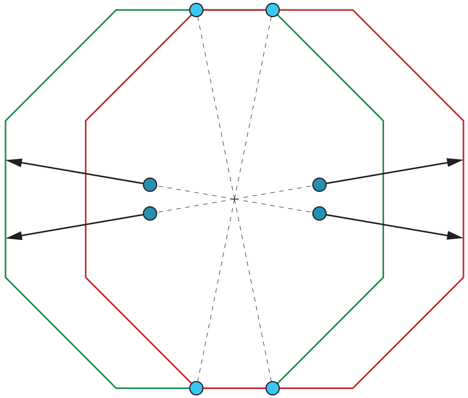
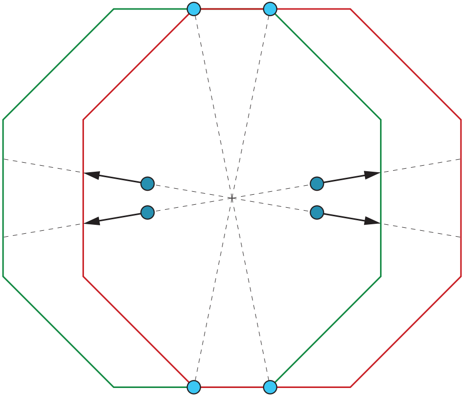
Accidental formation of Pre-regular configurations



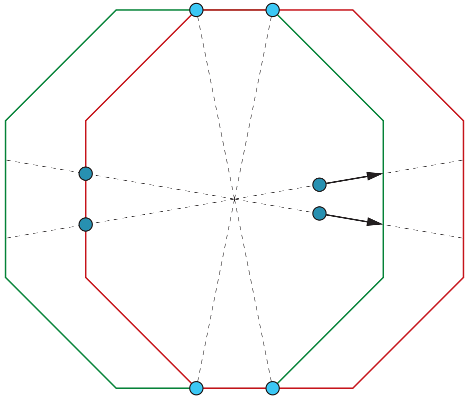
Corresponding critical points may lie on different Pre-regulars

Accidental formation of Pre-regular configurations



Even if robots wait for each other, they may get confused

Accidental formation of Pre-regular configurations



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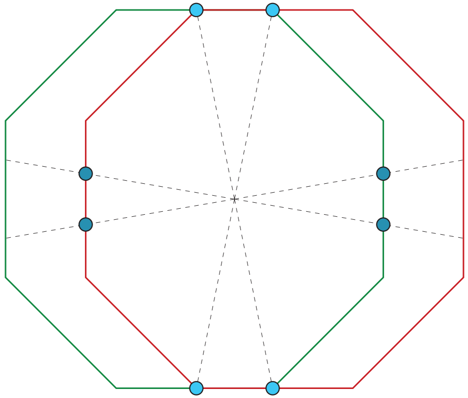

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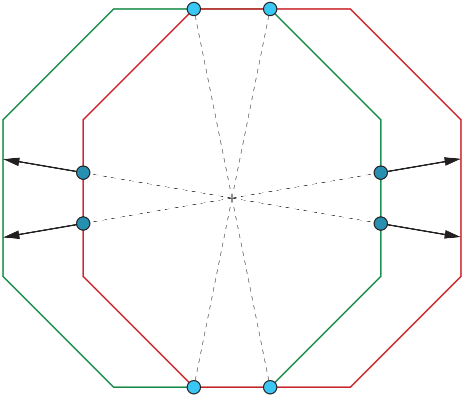

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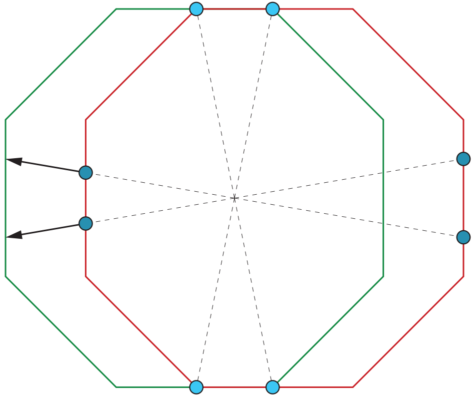

