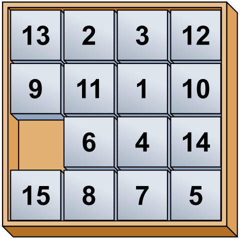
On girth and the parameterized complexity of token sliding and token jumping

Valentin Bartier, Nicolas Bousquet, Clement Dallard, Kyle Lomer, Amer E. Mouawad

G-SCOP, Grenoble, France

Reconfiguration

The 15 puzzle game:



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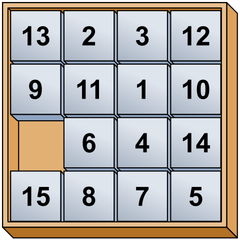

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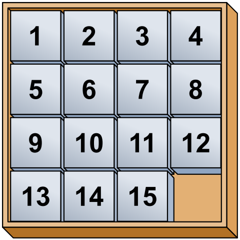

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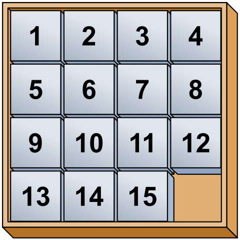

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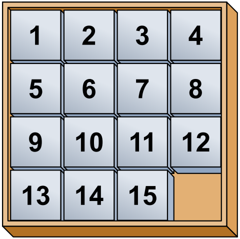

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Reachability for reconfiguration problems: initial configuration + pool of possible moves (rules). The goal is to reach a target configuration.

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Initial independent set I $\leftrightarrow$ Target independent set J.

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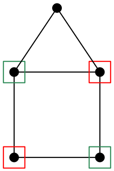

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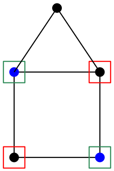

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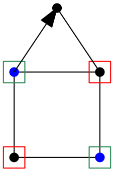

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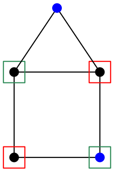

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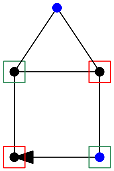

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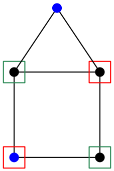

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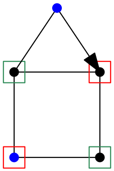

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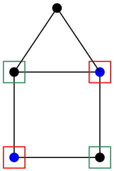

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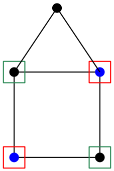

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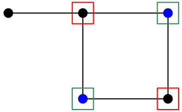

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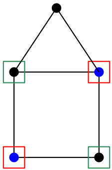

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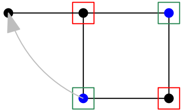

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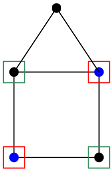

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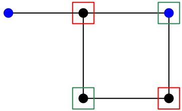

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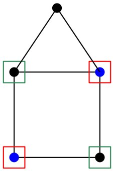

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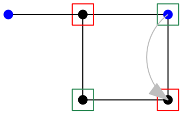

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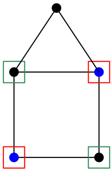

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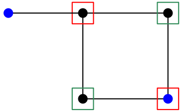

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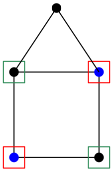

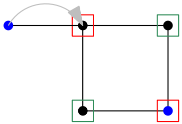

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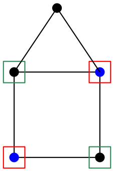

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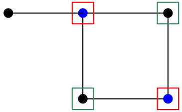

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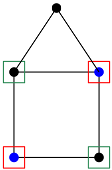

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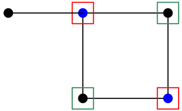

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$\rightarrow$ Similar definitions for Dominating Set Reconfiguration, Vertex Cover Reconfiguration, Vertex Separator Reconfiguration...

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$\rightarrow$ PSPACE-complete in general.

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- TOKEN JUMPING *is FPT on bounded-degree graphs when parameterized by* k*.*

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→ Let's see how it works for Token Sliding on bipartite and bipartite C_4-free graphs.

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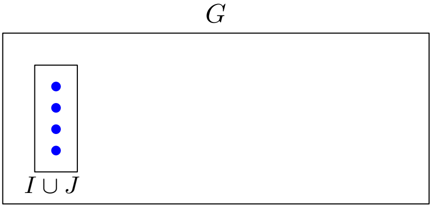

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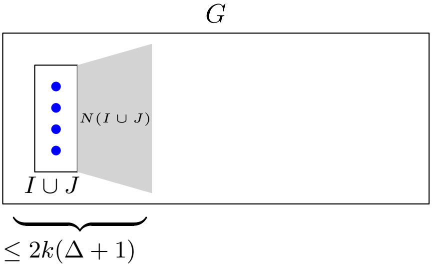

