Medical Imaging

MVA 2023-2024

http://www-sop.inria.fr/teams/asclepios/cours/MVA

X. Pennec

Medical Image registration

Epione team
2004, route des Lucioles B.P. 93
06902 Sophia Antipolis Cedex

http://www-sop.inria.fr/epione

Medical Image Analysis – MVA 2023-2024

- Tue Oct 3, ENSPS 2E30, Introduction to Medical Image Acquisition and Image Filtering, [HD]
- **Tue Oct 10, ENSPS 3E34, Medical Image Registration [XP]**
- **Tue Oct 17, ENSPS 2E30, Riemanian Geometry & Statistics [XP]**
- **Tue Oct 24, ENSPS 1B18, Basis of Image Segmentation [HD]**
- **Tue Nov 7, ENSPS 2E30, Image Segmentation based on Clustering and Markov Random Fields [HD]**
- **Tue Nov 14, ENSPS 3E34, Shape constrained image segmentation and Biophysical Modeling [HD]**
- **Tue Nov 21, ENSPS 1N82, Analysis in the space of Covariance Matrices [XP]**
- **Tue Nov 28, ENSPS 2E30, Diffeomorphic Registration end computational anatomy [XP]**
- **Tu Dec 5, VISI Exam [HD, XP]**

Course Exam

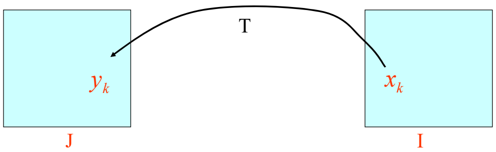
4 components :

- Scientific Article Study :
  - 10 min oral presentation
  - 10 min Questions & Answers
  - 5-6 page report presenting the paper and putting it in perspective.
  - Implementation (optional)
  - May be performed in pairs or triplets depending on class size
- Multiple choice Quizz : 10-15 questions

Goals of Registration

A dual problem

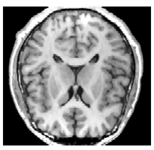
- Find the point y of image J which is corresponding (homologous) to each points x of image I.
- Determine the best transformation T that superimposes homologous points

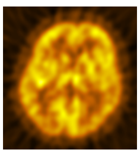


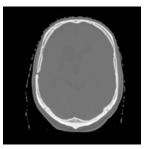
Principal Applications

- Fusion of multimodal images
- Temporal evolution of a pathology
- Inter-subject comparisons
- Superposition of an atlas
- Augmented reality

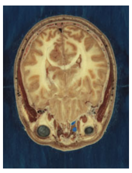
Fusion of Multimodal Images


MRI


PET

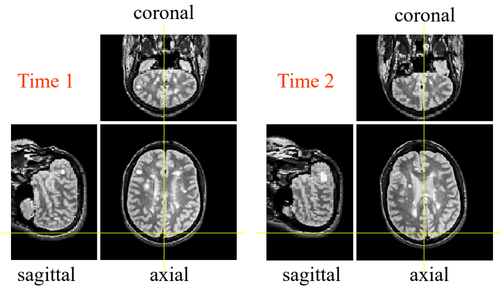

X-scan


US

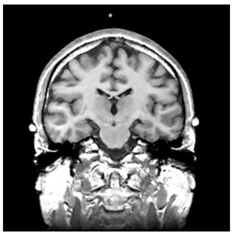

Histology

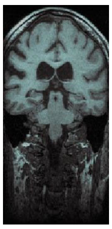
Visible Man

Temporal Evolution

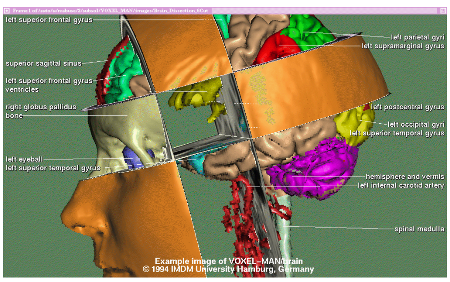


Inter-Subject comparison





Registration to an Atlas



Voxel Man

Augmented reality

Brigham & Women 's Hospital

E. Grimson

Classes of problems vs. applications

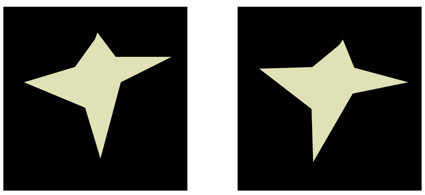
Temporal evolution	**Intra Subject – Monomodal**
Multimodal image fusion	**Intra Subject - Multimodal**
Inter-subject comparison	**Inter Subject - Monomodal**
Superposition on an atlas	**Inter Subject - Multimodal**

***Intra Subject*: Rigid or deformable**

***Inter Subject*: deformable**

Intuitive Example

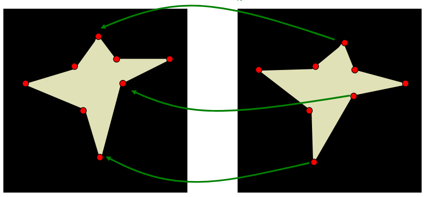
How to register these two images?



