

Further Connections between Contract–Scheduling and Ray–Searching Problems

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Outline and motivation

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one from OR/TCS, the other from AI

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Musk, Hawking Call to Ban Robots That Could Kill You All by Themselves

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By Tom Risen | July 27, 2015 | 5:52 p.m. EDT



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Robots in this presentation
are benign!*

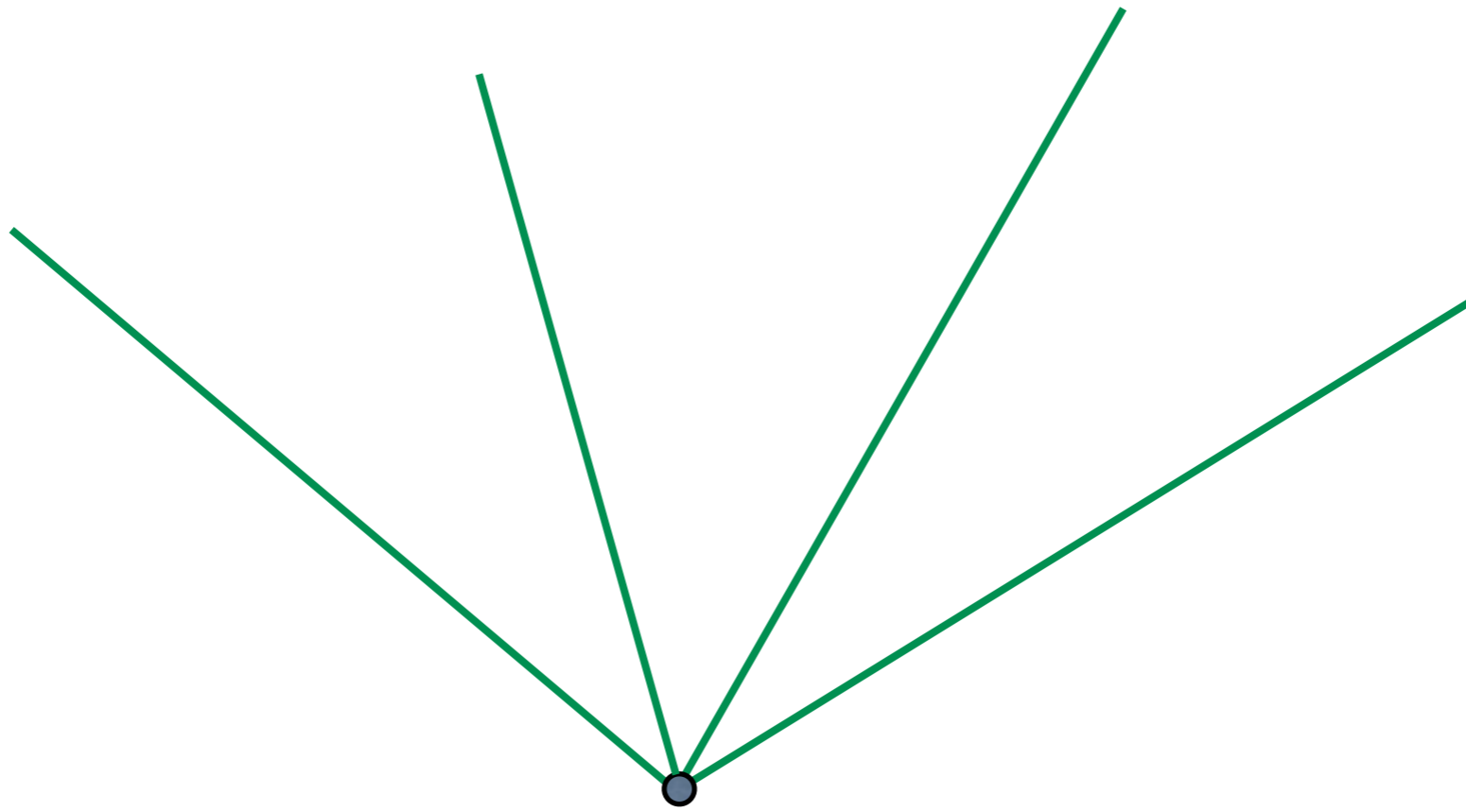
*certain conditions may apply



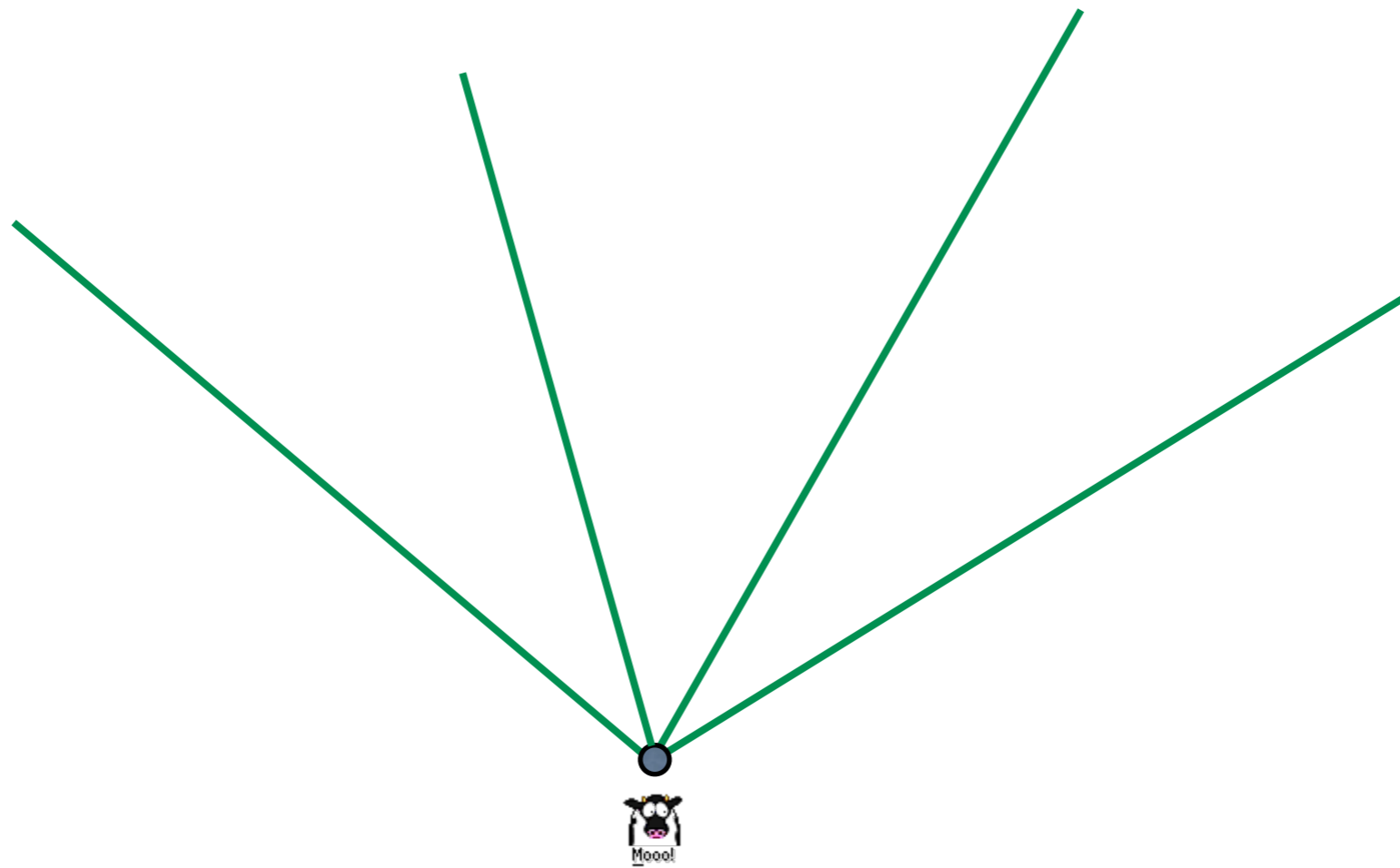
The screenshot shows the top portion of a news article on the U.S. News website. The header includes the U.S. News logo and navigation tabs for News, Opinion, National Issues, Special Reports, Cartoons, Photos, and The Report. Below the header, there are links to various content pieces like 'Ken Walsh's Washington', 'Newsgram', 'Washington Whispers', 'At the Edge', 'Data Mine', and 'The Run 20'. The main headline is 'Musk, Hawking Call to Ban Robots That Could Kill You All by Themselves', with a sub-headline: 'A massive coalition of leading thinkers and innovators has a chilling warning about the future of warfare.' Below the text is a photograph of an unmanned Predator drone in flight against a clear blue sky. Underneath the photo is a caption: 'An unmanned Predator drone flies over Afghanistan in 2009. A group of thought leaders wants to prevent the development of artificially intelligent machines that could attack with much more autonomy.' Below the caption is the author's name 'By Tom Risen' and the date 'July 27, 2015 | 5:52 p.m. EDT'. There are also social media sharing icons for Facebook, Google+, and Twitter. At the bottom of the article, there is a paragraph: 'The U.S. military is investing heavily in drones as the future of war, but a group of 1,000 scientific leaders wants to draw the line at developing weapons that essentially can think for themselves.' A red box highlights a paragraph at the very bottom: 'Futurists like Elon Musk, scientists like Stephen Hawking and innovators like Apple co-founder Steve Wozniak have signed an open letter calling for a ban on the use of autonomous weapons, with the petition scheduled to be unveiled at the opening of this week's International Joint Conference on Artificial Intelligence in Argentina. Hawking and Musk have previously urged scientists to be wary while developing artificial intelligence, cautioning that autonomous machines may not be able to understand the possible good and evil consequences of their actions.'