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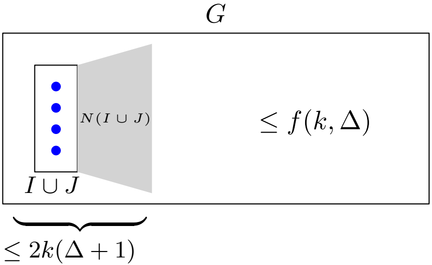

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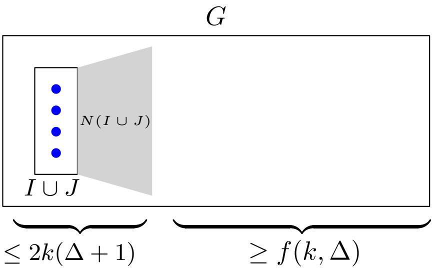

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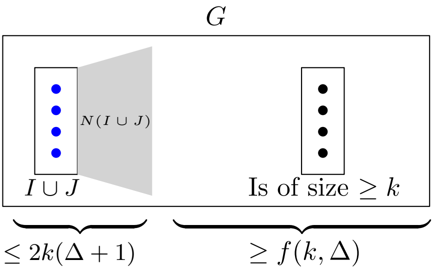

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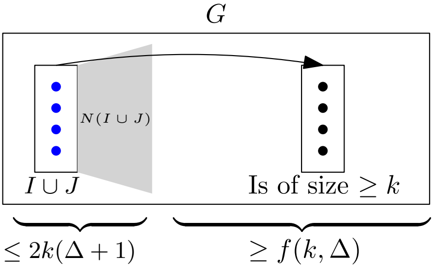

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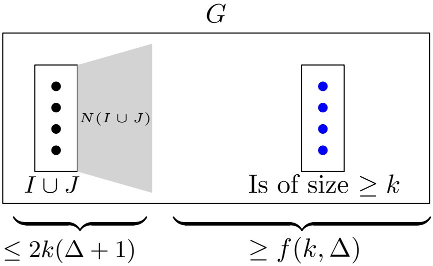

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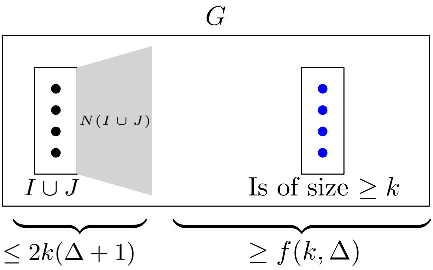

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$\rightarrow$ Does not work for TOKEN SLIDING

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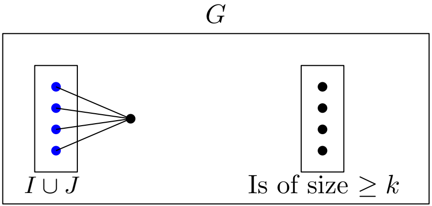

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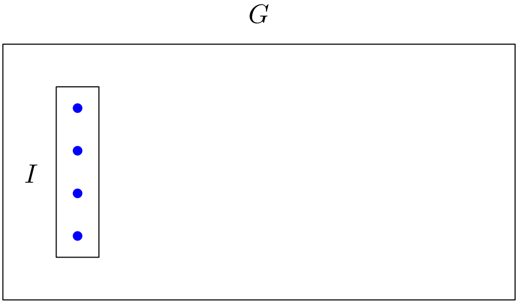

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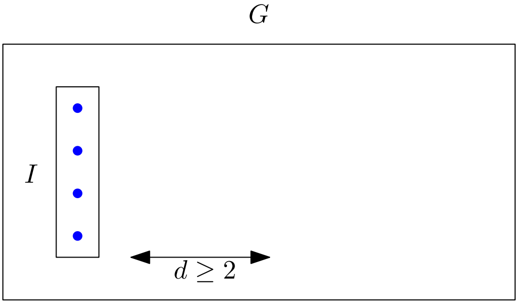

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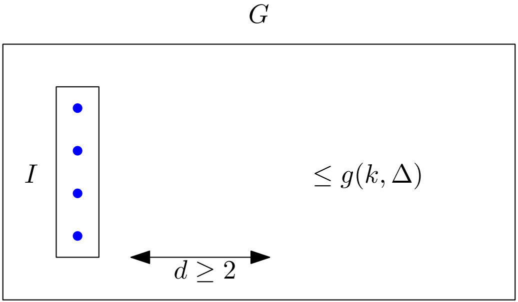

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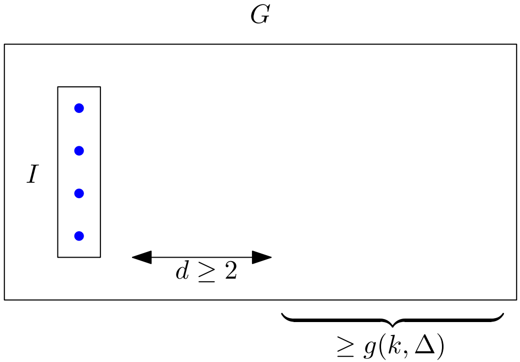