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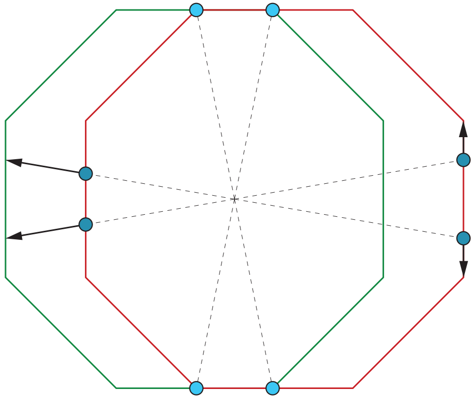

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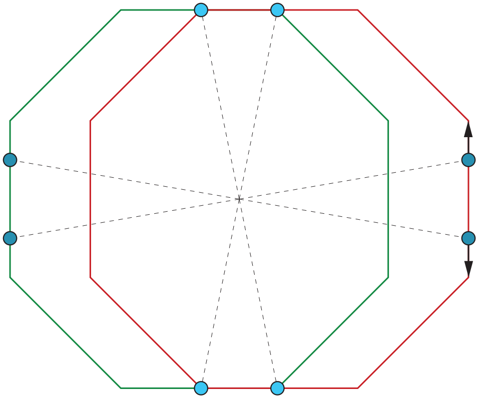

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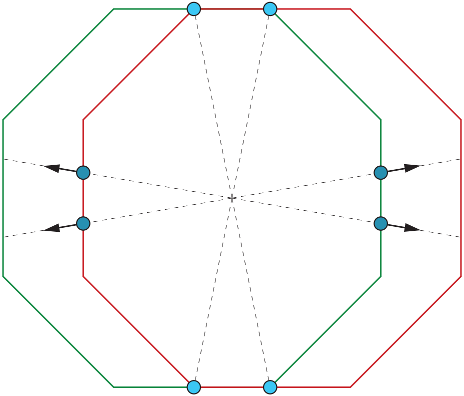
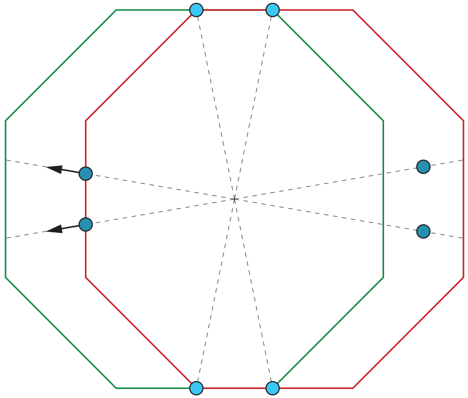


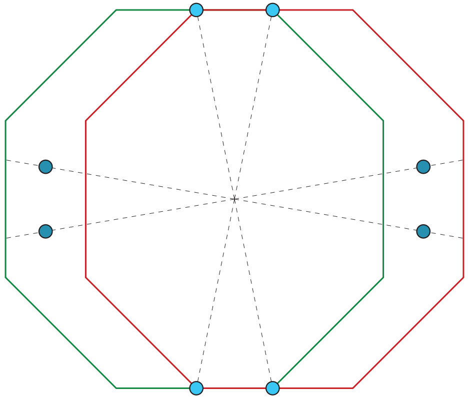
Strategy: add extra critical points between Pre-regulars

Accidental formation of Pre-regular configurations



With this extra step, no Pre-regular is formed

Accidental formation of Pre-regular configurations



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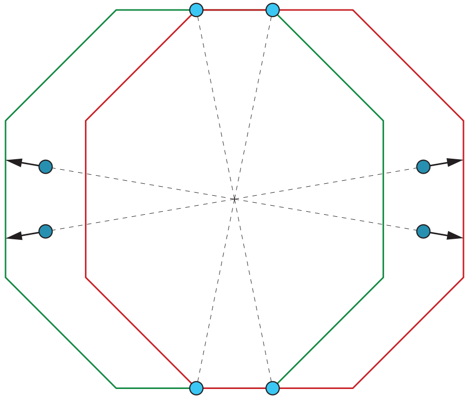

