Collaborative Spatial Decision Making with Qualitative Constraints

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Abstract: Usually spatial planning problems involve a large number of decision makers with different backgrounds and interests. The process of Collaborative Spatial Decision Making (CSDM) has to reconcile the individual approaches and lead to solutions that satisfy all participants. In this paper we deal with a particular instance of CSDM that involves qualitative constraints. Qualitative CSDM is very important for practical applications where decision makers communicate spatial knowledge using relations in space rather than absolute coordinates. It is also an interesting topic from the scientific point of view, because it combines several research areas related to Geographic Information Systems such as Spatial Query Languages, Constraint Satisfaction Mechanisms, Heterogeneous Databases, Spatial Access Methods and Group Decision Making. We describe the problems that need to be dealt with, we outline solutions, and we propose a computational model that can automate a large party of Qualitative CSDM.

Topics: GIS and cooperative work, Requirements analysis for GIS applications, Interoperability among heterogeneous GIS, GIS meta data

1 Introduction

Numerous Geographic Information Systems (GISs) have been developed, both in academia and industry, during the last decade. In spite of the fact that the use of computer science in spatial studies has certainly led to the implementation of advanced systems, there is still a lot of work to be done on *Collaborative Spatial Decision Making* (CSDM) systems. Such systems should provide an interactive, real-time environment for various groups, usually with different interests, authorities and interpretations on a *spatial planning problem*, to manipulate decision making issues. The well-tried interaction of decision makers (thereafter, the terms "decision maker" and "agent" are used interchangeably) with spatial analysis tools has to be enhanced with: (1) tools providing the appropriate group-based user interfaces, (2) methods to elicit and capture data from related GISs, Data and Knowledge Bases, and (3) methods for a fair, rational and efficient handling of debates in a group decision-making environment.

This paper deals with a particular instance of CSDM that involves qualitative constraints. Qualitative CSDM is of great interest since very often people communicate spatial knowledge in the form of spatial relations. As a result the qualitative representation and processing of spatial knowledge has recently gained a lot of attention in the GIS and Database community (for an extensive discussion see [Hernandez, 1994]). We propose a computational model which can efficiently handle such constraints and automate a large part of the decision making process. Throughout this paper we shall refer to a spatial planning problem illustrated in Figure 1 (the example was motivated by [Leitner, 1995]). The agents are given an area with wetlands, national parks, mountainous areas, urban areas, and railways, and they want to decide a potential site for a new airport. The agents do not necessarily have a scientific background in GISs; they may have different interests and consequently different, often conflicting priorities (e.g., environmental, political, commercial etc.).

The general constraint (assumed by all the agents) is that the airport cannot be in a mountainous terrain, in a wetland, or in the park. This translates into three topological constraints that must hold true: *disjoint(airport,mountain)* and *disjoint(airport,park)* and *disjoint(airport,wetlands)*. In addition, the first agent asserts that the airport should be somewhere *east* of the lake, *north* of the major urban area and *west* of the state park, so visitors can have easy access to all these places (direction constraints: *east(airport,lake)*, *north(airport,major_city)*, *west(airport,park)*). The second agent weighs only the position of the airport with respect to the major urban area. He/she asserts that the airport should not be very near (for safety reasons) or very far from the downtown of the major urban area (distance constraint: *not_very_near(airport,major_city)* and *not_very_far(airport,major_city)*). The third agent imposes that the airport should be accessible using the train routes (topological constraint: *intersects(railway,airport)*). A potential site that satisfies the above constraints is also illus-

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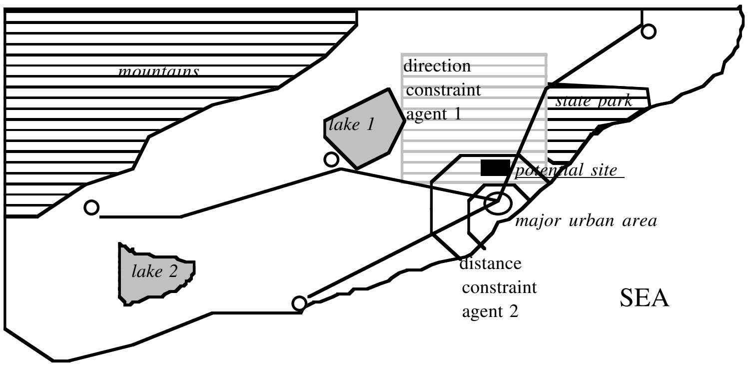

Figure 1: A qualitative spatial planning problem.

trated in Figure 1.