Feature-based/Image-based approach

Feature detection (here, points of high curvature)

Measure: for instance $S(T)=\sum_k \|T(\mathbf{x}_k)-\mathbf{y}_k\|^2$



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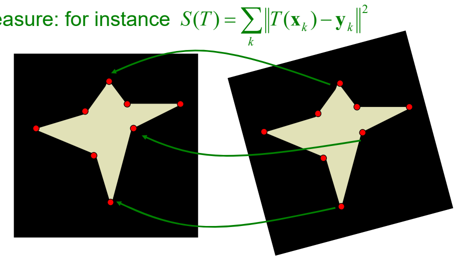
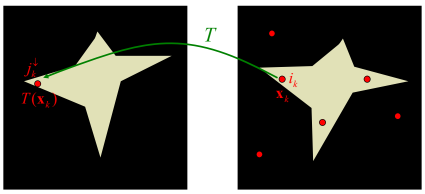


Feature-based/Image-based approach

No segmentation!

Measure: e.g. $S(T)=\sum_k (i_k - j_k^{\downarrow})^2$ Interpolation: $j_k^{\downarrow} \equiv J(T(\mathbf{x}_k))$



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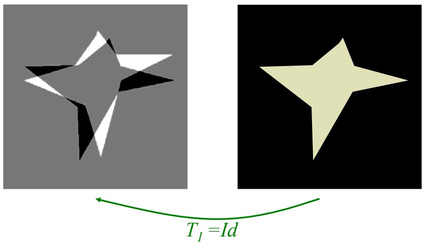

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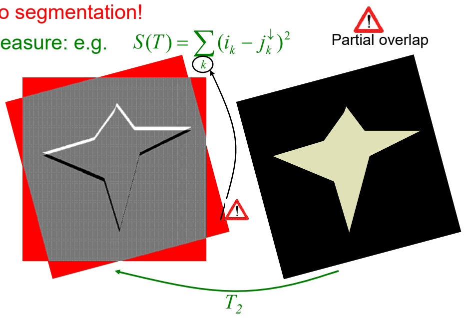
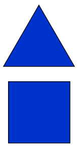


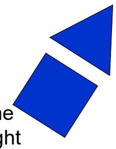
Classes of Transformations T

- Rigid (displacement)
- Similarities
- Affine (projective for 2D / 3D)
- Polynomials
- Splines
- Free-form deformations

Classes of Transformations T

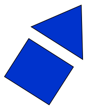


Rigid: $T(x) = Rx + t$



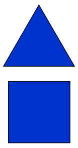
- Rotation and translation
- 6 parameters: (R: 3; t: 3)
- invariants: distances (isometry), frame orientation, curvatures, angles, straight lines

Similarities: $T(x) = s.Rx + t$

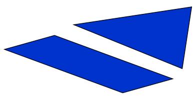


- Add a global scale factor
- 7 parameters
- invariants: ratio of distances, orientation, angles, straight lines

Classes of Transformations T



Affine: $T(x) = Bx + t$



- 3x3 matrix B
- 12 parameters: (B: 9; t: 3)
- invariants: straight lines, parallelism

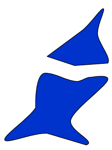
Quadratic: $T(x)^k = a^k_{ij}\, x_i x_j + b^k_i x_i + t^k$

- Add a symmetric 3x3 matrix A per axis
- 30 parameters (A: 18; B: 9; t: 3)
- invariants: do not preserve straight lines **any more**

Classes of Transformations T

Splines:

- Local polynomials of degree d, with a global continuity of degree C(d-1).
- number of parameters: depend on the number of control points (knots)
- locally affine: simplified version



Free form transformations: $T(x) = x + u(x)$

- a vector $u(x)$ is attached to each point x
- parameters: at most 3 times the number of voxels
- regularization: constrain to homeomorphisms (diffeomorphisms)

Classification of registration problems

Type of transformation

- Parametric
  - Rigid (displacement), similarity, affine, projective
- Deformables
  - Polynomial, spline, free-form deformations

Type of acquisition

- Monomodal
- Multimodal

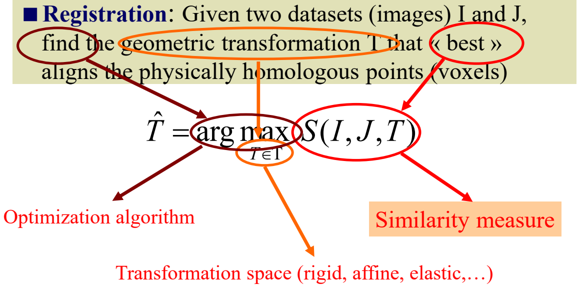
Homology of observed objects

- Intra-subject (generally a well posed problem)
- Inter-subject (one-to-one correspondences, regularization ?)