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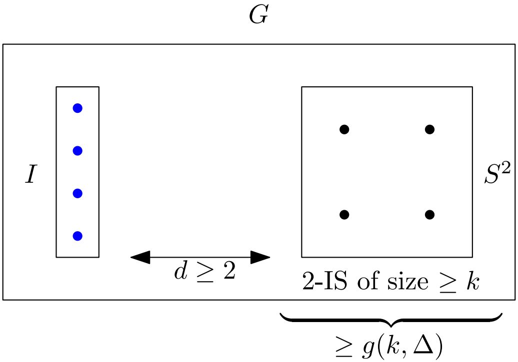

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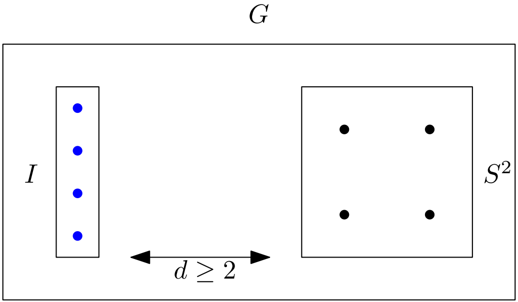

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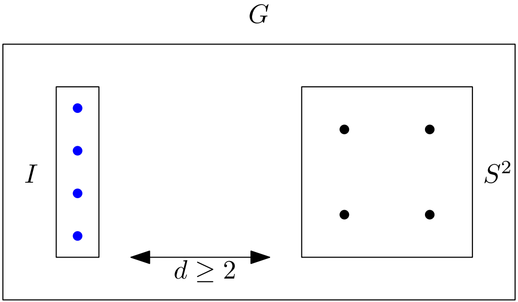

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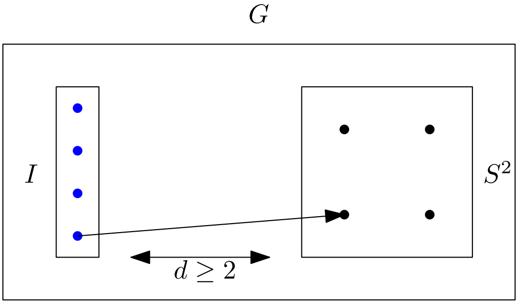

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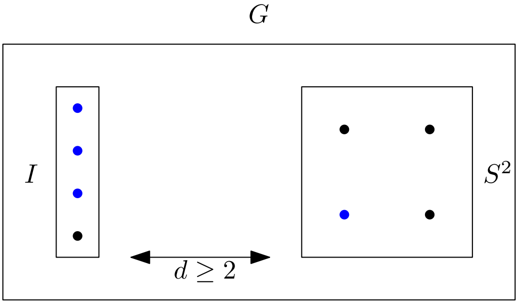

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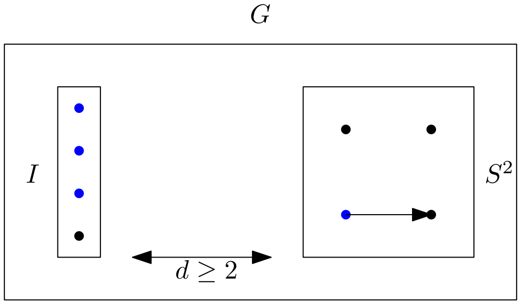

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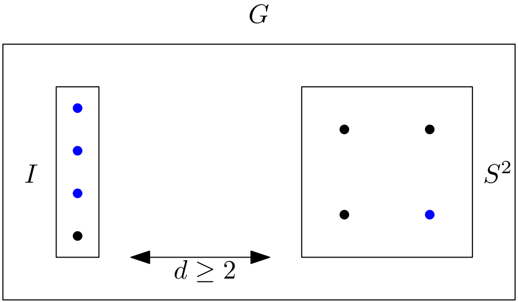

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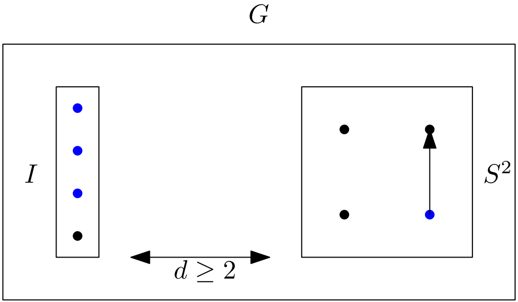