We use the above example only for demonstration purposes; real applications can certainly be much more complicated. A large number of explicit or implicit constraints involving spatial and non-spatial information may be involved (e.g., the total cost of the land should be less than a predefined threshold; land parcels occupied by industry are excluded, etc.). However, one point that we want to make here is that the agents, as often happens in real life with people who are responsible for decision making (politicians, managers), are not necessarily expert users of GISs. They probably use linguistic terms to express their constraints and they need intuitive, easy-to-use tools that facilitate the decision making process without having to interpret large amounts of spatial data themselves. Classical spatial planning systems have been developed for single-user applications and assume the existence of certain and complete information. A variety of impediments has to be surmounted in order to achieve the Qualitative Collaborative Spatial Decision Making.

This paper presents in detail the sub-goals involved in resolving the problem and suggests mechanisms for the solution of the computational issues that arise. Section 2 provides a description of Qualitative CSDM and outlines the involved subgoals. Section 3 discusses the lack of universally accepted semantics for spatial relations and the corresponding problem of translating constraints into a unified semantic model. Section 4 discusses spatial inference mechanisms that can detect inconsistencies due to the nature of spatial constraints. Section 5 presents the concept of a mediator mechanism, suitable for the regulation of a qualitative spatial planning debate. Section 6 describes the concept of distributed spatial access methods needed for the efficient retrieval of qualitative constraints. Section 7 concludes with comments and directions for future work.

2 Qualitative Collaborative Spatial Decision Making

The first sub-goal in a qualitative spatial problem is the interpretation of constraints provided by the agents. Figure 2a illustrates explicitly the direction constraints imposed by the first agent. In this figure we assume a *projection-based* notion of directions, that is, direction relations are defined using parallel lines to some predefined coordinate axes. However, if the agent assumes *cone-shaped* directions [Frank, 1992], the acceptance area imposed by the constraint is significantly different (Figure 2b). In general, there are no widely accepted definitions for spatial constraints and semantically different concepts are sometimes attached to the same linguistic terms. Thus the several constraints imposed by the agents have to be translated into a unified model capable of expressing the various perspectives in a coherent manner.

After the constraints are expressed in a unified model there is the second problem, namely to devise tractable algorithms that efficiently detect inconsistencies in the imposed constraints. Such inconsistencies depend only on the nature of the constraints; their early detection helps avoid unnecessary extensive access to stored data. If for example, an agent asserts that the airport should be *east* of the park and another *west* of the park, this leads to an inconsistency independently of the position of the park (assuming that the relations *east* and *west* are mutually exclusive). In real applications that involve a large number of constraints imposed by numerous agents the detection of inconsistencies requires sophisticated spatial inference mechanisms.

In case that some inconsistencies have been detected the agents will have to revise their constraints, which is in essence a negotiation process. The nature of this process is highlighted in [Golay, 1995] who proposes that *distributed cognition* can be achieved only if one considers the participants in a spatial planning debate as *conversation makers*, instead of classical decision makers. This shifted role of participants would be a remedy to the lack of social and behavioral attributes in most current information systems. In other words, flexibility for representing human desire and purposefulness cannot be achieved by following highly normative models, but only by emphasizing the role of the human conversation maker and letting computers do the processing tasks for which the human abilities are limited. CSDM problems require specialized Spatial Debate Mediators to provide information, assist and direct the negotiation process.