Mathematical Formulation of registration (Brown, 1992)

- **Registration**: Given two datasets (images) I and J, find the geometric transformation T that « best » aligns the physically homologous points (voxels)

$$\hat{T} = \arg\max_{T \in \Gamma} S(I, J, T)$$



Course overview

Feature-based registration

Multimodal Intensity-based Registration

Deformable intensity-based Registration

Geometric methods

Extract geometric features

- Invariant by the chosen transformations
  - Points
  - Segments
  - Frames

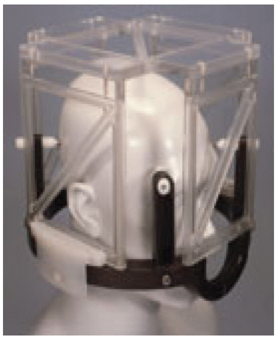
Given two sets of features, registration consists in:

- Feature identification (similarity): Match homologous features
- Localization: Estimate the transformation T

Algorithms

- Interpretation trees
- Alignement
- Geometric Hashing
- ICP

Artificial markers

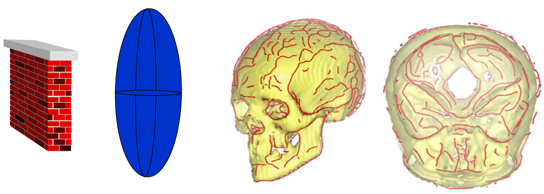


Stereotactic frame

- Invasive
- External markers
- Motion
- Short time use

Anatomical markers

Find geometric invariants to characterize a small number of singular points on anatomical surfaces

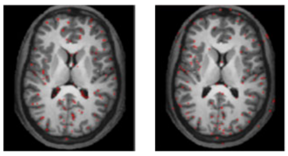


Generalization of edges and corner points to differentiable surfaces

Anatomical markers

Find geometric invariants to characterize a small number of singular points on anatomical surfaces

- Multiscale determinant of Hessian function (numerator of Gaussian curvature)
- 3D Harris detector [Rohr 99, Ruiz-Alzola et al 2001] based on the local correlation matrix C= $E(\nabla I \,.\, \nabla I^t)$



Detected salient points in a axial slice of a brain. In a) Beaudet/Thirion curvature based detector, in b) the Harris/Rohr correlation based method is shown. [From Lloyd, Szekely, Kikinis, Warfield 2005]

Iterative closest point (ICP)

One global criterion, $C(A,T) = \sum_i dist(T(P_i), A(P_i))^2$

alternatively minimized over

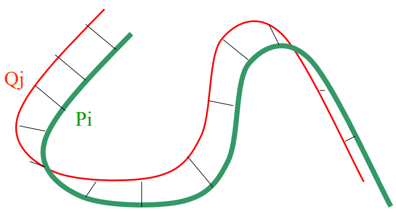
- Step 1: matches $A^* = \text{Argmin}_A \sum_i dist(T(P_i), A(P_i))^2$
- Step 2: transformation $T^* = \text{Argmin}_T \sum_i dist(T(P_i), A(P_i))^2$

Positive and decreasing criterion: convergence

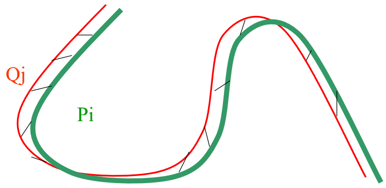
Robustness w.r.t. outliers: robust distances

$$dist(x,y)^2 = \Phi(\| x - y \|) \quad \begin{cases} \Phi(x) = \| x \|^2 & \text{(standard mean)} \\ \Phi(x) = \min\left(\| x \|^2, \chi^2\right) & \text{(saturated mean)} \\ \Phi(x) = \| x \| & \text{(median)} \end{cases}$$

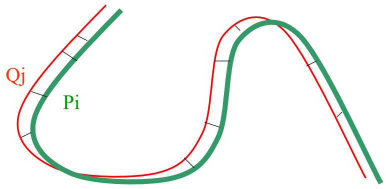
1. Optimization for matches



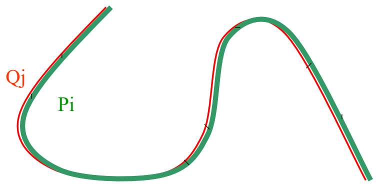
2. Optimization for T



1-bis Optimization for matches



2-bis Optimization for T



Augmented reality guided radio-frequency tumor ablation

Collaboration with IRCAD (Strasbourg, France)

