Space-Time Domain Decomposition Methods for Transport Problems in Porous Media

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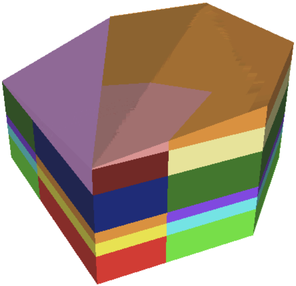
informatics mathematics

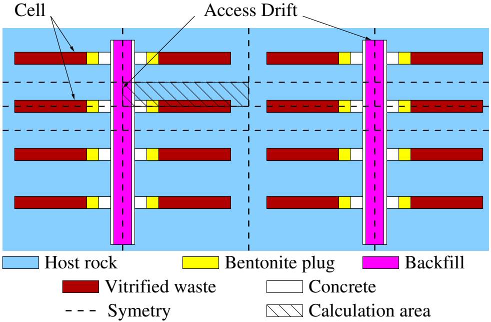
ANDRA La maîtrise des déchets radioactifs

MAISON DE LA SIMULATION

Simulation of the transport of radionuclides around a repository



Far-field simulation



Near-field simulation

Challenges

- Different materials $\rightarrow$ strong heterogeneity, different time scales.
- Large differences in spatial scales.
- Long-term computations.

$\rightarrow$ How to simulate efficiently & accurately?

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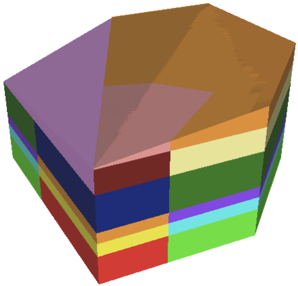

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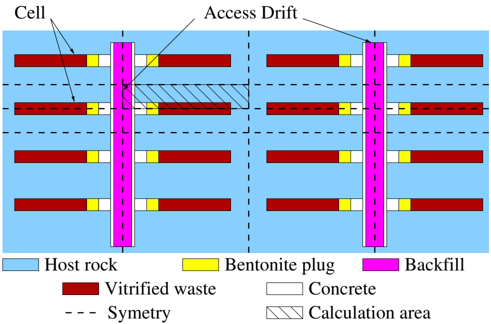

Near-field simulation

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- Long-term computations.

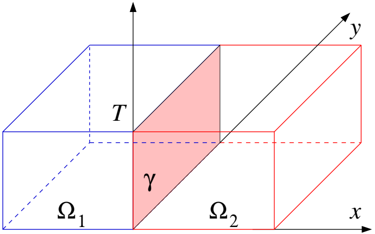
→ How to simulate efficiently & accurately?

⇒ Domain Decomposition methods
Global in Time

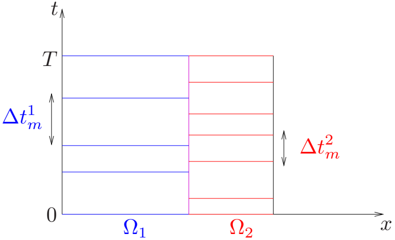
Multi-domain mixed formulation for transport equation

$$\begin{aligned} \omega \partial_t c + \operatorname{div}(\mathbf{u}c - \mathbf{r}) &= f, \quad \text{in } \Omega \times (0,T), \\ \mathbf{D}^{-1}\mathbf{r} + \nabla c &= 0, \quad \text{in } \Omega \times (0,T) + \text{ BCs, IC,} \end{aligned}$$

Decomposition into non-overlapping subdomains.



