Enumerating minimal dominating sets in the incomparability graphs of bounded dimension posets

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JGA 2020

November 16, 2020

Enumeration problems

Typical question:

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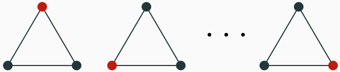
Remark: possibly many objects!
$3^{n/3} \approx 1.4422^n$



Two perspectives about complexity

Input-sensitive: in terms of input size

Theorem (Fomin, Grandoni, Pyatkin, and Stepanov, 2008)

There is an $O(1.7159^n)$-time algorithm enumerating all minimal dominating sets in n-vertex graphs.

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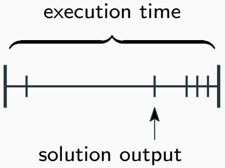
$\rightarrow$ *many techniques (reverse search, flashlight search, ordered generation, proximity search, etc.)*

Let n be input size, e.g., number of vertices of a graph G
Let d be output size, e.g., number of maximal cliques in G



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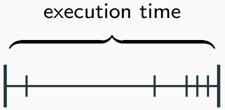

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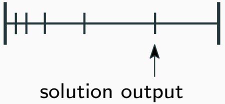

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outputs i^{th} *solution in* poly$(n+i)$*-time*

"Fast" output-sensitive algorithms

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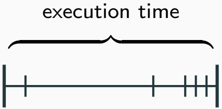

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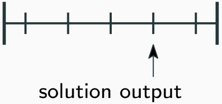
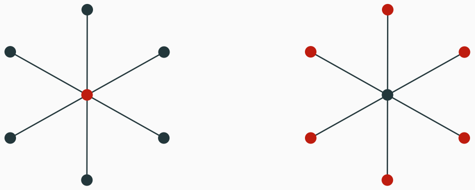


polynomial-delay
poly(n)*-time between two cons. outputs*

Minimal dominating sets

- $N(v)$: neighborhood of vertex v, $N[v] = N(v) \cup \{v\}\}$
- dominating set (DS): $D \subseteq V(G)$ s.t. $V(G) = D \cup N(D)$

  "D can see everybody else"
- minimal dominating set: inclusion-wise minimal DS



- $N(S) = \bigcup_{v \in S} N(v) \setminus S$: neighborhood of vertex set S
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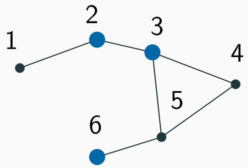
- minimal dominating set: inclusion-wise minimal DS
- private neighbors $\mathrm{Priv}(D, v)$ of $v \in D$:

  vertices that are $\begin{cases} \text{dominated by } v, \text{ and} \\ \text{not dominated by } D \setminus \{v\} \end{cases}$ (possibly v)

- irredundant set: $S \subseteq V(G)$ s.t. every $x \in S$ has a priv. neighbor



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Observation ★

A DS is minimal if and only if it is irredundant.

if all its vertices have a private neighbor.

if $\mathrm{Priv}(D, v) \neq \emptyset$ *for all* $v \in D$

Minimal DS Enumeration (Dom-Enum)

input: a n-vertex graph G.

output: the set $\mathcal{D}(G)$ of minimal DS of G.

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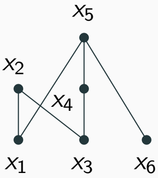
Posets & (In)comparability graphs

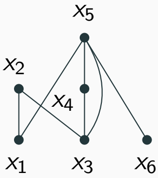
- poset $P = (V, \leq)$: refl., trans., antisymmetric relation $\leq$ on V

  $x \leq x \qquad x \leq y \wedge y \leq x \implies x = y$
- comp(P): graph on V s.t. $uv \in E$ if $u \leq v$ or $v \leq u$
- incomp(P): complementary of comp(P)

  i.e., an edge for every incomparable pair of elements



$P = (V, \leq)$



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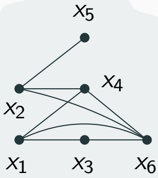

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Hard case: co-bipartite graphs, hence incomparability graphs

Known cases: ⚠

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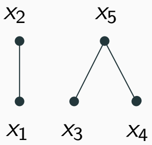
Poset dimension of $P = (V, \leq)$:

