INRIA Project: Kinetic models AppLIed for Future of Fusion Energy
KALIFFE

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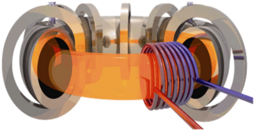
Aim and Contents of the Presentation

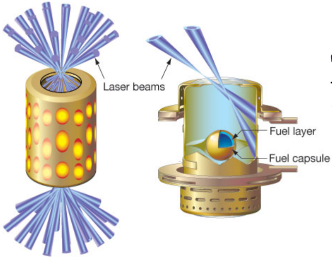
1. Plasma Physics: Controled Fusion Energy
2. Which models for Plasmas?
3. Which numerical methods for such models?
4. Two main objectives for KALIFFE
   - Part I : Approximation of kinetic models
   - Part II: Applications (collisional & multiscale problems)
   - Part II : Applications to plasma physics (Hybrid methods)

Plasma Physics: Controled Fusion Energy

Controled fusion energy is one of the major prospects for
a long term source of (clean) energy and two main research directions are studied

✔ Magnetic fusion: the plasma is confined in tokamaks using a large external magnetic field. The international project ITER is based on this idea and aims to build a new tokamak which could demonstrate the feasibility of the concept.





✔ Inertial fusion: the target containing the Deuterium and Tritium atoms is confined thanks to intense laser or particle beams.
For instance, the Laser Megajoule which is being built at CEA in Bordeaux will be used for experiments using this approach.

➪ **Two issues:** *Material/fluid (plasma) interactions* & *Time confinement*.

Which models for Plasmas?

Microscopic description : particle interactions

$$\begin{cases} \dfrac{dx_i}{dt} = v_i, \quad 1 \le i \le N \\ m\,\dfrac{dv_i}{dt} = F_i(t, X, V) \end{cases} \quad \rightarrow$$

Macroscopic description : Euler, Navier-Stokes equations

$$\begin{cases} \rho(t,x) = \text{density} \\ U(t,x) = \text{velocity} \\ T(t,x) = \text{temperature} \\ P(t,x) = \text{pressure} \end{cases}$$

$\searrow$ **Mesoscopic description**: Kinetic Vlasov-Boltzmann equation for gas & plasmas, $f(t,x,v) \ge 0$ $\nearrow$

$$\frac{\partial f}{\partial t} + v \cdot \nabla_x f + F(t,x,v) \cdot \nabla_v f = \frac{1}{\varepsilon}\, \mathcal{Q}(f,f).$$

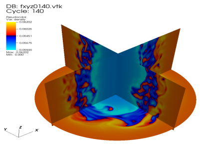
- Theoretical works: C. Cercignani, C. Bardos, R. DiPerna & P.-L. Lions, D. Levermore, C. Villani, F. Golse & L. Saint-Raymond.
- Numerical simulations: P. Degond, B. Perthame, E. Tadmor, S. Jin.

Which numerical methods for such models?

Two different approaches

- Particle methods : the gas is approximated by "particles", Mol. Dyn. Simulations
- Deterministic methods : resolution in phase space $(x, v) \in \mathbb{R}^6$.

➪ I have developed[1] and analyzed numerical methods (PFC, Fast spectral algorithms) and softwares (VADOR).



✔ Fast algorithms (FFT, multigrid method).

✔ Remove small physical scales of $f(t, x, v)$.

✔ High order reconstruction and damping of spurious numerical oscillations (PFC,WENO).

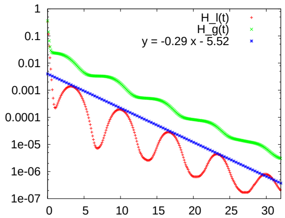
✔ Rigorous stability mathematical analysis.

➪ Applications:

✔ transport problems in plasma physics,

✔ relative entropy oscillations (L. Desvillettes & C. Villani, **Inventiones Mathematicae** (2006), C. Villani, **ICM'06**).

✔ in the framework of the **ERC-grant NUSIKIMO**: MEMS, charged particle beams.



[1] **JCP (2001), SIAM JSC (2006), SIAM JNA (2005), AMS Math. Comp.(2008)**

Two main objectives for KALIFFE

➪ Approximation of kinetic models:

✔ development and analysis of new finite volume schemes for kinetic equations (extensive development in CFD),

✔ design and analysis of asymptotic preserving schemes for an accurate treatment of the different scales inherent to kinetic equations.