The first problem: Ray searching

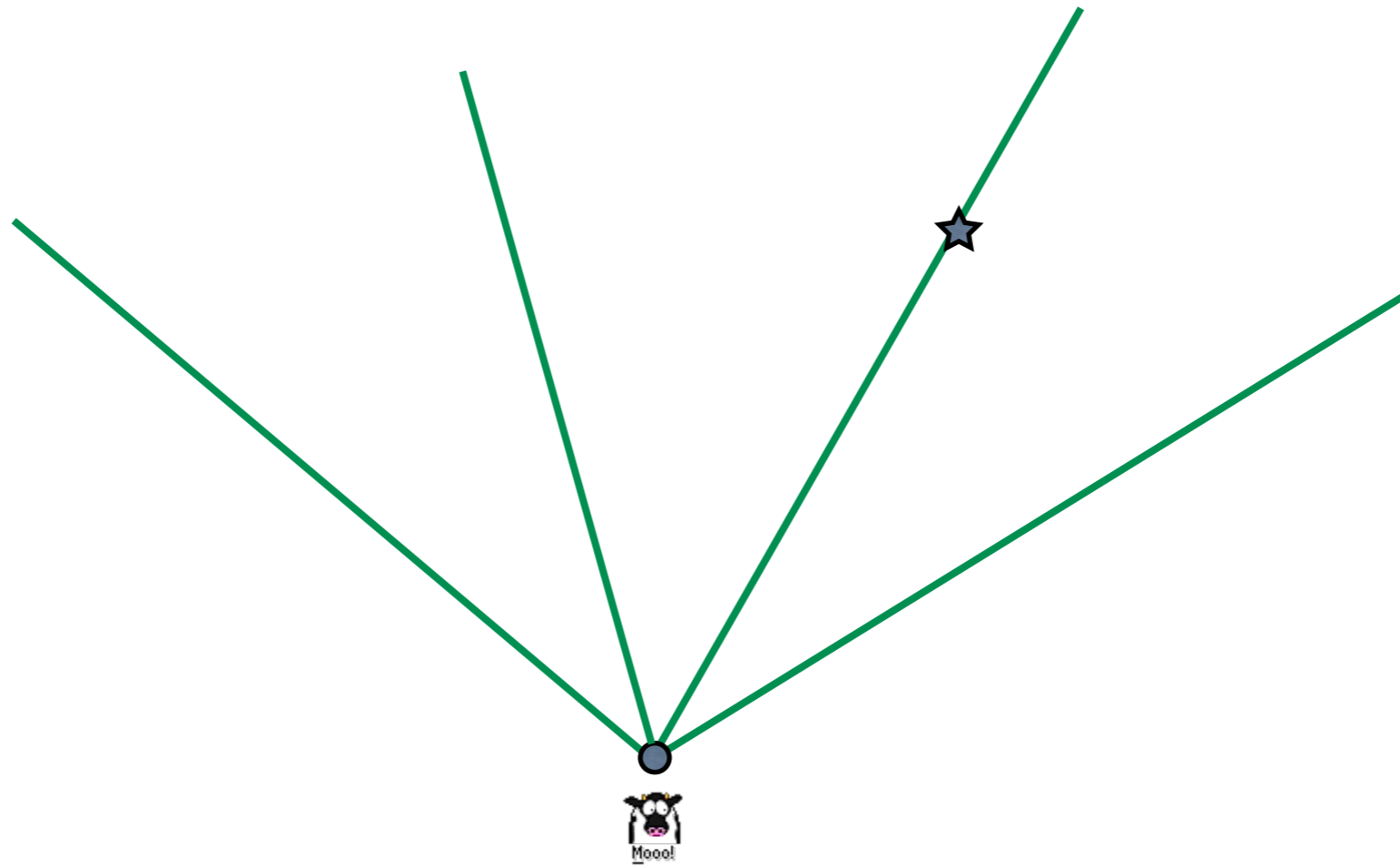
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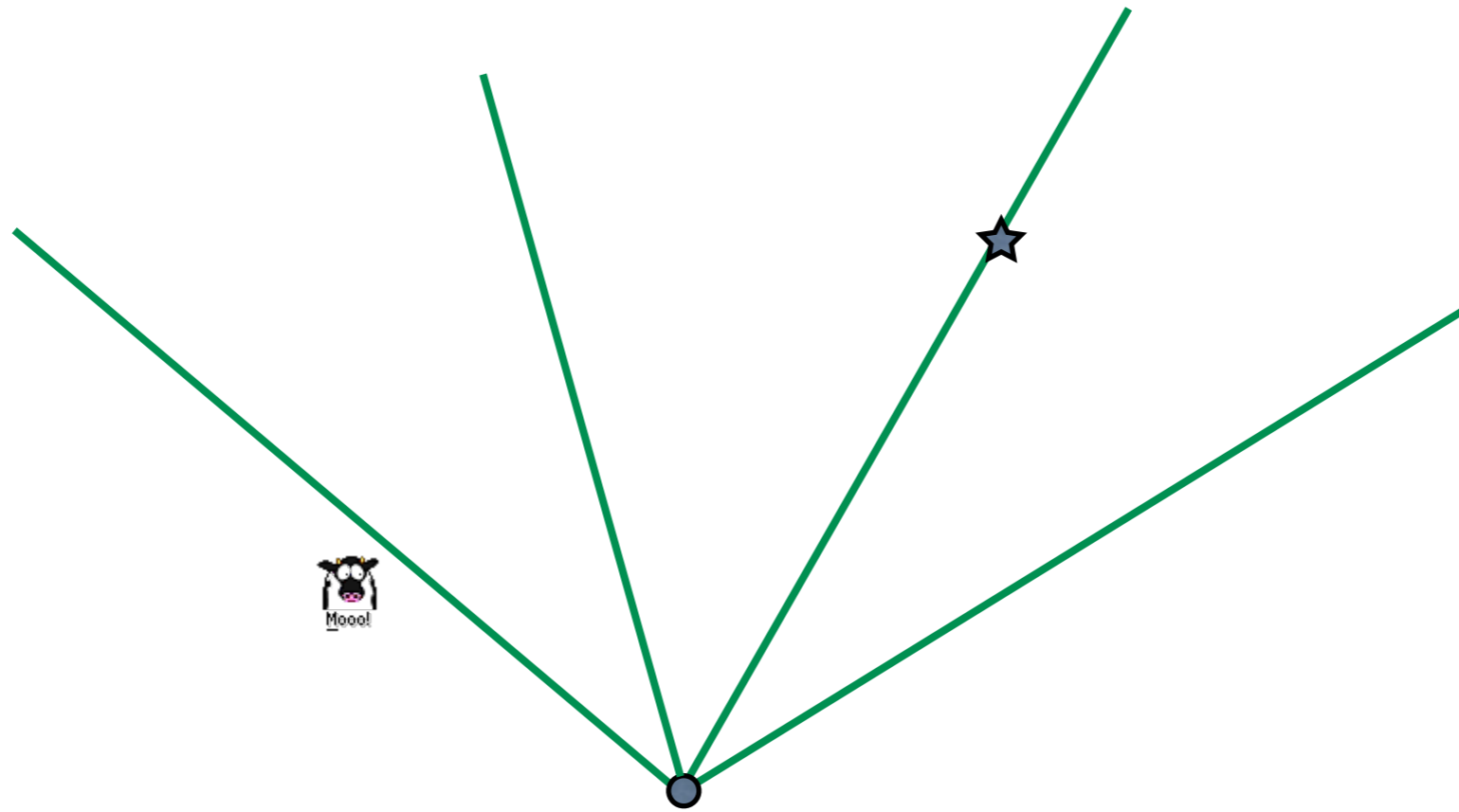
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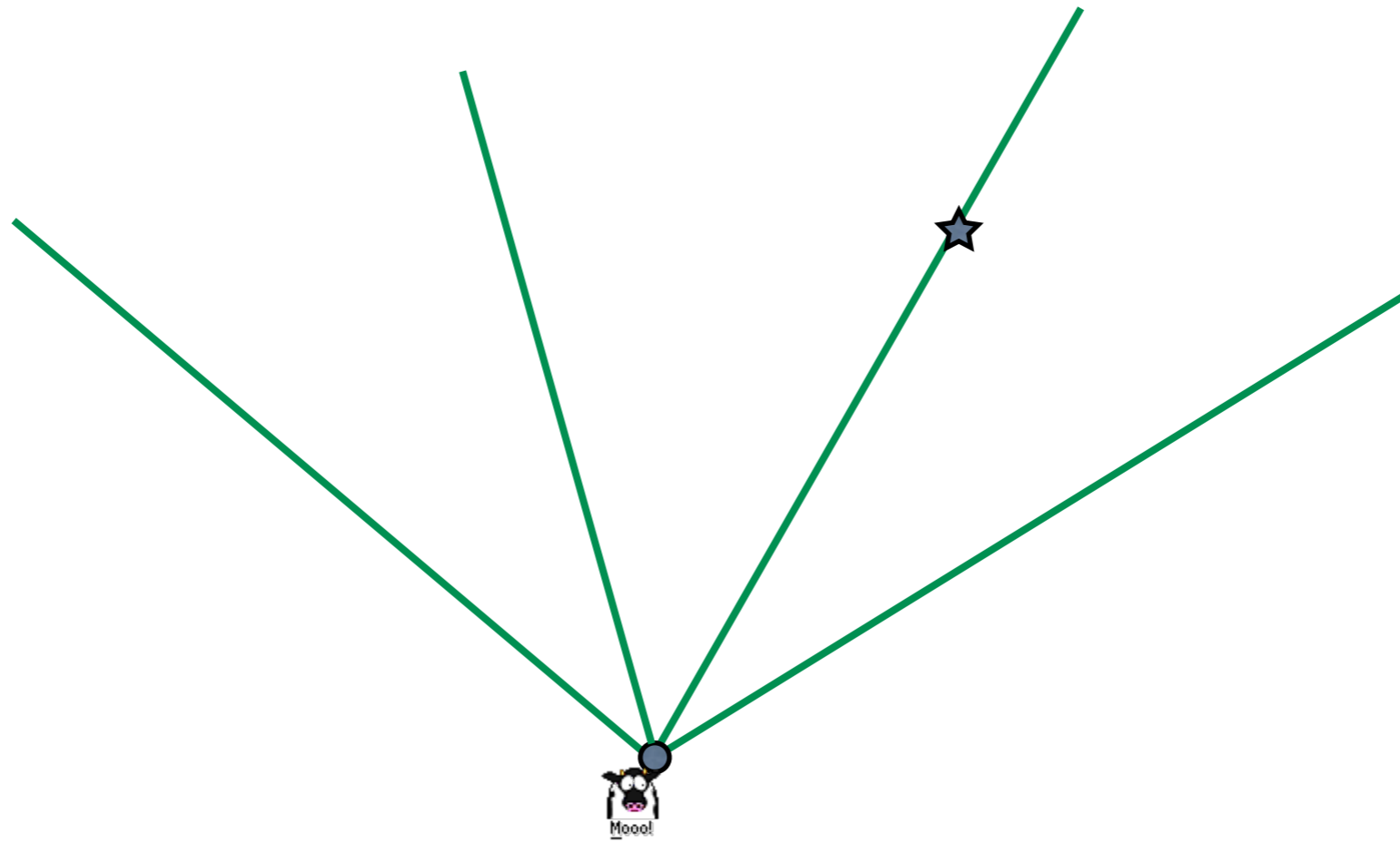