- Per-operative CT "guidance"
- Respiratory gating
  - Marker based 3D/2D rigid registration

S. Nicolau, X.Pennec, A. Garcia,L. Soler, N. Ayache IRCAD

Increase 3D/2D registration accuracy: A new Extended Projective Point Criterion

Standard criterion: $\sum_{l=1}^{M} \sum_{i=1}^{N} \left\| P^l(T*M_i) - m_i^l \right\|^2$

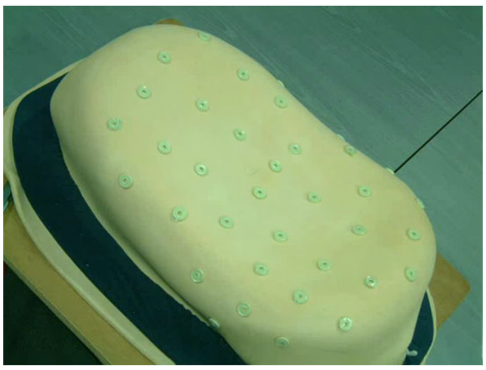
- image space minimization (ISPPC)
- noise only on 2D data

Complete statistical assumptions + ML estimation

- Gaussian noise on 2D *and* 3D data
- Hidden variables M_i (exact 3D positions)

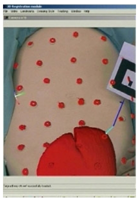
$$\sum_{l=1}^{M} \sum_{i=1}^{N} \frac{\left\| P^l(T*M_i) - \tilde{m}_i^l \right\|^2}{2\sigma_{2D}^2} + \sum_{i=1}^{N} \frac{\left\| M_i - \tilde{M}_i \right\|^2}{2\sigma_{3D}^2}$$

Registration for Augmented reality

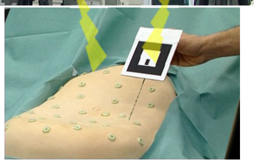


Whole loop accuracy evaluation

Left monitor with AR & needle tracking

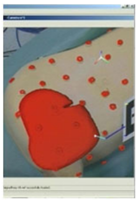


Stereoscopic HD video acquisition



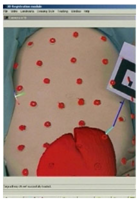
Real scene: phantom + needle

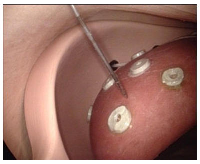
Right monitor with AR & needle tracking



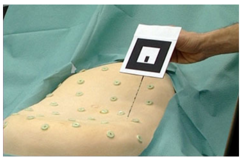
Whole loop accuracy evaluation

Left monitor with AR & needle tracking



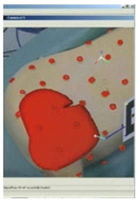


Endoscopic control

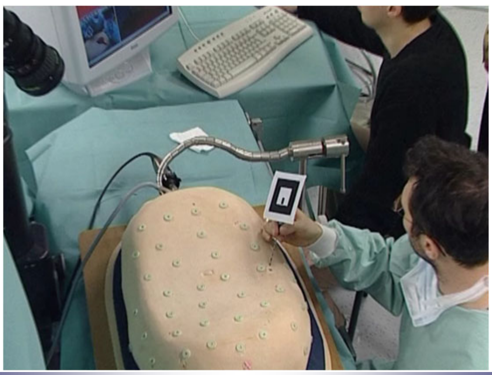


Real scene: phantom + needle

Right monitor with AR & needle tracking



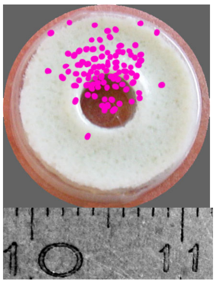
Whole loop accuracy evaluation



Whole loop accuracy evaluation

Experimental setup

- Two participants (comp. sci. + surgeon)
- 100 needle targeting



Measures

- Distribution of hits
  (endoscopic view, video recording)
- Average deviation from target
  2.8 mm ± 1.4
- Average targeting time:
  46.6 sec. ± 24.64

[S. Nicolau, A. Garcia et al., Aug. & Virtual Reality Workshop, Geneva, 2003]

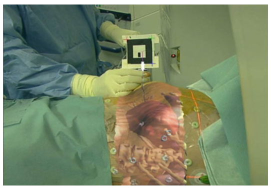
Liver puncture guidance using augmented reality

3D (CT) / 2D (Video) registration

- 2D-3D EM-ICP on fiducial markers
- Certified accuracy in real time

Validation

- Bronze standard (no gold-standard)
- Phantom in the operating room (2 mm)
- 10 Patient (passive mode): < 5mm (apnea)



[S. Nicolau, PhD'04 MICCAI05, ECCV04,, IS4TM03, Comp. Anim. & Virtual World 2005]

Course overview

Feature-based registration

Multimodal Intensity-based Registration

Deformable intensity-based Registration

Intensity-based methods

No geometric feature extraction

Advantages:

- Noisy images and/or low resolution
- Multimodal images

Drawbacks:

- All voxels must be taken into account

Compare multi-modal images?