➪ Application to Plasma Physics. Two projects in Physics: the Mega-Joule laser in Bordeaux (laser-plasma interaction), the ITER project (tokamak in Cadarache)

✔ development of numerical methods for collision operators, which are robust for different scales,

✔ numerical and mathematical analysis of multiscale problems: collisional problems, transport equations, effects of magnetic fields

✔ derivation of hybrid methods (N. Hadjiconstantinou, MIT).

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Part I: Approximation of kinetic models

✔ G.Q. Chen, T.P. Liu et C.D. Levermore, Hyperbolic conservation laws with stiff relaxation terms and entropy (1994)

✔ Shi Jin, Efficient asymptotic-preserving (AP) schemes for some multiscale kinetic equations (1999).

$$
\begin{array}{ccc}
\mathcal{P}_h^{\varepsilon} & \xrightarrow{\varepsilon \to 0} & \mathcal{P}_h^0 \\
h \downarrow 0 \;\Big\downarrow & & \Big\downarrow\; h \downarrow 0 \\
\mathcal{P}^{\varepsilon} & \xrightarrow[\varepsilon \to 0]{} & \mathcal{P}^0
\end{array}
$$

Recently together with S. Jin[2], we proposed a very promising Asymptotic Preserving method for kinetic equations: it is based on a splitting operator technique

$$
\frac{\partial f}{\partial t} + v \cdot \nabla_x f \; + F(t,x,v) \cdot \nabla_v f = \underbrace{\frac{\mathcal{Q}(f) - P(f)}{\varepsilon}}_{non\ stiff\ part} + \underbrace{\frac{P(f)}{\varepsilon}}_{stiff\ part},
$$

Expected results:

- reduce the computational cost and improve efficiency.

[2] JCP (2010)

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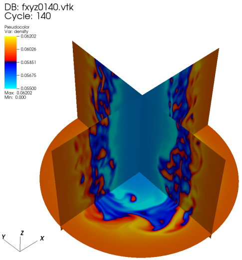
Part II: Applications (collisional effects & multiscale problems)

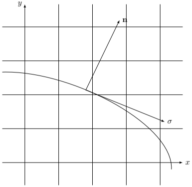
We want to solve numerically kinetic type equation (transport and collisions) on complex geometry.

- solve problems with solid boundary (walls)
- use an artificial boundary to connect different models (micro/meso)



Possible numerical methods :

- numerical algorithms based on unstructured meshes
  - ☹ very costly, not easy to parallelize
- numerical algorithms based on Cartesian meshes



☺ more efficient for computational cost

☺ easier for parallelization

- **how to capture B.C. ?**

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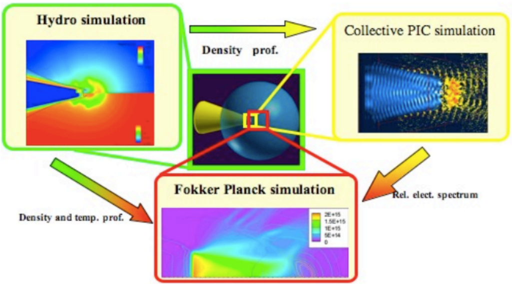
Part II: Applications to plasma physics (hybrid methods)

Example: Inertial Confinement Fusion (ICF).

✔Identify different scales
✔Take into account both transport and collisions
➪ Innovation: derivation of multiscale methods for hydrodynamic, collisional and transport problems.



Expected results:

- derivation of relevant macroscopic models, lower computational cost than kinetic ones (task 2),
- validation of this approach from kinetic solvers for different regimes: accuracy and physical relevance are the main criteria (task 1),
- validation of numerical results in collaboration with physicists, development of software, HPC issues (task 3).

Collaborations with other INRIA Projects and CEA

Collaboration with CEA.

- We already have some collaborations with R. Duclous (CEA Bruyères-le-Châtel) and the laboratory CELIA (V. Tikhonchuk, B. Dubroca) in Bordeaux (joint laboratory CEA-CNRS-University) on numerical simulations of multi-species charged particle beams.

Collaborations with INRIA.

- The CALVI Project in Nancy/Strasbourg led by E. Sonnendrucker.
- The IPSO Project at INRIA Rennes (P. Chartier) This project is interested in the design and the analysis of structure-preserving schemes for ODEs.
- The future CASTOR Project at INRIA Sophia (J. Blum, Université de Nice). This project will be devoted to Magneto-Hydro-Dynamic models for plasma physics.
- The future COFFEE Project at INRIA Sophia (Th. Goudon), is devoted to nonlinear kinetic models in various applications.