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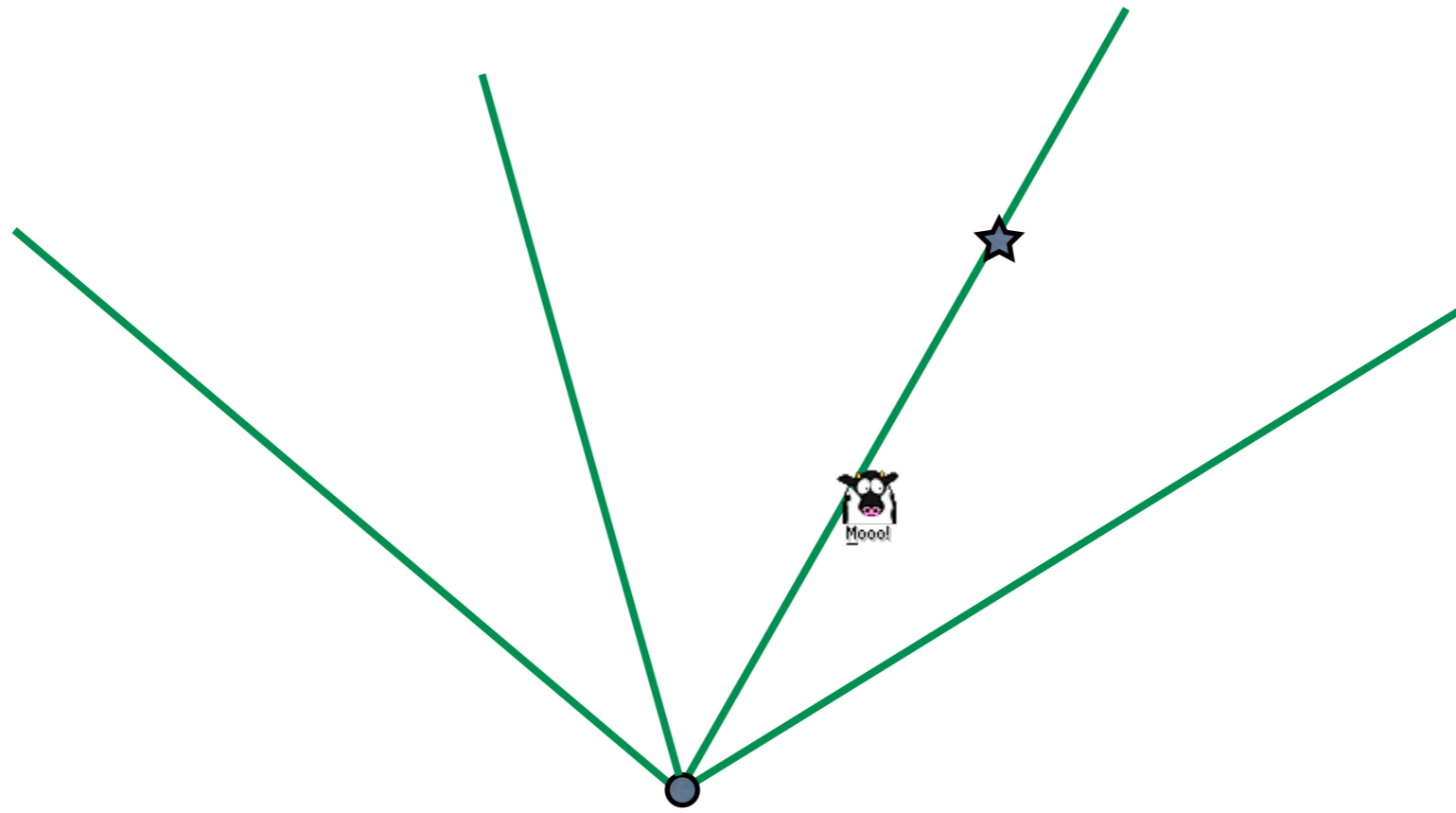
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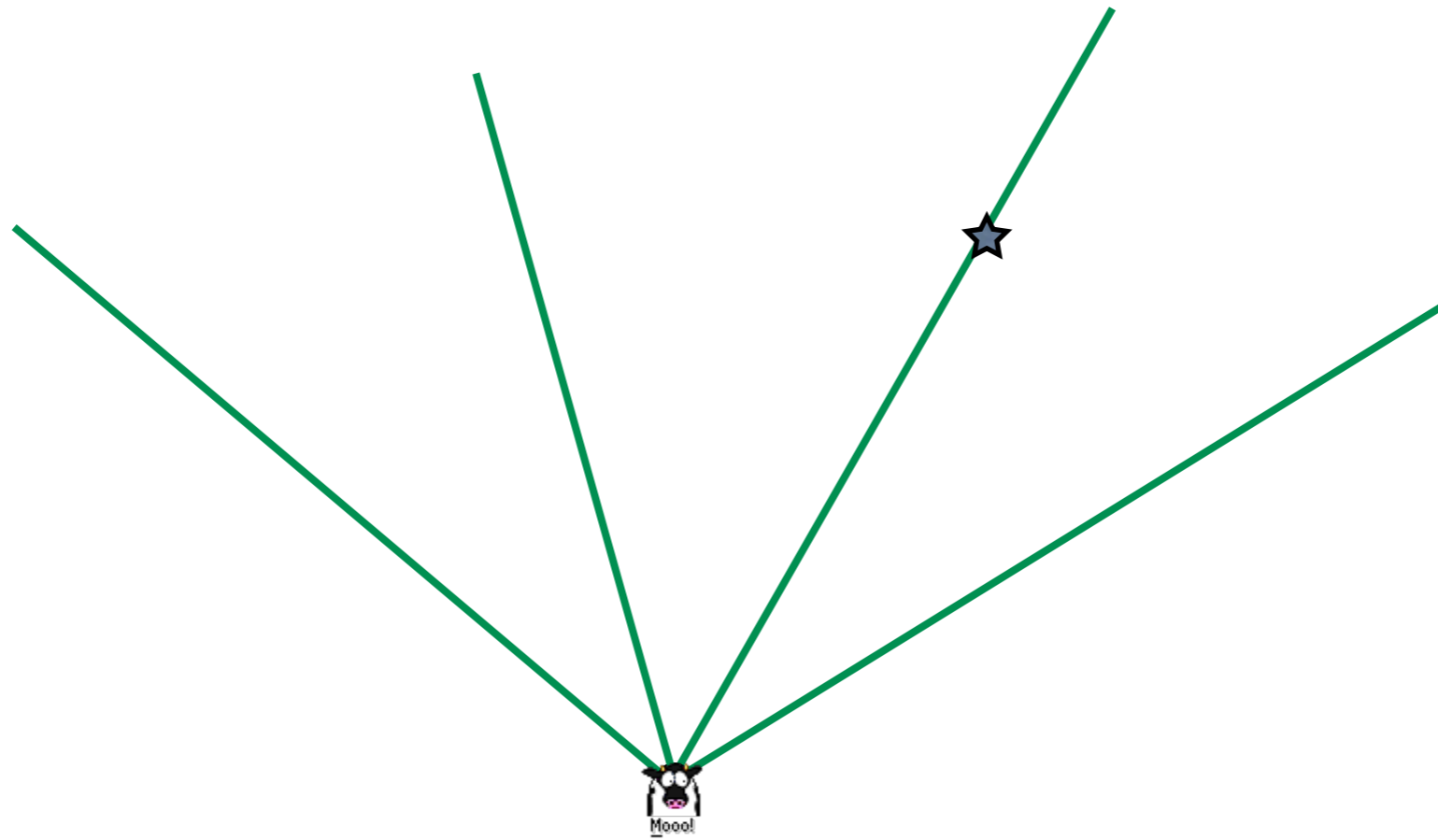
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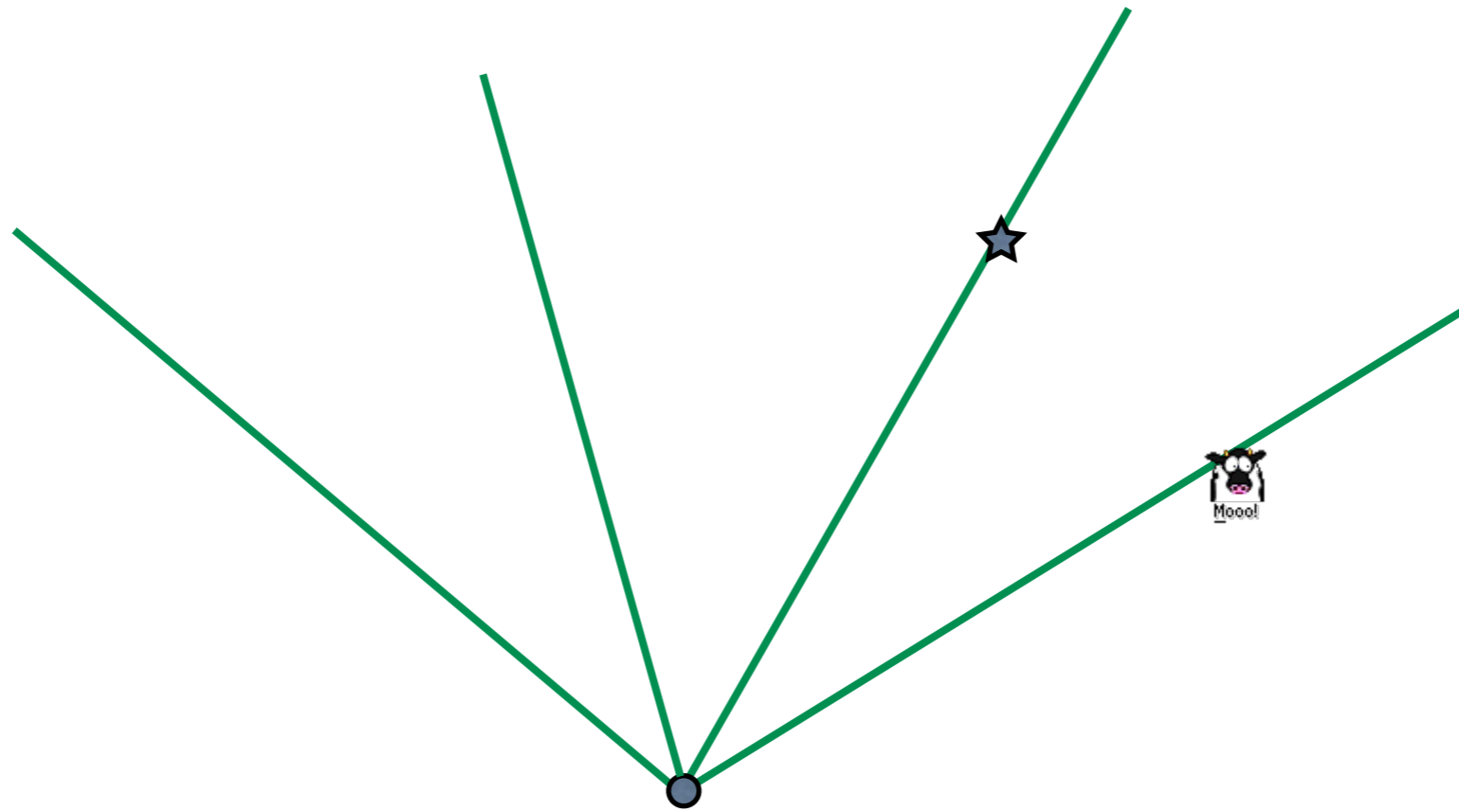
