Gathering robots on meeting-points: feasibility and optimality

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5th workshop on Moving And Computing (MAC)
7th workshop on GRAph Searching, Theory and Applications (GRASTA)
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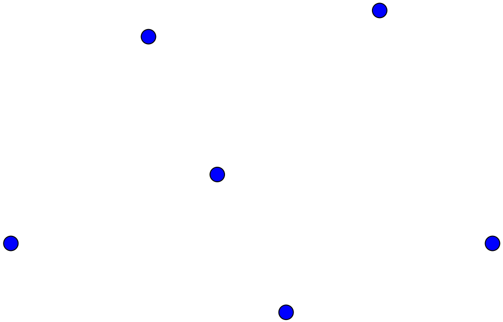
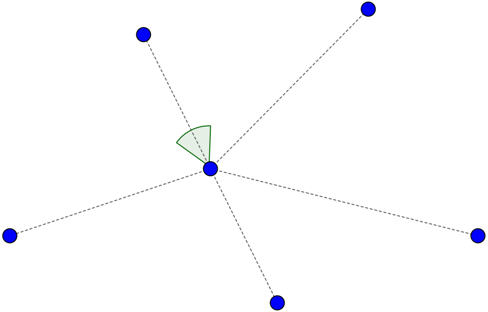
Gathering problem



- A configuration of anonymous & autonomous robots on the plane ...
- ... have to agree to meet at some location and remain in there

Gathering problem



- **sensing** the positions of other robots in its surrounding, ...

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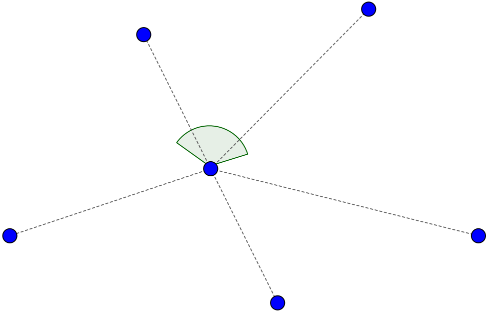

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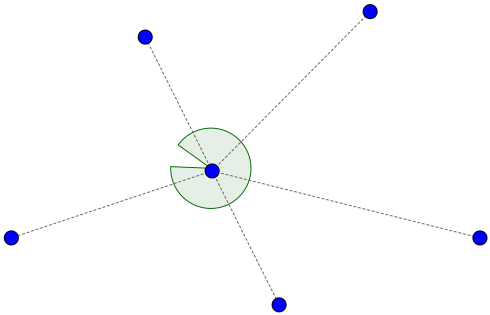

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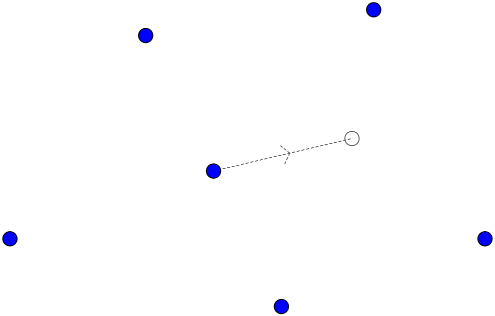
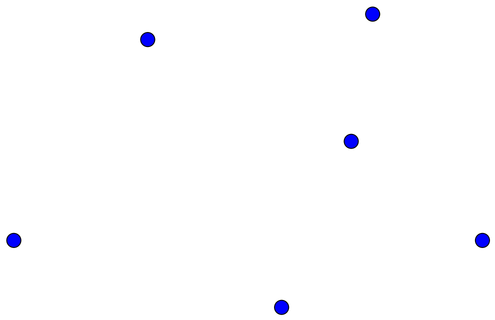


- **computing** a new position, ...

Gathering problem



- **moving** toward it accordingly, ...
- ...thus creating a **new** configuration of robots

Gathering problem

- **AIM**: all robots reach the same place, eventually, and do not move anymore

What is a robot?