Which similarity criterion ?

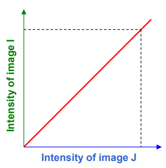
- Many available criteria:
  - SSD, Correlation, Mutual Information...?
- Variable costs and performances
- Where is the optimum ?

Maintz & Viergever, Survey of Registration Methods, Medical Image Analysis 1997

Classification of existing measures

Assumed relationship

Intensity conservation



Adapted measures

Sum of Square Differences
Sum of Absolute value of Differences
Measures of intensity differences (Buzug, 97)

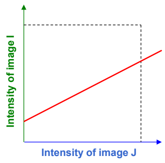
$$S(T) = \sum_k (i_k - j_k^{\downarrow})^2$$

Interpolation: $j_k^{\downarrow} \equiv J(T(\mathbf{x}_k))$

Classification of existing measures

Assumed relationship

Affine



Adapted measures

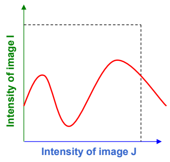
Correlation coefficient

$$\rho_{IJ}(T) = \frac{1}{n\sigma_I\sigma_J}\sum_k (i_k - \bar{I})(j_k^{\downarrow} - \bar{J})$$

Classification of existing measures

Assumed relationship

Functional



Adapted measures

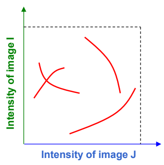
Woods' criterion (1993)
Woods' variants (Ardekani, 95; Alpert, 96; Nikou, 97)
Correlation ratio (Roche, 98)

$$\eta^2 = \frac{Var[E(I \mid J(T))]}{Var(I)}$$

Classification of existing measures

Assumed relationship

Statistical



Adapted measures

Joint Entropy (Hill, 95; Collignon, 95)
Mutual Information (Collignon, 95; Viola, 95)
Normalized Mutual Information (Studholme, 98)

$$MI(I,J) = H(I) + H(J) - H(I,J) = \sum_i \sum_j P(i,j) \log \frac{P(i,j)}{P(i)P(j)}$$

A general framework

- A. Roche proposed a unifying maximum likelihood framework
- Physical and statistical modeling of the image acquisition process
- Create a hierarchy of criteria

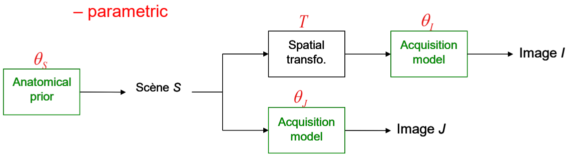
A. Roche, G. Malandain and N.Ayache : *Unifying maximum likelihood approaches in medical image registration.* International Journal of Imaging Systems and Technology : Special Issue on 3D Imaging 11(1), 71-80, 2000.

• Based on pioneer works of (Costa et al, 1993), (Viola, 1995), (Leventon & Grimson, 1998), (Bansal et al, 1998)

Principle of the method [A. Roche]

Generative model of images

– probabilistic
– parametric



Maximum likelihood Inference

$$\max_{T,\theta_I,\theta_J,\theta_S} P(I,J \mid T,\underbrace{\theta_I,\theta_J,\theta_S}_{\text{Auxiliary variables: } q})$$

Generic model

Scene: discrete random field (segmentation)

Assumptions:
– Spatial independence
– Stationarity

$$\Rightarrow P(S) = \prod_l \pi(s_l)$$

Generic model

Images: noisy measures of the scene

$$\begin{cases} i_k = f(s_k^\downarrow)+\varepsilon_k \\ j_l = g(s_l)+\eta_l \end{cases} \qquad s_k^\downarrow \equiv s(T(x_k))$$

Assumptions on ε and η :

- white (spatial indep.)
- Stationary
- Gaussian
- Additive
- Independent of each other

$$\Rightarrow P(I,J \mid S) = \prod_k G_{\sigma_\varepsilon}(i_k - f(s_k^\downarrow)) \times \prod_l G_{\sigma_\eta}(j_l - g(s_l))$$

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Likelihood function

Likelihood = joint law of images

$$P(I,J) = \int P(I,J \mid S) P(S) dS \equiv L(T,\theta) \quad \text{with} \quad \theta \equiv (\pi, f, g, \sigma_\varepsilon, \sigma_\eta)$$