Least integer d such that elements of P can be embedded into $\mathbb{R}^d$ in such a way that $x \leq y$ in P if and only if the point of x is below the point of y with respect to the product order of $\mathbb{R}^d$



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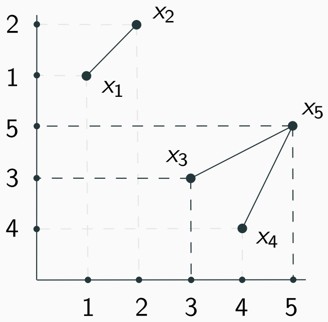
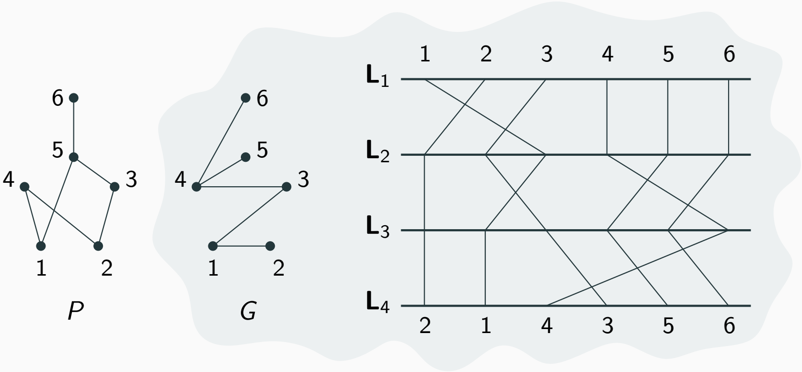


Theorem (Golumbic, Rotem, and Urrutia, 1983)

A graph G is the incomparability graph of a poset of dimension d if and only if it is the intersection graph of the concatenation of d permutation diagrams.



General idea of the algorithm

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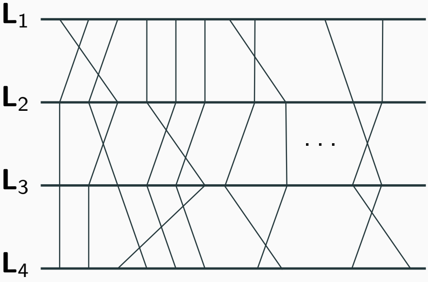
if all its vertices have a private neighbor.

- make grow irredundant sets to minimal dominating sets
- ensure that each constructed partial set leads to a solution



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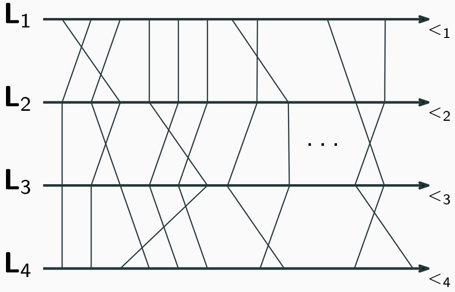

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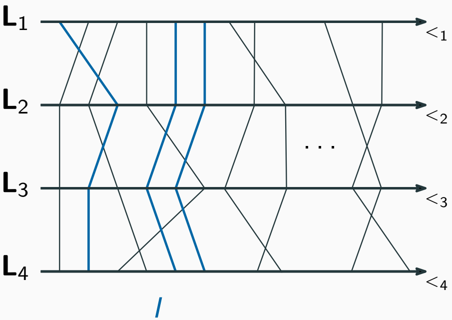

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- $\mathsf{R}(I) = \{v \in V \setminus I : \exists j,\ \forall u \in I,\ u <_j v\}$

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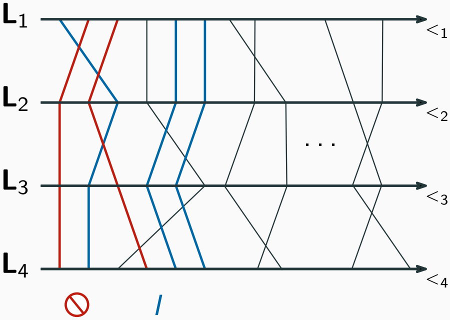

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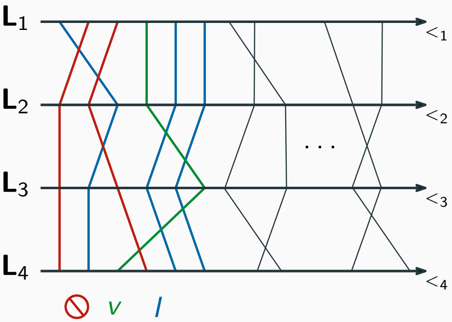