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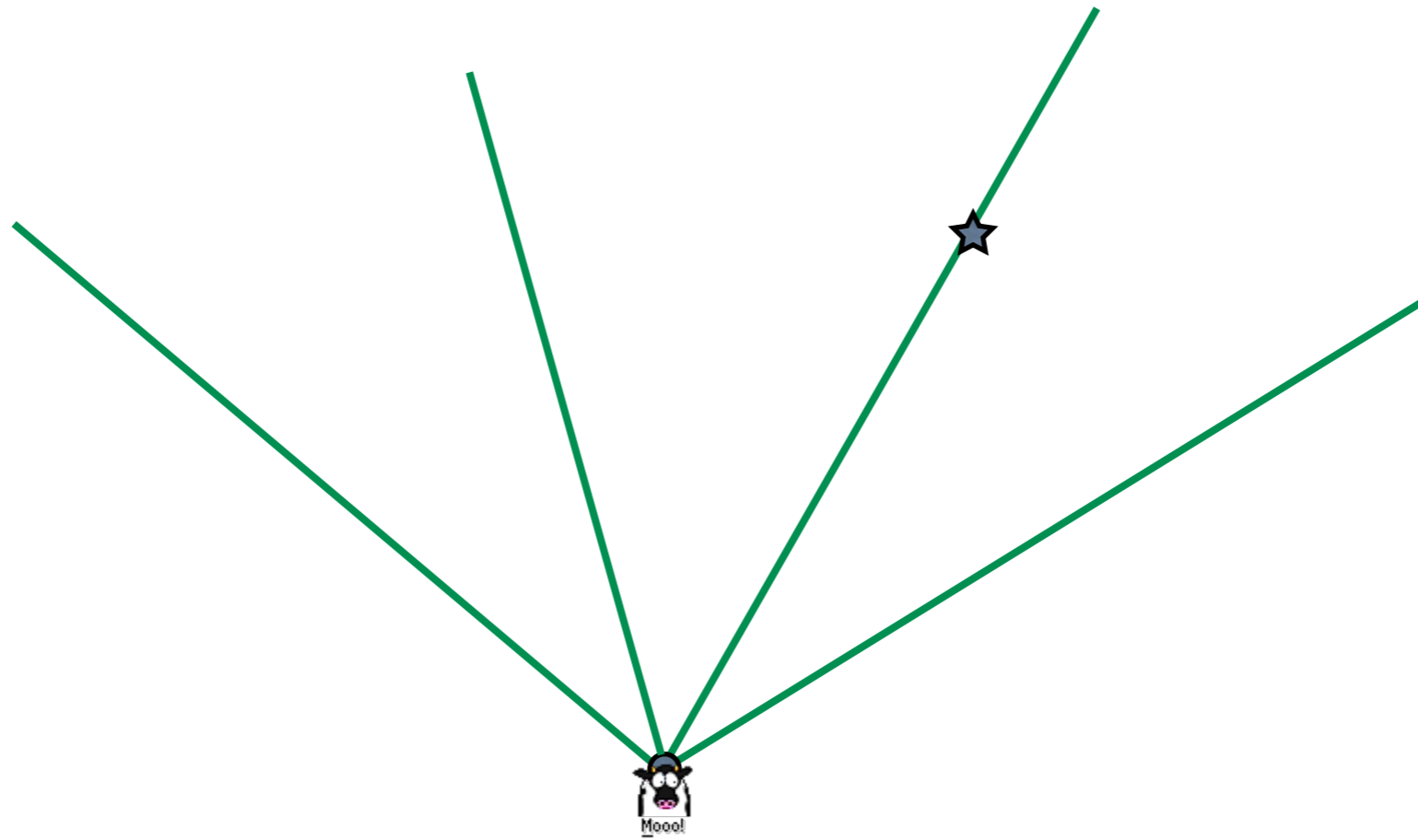
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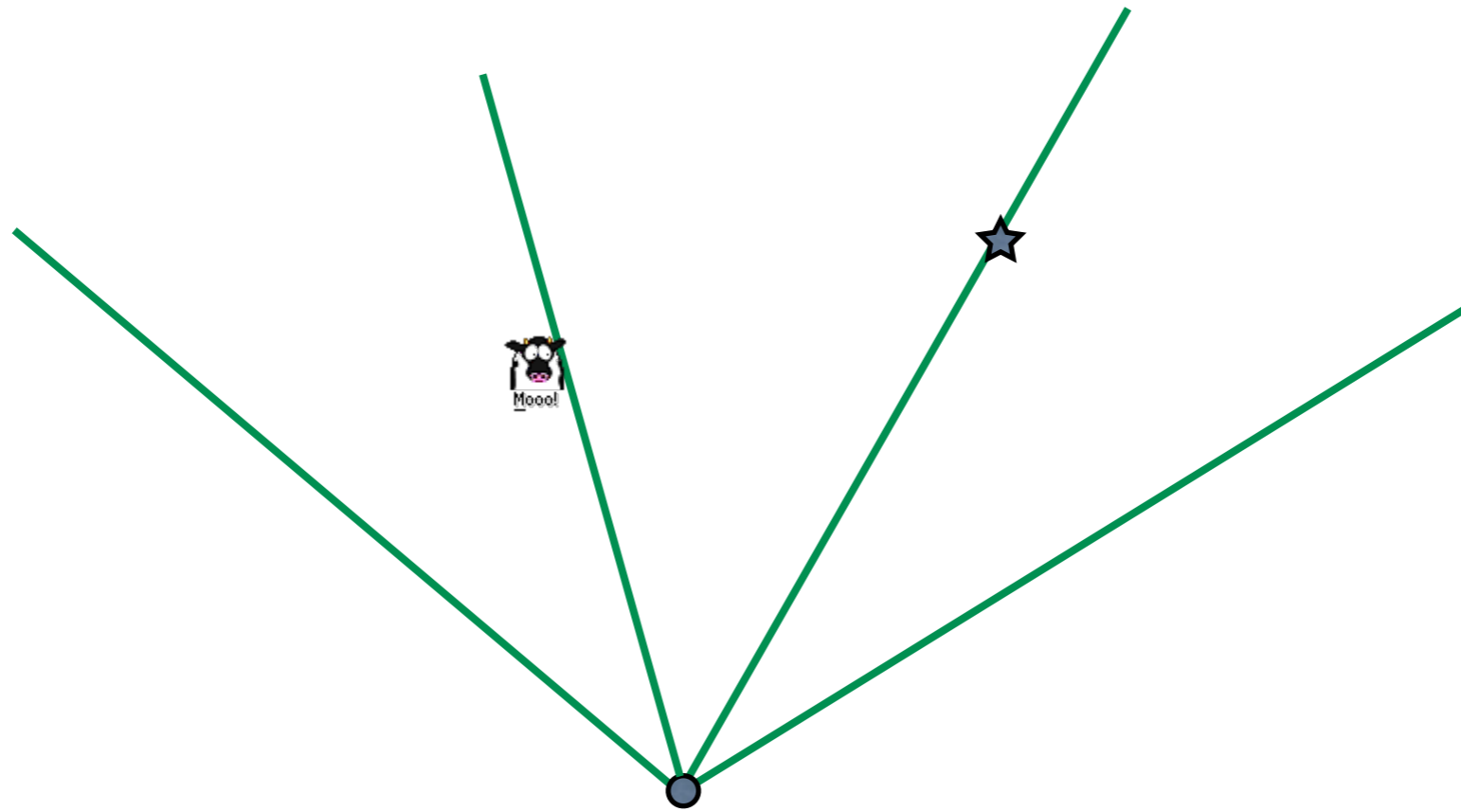
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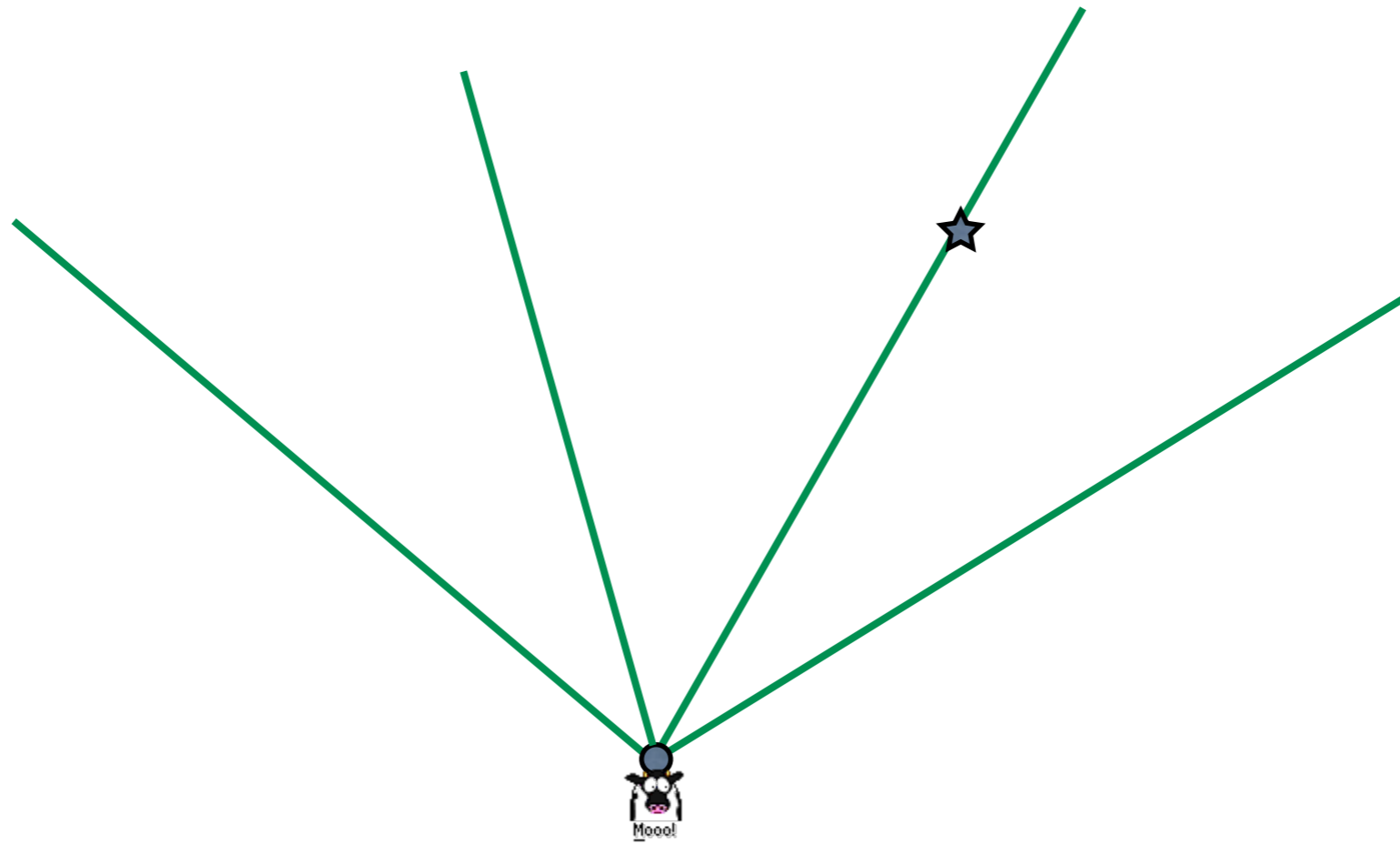