Each robot is a **computational unit** that repeatedly cycles through 4 states:

- **Wait**: the robot is idle – a robot cannot stay indefinitely idle – initially, all robots are waiting
- **Look**: the robot observes the world using its sensors which return a configuration (set of points) of the relative positions of all other robots (*configuration view*)
- **Compute**: the robot performs a local computation according to a deterministic algorithm, which is the same for all robots – the result of this phase is a destination point
- **Move**: Non-rigid movement, i.e. there exists δ such that the robot is guaranteed to move of at least of δ unless it wants to move less

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Robots are

- **Dimensionless** – robots are modeled as geometric points in the plane
- **Anonymous** – no unique identifiers
- **Homogeneous** – all the robots execute the same algorithm
- **Autonomous** – no centralized control
- **Oblivious** – no memory of past events
- **Silent** – no explicit way of communicating – the only mean is to move and let others observe
- **Asynchronous** – there is no global clock ...
  - each phase may have any finite duration, and different robots executions are completely independent
  - fair scheduling: every robot wakes up within finite time, infinitely often
- **Unoriented** – robots do not share a common coordinate system
  - no common compass
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Gathering problem: known results

Impossibility of gathering [P'07]

In the asynchronous setting, there exists no deterministic algorithm that solves the gathering problem in finite time, for a set of $n \geq 2$ oblivious robots.

To solve it, we need to add some additional capabilities to robots...

Recent positive result [CFPS'12]

In the asynchronous setting, there exists a deterministic algorithm that solves the gathering problem in finite time, for a set of $n > 2$ oblivious robots with **multiplicity detection**.

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Optimal Gathering problem

- We want to add optimality constraints
- E.g.: minimize the total distance covered by all robots to finalize gathering
- In 2D Euclidean space, it equals to compute the so called **Weber-point**, and move the robots toward it
- **Good news**: there exists one unique Weber-point (unless robots are all collinear)
- **Bad news**: the Weber-point is computationally intractable (already for 5 robots!)

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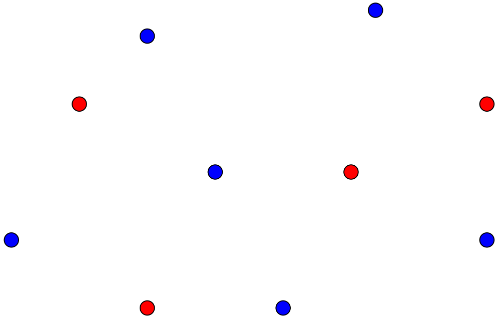
A new challenge...

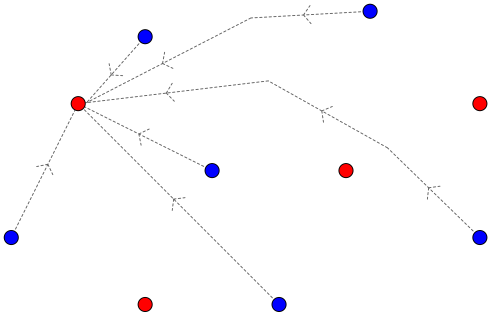


A configuration $\mathcal{C}$ consisting of ...

- a set R of anonymous robots on the plane
- a set M of fixed meeting points

As for the classical gathering, initial configurations are assumed to not contain multiplicities

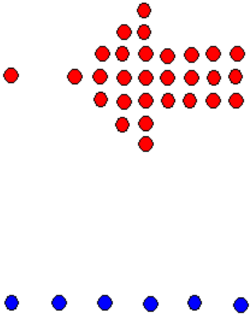
A new challenge...



Problem: GATHERING OVER MEETING POINTS (GMP)

- design an algorithm able to gather all robots on a meeting point in M (**Algosensors'15**)

Dealing with Meeting points



- Meeting points can sometimes help in designing a gathering algorithm...
- ...while sometimes they can play for the adversary.

