A computer-assisted discharging procedure: application to 2-distance coloring

Hoang LA, Petru VALICOV

LIRMM, University of Montpellier

November, 2020
2-distance coloring

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<thead>
<tr>
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$\chi^2(G) = 6$. 

Not a 2-distance coloring. 

An optimal 2-distance 6-coloring.
Observation

For any graph $G$ with maximum degree $\Delta$, $\Delta + 1 \leq \chi^2(G) \leq \Delta^2 + 1$. 

2-distance coloring
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For any graph $G$ with maximum degree $\Delta$, $\Delta + 1 \leq \chi^2(G) \leq \Delta^2 + 1$.

\[\chi^2(G) = \Delta^2 + 1.\]
Wegner’s conjecture, 1977

Let $G$ be a planar graph. Then,

$$\chi^2(G) \leq \begin{cases} 
7, & \text{if } \Delta \leq 3, \\
\Delta + 5, & \text{if } 4 \leq \Delta \leq 7, \\
\left\lfloor \frac{3\Delta}{2} \right\rfloor + 1, & \text{if } \Delta \geq 8.
\end{cases}$$
2-distance coloring of planar graphs

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Results on planar graphs with high girth:

If $G$ is a planar graph with girth $g \geq g_0$ and maximum degree $\Delta \geq \Delta_0$, then $\chi^2(G) \leq \Delta(G) + c_0$. 

Bu et al., 2015

If $G$ is a planar graph with $g \geq 8$ and $\Delta = 5$, then $\chi^2(G) \leq \Delta + 3$. 

La and Valicov, 2020

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Hoang LA (LIRMM)
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### 2-distance coloring of planar graphs

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<th>$\Delta + 3$</th>
<th>$\Delta + 4$</th>
<th>$\Delta + 5$</th>
<th>$\Delta + 6$</th>
<th>$\Delta + 7$</th>
<th>$\Delta + 8$</th>
</tr>
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<tbody>
<tr>
<td>3</td>
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<td>$\Delta = 3$</td>
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</tr>
<tr>
<td>5</td>
<td>$\Delta \geq 10^7$</td>
<td>$\Delta \geq 339$</td>
<td>$\Delta \geq 312$</td>
<td>$\Delta \geq 15$</td>
<td>$\Delta \geq 12$</td>
<td>$\Delta \neq 7, 8$</td>
<td>all $\Delta$</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$\Delta \geq 17$</td>
<td>$\Delta \geq 9$</td>
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</tr>
<tr>
<td>7</td>
<td>$\Delta \geq 16$</td>
<td>$\Delta = 4$</td>
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<tr>
<td>8</td>
<td>$\Delta \geq 9$</td>
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<tr>
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<td>11</td>
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<td>14</td>
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<td></td>
<td></td>
<td>$\Delta \geq 4$</td>
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<td>22</td>
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<td></td>
<td></td>
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Results from almost 20 different papers.
The discharging method (on planar graphs):

1. Suppose that there exists a counter-example $G$ and suppose that $G$ has the smallest number of vertices.
2. Study the structural properties of $G$.
3. Assign charges to vertices and faces so that the sum of all charges is negative thanks to Euler's formula ($|V| - |E| + |F| = 2$).
4. Redistribute the charges without changing the total sum, and show that we obtain a non-negative final amount, thanks to the structural properties, which is a contradiction.
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The discharging method: Step 1

**Theorem**
If $G$ is a planar graph with girth $g \geq 8$ and maximum degree $\Delta = 3$, then $\chi^2(G) \leq 6$.

**Step 1:** Take a minimal counter-example $G$, with $\Delta = 3$, $g \geq 8$, and $\chi^2(G) \geq 7$. 
The discharging method: Step 2

**Step 2:** Structural properties of $G$. 

Some trivial properties:
- $G$ is connected
- $G$ has no
- And almost 50 other complicated reducible configurations...
The discharging method: Step 2

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Step 3: Assign charges to vertices.

Charge assignment

\[ \mu : v \mapsto 3.5d(v) - 9 \quad \text{and} \quad \mu : f \mapsto d(f) - 9 \]
Step 3: Assign charges to vertices.

Charge assignment

\[ \mu : v \mapsto 3.5d(v) - 9 \text{ and } \mu : f \mapsto d(f) - 9 \]

Since \(|V| - |E| + |F| = 2\),

\[ \sum_{v \in V} (3.5d(v) - 9) + \sum_{f \in F} (d(f) - 9) = -18 < 0 \]
### Comparison

<table>
<thead>
<tr>
<th>Reducible configurations</th>
<th>&quot;Human&quot; proof $g \geq 9$</th>
<th>&quot;Human&quot; proof? $g \geq 8$</th>
<th>Our proof $g \geq 8$</th>
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<td>Few &quot;local&quot; configs</td>
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### Charge distribution

<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td></td>
</tr>
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<td>≥0</td>
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\[
\mu(v) = 3.5d(v) \\
\mu(f) = d(f) - 9
\]
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<td>( \mu(v) = 3.5d(v) )</td>
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\( \mu'(v) = 3d(v) \) for \( g \geq 8 \)
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<tr>
<td>( \bullet ) : -2</td>
<td></td>
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<td></td>
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<tr>
<td>( \nearrow \bullet ) : 1.5</td>
<td></td>
<td>( \nearrow \bullet ) : 1</td>
<td></td>
</tr>
<tr>
<td>face ( \geq 9 ) : ( \geq 0 )</td>
<td></td>
<td>face ( \geq 8 ) : ( \geq 0 )</td>
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\[
\mu(v) = 3.5d(v) \]

\[
\mu(f) = d(f) - 9
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\[
\mu'(v) = 3d(v) \]

\[
\mu'(f) = d(f) - 8
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### Diagrams