Log likelihood

$$\log L(T,\theta) = \underbrace{\sum_{k \in A} \log \frac{P_\theta(i_k, j_k^\downarrow)}{P_\theta(i_k) P_\theta(j_k^\downarrow)}}_{(T,\theta)} + \underbrace{\sum_k \log P_\theta(i_k) + \sum_l \log P_\theta(j_l)}_{(\theta)}$$

$$\text{with} \quad P_\theta(i,j) = \sum_{p=1}^{K} \pi_p G_{\sigma_\varepsilon}(i - f_p) G_{\sigma_\eta}(j - g_p)$$

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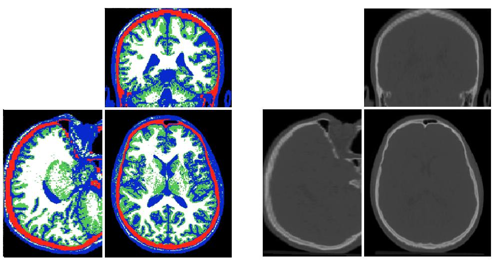
Example: X-Scan / MRI rigid registration

The estimation of θ allows the a posteriori estimation of the scene



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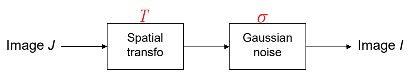
Limitations

- In general, the maximization w.r.t. θ is costly (EM segmentation algorithm)
- With additional assumptions on θ, one can approach the solution analytically

Particular case 1

Assumptions

$$\begin{cases} f = g \\ |\eta| << |\varepsilon| \end{cases} \Rightarrow i_k \approx j_k^{\downarrow} + \varepsilon_k$$



Maximum Likelihood

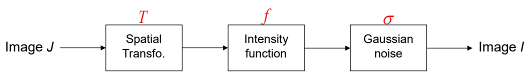
$$\max_{\theta} L(T,\theta) \propto -n \log\left(\frac{c}{n^2} \sum_k (i_k - j_k^{\downarrow})^2 \right) + E_{\text{cte}}$$

SSD

Particular case 2

Assumptions

$$\begin{cases} g \text{ injective} \\ |\eta| << |\varepsilon| \end{cases} \Rightarrow i_k \approx f(j_k^{\downarrow}) + \varepsilon_k, \quad f \text{ constant by part}$$



Maximum Likelihood

$$\max_{\theta} L(T,\theta) \propto -n \log\left(\frac{c}{n^2} \min_f \sum_k (i_k - f(j_k^{\downarrow}))^2 \right) + E_{\text{cte}}$$

Squared fit error

Particular case 2

Limit case: correlation ratio (CR)

$$\eta^2_{I|J}(T) \equiv \frac{\min_f \sum_k (i_k - f(j_k^\downarrow))^2}{n\,\mathrm{Var}(I)}$$

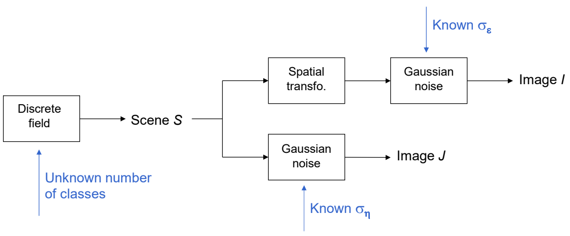
Squared fit error

Different normalization of the likelihood criteria

Equivalence ML / CR

Particular case 3

Assumptions



Particular case 3

Maximum Likelihood

$$\hat{P}(i,j) \approx \frac{1}{n}\sum_k G_{\sigma_\varepsilon}(i - i_k)\, G_{\sigma_\eta}(j - j_k^\downarrow) \quad \text{(Parzen)}$$

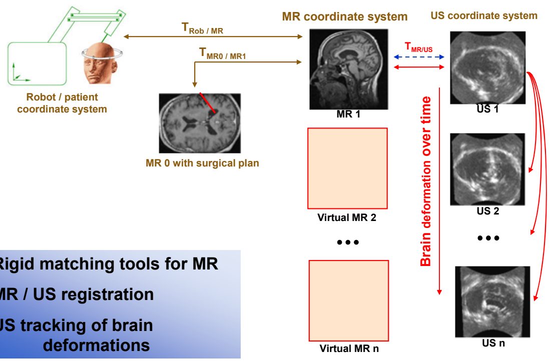
$$\max_P L(T,P) \propto \sum_k \log \frac{\hat{P}(i_k, j_k^\downarrow)}{\hat{P}(i_k)\hat{P}(j_k^\downarrow)} + \text{corrective terms}$$

