Statistical estimation on Riemannian and affine symmetric spaces with applications to the statistical survey of the brain anatomy

At the interface of geometry, statistics, image analysis and medicine, computational anatomy aims at analyzing and modeling the biological variability of the organs shapes and their dynamics at the population level. The goal is to model the mean anatomy, its normal variation, its motion / evolution and to discover morphological differences between normal and pathological groups. Since shapes and deformations live in non-linear spaces, this requires a consistent statistical framework on manifolds and Lie groups, which has motivated the development of Geometric Statistics during the last decade. To consolidate the mathematical bases of geometric statistics, it is now essential to explore geometric structures beyond the classical Riemannian framework. In computational anatomy, the deformable template theory considers statistics on groups of diffeomorphisms. These groups can be endowed with a right-invariant Riemannian metric in the large deformation diffeomorphic metric mapping (LDDMM) framework (Miller & Younes 2001) or with the bi-invariant Cartan-Schouten connection (Pennec and Arsigny, 2012). However, in these affine connection spaces, one can locally define means using exponential barycenters but more general statistical tools and results remains to be established.

The PhD will explore the extension of the statistical estimation theory from Riemannian manifolds to affine symmetric spaces. This class of spaces include non-metric spaces like Lie groups with the bi-invariant Cartan-Schouten connection. The change of paradigm from the metric to the affine connection can potentially be generalized to homogeneous spaces that have invariant connection but no invariant metric, like reductive homogeneous manifolds (Alekseevsky, personal communication, 2017). The setting of affine symmetric spaces appears to be a very powerful non-Riemannian framework for geometric statistics where we will exemplify the impact of curvature on the non-asymptotic estimation. The second goal of the PhD will illustrate the statistical estimation methodology on dimension reduction problems for brain anatomodesy (the statistical survey of the brain anatomy). Large database of medical images are now available to study the variability of the anatomy in relation to clinical variables and outcomes at the scale of a population (1,800+ subjects in ADNI, 5,000+ in UK biobank). For complex shapes such as the brain, the classical unimodal Gaussian setting of statistical shape analysis with a few deformation modes around a mean shape model needs to be revisited. The PhD will investigate dimension reduction methods like barycentric subspace analysis (generalizing PCA on manifolds) and other generalization of PLS/CCA on the space of deformations seen as an affine symmetric space. In particular, the PhD will explore the formalization of multi-atlas methods for the segmentation of the brain based on barycentric subspaces in the space of diffeomorphisms.

References

- Xavier Pennec. Intrinsic Statistics on Riemannian Manifolds: Basic Tools for Geometric Measurements. Journal of Mathematical Imaging and Vision, 25(1):127–154, 2006.
- Xavier Pennec and Vincent Arsigny. Exponential Barycenters of the Canonical Cartan Connection and Invariant Means on Lie Groups. In Matrix Information Geometry, p.123–168. Springer, 2012.
- Xavier Pennec. Barycentric Subspace Analysis on Manifolds. Annals of Statistics, to appear in 2018. arXiv:1607.02833
- Sebastian S. Koh. On affine symmetric spaces. Transactions of the American Mathematical Society 119(2): 291-309, 1965.
- M. Lorenzi and X. Pennec. Geodesics, Parallel Transport & One-parameter Subgroups for Diffeomorphic Image Registration. International Journal of Computer Vision, 105(2):111–127, November 2013.
- Juan Eugenio Iglesias, Mert R. Sabuncu. Multi-atlas segmentation of biomedical images: A survey, Medical Image Analysis 24(1):205-219, 2015.
- M. Miller, L. Younes. Group Actions, Homeomorphisms, and Matching: A General Framework. Int. J. of Computer Vision 41(1-2):61-84, 2001.

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G-Statistics

Localization and context

This PhD will take place within the Epione team at Inria Sophia Antipolis under the direction of Xavier Pennec (Xavier.pennec@inria.fr, http://www-sop.inria.fr/members/Xavier.Pennec/), Senior Research Scientist (Directeur de Recherches) at Inria.

The Inria Sophia Antipolis – Méditerranée research center has about 600 people working in 37 research teams and 9 services to sustain research. It is localized in Sophia Antipolis, close to Antibes on the French Riviera. Within this research center, the Epione team aims at contributing to the development of what we call the e-patient (digital patient) for e-medicine (digital medicine). The e-patient (or digital patient) is a set of computational models of the human body able to describe and simulate the anatomy and the physiology of the patient's organs and tissues, at various scales, for an individual or a population.

The PhD proposal is part of the **ERC G-Statistics advance grant** # 786854 (2018-2023) from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program. G-statistics aims at exploring the consequences of the non-linearity of data spaces on the statistical estimation through geometry. Estimate the location (mean, median) and the concentration (covariance) of a random variable in a Riemannian manifold is now reasonably understood. There are also mathematical results for some classes of less smooth spaces, for instance length spaces of non-positive curvature. One of the objectives of the **G-Statistics project** is to unify these methods and to extend them to other non-Riemannian geometric structures like spaces with singularities and changes of dimension (stratification), affine connection, quotients or stratified spaces. These geometric structures appear in practical life sciences applications, as for example diffeomorphisms (invertible transformations of space) acting on images used in the registration of medical images, phylogenetic trees or shape spaces.

Required Skills

- Master degree with strong competences in mathematical modeling, statistical learning as well as some knowledge in medical imaging, signal and image processing (Master 2 level).
- Solid programming and IT skills are necessary (Python and C++, bash scripting, version control systems).
- Strong communication abilities
- Fluent English (written and spoken)

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