HPC & Castor Team

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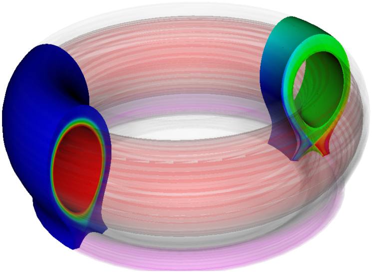
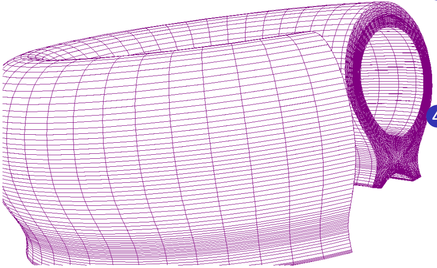
.

Numerical Strategies for MHD and Reduced MHD : Toroidal geometries (Tokamaks).





1. Spectral Finite Element/Spectral method for the two fluid Braginskii model (S. Minjeaud , R. Pasquetti)
2. C1-Finite Element/Spectral method. (J. Costa, G. Huijsmans, B. Nkonga)
3. Finite Volume/Finite Volume (H. Guillard, B. Nkonga, A. Sangam, G. Vides, E. Audit)
4. C1-Finite Element/C1-Finite Element. (M. Bilanceri, J. Costa, H. Guillard, B. Nkonga, M. Martin)

Goals.

1. Developed and improved numerical tools (JorekX/PlaTo $-- >$ Anemos?)
2. in order to simulate physical mechanisms of Elm's
3. and qualified some strategies for their control.
4. Design numerical Strategies that are efficient on the most advanced computers available
5. such as to contribute to the science base underlying of proposed burning plasma
6. tokamak experiments such as ITER

ANEMOS 2011-2015 : Toward Elm's control.

Main tasks.

1. Elm's control by Resonant Magnetic Perturbations.
   (IRFM)
2. Modeling of pellets Elm's pacing.
   (MDS & INRIA)
3. Numerical algorithm optimizations and parallel scaling.
   (BACCHUS & IRFM & JAD)
4. Higher-order-of-continuity advanced meshing.
   (JAD & INRIA)
5. Variational multiscale stabilization for compressible MHD.
   (JAD & ITER)
6. Visualization, data management and software integration.
   (MDS & INRIA)

Fluid Models for MHD

1. Single Fluid MHD: $\mathbf{E} \simeq -\boldsymbol{v} \times \mathbf{B} + \eta \mathbf{J} + \cdots$, $\mathbf{J} \simeq \nabla \times \mathbf{B}$

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \boldsymbol{v}) & = 0 \\ \partial_t \boldsymbol{m} + \nabla \cdot (\boldsymbol{m} \otimes \boldsymbol{v} + p\mathbf{I} + \pi \mathbf{I} - \mathbf{B} \otimes \mathbf{B}) & = \mu \nabla \cdot \underline{\boldsymbol{\tau}} + \cdots \\ \partial_t p + \boldsymbol{v} \cdot \nabla p + \rho c^2 \nabla \cdot \boldsymbol{v} & = 0 \\ \partial_t \mathbf{B} + \nabla \times \mathbf{E} & = 0 \end{cases}$$

2. Reduded MHD : $\partial_t \rho \simeq 0$, $\partial_t p \simeq 0$
   $\mathbf{B} \simeq F \nabla \phi + \nabla \psi \times \nabla \phi$, $\quad \boldsymbol{v} \simeq \mathrm{R}^2 \nabla \phi \times \nabla \mathrm{U}$,

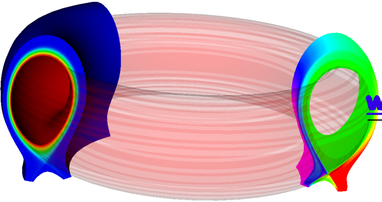
General formulation

$$\boldsymbol{\mathcal{M}}(\boldsymbol{\partial}, \partial_t \mathbf{w}) + \boldsymbol{\mathcal{L}}(\boldsymbol{\partial}, \mathbf{w}) = 0 \quad \text{for} \quad t > 0. \quad \boldsymbol{x} \in \boldsymbol{\Omega}_x \subset \mathbb{R}^d$$

Approximations, Poloidal/Toroidal : $\underline{\boldsymbol{\psi}}_h(\xi,\phi) = \boldsymbol{\varphi}_h(\xi) \otimes \mathbf{T}_h(\phi)$