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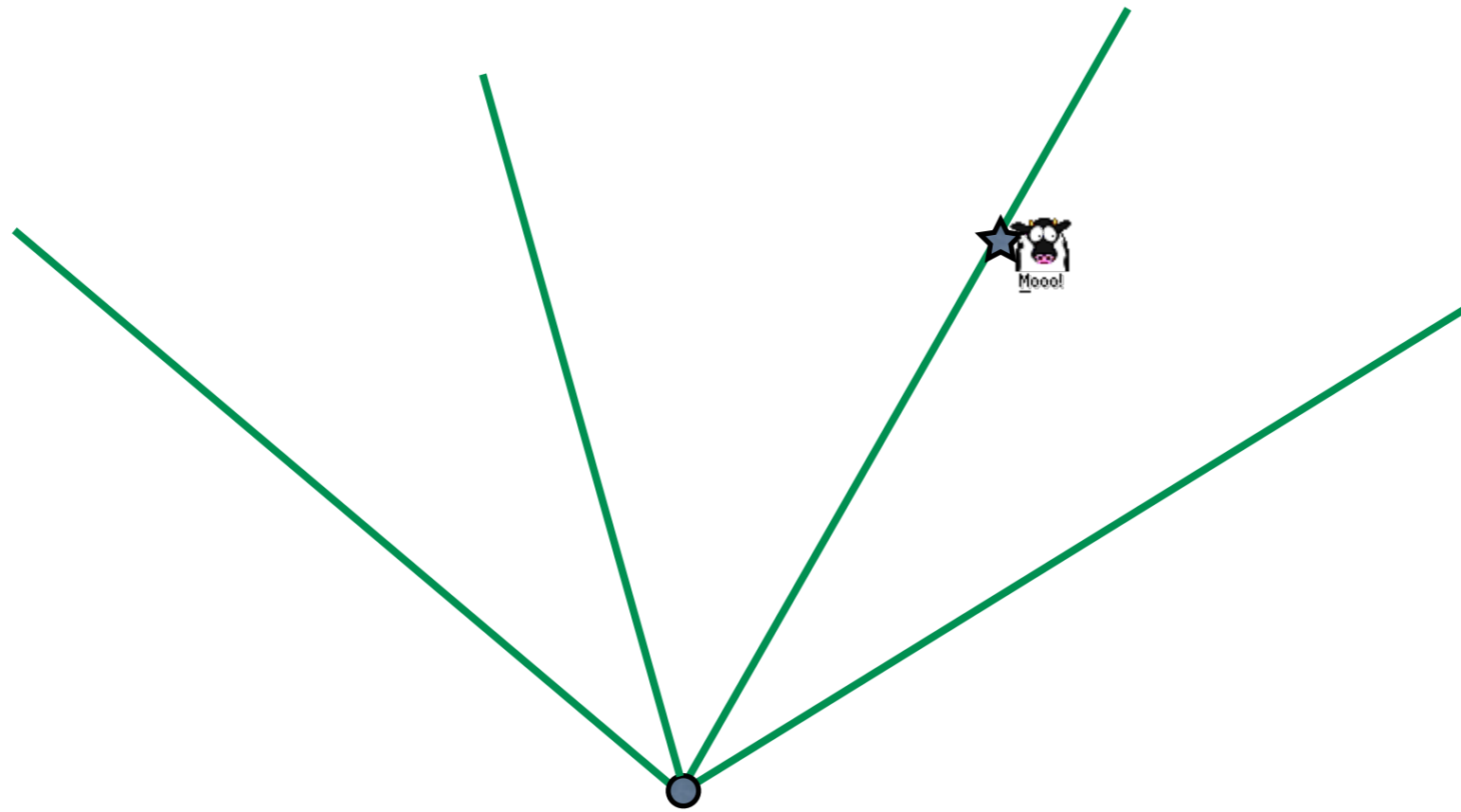
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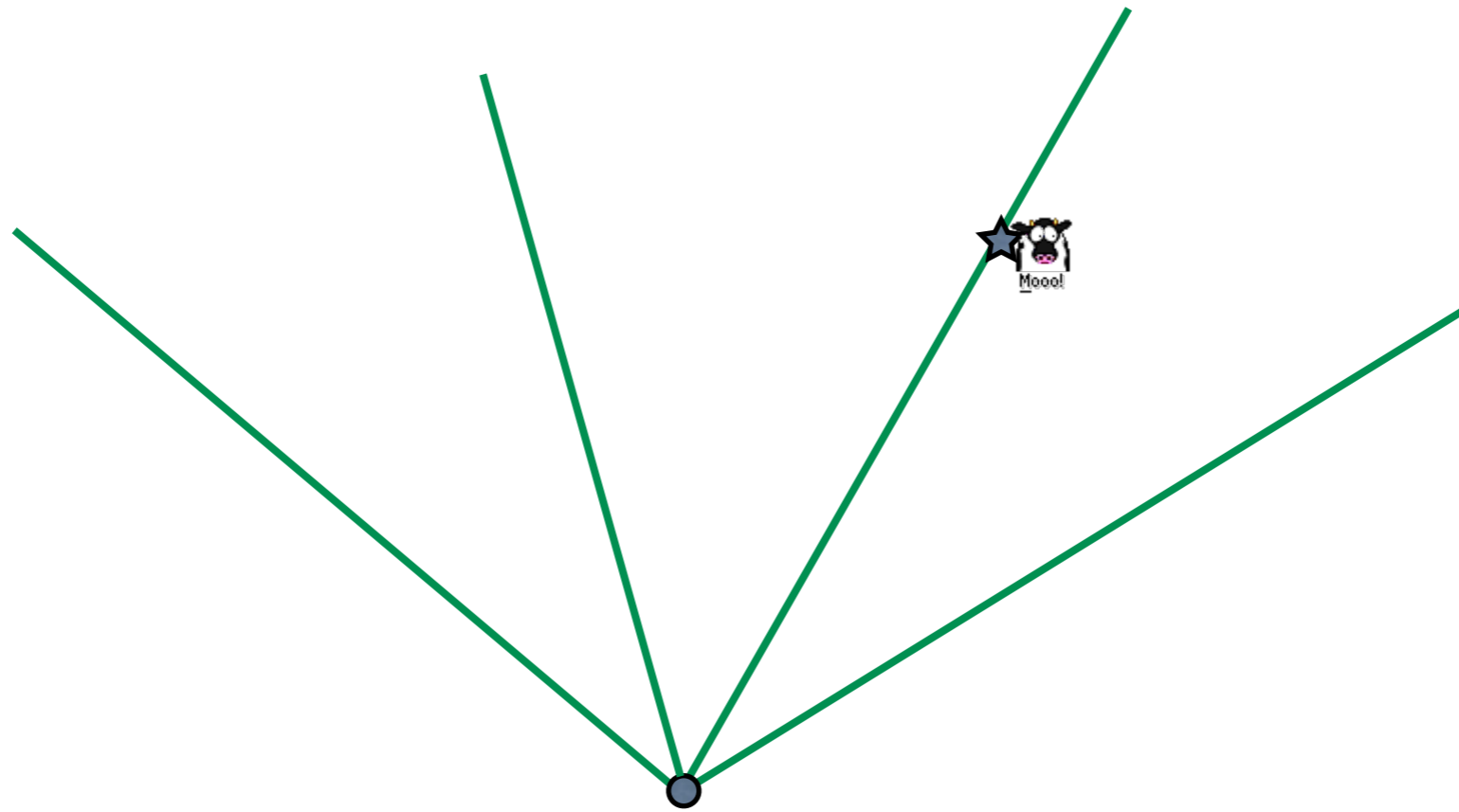
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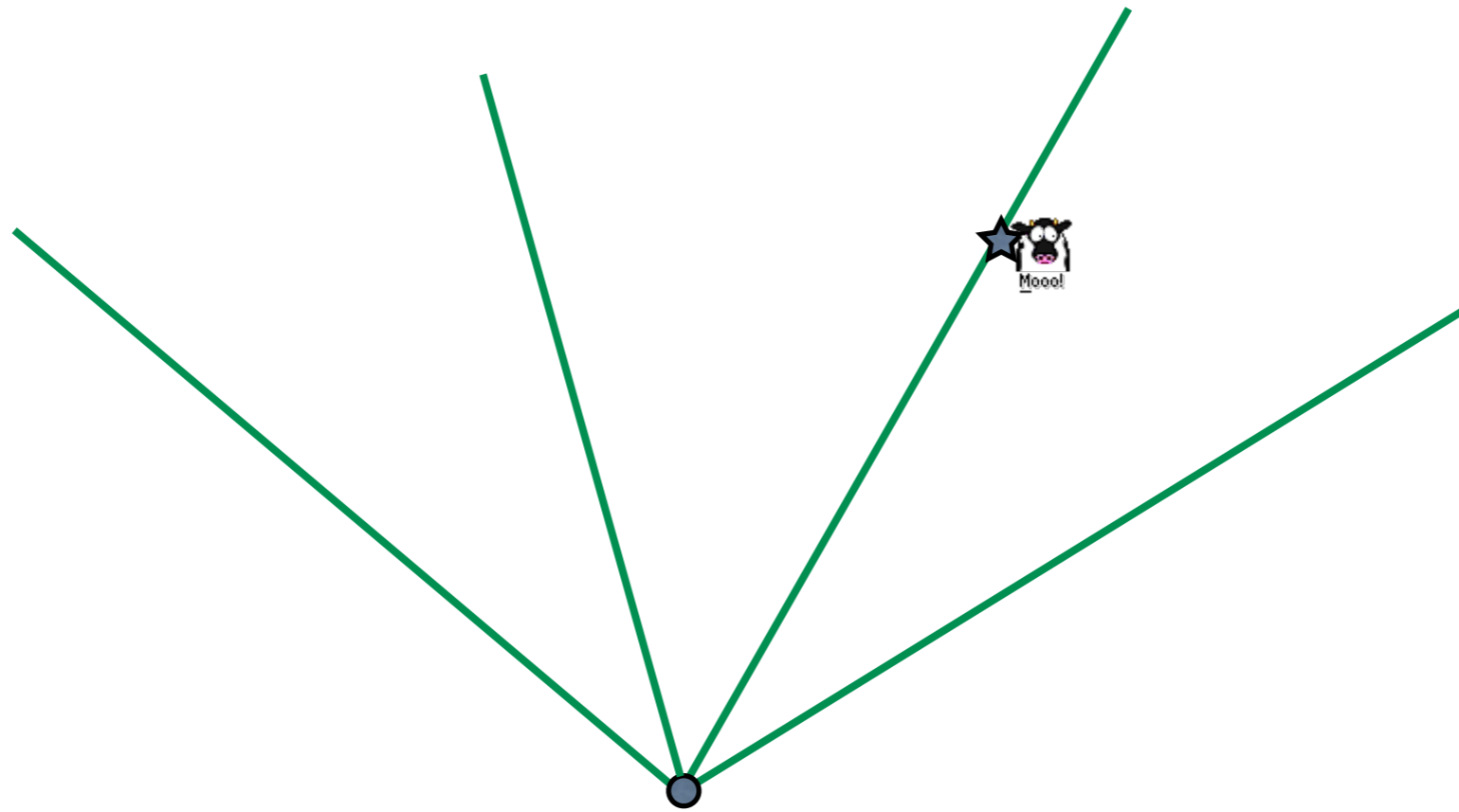


