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Postdoctoral project proposal

High order hybridized DG method for the 3d time-harmonic Maxwell equations with application to radar cross section evaluation

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Nachos project-team
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Electromagnetic devices are ubiquitous in present day technology. Indeed, electromagnetism has found and continues to find applications in a wide array of areas, encompassing both industrial and societal purposes. Applications of current interest include (among others) those related to communications (e.g transmission through optical fiber lines), to biomedical devices and health (e.g tomography, power-line safety, etc.), to circuit or magnetic storage design (electromagnetic compatibility, hard disc operation), to geophysical prospecting, and to non-destructive evaluation (e.g crack detection), to name but just a few. Equally notable and motivating are applications in defense which include the design of military hardware with decreased signatures, automatic target recognition (e.g bunkers, mines and buried ordnance, etc.) propagation effects on communication and radar systems, etc. Although the principles of electromagnetics are well understood, their application to practical configurations of current interest, such as those that arise in connection with the examples above, is significantly complicated and far beyond manual calculation in all but the simplest cases. These complications typically arise from the geometrical characteristics of the propagation medium (irregular shapes, geometrical singularities), the physical characteristics of the propagation medium (heterogeneity, physical dispersion and dissipation) and the characteristics of the sources (wires, etc.). The significant advances in computer performances that have taken place over the last two decades have been such that nowadays the design of electromagnetic devices heavily relies on computer simulation. The present project is concerned with the numerical modeling of electromagnetic wave interaction with irregularly shaped structures and complex materials, with application to radar cross section evaluation. In practice, one has to solve the system of 3d time-harmonic Maxwell equations coupled to appropriate material models, in exterior (theoretically unbounded) domains. For that purpose, the underlying numerical methodology should ideally match several features:

- high order accuracy, in particular in view of dealing with high frequency incident fields,
- flexibility with regards to the spatial discretization (meshing) of the scattering structures,
- accurate and efficient treatment of the artificial truncation of the propagation domain,
- algorithmic adaptation to hardware characteristics of modern massively parallel computers in view of simulating large-scale problems.

During the last 10 years, discontinuous Galerkin (DG) methods have been extensively considered for obtaining approximate solution of Maxwell's equations modeling electromagnetic wave propagation. Thanks to the discontinuity of the approximation, this kind of methods has many advantages, such as adaptivity to complex geometries through the use of unstructured possibly non-conforming meshes, easily obtained high order accuracy, *hp*-adaptivity and natural parallelism. However, despite these advantages, DG methods have one main drawback particularly sensitive for stationary problems: the number of globally coupled degrees of freedom (DOFs) is much greater than the number of DOFs required by conforming finite element methods for the same accuracy. Consequently, DG methods are expensive in terms of both CPU time and memory consumption, especially for time-harmonic problems. Hybridization of DG methods [CGL09] is devoted to address this issue while keeping all the advantages of DG methods. Such a hybridizable discontinuous Galerkin (HDG) method for the discretization of the system of 3d time-harmonic Maxwell's equations will be at the heart of the present project.

HDG methods introduce an additional *hybrid* variable on the faces of the elements, on which the definition of the local (element-wise) solutions is based. A so-called *conservativity condition* is imposed on the numerical trace, whose definition involved the hybrid variable, at the interface between neighboring elements. As a result, HDG methods produce a linear system in terms of the DOFs of the additional hybrid variable only. In this way, the number of globally coupled DOFs is reduced. The local values of the electromagnetic fields can be obtained by solving local problems element-by-element. Recently, we have studied such HDG formulations for the systems of 2d and 3d time-harmonic Maxwell equations [LLP13]-[LLP14]. In the 3d case, we have proposed a HDG formulation taking the tangential component of the magnetic field as the hybrid variable, and we have shown that the reduced system of the hybrid variable has a wave-equation-like characterization, and the tangential components of the numerical traces for both electric and magnetic fields are single-valued. Very promising results have been obtained, however, the development of the method presented in [LLP14] has been limited so far to a piecewise linear interpolation of the electromagnetic field components and the use of conforming tetrahedral meshes. Extension to higher order accuracy combined to a local (i.e. element-wise) definition of the interpolation order, and a local adaptation (i.e. refinement) of the mesh in a non-conforming way (i.e. with hanging nodes) will be a first objective of the study proposed here.