$\mathcal{M}(\boldsymbol{\partial}, \partial_t \mathbf{w}) + \mathcal{L}(\boldsymbol{\partial}, \mathbf{w}) = 0$

$$\mathbf{w}_h(t,\xi,\phi) = \sum_{\ell'=1}^{N_\phi} \sum_{j=1}^{N_p} \underline{\mathbf{w}}_{j\ell'}(t) \star \left(\boldsymbol{\varphi}_j(\xi) \otimes \mathbf{T}_{\ell'}(\phi)\right) = \underline{\underline{\mathbf{w}}}_h(t) \star \underline{\boldsymbol{\psi}}_h(\xi,\phi)$$



$$\underline{\underline{\mathbf{w}}}_h(t) \equiv \begin{pmatrix} \underline{\mathbf{w}}_{11} & \cdots & \underline{\mathbf{w}}_{1\ell'} & \cdots & \underline{\mathbf{w}}_{1N_\phi} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ \underline{\mathbf{w}}_{j1} & \cdots & \underline{\mathbf{w}}_{j\ell'} & \cdots & \underline{\mathbf{w}}_{jN_\phi} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ \underline{\mathbf{w}}_{N_p 1} & \cdots & \underline{\mathbf{w}}_{N_p\ell'} & \cdots & \underline{\mathbf{w}}_{N_p N_\phi} \end{pmatrix}$$

$\boldsymbol{\varphi}_j(\xi) \in \mathbb{R}^{\vartheta_p}, \quad \mathbf{T}_{\ell'}(\phi) \in \mathbb{R}^{\vartheta_\phi}, \quad \underline{\mathbf{w}}_{j\ell'} \in \mathbb{R}^{N_v} \times \mathbb{R}^{\vartheta_p} \times \mathbb{R}^{\vartheta_\phi}$

Time Integration , $\mathcal{M}(\partial, \partial_t \mathbf{w}) + \mathcal{L}(\partial, \mathbf{w}) = 0$

Find a function $\underline{\underline{\mathbf{w}}}_h(t)$ such that

$$\underline{\underline{\mathbf{M}}}_h\left(\frac{d\underline{\underline{\mathbf{w}}}_h}{dt}\right) + \underline{\underline{\mathbf{K}}}_h\left(\underline{\underline{\mathbf{w}}}_h\right) = \underline{\underline{\boldsymbol{b}}}_h$$

with $\underline{\underline{\mathbf{w}}}_h(t=0)$ given.
$N_G = N_v \star (N_p \star \vartheta_p) \star (N_\phi \star \vartheta_\phi)$,

Time step : $\delta\underline{\underline{\mathbf{w}}}_h = \underline{\underline{\mathbf{w}}}_h^{n+1} - \underline{\underline{\mathbf{w}}}_h^n$ and $\delta t = t^{n+1} - t^n$

$$\frac{1}{\delta t}\underline{\underline{\mathbf{M}}}_h\left(\delta\underline{\underline{\mathbf{w}}}_h\right) + \underline{\underline{\mathbf{K}}}_h\left(\theta\delta\underline{\underline{\mathbf{w}}}_h + \underline{\underline{\mathbf{w}}}_h^n\right) = \underline{\underline{\boldsymbol{b}}}_h$$

with $\underline{\underline{\mathbf{w}}}_h^n = \underline{\underline{\mathbf{w}}}_h(t^n)$.

Usually $\underline{\underline{\mathbf{M}}}_h$ is linear and $\underline{\underline{\mathbf{K}}}_h$ is nonlinear.

Resumed Linearized time step for given $\underline{\underline{\mathbf{w}}}_h^n$

1. Elements contributions :
   dependent to physical models and numerical strategies
   - $\underline{\forall i \in \boldsymbol{\tau},\ \forall \ell \in \gamma}$ Compute $\left[\underline{\underline{\mathbf{K}}}_{i\ell}^{\boldsymbol{\tau}\gamma}\right]^n$
     - $\underline{\forall i' \in \boldsymbol{\tau},\ \forall \ell' \in \gamma.}$ Compute $\underline{\underline{\mathbf{M}}}_{i\ell,i'\ell'}^{\boldsymbol{\tau}\gamma}$ and $\left[\frac{\partial \underline{\underline{\mathbf{K}}}_{i\ell}^{\boldsymbol{\tau}\gamma}}{\partial \underline{\underline{\mathbf{w}}}_{i'\ell'}}\right]^n$.
2. Assembling

$$\underline{\underline{\mathbf{M}}}_h = \biguplus_{\gamma} \biguplus_{\boldsymbol{\tau}} \underline{\underline{\mathbf{M}}}_{i\ell,i'\ell'}^{\boldsymbol{\tau}\gamma}, \quad \left[\underline{\underline{\mathbf{K}}}_h\right]^n = \biguplus_{\gamma} \biguplus_{\boldsymbol{\tau}} \left[\underline{\underline{\mathbf{K}}}_{i\ell}^{\boldsymbol{\tau}\gamma}\right]^n, \quad \left[\frac{\partial \underline{\underline{\mathbf{K}}}_h}{\partial \underline{\underline{\mathbf{w}}}_h}\right]^n = \biguplus_{\gamma} \biguplus_{\boldsymbol{\tau}} \left[\frac{\partial \underline{\underline{\mathbf{K}}}_{i\ell}^{\boldsymbol{\tau}\gamma}}{\partial \underline{\underline{\mathbf{w}}}_{i'\ell'}}\right]^n$$