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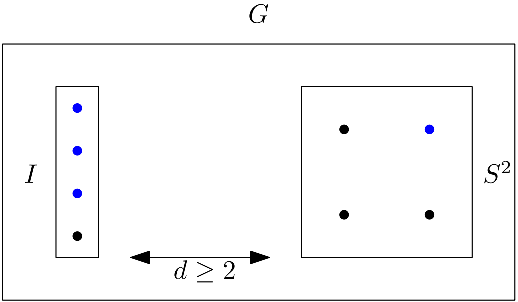

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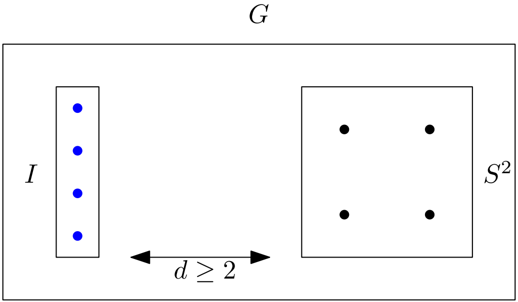

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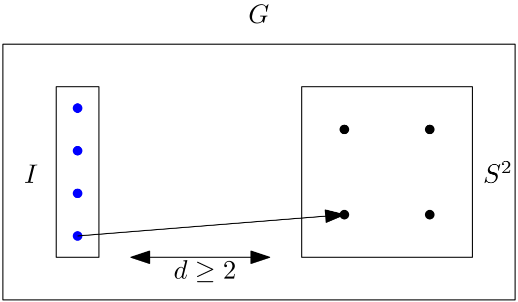

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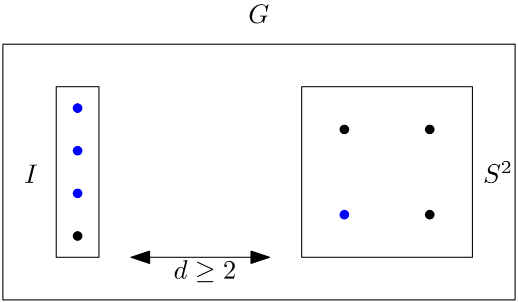

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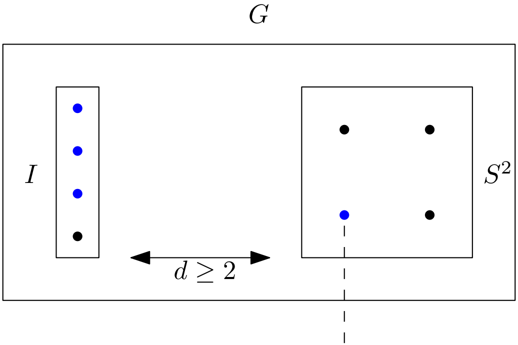

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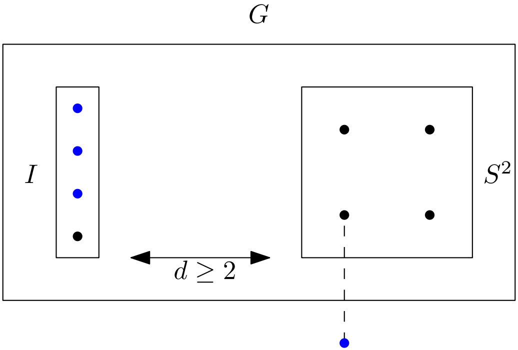

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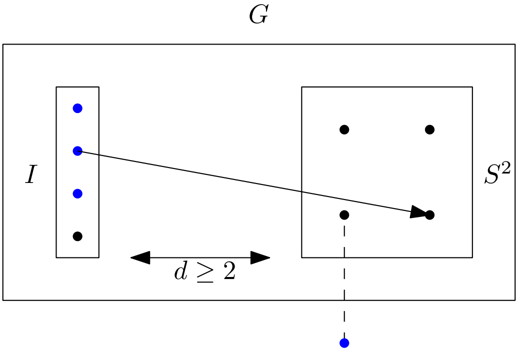

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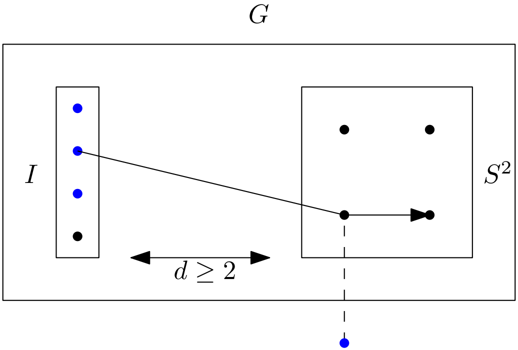

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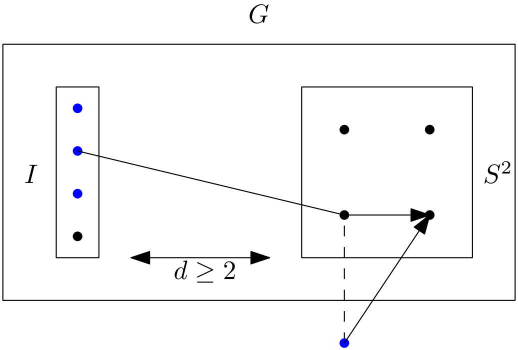