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Performance evaluation: **Competitive ratio**

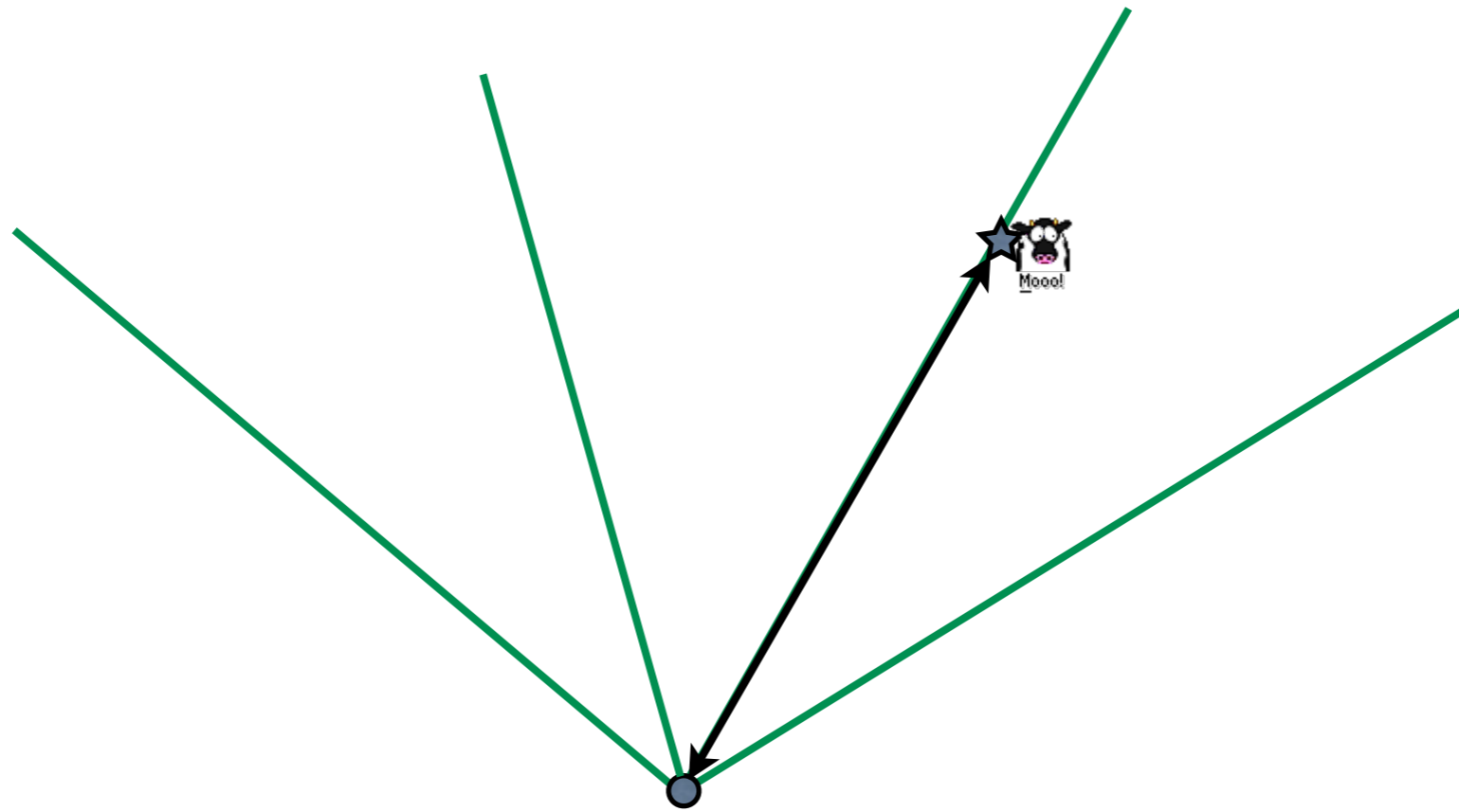
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Contract algorithms