Assume now that the agents have reached an agreement and a satisfiable set of constraints has been found. Then the actual land parcels that will be used for the new airport site have to be retrieved. This information may be stored in distributed repositories, which include heterogeneous GISs, Databases, and Knowledge-Based Systems, designed in the past only for single-user workstations. Consequently, spatial data are mainly stored in proprietary formats and *interoperability* between systems should be achieved with appropriate converters. Specification of international standard formats plays a central role here. Having the above problems solved, efficient spatial access method have to efficiently access large amounts of geographic data in order to retrieve the potential land parcels. The solutions are then returned to the agents

Figure 2: Two different concepts of directions.

for evaluation and approval.

Figure 3 illustrates a Qualitative CSDM System that incorporates the previous phases. In the next sections we describe each phase in detail and we propose mechanisms for the solution of the computational problems that arise.

3 Translation into a unified semantic model

Frequently people communicate spatial information using *qualitative relations (constraints)* between distinct objects rather than absolute coordinates. The importance of qualitative spatial constraints has led to their application in several domains related to space. The relations that have been used in the GIS literature are *topological* (e.g., disjoint, overlap), *direction* (e.g., north, east) and *distance* constraints (e.g., near, far) [Papadias and Sellis, 1994]. Existing GISs however are based on conventional data models which are of general purpose and do not encapsulate spatial semantics effectively. In addition, different agents may assume different models of space. The lack of semantic spatial data models has a great impact on the geospatial data processing and computations, especially in the modelling of complex problems, leading to an inevitable loss of information.

As an example consider the direction constraints imposed by the first agent (Figure 2). Similar differences may occur in the case of topological relations. For instance, in Spatial Access Methods (e.g., *window queries*) the term *overlap* has been used to denote any configuration of objects that share some common point(s). This includes objects totally inside one another, objects that intersect at the boundaries but not at the interiors, and so on. On the other hand, the intersection models [Egenhofer, 1991], the main models used in the GIS literature, differentiate between these sub-cases resulting in a set of seven pairwise refinements of *overlap* as it is used in access methods terminology. One of these refinements is again called *overlap*. In [Grigni et al., 1995], several sets of topological relations are defined to reason in multiple levels of qualitative resolution. The same linguistic term has a different interpretation according to the resolution level; at the lowest level *overlap* has its access methods meaning, while at the highest resolution it assumes the 4-intersection semantics.

Obviously, in the case of distance relations there can also be significant differences in the notion that each agent has for *near*, *far*, etc. Such differences arise from subjective criteria, different metrics used (e.g., Euclidean vs. block worlds), and distortion caused by mental representations of space (for such examples see [Hirtle and Jonides, 1985]). Only recently, work has focused on the formalization of the spatial relations that people evoke in everyday reasoning [Mark and Egenhofer, 1994].

An important issue in a Qualitative CSDM is to identify the differences between the semantics of each agent. Since a universally accepted model for all spatial relations does not exist at this point, agents should be provided with guidelines about the use of spatial relations. This could be achieved by *Spatial Query Languages* that permit only certain kinds of spatial constraints with well-defined semantics [Egenhofer, 1994]; [Papadias and Sellis, 1995].

4 Spatial Inference Mechanisms

Spatial inference mechanisms should remove inconsistencies and provide a set of constraints that corresponds to actual configurations of objects in space. They may have multiple goals: find one solution that satisfies all constraints, find all possible solutions, a number of possible solutions etc. It is difficult to develop a model that can express all types of spatial constraints and is capable of performing efficient inconsistency detection. Work on spatial constraint satisfaction problems has concentrated mostly on *homogeneous constraints*, that is constraints of the same form (only topological or only direction relations). Even for such cases, a large class of problems is intractable (*NP-Complete*).

The first step of a topological inference mechanism is *path consistency checking* using some composition table. *Composition* infers the relation between two objects X and Y when their relation with a third object Z is known. If, for example, X is *inside* Z and Z is *north* of Y, we can conclude that X is *north* of Y. The path consistency algorithm explicates all constraints between every pair of objects and removes relations that yield inconsistencies. Work in the area of spatial reasoning has produced composition tables for homogeneous spatial constraints (e.g., topological in [Egenhofer, 1991], direction in [Papadias et al., 1994], distance in [Topaloglou, 1994]) and for combinations of several types of constraints (topological and direction in [Hernandez, 1994]; direction and distance in [Frank, 1992]).