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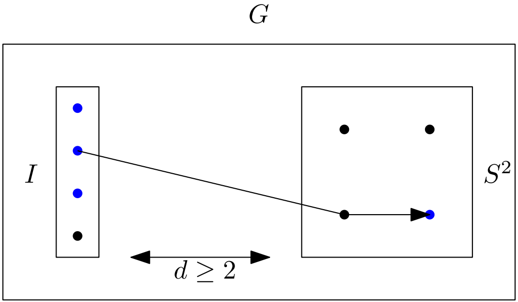

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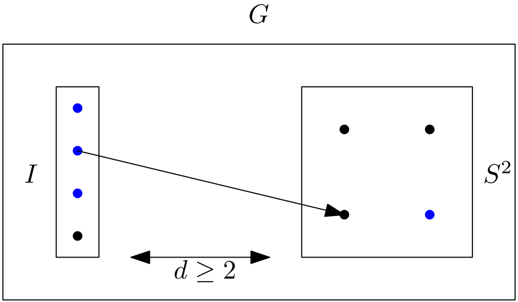

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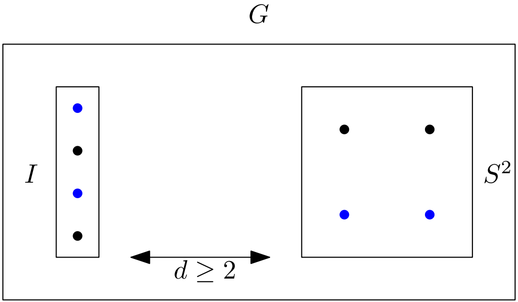

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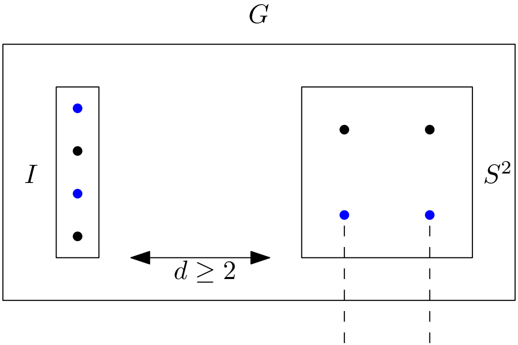

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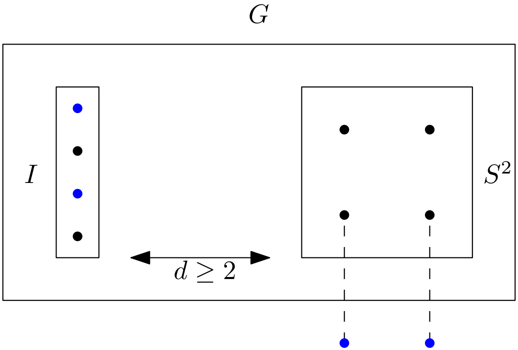

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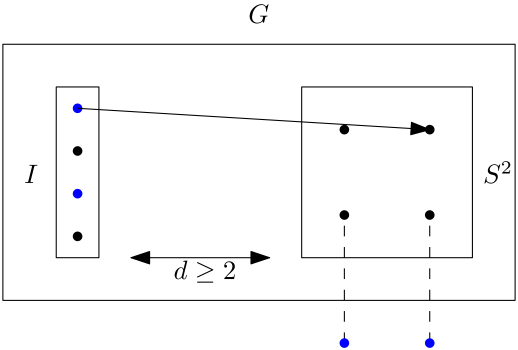

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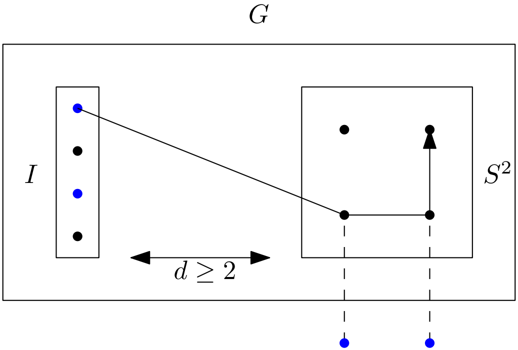

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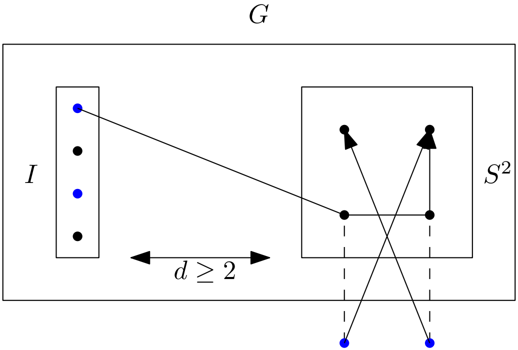