- execution time given as input
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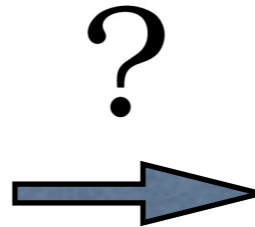
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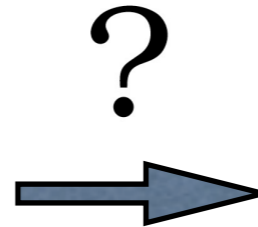
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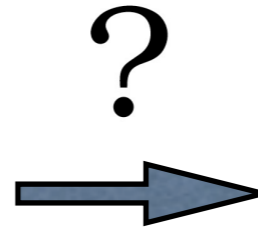
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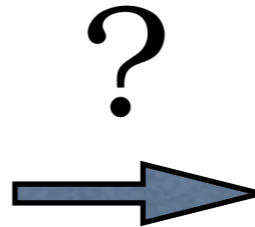
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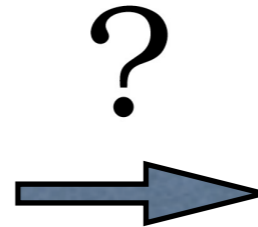
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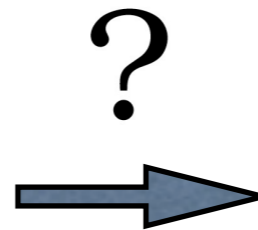
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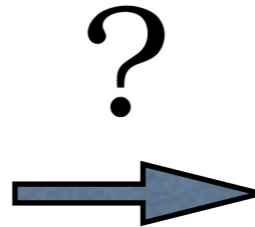
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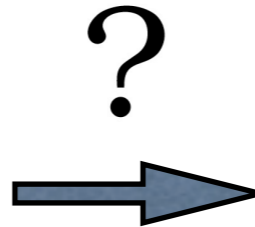
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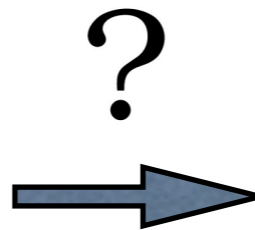
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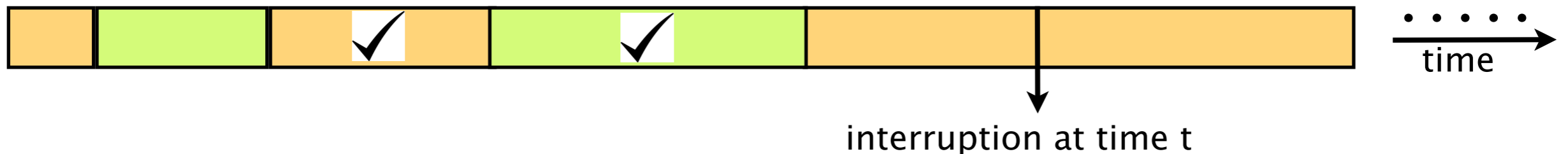
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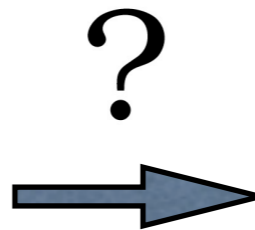
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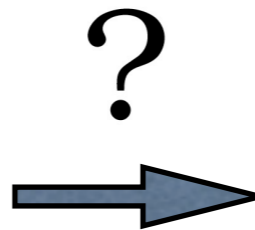
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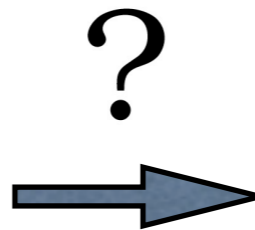
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Performance evaluation: **Acceleration ratio** $\beta = \sup_t \frac{\text{interruption time } t}{\text{length of } \text{■}}$

Some related previous work

Ray searching

m rays



- Early work by Bellman, Beck and Newman for $m = 2$
- Optimal strategies by [Gal 74]
- Re-discovered in CS context [Baeza-Yates et al. 93]
- Several other settings:
 - Randomization [Kao et al. 96]
 - Multiple searchers [Lopez-Ortiz and Schuierer 04]
 - Turn cost [Demaine et al. 06], [A. et al. 14+]
 - New measures [Kirkpatrick 09]