Path consistency algorithms have a polynomial complexity [Smith and Park, 1992]; [Egenhofer and Sharma, 1993] but in general do not suffice for achieving satisfiability. They reveal only local inconsistencies and search-based constraint satisfaction algorithms (such as backtracking) have to be applied to locally consistent networks in order to ensure global satisfiability. Although such algorithms have a worst case complexity exponential in the number of objects, efficient heuristics provide good results in most cases [Dechter, 1990].

Different semantics used by different agents can further complicate the problem solving and render a problem from tractable to NP-Complete. As an example consider a set of agents reasoning about a set of contiguous regions without holes. Each agent imposes a simple topological constraint for some pairs of objects (e.g., one of the eight topological relations of the 4-intersection model). The unified model that expresses all agents' constraints contains for each pair of objects either:

- a null constraint (if no constraint for this pair has been imposed by any agent),

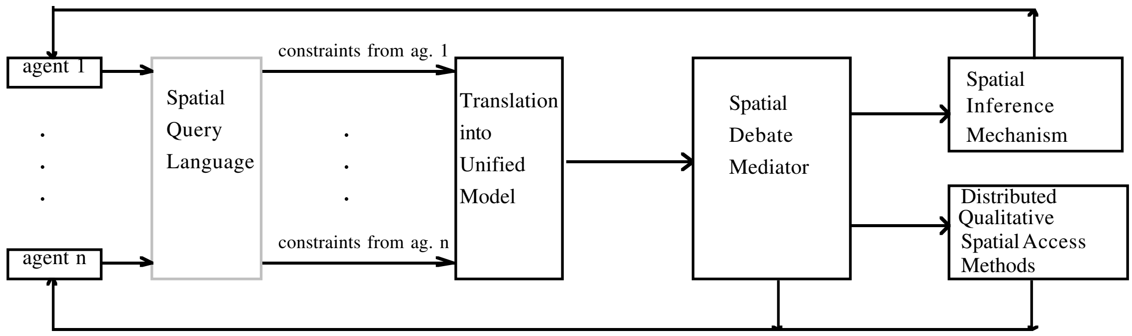

Figure 3: Translation into a unified semantic model.

- a single constraint (if only one agent has imposed a constraint for this pair, or if multiple agents have imposed the same constraint),
- a conjunction of different constraints imposed by different users (in which case we have an inconsistency).

[Grigni et al., 1995] have shown that the above problem is solvable by a polynomial path consistency algorithm. On the other hand, if some of the agents assume low resolution relations (e.g., they use the term *overlap* according to spatial access methods terminology) the same problem becomes exponential because the new meaning of *overlap* should be expressed by a disjunction of seven topological relations.

Satisfiability does not necessarily imply realizability in space. In graph theory it is known that we cannot draw a complete graph of more than four vertices in a way that no two edges intersect each other. This is called the *planarity problem* and a similar case exists for objects in 2D space; a set of spatial constraints may be satisfiable, but still there may not exist a planar model of arbitrary objects that satisfies the given constraints. Although checking planarity belongs to NP-Hard, [Grigni et al., 1995] suggest that heuristics have very good chances of recognizing planarity problems.

A Query Optimizer can be incorporated as the last part of the inference mechanism. After it has been ensured by the first steps that the imposed constraints can correspond to some configuration of arbitrary objects in two-dimensional space, the optimizer generates efficient query plans that search the database for the actual parcels. Figure 4 illustrates the parts of the spatial inference mechanism; the first three parts are in non-decreasing order of computational complexity.

Although a large part of the spatial inference mechanism involves problems that are in general exponential, the specific structure of the Spatial Domain facilitates the development of efficient heuristics, and leads to polynomial algorithms for certain sub-cases.