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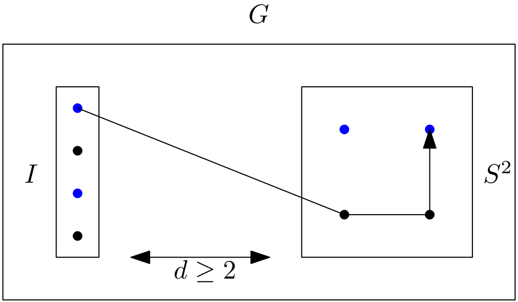

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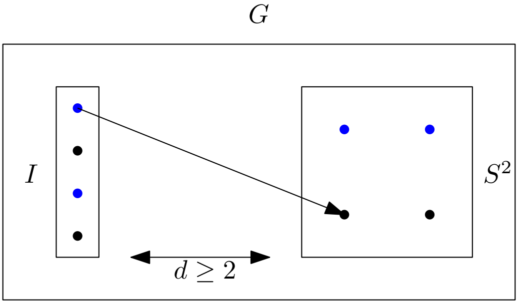

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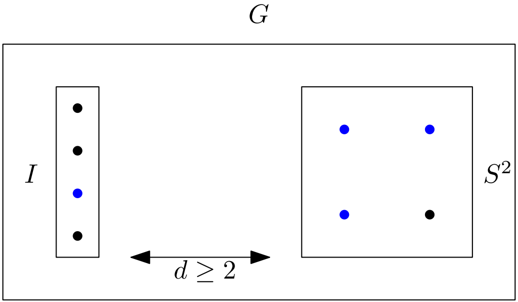

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Multicolored Independent Set: graph G, integer k and partition $\{V_1, \ldots, V_k\}$ of $V(G)$. Is there an IS $\{u_1, \ldots, u_k\}$ of G s.t. $u_i \in V_i$ $\forall i$?

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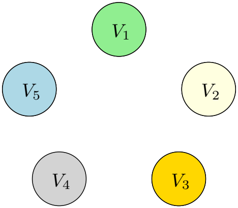

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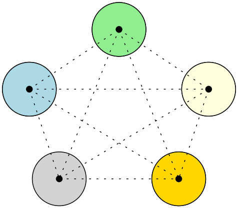

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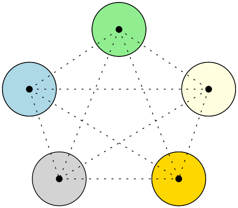

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Multicolored independent set *is $W[1]$-hard when parameterized by k.*

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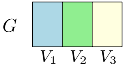

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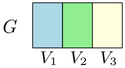

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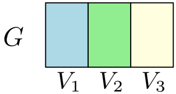

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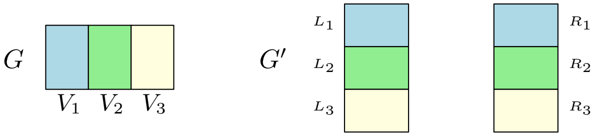

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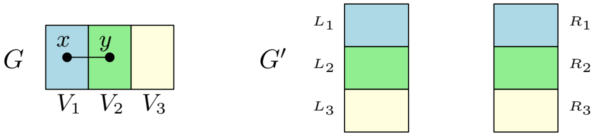

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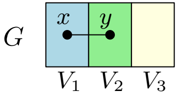

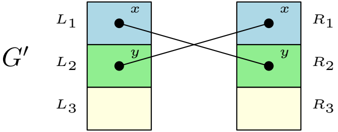

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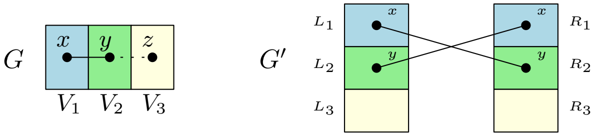

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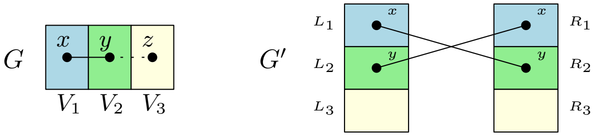

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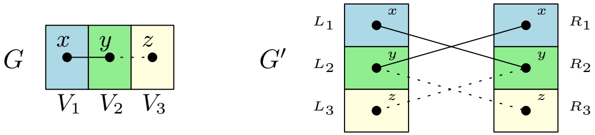

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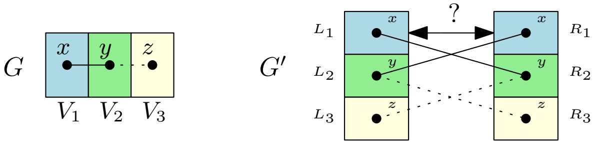

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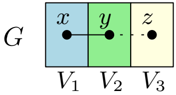