Contract scheduling

n problems



- [Russell and Zilberstein 91]: $n = 1$
- [Bernstein et al. 02]: general n
- [Zilberstein et al. 03]: $n = 1$, multiple processors
- [Bernstein et al. 03], [Lopez-Ortiz et al. 06] general n , multiple processors
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[Bernstein et al. 03]: Connections between **cyclic strategies** for the two problems

Results

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Contract scheduling
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Stochastic
setting

Setting: Target detection
with probability p

Results: Strategy with
competitive ratio $\Theta(m/p^2)$

+

No strategy is better than
 $m/(2p)$ -competitive

Setting: Contracts are
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with success prob. p

Results: Schedule with
accel. ratio $(e(n+1))/p$

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Fault
tolerance /
redundancy

Setting: Target detection on the r -th visit

Results: Strategy with competitive ratio

$$r(m-1) \left(\frac{m}{m-1} \right)^m + 2 - r$$

+ no strategy better than $rm/2$ -competitive



Setting: Output the r -th smallest contract

Results: Strategy with acceleration ratio

$$r(n+1) \left(1 + \frac{1}{rn} \right)^{rn}$$

+ no strategy better than rn -competitive

Results

	<p>Ray searching </p> <p>m rays</p>	<p>Contract scheduling </p> <p>n problems</p>	<p>Methodology</p>
<p>Stochastic setting</p>	<p>Setting: Target detection with probability p</p> <p>Results: Strategy with competitive ratio $\Theta(m/p^2)$</p> <p style="text-align: center;">+</p> <p>No strategy is better than $m/(2p)$-competitive</p>	<p>Setting: Contracts are Monte Carlo algorithms with success prob. p</p> <p>Results: Schedule with accel. ratio $(e(n+1))/p$</p> <p style="text-align: center;">+</p> <p>No schedule better than n/p-acceleration</p>	<p>Non-trivial analysis of cyclic strategies</p>
<p>Fault tolerance / redundancy</p>	<p>Setting: Target detection on the r-th visit</p> <p>Results: Strategy with competitive ratio $r(m-1) \left(\frac{m}{m-1}\right)^m + 2 - r$</p> <p>+ no strategy better than $rm/2$-competitive</p>	<p>Setting: Output the r-th smallest contract</p> <p>Results: Strategy with acceleration ratio $r(n+1) \left(1 + \frac{1}{rn}\right)^{rn}$</p> <p>+ no strategy better than rn-competitive</p>	<p>Non-cyclic strategies that improve upon the best cyclic ones</p>

Results (continued)

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m rays



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Randomized scheduling

Known: Randomization helps improve the competitive ratio [Kao et al. 96]

Result: Randomized schedule of acceleration ratio about 0.6 times the deterministic acceleration ratio

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Trade offs between performance and turns / executions

Setting: we are interested in the # of **turns**

Results: Optimal trade-offs between competitive ratio and # of turns

Setting: we are interested in the # of **executions**

Results: Optimal trade-offs between acceleration ratio and # of executions

Results (continued)

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Similar strategies but different analysis (no closed form in the case of contract scheduling)

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Combination of uniform and exponentially increasing strategies

The stochastic setting

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In the resulting expression, there is a term of the form $\sum_{j=2}^{\infty} ((b^m(1-p))^{j-1})$

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4. Applying some calculus we show that $\alpha \leq 1 + 8m/p^2$

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Algorithm

1. Choose a random permutation of the n problems and random $\epsilon \in (0, 1)$
2. In the i -th step schedule a contract for problem $i \bmod n$ and of length $b^{i+\epsilon}$

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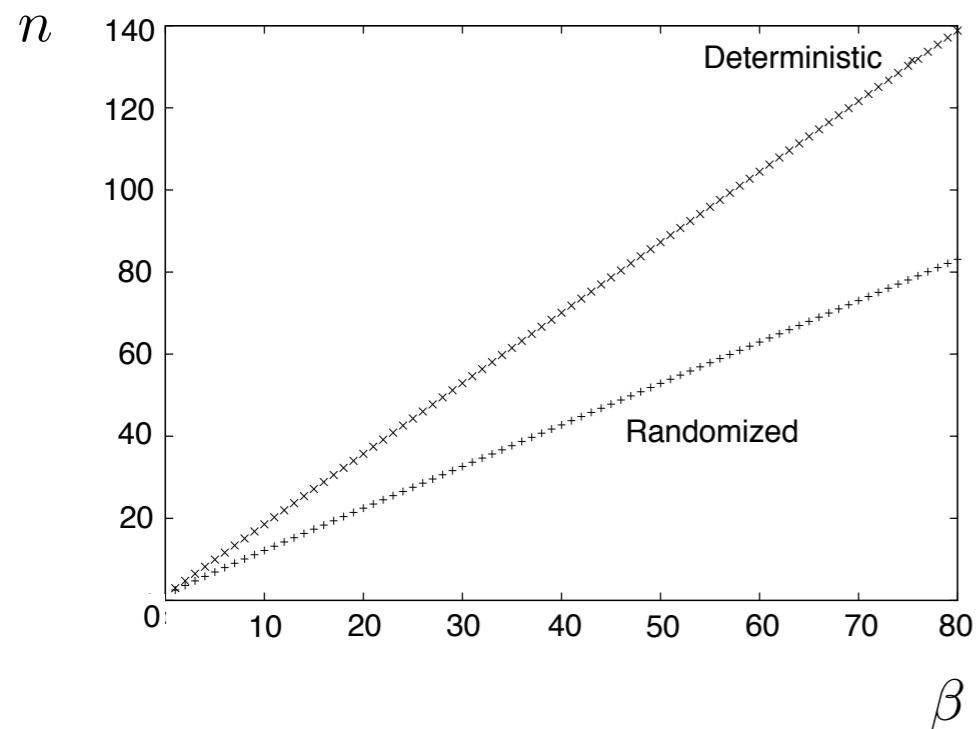
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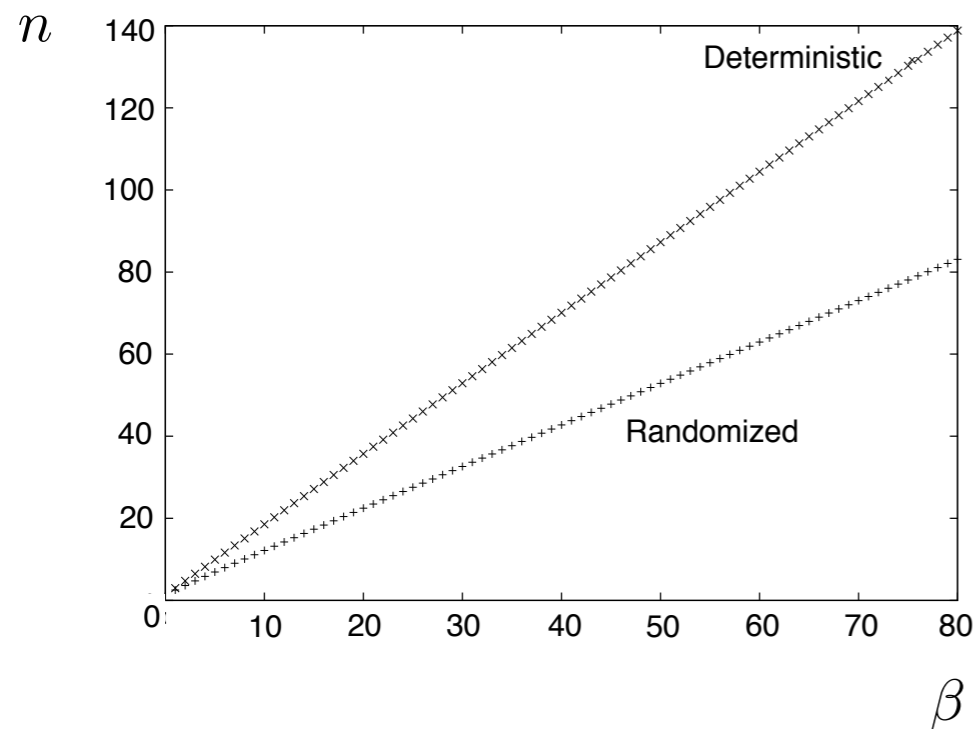


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Inspired by the randomized ray-searching algorithm of [Kao et al. 95]



- A closed formula does not appear to exist
- We can give analytical bounds for $n \rightarrow \infty$

Tradeoffs between performance and turns/executions

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Ray searching

m rays



Contract scheduling

n problems



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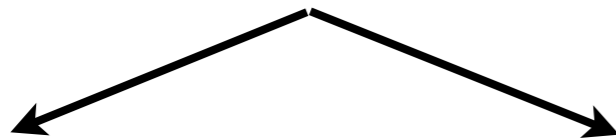
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pathwise search

expanding search

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Theorem: For any strategy with acceleration ratio $n(b+1) - \epsilon$, for $b > 1$ and $\epsilon > 0$ there exists t such that the # of preemptions up to t is at least $n \log_b \left(\frac{t(b-1)}{n} + 1 \right) - n$.

Furthermore, there is a strategy with acceleration ratio $n(b+1)$ and at most $n \log_b \left(\frac{t(b-1)}{n} + 1 \right) + n$ preemptions.

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