$n \times$ Mutual Informatio n

Choice of a criterion

- Choosing a criterion imposes to deeply understand the physical image acquisition process
- Whenever several models are known, choosing the one with the smallest number of degrees of freedom increase the robustness of the approach.
- Current trend: learn it.
  Pitfall: no idea of #dof and local minima

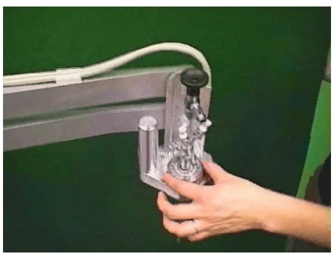
Roboscope



- Rigid matching tools for MR
- MR / US registration
- US tracking of brain deformations

Manipulator

Steady Hand Motion Compensation
Active Motion Constraints



Courtesy B. Davies & S. Starkie

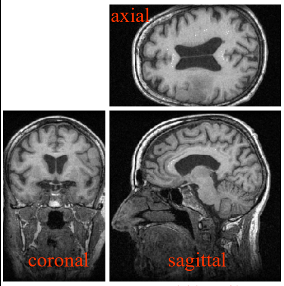
Manipulator

Steady Hand Motion Compensation
Active Motion Constraints

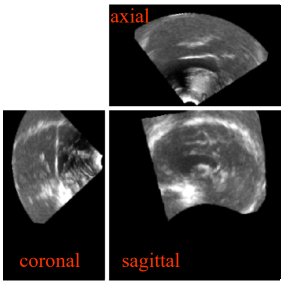
Courtesy B. Davies & S. Starkie

MR-US Images

Pre - Operative MR Image



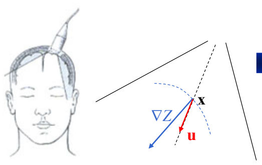
Per - Operative US Image



Acquisition of images : L. & D. Auer, M. Rudolf

Ultrasound image / MRI registration

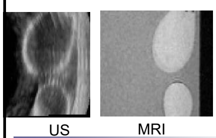
Elementary principles of US imagery



$$I_{rf}(\mathbf{x}) = |\nabla Z.\mathbf{u}\ (\mathbf{x})| \times \xi(\mathbf{x})$$

Logarithmic compression

$$I(\mathbf{x}) \approx A \log|\nabla Z.\mathbf{u}\ (\mathbf{x})| + B + \varepsilon(\mathbf{x})$$



Ultrasound image / MRI registration

Assumption: acoustic impedance is a function of the MR signal (denote by *J*)

$$Z(\mathbf{x}) = g(J(\mathbf{x})) \quad \Rightarrow \quad \nabla Z(\mathbf{x}) = g'(J(\mathbf{x})) \times \nabla J(\mathbf{x})$$

Relation between US and MR signals

$$I(\mathbf{x}) = f\left[J(\mathbf{x}), |\nabla J^t \mathbf{u}(\mathbf{x})|\right] + \varepsilon(\mathbf{x})$$

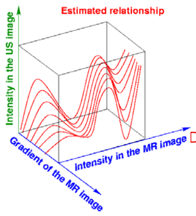
In practice, the influence of orientation is neglected

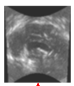
$$I(\mathbf{x}) \approx f\left[J(\mathbf{x}), \|\nabla J(\mathbf{x})\|\right] + \varepsilon(\mathbf{x})$$

Bivariate Correlation Ratio

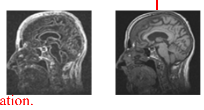
Intensity = function of 2 variables

$$I = f(J, |\nabla J|)$$





Dependency Hypothesis



2 iterated stages

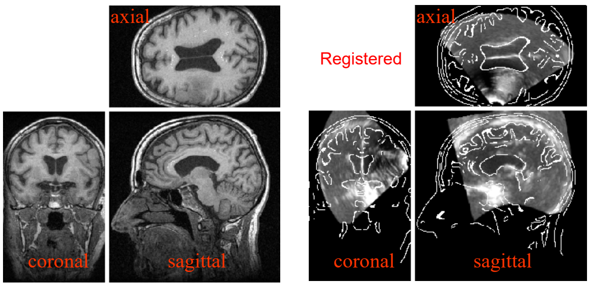
- Robust polynomial approx. of f
- Estimation of T:

$$\hat{T} = \arg\min_T \sum_k \left(I(x_k) - \hat{f}(J(T(x_k)), |\nabla J(T(x_k))|) \right)^2$$

A. Roche, X. Pennec, G. Malandain, and N. Ayache
Rigid Registration of 3D Ultrasound with MR Images:
a New Approach Combining Intensity and Gradient Information.
IEEE Transactions on Medical Imaging, 20(10):1038--1049, October 2001.