Different time steps for different subdomains



L^2 projections from piecewise constant functions on $\mathcal{T}_i$ onto piecewise constant functions on $\mathcal{T}_j$

The Steklov-Poincaré interface equation

Natrual transmission conditions

$$\begin{aligned} c_1 &= c_2 \\ (\mathbf{u}_1 \cdot \mathbf{n}_1 c_1 + \mathbf{r}_1 \cdot \mathbf{n}_1) + (\mathbf{u}_2 \cdot \mathbf{n}_2 c_2 + \mathbf{r}_2 \cdot \mathbf{n}_2) &= 0 \end{aligned} \qquad \text{on } \gamma \times (0, T).$$

- Use Dirichlet data on the interface

  $$c_i = \lambda \qquad \text{on } \gamma \times (0, T), \text{ for } i = 1, 2,$$

  and solve subdomain problem:

  $$(\lambda, f, c_0) \mapsto (c_i(\lambda, f, c_0), \mathbf{r}_i(\lambda, f, c_0)).$$

- Transmission conditions reduced to flux equality:

  $$\mathcal{S}\lambda \equiv \mathbf{r}_1(\lambda, f, c_0) \cdot \mathbf{n}_1 + \mathbf{r}_2(\lambda, f, c_0) \cdot \mathbf{n}_2 = 0, \qquad \text{on } \gamma \times (0, T),$$

- Interface problem $\mathcal{S}\lambda = \chi \qquad \text{on } \gamma \times (0, T)$.
- Solved iteratively (e.g., with GMRES).
- Apply Neumann-Neumann preconditioner with weights

Optimized Schwarz Waveform Relaxation (OSWR) method

- Equivalent Robin transmission conditions on $\gamma \times (0, T)$:

$$-(\mathbf{u}_1 \cdot \mathbf{n}_1 c_1 + \mathbf{r}_1 \cdot \mathbf{n}_1) + \alpha_{1,2} c_1 = -(\mathbf{u}_2 \cdot \mathbf{n}_1 c_2 + \mathbf{r}_2 \cdot \mathbf{n}_1) + \alpha_{1,2} c_2$$
$$-(\mathbf{u}_2 \cdot \mathbf{n}_2 c_2 + \mathbf{r}_2 \cdot \mathbf{n}_2) + \alpha_{2,1} c_2 = -(\mathbf{u}_1 \cdot \mathbf{n}_2 c_1 + \mathbf{r}_1 \cdot \mathbf{n}_2) + \alpha_{2,1} c_1$$

- UseRobin data on the interface

$$-(\mathbf{u}_i \cdot \mathbf{n}_i c_i + \mathbf{r}_i \cdot \mathbf{n}_i) + \alpha_{i,j} c_i = \xi_i \text{ on } \gamma \times (0, T)\,,$$

  and solve subdomain problem with

$$\mathcal{R}_i : (\xi_i, f, c_0) \mapsto (c_i(\xi_i, f, c_0)\,, \mathbf{r}_i(\xi_i, f, c_0)).$$

- Interface problem: global in space and time with 2 interface concentrations

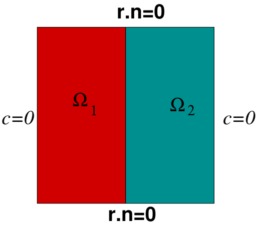
$$\mathcal{S}_R \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \chi_R \quad \text{on } \gamma \times (0, T)\,.$$

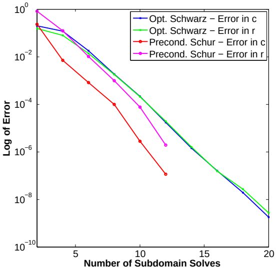
- Solve iteratively (Jacobi = OSWR, GMRES, etc.).

Test case 1: diffusion, 2 subdomains, varying contrast

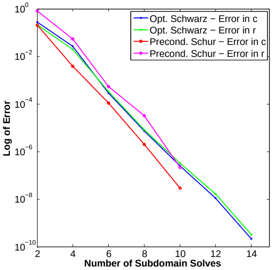
- Matching meshes in space $\Delta x = 1/200$.
- **Non-conforming time grids**

Contrast	d_1	$1/\Delta t_1$	$1/\Delta t_2$
10	0.02	150	200
100	0.002	50	200
1000	0.0002	20	200