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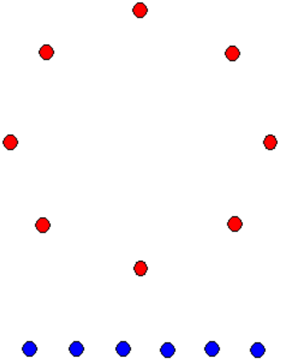

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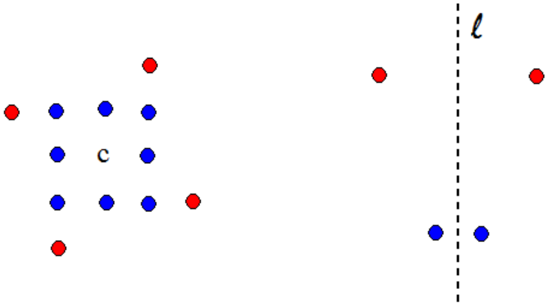
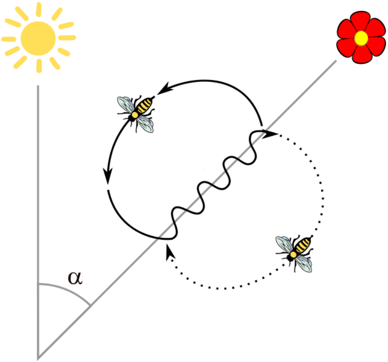
Ungatherable configurations



1. $\mathcal{C}$ admits a rotation with center c, and there are neither robots nor meeting-points on c;
2. $\mathcal{C}$ admits one axis of symmetry ℓ, and there are neither robots nor meeting-points on ℓ.

Gathering on meeting-points: stigmergy



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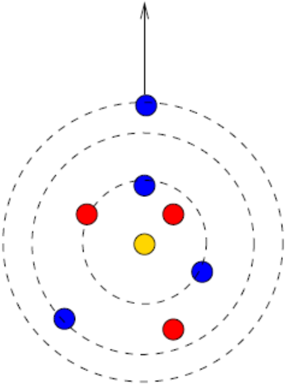

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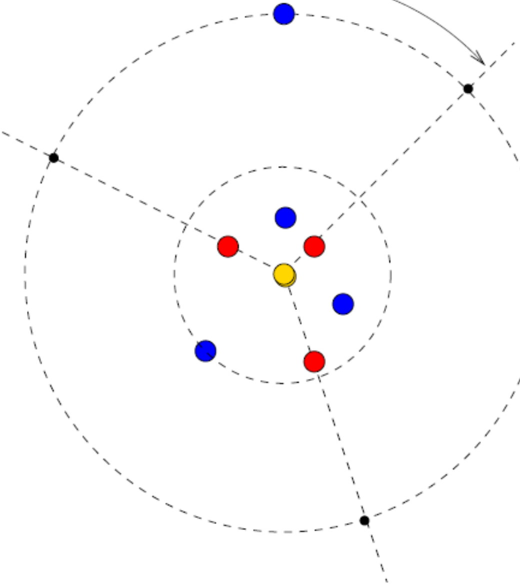

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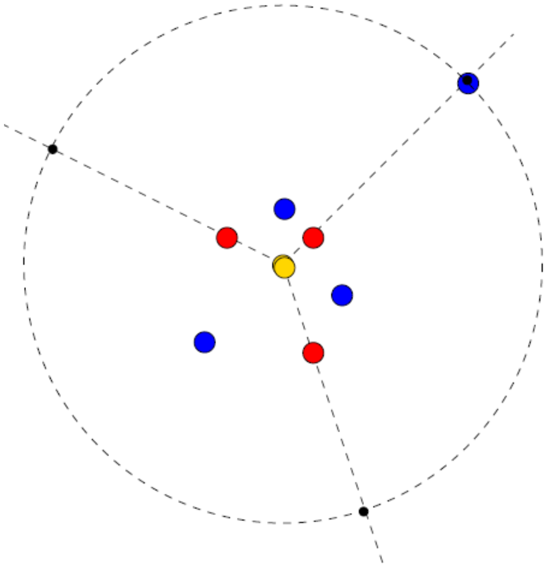
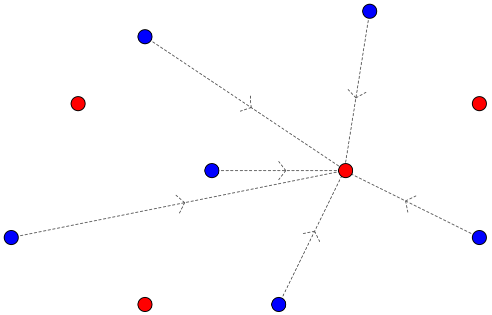


Optimization versions



Problem GMP is addressed with an additional optimality constraint:

1. the robots must cover the minimum total travel distance to finalize the gathering (**Algosensors'14**)
2. the maximum distance traveled by a single robot must be minimized (**CIAC'15**)

Optimal gathering for case 1

Let $\mathcal{C} = (R, M)$ be a configuration,

$m \in M$ is a **Weber-point** of $\mathcal{C}$ if m minimizes $\sum_{r \in R} d(r, m)$

Let Δ be the above quantity, Δ **is a lower bound for each gathering algorithm**

Definition

- A gathering algorithm is **optimal** if it achieves the gathering with a total distance equal to Δ.