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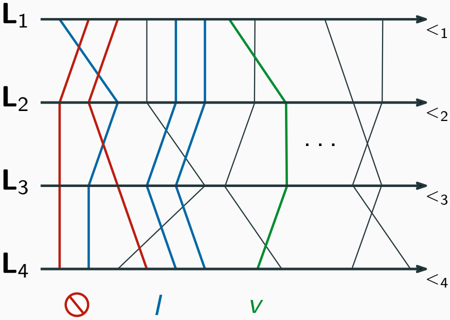

Right-Extension Problem:

Given $I \subseteq V(G)$, decide whether I can be *extended to the right into a min DS*, i.e., whether $\exists D \in \mathcal{D}(G)$ s.t. $I \subseteq D$ and $D \setminus I \subseteq \mathrm{R}(I)$

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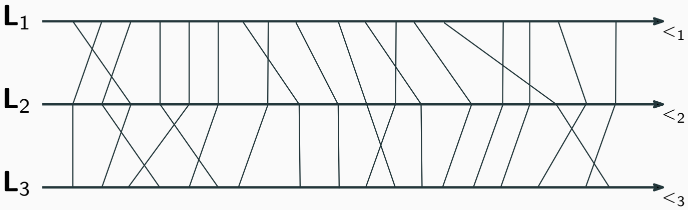
Set $I = \{u_1, \ldots, u_p\}$ *can be extended to the right iff*

$\exists v_1, \ldots, v_p \in \mathrm{Priv}(I, u_1) \times \cdots \times \mathrm{Priv}(I, u_p)$

s.t. $R(I) \setminus N[v_1, \ldots, v_p]$ *dominates* $G - N[I]$



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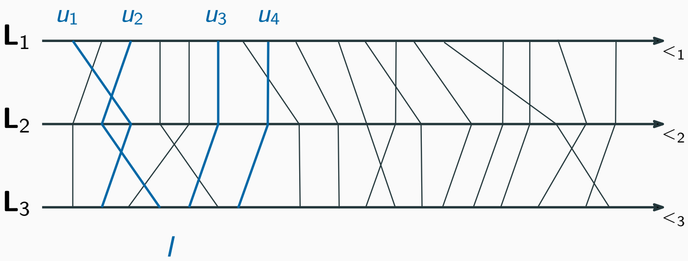

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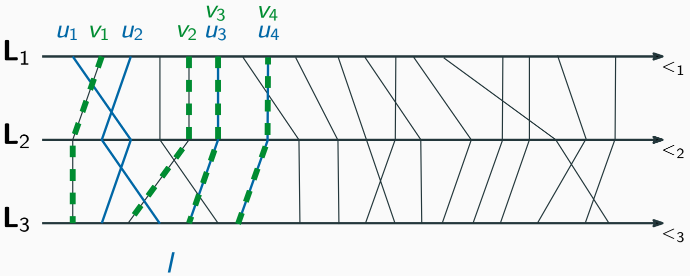

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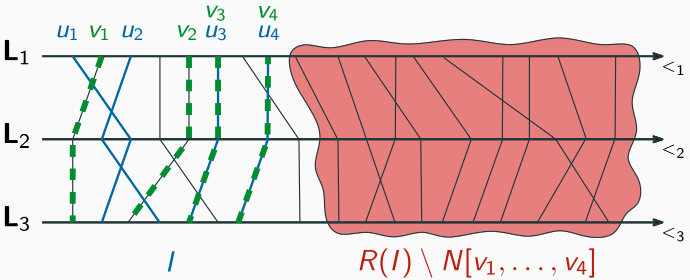

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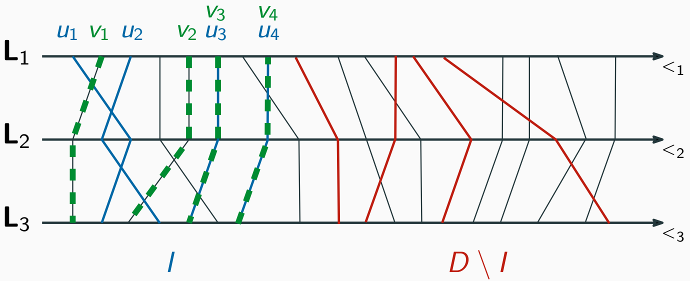