- **"Human" proof \( g \geq 9 \):**
  - \( \mu(v) = 3.5d(v) \)
  - \( \mu(f) = d(f) - 9 \)

- **"Human" proof ? \( g \geq 8 \):**
  - \( \mu'(v) = 3d(v) \)
  - \( \mu'(f) = d(f) - 8 \)

- **Our proof \( g \geq 8 \):**
  - \( \mu(v) = 3.5d(v) \)
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\[
\mu(v) = 3.5d(v) \\
\mu(f) = d(f) - 9 \\
\mu'(v) = 3d(v) \\
\mu'(f) = d(f) - 8 \\
\]

Face condition:

- \( \mu(v) = \sum_{f \text{ incident to } v} \mu'(f) \)
- \( \mu(f) = \sum_{v \text{ incident to } f} \mu'(v) \)

For face \( \geq 9 \):

- \( \mu(v) \geq 0 \)
- \( \mu'(v) \geq 0 \)
- \( \mu' = 3d - 9 \)
- \( \mu = 3d - 8 \)

For face \( \geq 8 \):

- \( \mu(v) \geq 0 \)
- \( \mu'(v) \geq 0 \)
- \( \mu' = 3d - 9 \)
- \( \mu = 3d - 8 \)

For face \( = 8 \):

- \( \mu(v) = -1 \)
- \( \mu'(v) = -1 \)

\( \mu(v) = 3.5d(v) \)

\( \mu(f) = d(f) - 9 \)
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<td>Lots of &quot;local&quot; configs</td>
</tr>
<tr>
<td>Charge distribution</td>
<td><img src="graph1.png" alt="Graph" /> ( \mu(v) = 3.5d(v) ) ( \mu(f) = d(f) - 9 )</td>
<td><img src="graph2.png" alt="Graph" /> ( \mu'(v) = 3d(v) ) ( \mu'(f) = d(f) - 8 )</td>
<td><img src="graph3.png" alt="Graph" /> ( \mu'(v) = 3d(v) ) ( \mu'(f) = d(f) - 8 )</td>
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\( \mu'(v) = d(f) - 8 \) \( \mu'(f) = d(f) - 8 \)
**Step 4:** Redistribute the charges to obtain a non-negative sum (via discharging rules).
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Choose rules to assure the vertices have non-negative charges.

For example, here, we can reuse the same rules as Cranston and Kim for $g \geq 9$.

Choose rules to assure the 8-faces have non-negative charges.

\[-2 : \quad 1.5 \quad \text{face } \geq 9 : \quad \geq 0 \quad \text{face } = 8 : \quad = -1\]
The discharging method: Step 4

**Step 4:** Redistribute the charges to obtain a non-negative sum (via discharging rules).

- \(-2\):
- \(1.5\):
- \(\geq 9\):
- \(\geq 0\):
- \(\text{face } = 8\):
- \(= -1\)

Choose rules to assure the vertices have non-negative charges.
Step 4: Redistribute the charges to obtain a non-negative sum (via discharging rules).

Choose rules to assure the vertices have non-negative charges.

For example,
Step 4: Redistribute the charges to obtain a non-negative sum (via discharging rules).

Choose rules to assure the vertices have non-negative charges.

For example, Here, we can reuse the same rules as Cranston and Kim for $g \geq 9$. 
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Choose rules to assure the vertices have non-negative charges.

1. For example,

2. Here, we can reuse the same rules as Cranston and Kim for \( g \geq 9 \).

Choose rules to assure the 8-faces have non-negative charges.
Identify the 3-vertices.

Count the 2-vertices in between.

Encode the neighborhood of each 3-vertex.

We obtain: 1a1a0b0c0a0c.
Identify the 3-vertices. Count the 2-vertices in between.
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Computer Assistance: Encoding Cycles

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We obtain: \textit{1a1a0b0c0a0c}.
Encode a tree from all incident faces
Encode a tree from all incident faces
Encode a tree from all incident faces:

- \( f_0, f_2 : 1c0c1 \)
Encode a tree from all incident faces

- \( f_0, f_2 : 1c0c1 \)
- \( f_1, f_3 : 1b1 \)
How to verify our proof with the computer assistance:

1. Generate all possible words on \{0, 1, a, b, c\} corresponding to the 8-faces (more than 10,000 words).
2. Filter the list with encoded reducible configurations.
3. Calculate the charges of the remaining faces/words.
4. Define a dictionary of charges for each subword.
5. Calculate the charge of each word by its subwords.
How to verify our proof with the computer assistance:

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- Filter the list with encoded reducible configurations.

- Calculate the charges of the remaining faces/words.

- Define a dictionary of charges for each subword.

- Calculate the charge of each word by its subwords.
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- Solve the problem of verifying a huge amount of configurations.
- Find problematic configurations (non-reducible and non-dischargeable) immediately.
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