5 Spatial Debate Mediator

Collaborative spatial planning has to be performed through debates, negotiations, and arguments between various agents. Additionally, mechanisms to express beliefs and goals are needed. Predominant conditions in real world spatial planning paradigms, which usually have to be addressed, are:

- Planning agents assert arguments supporting or against alternative solutions. Conflicts are rather unavoidable and a rational planning framework is required.
- Reasoning is defeasible. Further information may cause another alternative to be more preferable than what seems optimal at the moment.
- Factual knowledge is not always sufficient. Usually, value judgments on aspects like weighing of attributes that alternative solutions pose, appear as the most critical issues. In addition, these judgments should be derivable and subject to debate.
- Agents have to manage the existence of both not enough and too much information, as well as of limited resources for finding a solution.

The ubiquitous task in all planning problems is the identification and selection among alternative courses of action. CSDM systems require adequate tools for supporting and reasoning about rational and effective decision-making, when information is limited and uncertain, and conflicts of interest and opinion are common. Computational models of norms of rational discourse are needed, taking these considerations into account [Gordon, 1994]; [Karacapilidis and Gordon, 1995]. Such a *mediating system* should support argumentation, negotiation and decision making in groups, where the above conditions prevail. The system should (1) manage the dependencies between arguments, claims, positions and issues, (2) inform agents about their rights and obligations in a planning problem, and (3) provide access to procedures for negotiation and conflict resolution. The system should act as an assistant and advisor, by recommending solutions and leaving the final enforcement of decisions and actions to the agents.

We propose a Spatial Debate Mediator which is based on the mediating system first described in [Karacapilidis and Gordon, 1995]. The system consists of various layers:

- a *Logic Layer*, that defines the notions of *necessary consequence* and *contradiction*;
- an *Argumentation Framework Layer*, that specifies the concepts of *proposition*, *supporting argument*, *counter-argument*, and *issue* as well as linguistic constructs for arguing about priority relationships among competing arguments;
- a *Speech Act Layer*, that defines the possible kinds of actions agents may follow during the planning procedure;

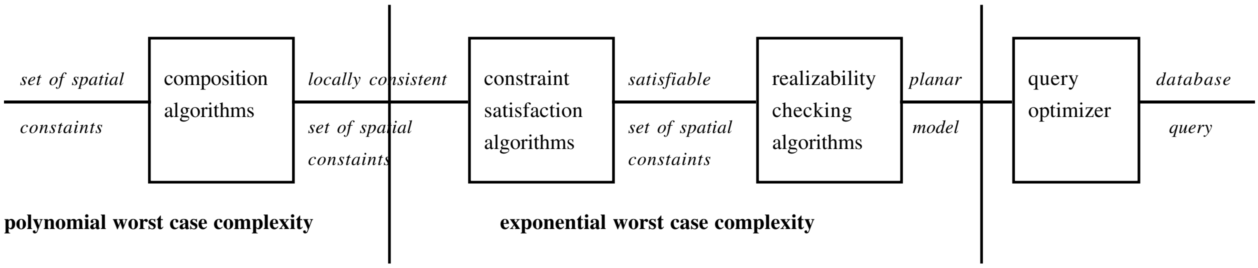

Figure 4: Spatial inference mechanism.

- a *Protocol Layer* that specifies norms about the duties and the rights of the agents to perform actions defined at the previous layers, depending on their roles, and/or the evolution of the planning procedure so far.

A proposition is considered to be the lowest level of granularity in a spatial debate framework. Any kind of data an agent wants to assert during the planning procedure can be used in order to represent a proposition. An issue consists of the set of alternative propositions and the set of the related constraints. In other words, an issue refers to which alternative to prefer, if any. Arguments are assertions about the positions regarding their properties or attributes, which speak for or against them. By means of the mediator framework suggested here, decisions will automatically being reviewed at each stage of a plan's evolution to enable an agent to check that a decision taken towards a certain direction is still recommended.