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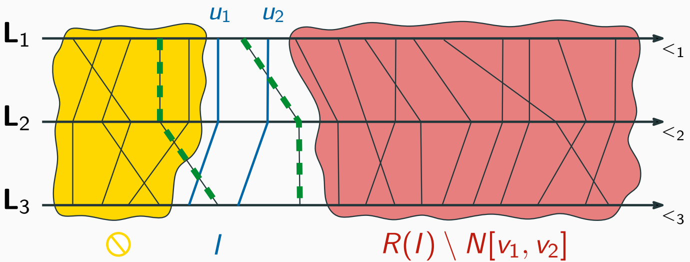

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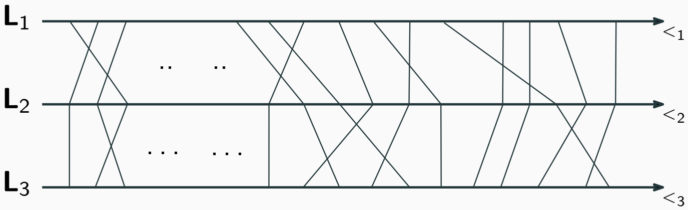

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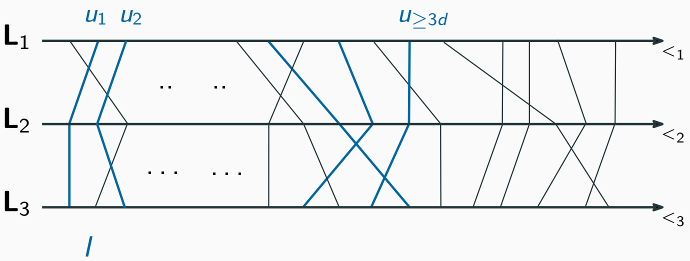

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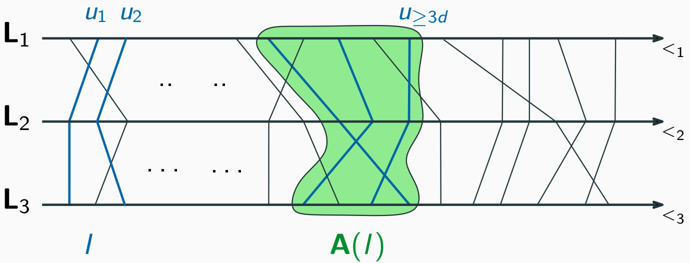

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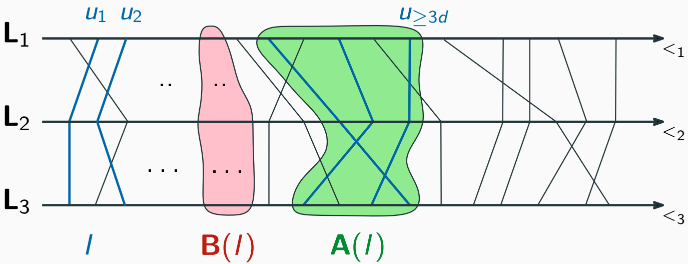

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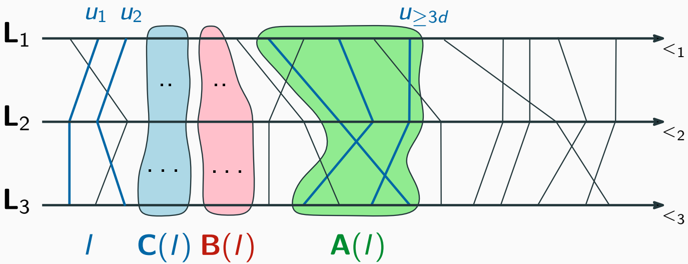

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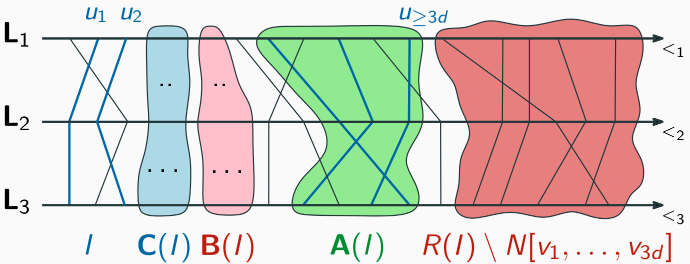
Theorem $\diamondsuit$