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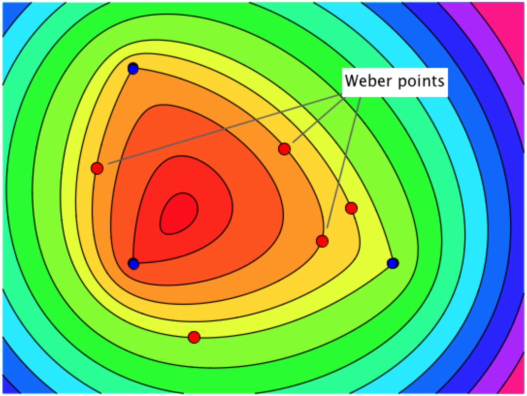
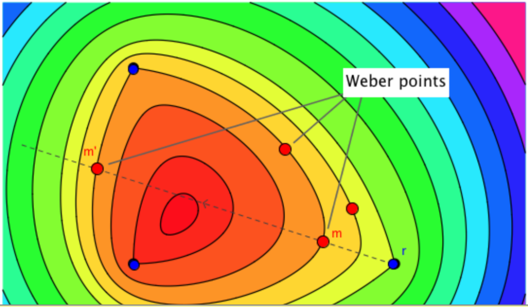
- **What we achieved**:
  1. characterize all the configurations for which an optimal algorithm exists, and ...
  2. ... define the optimal algorithm

Robots are the foci of a *k*-ellipse



- 3-ellipses with different radii
- *k*-ellipses are strictly convex curves

Robots are the foci of a k-ellipse



- **Lemma**:
  Let $\mathcal{C} = (R, M)$ be a configuration, robots in R are not collinear, $r \in R$ moves toward a Weber-point m and this move creates $\mathcal{C}' = (R', M)$. Then:
  - $\mathcal{C}'$ **contains one or two Weber-points only**: m and m' (if any) lies on $hline(r, m)$

- 3-
- k-

The strategy of the algorithm

- Select and move robots **straightly toward a Weber-point** m, so that
- ... after a certain number of moves,
- ... m remains the only Weber-point.
- Once only the Weber-point m exists, all robots move toward it!

This approach provides an optimal algorithm for some special cases:

- $\mathcal{S}_1$: conf's s.t. there is exactly one multiplicity on a meeting-point
- $\mathcal{S}_2$: conf's s.t. there is exactly one Weber-point
- $\mathcal{S}_3$: conf's s.t. a Weber-point m lies on $cg(M)$, the center of gravity of M

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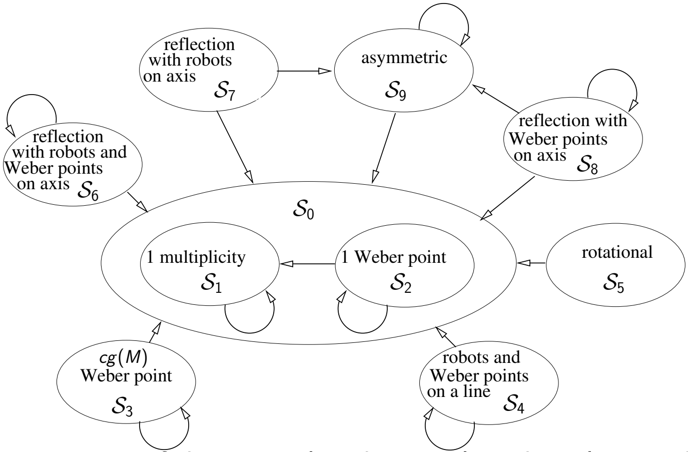
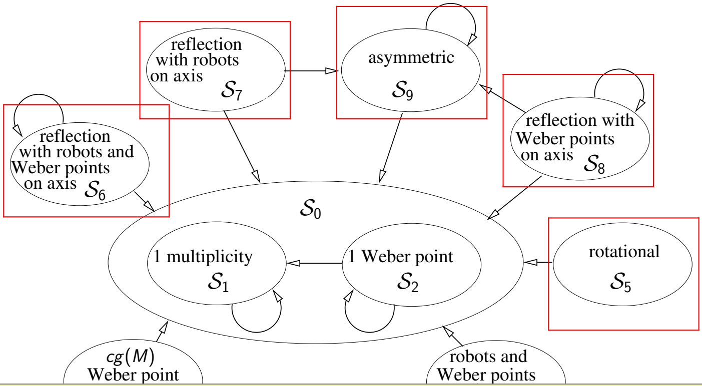
Partitioning all the configurations