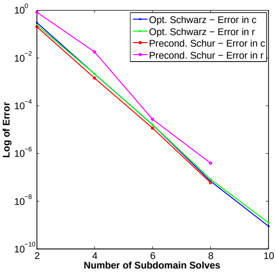




Contrast = 10

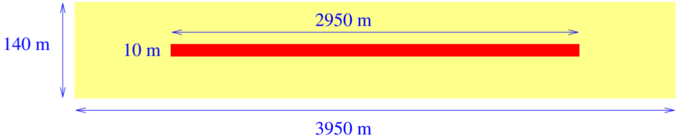


Contrast = 100

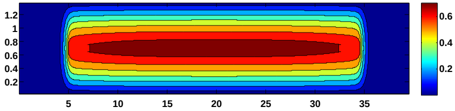


Contrast = 1000

Test case 2 (ANDRA) - Data



- Porosity $\omega = 0.05$ in clay layer and $\omega = 0.2$ in repository.
- Diffusion $d = 5\,10^{-12}$ m^2/s in clay, $d = 2\,10^{-9}$ m^2/s in repository.
- Source term $f = 0$ in clay, and $f = \begin{cases} 10^{-5} & \text{if } t \leq 10^5 \\ 0 & \text{if } t > 10^5 \end{cases}$ in repository.
- 9 rectangular subdomains. Non-uniform spatial mesh $\Delta x = 1/300$.
- Non-conforming time grids: $\Delta t = 2000$ (years) in the repository and $\Delta t = 10000$ (years) in the clay layer.

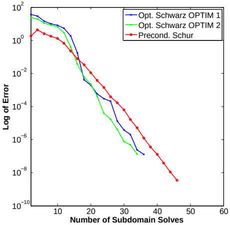


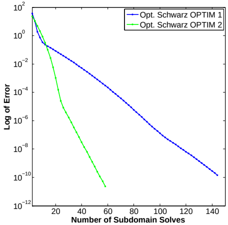
Snapshots of multi-domain solution at 1 million years

2 optimization techniques for computing parameters $\alpha_{i,j}$:

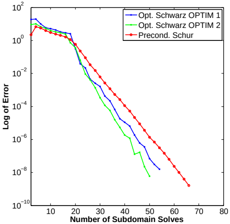
- Opt. 1: 2 half-space Fourier analysis.
- Opt. 2: taking into account the length of the domains

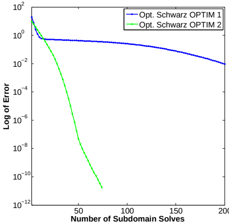
$T = 2\,10^5$ years





$T = 10^6$ years

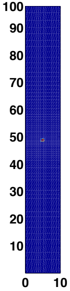


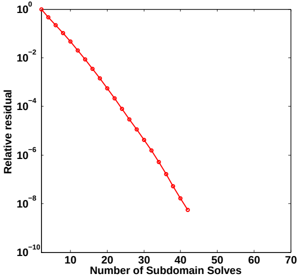


GMRES　　　　Jacobi

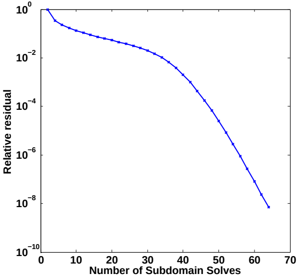
Near-field simulation with advection (CEA)

Material	Dims. (m)	Perm. ($m.s^{-1}$)	Porosity	Diff. ($m^2.\ s^{-1}$)
Host rock	10×100	10^{-13}	0.06	$6\,10^{-13}$
Repository	1×1	10^{-8}	0.1	10^{-11}





Precond. Schur



OSWR

Conclusion

Summary

- Diffusion problems
  - Two global-in-time methods: time-dependent Schur and OSWR.
  - Well-posedness of local problems in mixed form.
  - Performance of two methods for non-conforming time grids.
- Advection-Diffusion problems:
  - Formulation of interface problems with operator splitting.
  - Preliminary numerical results

Work in progress

- Fractures.
- Order 2 (Ventcel) transmission conditions.
- Non-matching grids in space.

T.T.P. Hoang, J. Jaffré, C. Japhet, MK and J.E. Roberts, **Space-Time Domain Decomposition Methods for Diffusion Problems in Mixed Formulations**, submitted to SINUM, hal-00803796