Set $I, |I| \geq 3d$ with $A \cup B \cup C = \{u_1, \ldots, u_{3d}\}$ can be ext. iff

$$\exists v_1, \ldots, v_{3d} \in \mathrm{Priv}(I, u_1) \times \cdots \times \mathrm{Priv}(I, u_{3d})$$

s.t. $R(I) \setminus N[v_1, \ldots, v_{3d}]$ *dominates* $G - N[I]$



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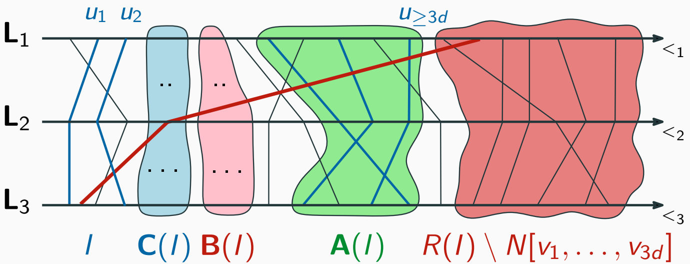

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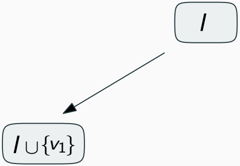

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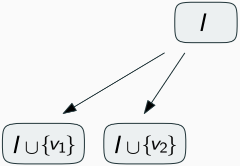

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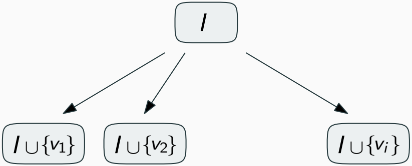

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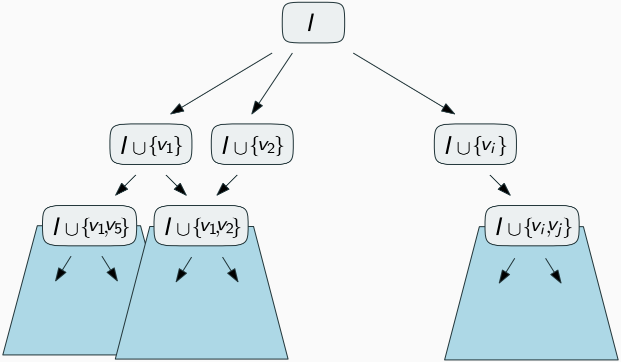

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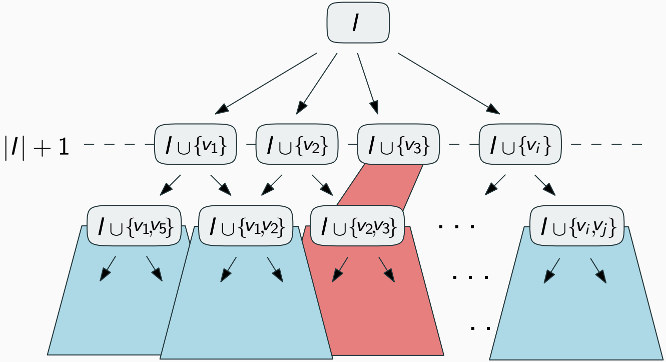

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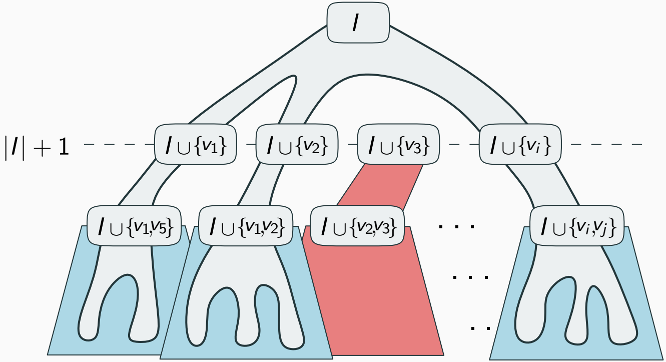

Main theorem and future work

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Thank you!