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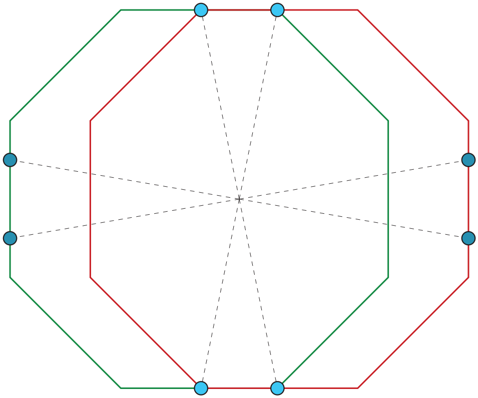

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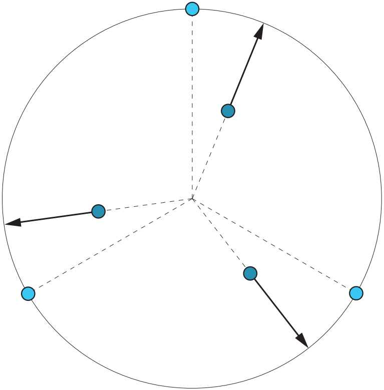
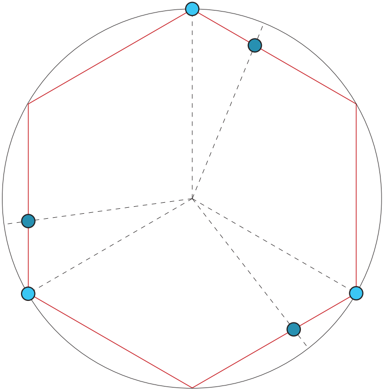


Formable Pre-regulars may be infinitely many!

Accidental formation of Pre-regular configurations



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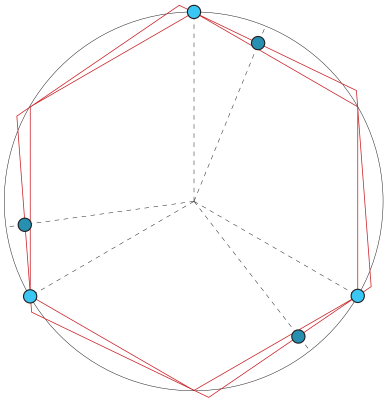

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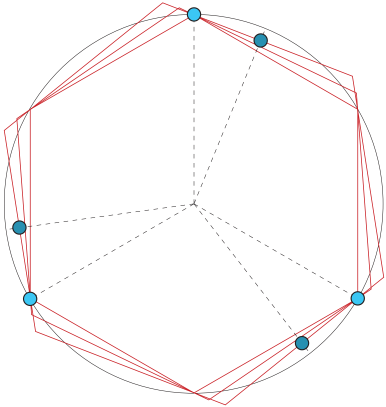

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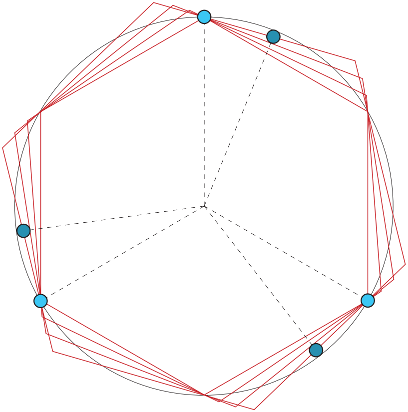

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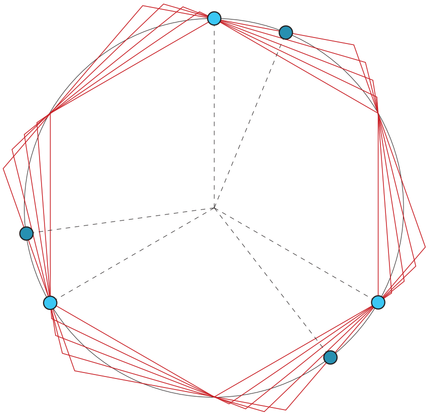