Furthermore the spatial debate mediator should have the ability to handle additional preference constraints. Assume, for instance, that three potential sites that satisfy the constraints imposed by the agents have been found. The goal at this stage of the planning debate is to select the best site according to some additional (possibly non-spatial) attributes and constraints on them. We consider that the important attributes here concern the ownership of the site (*state land parcel* or *private land parcel*), environmental issues (*environmentally sensitive zone* or *not*), and safety issues (*residential area* or *not*). Let the following asserted attributes for each of the alternative sites:

potential site 1: *state land parcel*
does not include residential areas
environmentally sensitive zone
potential site 2: *state land parcel*
not environmentally sensitive zone
potential site 3: *private land parcel*
includes residential areas
not environmentally sensitive zone

Notice that in this example, there is not complete linking between each alternative of an issue and every asserted attribute; there is no argumentation about the residential areas in the second site. This is common in such problems. Uncertainty is inherent in various instances of geospatial problems and the Spatial Debate Mediator should provide facilities for dealing with incomplete knowledge about the world. Furthermore it should supply mechanisms for indicating preferences among attributes.

Our model for expressing preferences is based on a qualitative approach for similar AI planning problems under uncertainty [Brewka and Gordon, 1994]; [Karacapilidis, 1995]. According to this approach, *constraints* provide a qualitative way to argue about preferences and value judgments by ranking the quality of alternative choices. An axiomatization of the integer operations $+$ and $-$, and the predicates $=$, $>$ and $<$ has been assumed, in order to express preferences between arguments. In our spatial planning example preferences could be expressed by the following constraints:

environmentally sensitive zone $=$ *includes residential areas*, and
environmentally sensitive zone $+$ *includes residential areas* $>$ *state land parcel*

The first constraint states that a site's attribute of being an environmentally sensitive zone is considered as equally important with that of including residential areas. The second constraint states that a site's attribute of being an environmentally sensitive zone and a residential area is considered as more important of being a state land parcel. Such constraints provide only a partial ordering among the asserted attributes. The above constraints are classified in supporting arguments (e.g. *state land parcel*) and counterarguments (e.g. *environmentally sensitive zone*). Supporting arguments can be defeated by stronger counterarguments, or combinations of counterarguments (e.g. *environmentally sensitive zone* $+$ *includes residential areas* $>$ *state land parcel*).

Constraints comprise attributes of alternative solutions that are linked to the corresponding alternative via supporting and counterarguments. A solution to the above problem can be achieved by a method suggested in [Karacapilidis, 1995], which exploits abilities of constraint satisfaction programming languages on revealing constraint optimization problems (experiments with ECLiPSe constraint programming language are reported). Jointly regarding the constraints holding and the arguments asserted for alternative choices, four inference relations pertaining to the *optimistic*, *pessimistic*, *credulous*, and *skeptical* conclusion of an issue have been defined.

6 Distributed Data Retrieval and Conversion

As suggested above, the mediator mechanism should assist agents by indicating missing information, which possibly includes land parcels occupied by local industry, rivers and in general all areas inappropriate for airport sites. Spatial access method have to efficiently access large amounts of geographic data in order to retrieve additional information. For example, a search in some distributed GISs and/or Databases of the form "find all land parcels east of the lake, and north of the major urban area owned by the state" is needed. However, most of the work on Spatial

Access Methods has focused on window queries and does not support efficient processing of qualitative constraints. Only recently it has been shown how alternative indexing techniques can be used to efficiently retrieve objects that satisfy certain topological, direction and distance relations (for samples of this work see [Papadias et al., 1995]; [Papadias and Theodoridis, to appear]). This work provides the basis for database search in qualitative CSDM.

In our example, the result of the database search is a set of potential candidates out of which only a subset could be used for the new site. This is due to some unforeseen constraints that were not predicted by the agents because of *data quality* issues. For instance, the agents may have not been given all the information about the area, or they did not take it into account during the decision making process. One may argue that the decision making process could go directly to the database search phase without passing through spatial inference mechanisms. However, this is inappropriate because Spatial Databases and GISs usually contain large volumes of data and search is very expensive due to large I/O load. The satisfiability mechanisms basically provides a fast filter step that recognizes inconsistencies in the constraints themselves, without access to the database. In the extreme case where the database search fails and no potential sites are found, a new set of constraints should be used for the search. This is achieved by the Spatial Debate Mediator either by relaxing some (unimportant) constraints or by entering a new phase of negotiations.