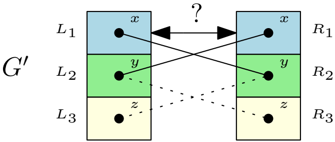

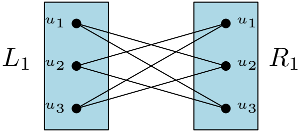

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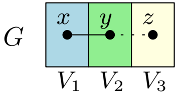

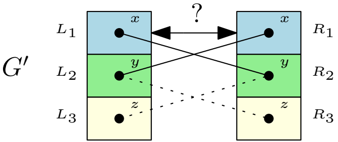

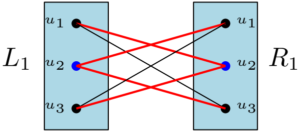

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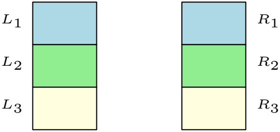

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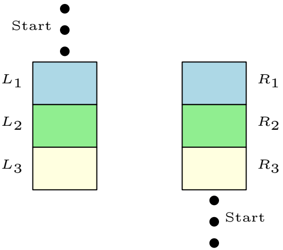

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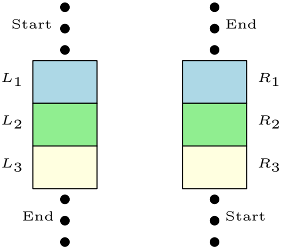

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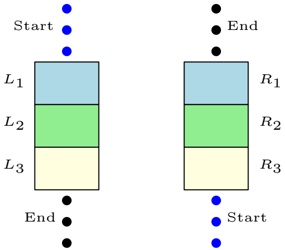

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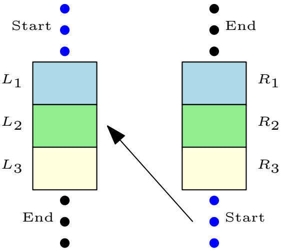

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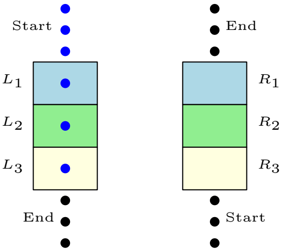

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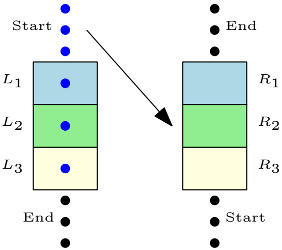

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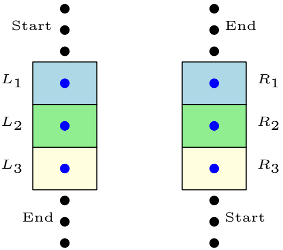

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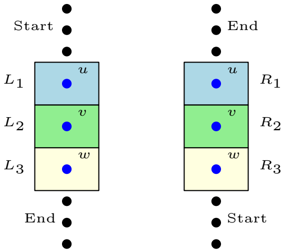

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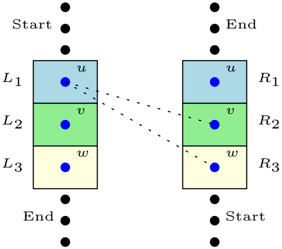

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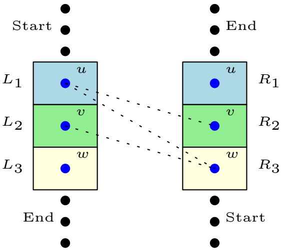

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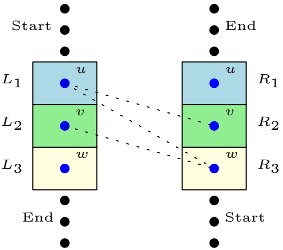

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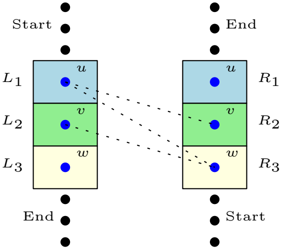

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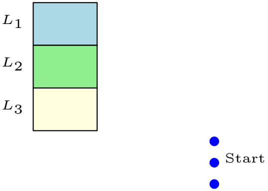

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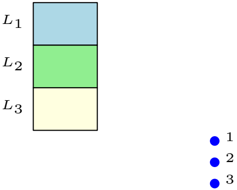

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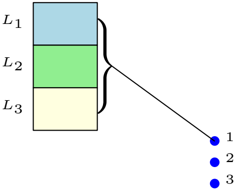

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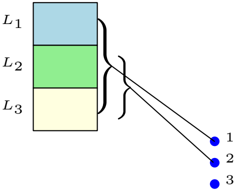

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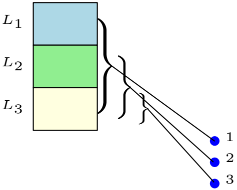