3. Other Contributions and BC : $\underline{\underline{\boldsymbol{b}}}_h$
4. Solve the linear system

$$\left(\frac{1}{\delta t}\underline{\underline{\mathbf{M}}}_h + \theta \left[\frac{\partial \underline{\underline{\mathbf{K}}}_h}{\partial \underline{\underline{\mathbf{w}}}_h}\right]^n\right) \delta \underline{\underline{\mathbf{w}}}_h = -\left[\underline{\underline{\mathbf{K}}}_h\right]^n + \underline{\underline{\boldsymbol{b}}}_h$$

5. Update : $\underline{\underline{\mathbf{w}}}_h^{n+1} = \underline{\underline{\mathbf{w}}}_h^n + \delta \underline{\underline{\mathbf{w}}}_h$

PlaTo/Jorek : Compressed Storage for $\boldsymbol{W}_h$ and $\underline{\mathbb{A}}_h^n$

According to Fortran storage

$$\boldsymbol{W}_h \equiv \left[1 : \vartheta_p, \quad 1 : \vartheta_\phi, \quad 1 : N_v, \quad 1 : N_p, \quad 1 : N_\phi\right]$$

$$\underline{\mathbb{A}}_h^n \equiv \left[1 : \vartheta_p, \quad 1 : \vartheta_\phi, \quad 1 : N_v, \quad 1 : N_p, \quad 1 : N_\phi\right]^2$$

Tipical Vectors sizes : $O\left(10^5\right)$, long term target $O\left(10^9\right)$.

$$\vartheta_p \simeq 4\text{--}6, \quad \vartheta_\phi \simeq 2\text{--}4, \quad N_v \simeq 2\text{--}10, \quad N_p \simeq 10^{4-6}, \quad N_\phi \simeq 10^{1-2}$$

Ordering : $(1 : N_p) \times (1 : N_\phi) \Longrightarrow [1 : (N_p * N_\phi)]$

Rearrangement and Storage needs for $\underline{\underline{\mathbb{A}}}_h^n$

Compressed Storage needs for $\underline{\underline{\mathbb{A}}}_h^n$

- FE/Fourier : $(N_v \times \vartheta_p \times \vartheta_\phi)^2 \otimes (2N_{seg} + N_p) \otimes (N_\phi)^2$
  Quadratic dependency to N_ϕ!
- FE/B-Splines : $(N_v \times \vartheta_p \times \vartheta_\phi)^2 \otimes (6N_{seg} + 2N_p) \otimes N_\phi$
  Linear dependency to N_ϕ.

$\underline{\underline{\mathbf{w}}}_h$ as a vector $\boldsymbol{W}_h$: $(\vartheta_\phi \times \vartheta_p \times N_v) \otimes (N_p \times N_\phi)$

- JOREK/PASTIX :: FE/Fourier.
  - $\underline{\underline{\mathbb{A}}}_h^n$ dense matrix of $\underline{\underline{\mathbb{A}}}_{\ell\ell'}$ blocks.
  - **$\underline{\underline{\mathbb{A}}}_{\ell\ell'}$ sparse matrix of dense blocks of size $\vartheta_\phi \times \vartheta_p \times N_v$**
- PLATO/????? :: FE/B-Splines.
  - **$\underline{\underline{\mathbb{A}}}_h^n$ tridiagonal block circulant matrix : $\underline{\underline{\mathbb{A}}}_{\ell\ell'}$ blocks**
  - **$\underline{\underline{\mathbb{A}}}_{\ell\ell'}$ sparse matrix of dense $\underline{\underline{\mathbb{A}}}_{\ell\ell'}^{ij}$ blocks.** of size $\vartheta_\phi \times \vartheta_p \times N_v$

Linearized Implicit scheme

$$\left(\frac{1}{\delta t}\underline{\underline{\mathbf{M}}}_h + \theta\left[\frac{\partial \underline{\underline{\mathbf{K}}}_h}{\partial \underline{\underline{\mathbf{w}}}_h}\right]^n\right)\delta\underline{\underline{\mathbf{w}}}_h = -\underline{\underline{\mathbf{K}}}_h\left(\underline{\underline{\mathbf{w}}}_h^n\right) + \underline{\underline{\boldsymbol{b}}}_h$$

After ordering

$$\underline{\mathbb{A}}_h^n \delta \boldsymbol{W}_h = -\boldsymbol{R}_h^n$$

Some questions to be addressed

1. How to "optimally" order $\underline{\underline{\mathbf{w}}}_h$ as a vector $\boldsymbol{W}_h$?
2. What is the corresponding structure and properties of $\underline{\mathbb{A}}_h^n$?
3. How to seed-up the resolution of the linear system $\underline{\mathbb{A}}_h^n \delta \boldsymbol{W}_h = -\boldsymbol{R}_h^n$?, Preconditioning $\underline{\mathbb{P}}^{-1}\underline{\mathbb{A}}_h^n \delta \boldsymbol{W}_h = -\underline{\mathbb{P}}^{-1}\boldsymbol{R}_h^n$?
4. How to manage "high order FE" data outputs?
5. Parallel and multilevel visualization.