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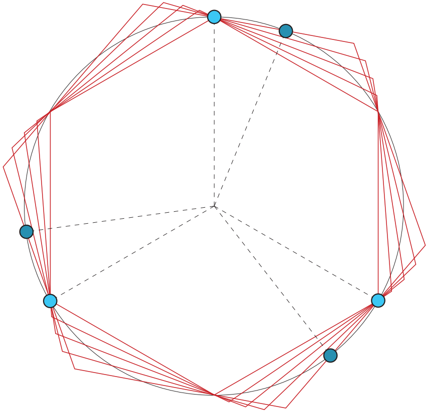
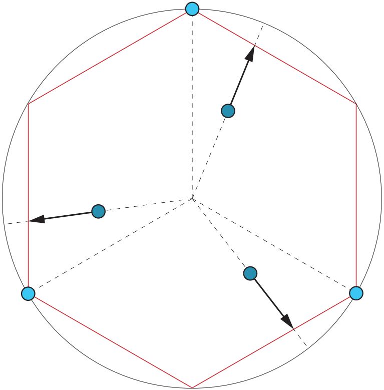


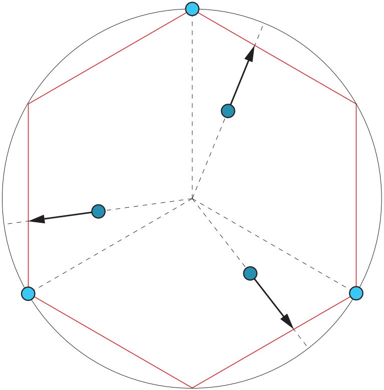
But we cannot have infinitely many critical points

Accidental formation of Pre-regular configurations



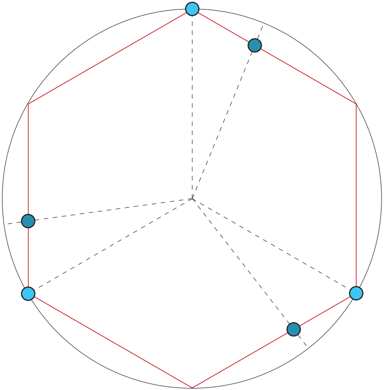
Lemma: a unique Pre-regular is formable before the others

Accidental formation of Pre-regular configurations



Hence finitely many critical points are sufficient

Accidental formation of Pre-regular configurations



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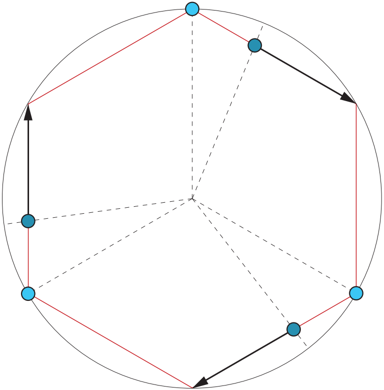

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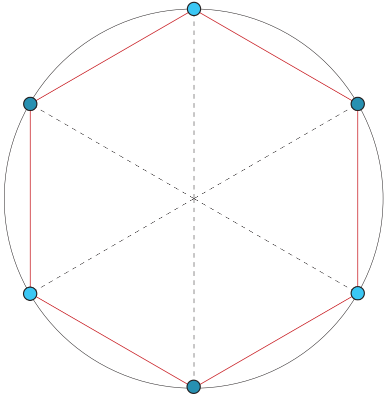
Accidental formation of Pre-regular configurations



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Concluding remarks

The only solvable Pattern Formation problems are:

- **Gathering** problem ($n \neq 2$)
- **Uniform Circle Formation** problem ($n \neq 4$)

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- robots are fully synchronous
- robots have a common notion of "clockwise" (chirality)
- robots always reach their destination (rigidity)

$\Longrightarrow$ For Pattern Formation problems, these features are computationally irrelevant!