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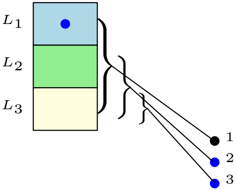

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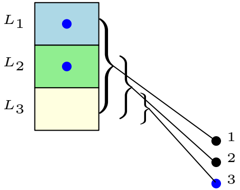

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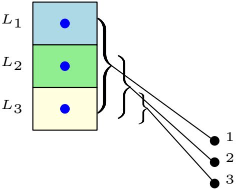

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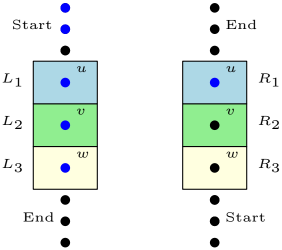

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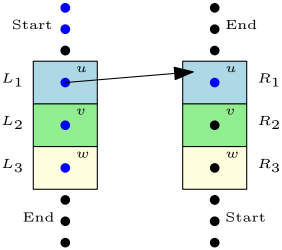

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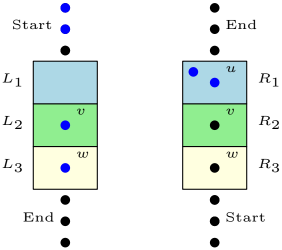

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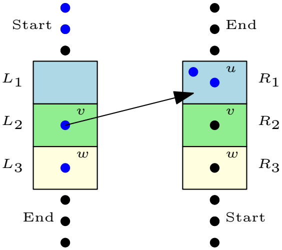

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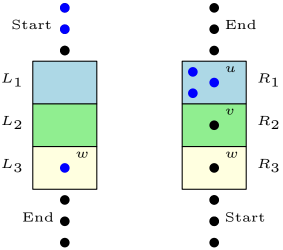

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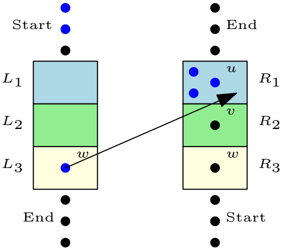

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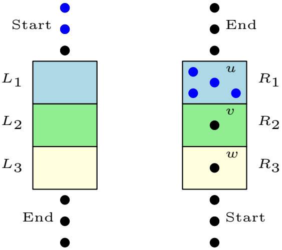

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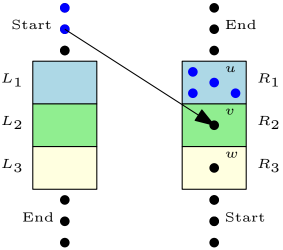

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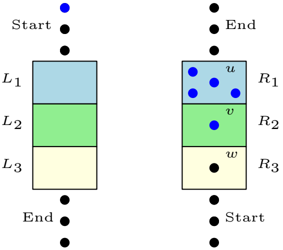

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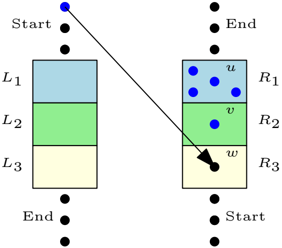

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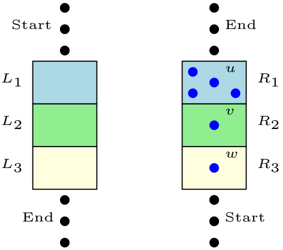

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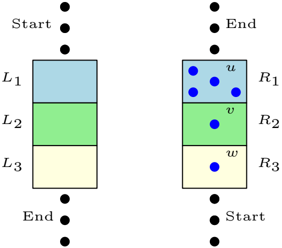

$\rightarrow$ Duplicate each L_i, R_i ($\Longrightarrow$ double the number of tokens)

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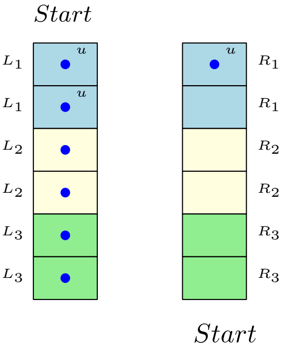

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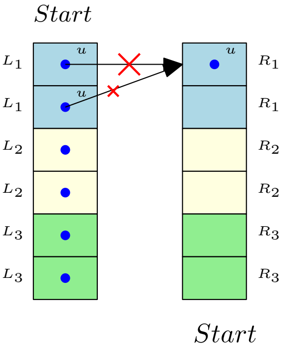

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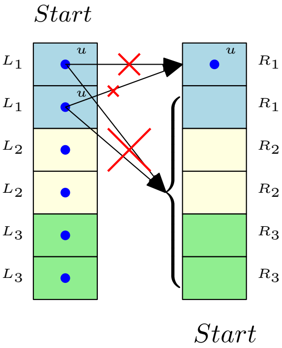

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