- Schematization of the optimal gathering algorithm along with priorities
- The general algorithm is divided into sub-procedures:
  - each sub-procedure is specific for configurations of a given class $\mathcal{S}_i$

Partitioning all the configurations



Several classes concern **isometries**:

- asymmetric configurations
- configurations with reflection
- configurations with rotation

- Sc
  pr
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Optimal gathering for problem 2

Let $\mathcal{C} = (R, M)$ be a configuration,

$m \in M$ is a **minmax-point** of $\mathcal{C}$ if m minimizes $\max_{r \in R} d(r, m)$

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- A gathering algorithm is **optimal** if it achieves the gathering by letting move each robot of at most $\Delta(\mathcal{C})$.

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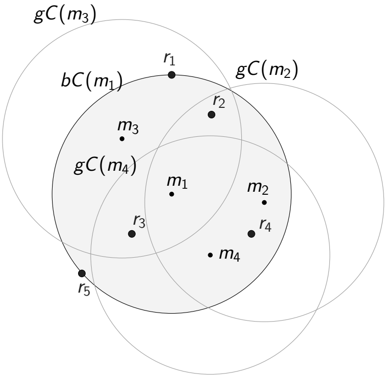
Some Notation



Black-Circle (bC): circle of radius $\Delta(\mathcal{C})$ centered on a meeting-point m containing all robots, hence m is a minmax-point

- **Border-robots**: wrt $bC(m_1)$: r_1, r_5
- **Internal-robots** wrt $bC(m_1)$: r_2, r_3, r_4

Grey-circle (gC): circle of radius $\Delta(\mathcal{C})$ centered on a meeting-point, not containing all robots

The strategy of the algorithm

- Select and move robots **straightly toward a Minmax-point** m, so that
- ...after a certain number of moves,
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- Once only the Minmax-point m exists, **still robots require special strategies to "safely" move toward it!**

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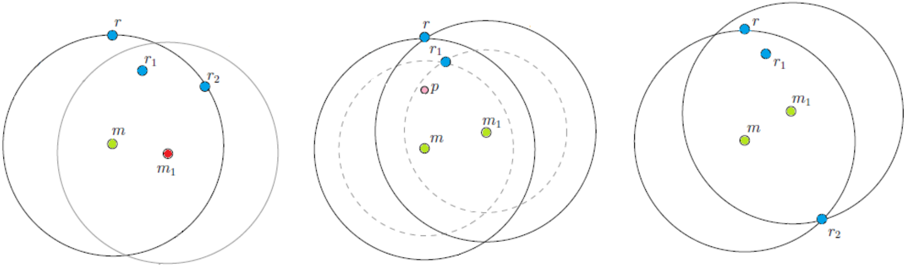

Characterizing configurations

- $mm_B(\mathcal{C}) \subseteq mm(\mathcal{C})$ is the set of minmax-points with minimum number of border-robots;
- $mm_W(\mathcal{C}) \subseteq mm_B(\mathcal{C})$ is the set of minmax-points in $mm_B(\mathcal{C})$ with minimal Weber distance;
- $mm_V(\mathcal{C}) \in mm_W(\mathcal{C})$ is the minmax-point in $mm_W(\mathcal{C})$ with minimal view.