Concluding remarks

The only solvable Pattern Formation problems are:

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This is true even if

- robots are fully synchronous
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$\Longrightarrow$ For Pattern Formation problems, these features are computationally irrelevant!

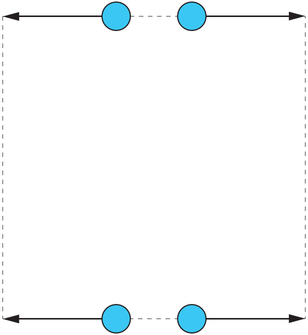
For $n = 4$, the Uniform Circle Formation problem is

- **solved** for semi-synchronous robots (Dieudonné–Petit, 2009)
- **solved** for robots with chirality (Fujinaga–Yamauchi–Kijima–Yamashita, 2012)
- **open** for asynchronous robots with no chirality

Open problem: $n = 4$

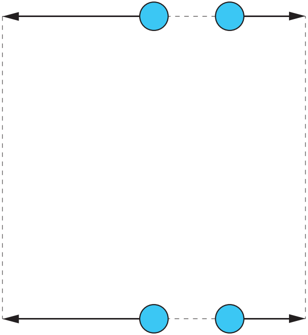
The general algorithm fails for $n = 4$

Open problem: $n = 4$



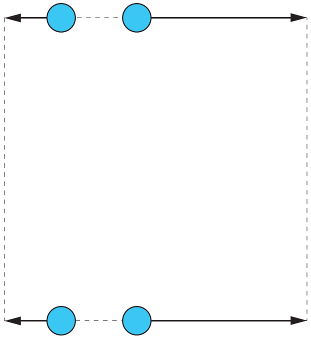
A rectangle is a Biangular configuration

Open problem: $n = 4$



But the supporting polygon is not unique!

Open problem: $n = 4$



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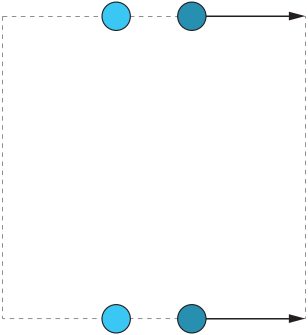
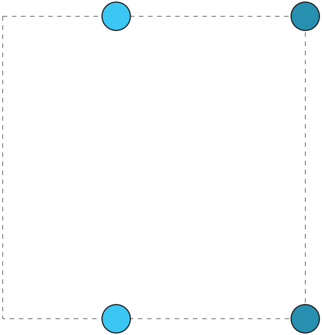


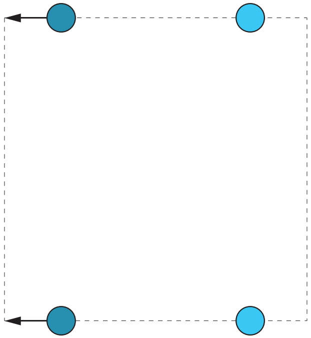
The "central" supporting polygon may be chosen...

Open problem: $n = 4$



...but asynchronous robots may never form a square

Open problem: $n = 4$



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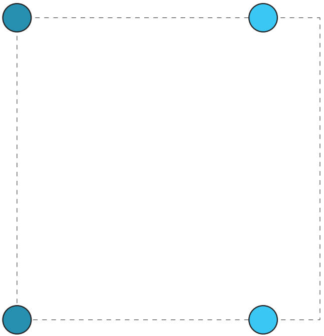

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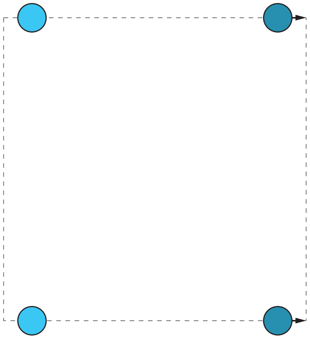

Conjecture: the Square Formation problem is unsolvable