Though the HDG method results in a smaller linear system than the one associated to a classical upwind flux-based DG method, the size of this system is often too large to be solved by a direct solver as soon as one considers realistic 3d problems. In addition, for very large-scale propagation problems, exploiting a multi-processor system is a mandatory path to reduce the solution time and have access to the required memory capacity. Therefore, as a second objective of the present work, we propose here to design a hybrid iterative-direct solution strategy for the HDG system, based on domain decomposition principles [SBG96]. Our previous works on domain decomposition methods for the solution of the time-harmonic Maxwell equations show promising results of Schwarz-type algorithms through many test problems in 2d and 3d [DLP08a, DLP08b]. A classical Schwarz algorithm which exchanges impedance data between subdomains is studied [DLP08a]. A similar algorithm has been adopted in [LLP14] allowing to exploit moderately parallel systems (a few hundreds of computing units) and further developments are necessary (both at the numerical and algorithmic levels) in order to scale to thousands of cores.

Finally, in order to deal with unbounded propagation domain, an appropriate strategy will be designed for coupling the proposed HDG method with a boundary element (BE) method for modeling exactly the propagation in the far field. This strategy will again exploit a domain decomposition principle, similarly to what is presented in [Stu01]. An existing BE solver for time-harmonic electromagnetic wave propagation will be used in a minimally invasive way. For the solution of the sparse linear systems of equations resulting from the discretization of the subdomain problems by the HDG method, state of the art direct and hybrid iterative-direct solvers will be used.

For this postdoctoral project, applications are invited from recently graduated Ph.D students in applied mathematics or scientific computing with emphasis on the numerical treatment of PDE systems. A first research experience in computational electromagnetics will be an asset. A good knowledge of high performance computing principles and a first experience in parallel programming is required, as is the ability to manipulate complex computer codes in a Linux/Unix environment. This project will be conducted in the Nachos project-team at Inria Sophia Antipolis - Méditerranée (Stéphane Lanteri) in collaboration with the Laplace CNRS laboratory (Ronan Perrussel), and with researchers from the Hipeacs project-team at Inria Bordeaux - Sud-Ouest for what concern the use of algebraic sparse linear solvers.

References

[CGL09] B. Cockburn, J. Gopalakrishnan, and R. Lazarov. Unified hybridization of discontinuous Galerkin, mixed, and continuous Galerkin methods for second order elliptic problems. *SIAM J. Numer. Anal.*, 47(2):1319–1365, 2009.

[DLP08a] V. Dolea, S. Lanteri, and R. Perrussel. A domain decomposition method for solving the three-dimensional time-harmonic Maxwell equations discretized by discontinuous Galerkin methods. *J. Comput. Phys.*, 227(3):2044–2072, 2008.

[DLP08b] V. Dolean, S. Lanteri, and R. Perrussel. Optimized schwarz algorithms for solving time-harmonic Maxwell's equations discretized by a discontinuous galerkin method. *IEEE Trans. Magn.*, 44(6):954–957, 2008.

[LLP13] L. Li, S. Lanteri, and R. Perrussel. Numerical investigation of a high order hybridizable discontinuous Galerkin method for 2d time-harmonic Maxwell's equations. *COMPEL*, 2(3):1112–1138, 2013.

[LLP14] L. Li, S. Lanteri, and R. Perrussel. A hybridizable discontinuous Galerkin method combined to a Schwarz algorithm for the solution of 3d time-harmonic Maxwell's equations. *J. Comput. Phys.*, 256:563–581, 2014.

[SBG96] B. Smith, P. Bjorstad, and W. Gropp. *Domain decomposition and parallel multilevel methods for elliptic partial differential equations.* Cambridge University Press, 1996.

[Stu01] B. Stupfel. A hybrid finite element and integral equation domain decomposition method for the solution of the 3-D scattering problem. *J. Comput. Phys.*, 172(2):451–471, 2001.