Most existing GISs are vector-based with limited capabilities of dealing with satellite images. With the availability of more frequent multispectral satellite images which have higher resolutions, provide world-wide coverage, and are inexpensive, more complex problems in areas such as environmental, transportation, natural resource, and emergency management can be solved. GISs must allow the incorporation of satellite images, and they should be able to use them in conjunction with other geospatial, vector, and ancillary data. In addition, they must allow the use of high-resolution images and temporal data.

Furthermore, since existing GISs are based on proprietary data formats and/or structures, the conversion of data can be a very cumbersome procedure. To solve this problem, interfaces between existing GISs and international geospatial data standards, such as the *NSDI Q* geospatial metadata and spatial data transfer standards, need to be built [Karimi, 1995]. Heading to "open" GIS, interoperability issues between different platforms have also to be solved. The enormous growth of the GIS community advocates for establishing "open" approaches on issues such as spatial data models, system architectures, transmission protocols, user interfaces and interoperability.

The exploitation of Internet and World Wide Web (WWW) currently attracts the interest of many researchers (see for example [Gordon, 1995]; [Wang, 1995]), and can solve problems of platform independence (interoperability) and data transmission. WWW browsers (e.g., Mosaic, Netscape, etc.) are now among the most widespread communication tools all over the world. The document format used at present (HTTP-HyperText Transmission Protocol) allows only for standard GIF and JPEG graphic formats (bit maps). Vector and raster graphics should be supported in advanced spatial systems. The establishment of Open GIS will facilitate the decision making process by providing easy access to distributed data.

7 Conclusions

We believe that CSDM with qualitative constraints is significant for a number of reasons, the most important of which is the fact that people usually communicate spatial knowledge by use of qualitative relations rather than absolute coordinates. In this paper we propose a computational model which can deal with such constraints efficiently and can automate a large part of the decision making process. The paper describes several concepts that need to be dealt with, before qualitative CSDM becomes a reality:

- the lack of universally accepted semantics for spatial relations
- the development of spatial inference mechanisms that can detect inconsistencies due to the nature of spatial constraints
- the design of spatial debate mediators suitable for qualitative spatial planning
- the need for distributed spatial access methods used for the efficient retrieval of qualitative constraints from heterogeneous databases.

In an ideal environment all the processes will be automated and the only human intervention will be the positions and constraints imposed (and revised) by the agents. In the initial phases of such a project though, human intervention is necessary to organize the process and communicate potential inconsistencies to the individual agents. [Lemberg, 1995] suggests three different roles for the users in the negotiation process: the *Operator* (which acts as a technician operating the system, with no authority for participation in the discussion), the *Facilitator* (which acts as a consultant for the group), and the *Mediator* (who has a "leadership" role, generating alternatives and leading the discussion; he/she can question, moderate, or reject proposals or information that are inaccurate, distorted or biased).

Future work should focus on the availability of intelligent tools for spatial planning problems through a public-domain assistant application. In this way, any WWW browser would be sufficient for an agent to take part in spatial planning discussions. Advanced CSDM systems should also be supported with innovative representation mechanisms based on interactive maps and Computer Supported Co-operative Work technology. Such tools should provide a means for the elicitation of the spatial decision making process, enabling the agents to be aware of the evolution of the planning debate. It is widely argued that high visualization is the key for the efficient user participation in the determination of important spatial dependence parameters, especially in models of uncertainty (see for example [Goodchild et al., 1992]).

Qualitative CSDM is an interesting topic that involves several domains such as Query Languages, Constraint Satisfaction, Heterogeneous Databases, Spatial Access Methods, and Group Decision Making. Although solutions to all the problems at this time are not straightforward, we believe that its significance and the range of potential applications poses a challenge for future work on this topic.

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