$\mathcal{S}_1$: any configuration $\mathcal{C}$ such that $|mm(\mathcal{C})| = 1$;

$\mathcal{S}_2$: any configuration $\mathcal{C} \notin \mathcal{S}_1$ such that $|mm_V(\mathcal{C})| = 1$;

$\mathcal{S}_3$: initial configurations $\mathcal{C} \notin \bigcup_{1\leq i\leq 2} \mathcal{S}_i$ such that $\mathcal{C}$ admits a reflection with robots on the axis;

$\mathcal{S}_4$: initial configurations $\mathcal{C} \notin \bigcup_{1\leq i\leq 3} \mathcal{S}_i$ such that $\mathcal{C}$ admits a rotation with a robot as center;

$\mathcal{S}_5$: initial configurations $\mathcal{C} \notin \bigcup_{1\leq i\leq 4} \mathcal{S}_i$ such that $\mathcal{C}$ admits a reflection with minmax-points on the axis;

Characterizing configurations

- $mm_B(\mathcal{C}) \subseteq mm(\mathcal{C})$ is the set of minmax-points with minimum number of border-robots;
- $mm_W(\mathcal{C}) \subseteq mm_B(\mathcal{C})$ is the set of minmax-points in $mm_B(\mathcal{C})$ with minimal Weber distance;
- $mm_V(\mathcal{C}) \in mm_W(\mathcal{C})$ is the minmax-point in $mm_W(\mathcal{C})$ with minimal view.

$\mathcal{S}_1$: any configuration $\mathcal{C}$ such that $|mm(\mathcal{C})| = 1$;

$\mathcal{S}_2$: any configuration $\mathcal{C} \notin \mathcal{S}_1$ such that $|mm_V(\mathcal{C})| = 1$;

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Strategy for class $\mathcal{S}_1$

1. If $\mathcal{C} \in \mathcal{S}_1$ then $mm(\mathcal{C}) = \{m\}$;
2. All robots can move toward m without entering grey-circles $\rightarrow$ without creating new minmax-points
3. A robot evaluates the closest grey-circle on the direction toward m and moves by halving such a distance
4. Once all robots have moved (a *round* has completed), $\Delta(\mathcal{C})$ decreases
5. Eventually, all robots reach m as for each round we guarantee a minimum constant movement

Strategy for class $\mathcal{S}_2$

1. If $\mathcal{C} \in \mathcal{S}_2$ then $mm_V(\mathcal{C}) = \{m\}$;
2. All *border* robots move toward m without entering grey-circles
3. Once all such robots have moved $mm(\mathcal{C}) = \{m\} \rightarrow$ the configuration becomes of class $\mathcal{S}_1$

About the other classes

1. If $\mathcal{C} \in \mathcal{S}_3$ a robot on the axis is selected and moved in such a way the configuration becomes of class $\mathcal{S}_1$ or $\mathcal{S}_2$
2. If $\mathcal{C} \in \mathcal{S}_4$ the robot placed in the center of the rotation is moved in such a way the configuration becomes of class $\mathcal{S}_1$ or $\mathcal{S}_2$
3. For $\mathcal{C} \in \mathcal{S}_5$ some cases remain open. We prove that in general optimal gathering is impossible but perhaps there is a subclass where an optimal algorithm can be designed

Remark

- No *multiplicity detection* is required by the current strategy.

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  - Asynchronous Look-Compute-Move model
  - global weak multiplicity detection capability
- ... by introducing **restrictions on the places where to gather**
- Introduced also **optimality requirements** for the algorithm
  1. minimize the total distance covered by all robots
  2. minimize the maximum distance covered by a robot
- Provided non-gatherability characterizations
- Defined a fully characterizing algorithm for GMP
- Defined an optimal gathering algorithm for case 1 (case 2, resp.) dealing with all (almost all, resp.) possible configurations

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Future Work

- Study GMP without multiplicity detection
- Extend the analysis and the algorithms for the optimization problems to configurations where optimal gathering cannot be assured
- Use different objective functions
- Study GMP (with/without optimization) on graphs
- Study different tasks that may include meeting-points, e.g., pattern formation on specified points as in [FYOKY'15]

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[FYOKY'15] Fujinaga, Yamauchi, Ono, Kijima, Yamashita: *Pattern Formation by Oblivious Asynchronous Mobile Robots.* SIAM J. on Comp., 44(3) (2015)

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THANK YOU