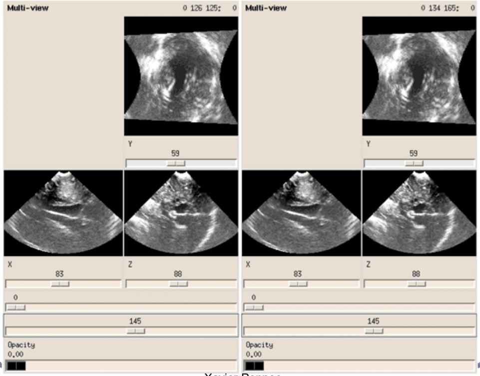
Typical Registration Result with Bivariate Correlation Ratio

Pre - Operative MR Image | Per - Operative US Image



Acquisition of images : L. & D. Auer, M. Rudolf

US Intensity
MR Intensity and Gradient



Course overview

Feature-based registration

Multimodal Intensity-based Registration

Deformable intensity-based Registration

The deformable Registration Landscape in 1995

Optical flow

- Horn and Schunck, Artif. Intell. 17, 1981;
- Aggarwal and Nandhakumar, Proc. IEEE 76: 917–935,1988;
- Barron *et al., 1994.*

Linear elastic deformation

- Broit, PhD 1981.
- Bajcsy and Kovacic CVGIP 46, 1989
- Gee, Reivich, Bajcsy, *J. Comp. Assis.Tom. 17,* 1993.

Fluid (images & surface)

- Christensen, Rabbitt, Miller, *Phys. Med. Biol. 39,* 1994.
- Christensen, Rabbitt, Miller.IEEE Trans. Im. Proc. 5(10), 1996.
- Thompson and Toga, IEEE TMI 15(4), 1996.

Mechanical deformations

T is a deformation endoded by its displacement vector field: $x_i \mapsto T(x_i) = x_i + u(x_i)$

Similarity measure is the SSD

$$C = \sum (I(x) - J(x+u(x))^2$$

The differential of this energy is considered as a force:

$$F(x,u) = -(I(x) - J(x+u))\nabla J(x+u) \qquad (1)$$

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Mechanical deformations

The force F is applied to the image considered

- Either as a linear elastic material (Lamé Coef.)

$$\mu \nabla^2 u + (\mu + \lambda)\nabla(div(u)) = F \qquad (2)$$

- Or as a viscous fluid (Navier-Stokes, Viscosity Coef.)

$$\mu \nabla^2 v + (\mu + \lambda)\nabla(div(v)) = F \qquad (3)$$

$$\frac{\partial u}{\partial t} = v - (\nabla u)\, v \qquad (4)$$

Equations (2) and (3) are iteratively solved with F computed by (1).
u is computed by integrating equation (4).

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Difficulties

- Differential equations are costly to solve
- Regularity of T?
- Small time steps, many iterations
- Very high computation time...

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Demon

- **Computer Science**
  A program or process that sits idly in the background until it is invoked to perform its task.
- **A person who is part mortal and part god**
  Demigod, deity, divinity, god, immortal - any supernatural being worshipped as **controlling some part of the world** or some aspect of life or who is **the personification of a force**
- **Maxell's demon**
  An imaginary creature who is able to sort hot molecules from cold molecules without expending energy, thus bringing about a general decrease in entropy and violating the second law of thermodynamics.

Demons' algorithm (MRCAS 95, CVPR96, Media98)

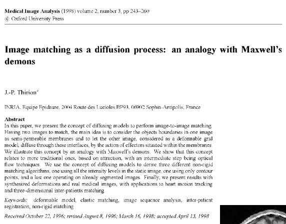
Medical Image Analysis (1998) volume 2, number 3, pp 243–260
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Image matching as a diffusion process: an analogy with Maxwell's demons

J.-P. Thirion*

INRIA, Equipe Epidaure, 2004 Route des Lucioles BP93, 06902 Sophia-Antipolis, France

Abstract
In this paper, we present the concept of diffusing models to perform image-to-image matching. Having two images to match, the main idea is to consider the objects boundaries in one image as semi-permeable membranes and to let the other image, considered as a deformable grid model, diffuse through these interfaces, by the action of effectors situated within the membranes. We illustrate this concept by an analogy with Maxwell's demons. We show that this concept relates to more traditional ones, based on attraction, with an intermediate step being optical flow techniques. We use the concept of diffusing models to derive three different non-rigid matching algorithms, one using all the intensity levels in the static image, one using only contour points, and a last one operating on already segmented images. Finally, we present results with synthesized deformations and real medical images, with applications to heart motion tracking and three-dimensional inter-patients matching.

Keywords: deformable model, elastic matching, image sequence analysis, inter-patient registration, non-rigid matching

Received October 22, 1996; revised August 8, 1996; March 16, 1998; accepted April 13, 1998

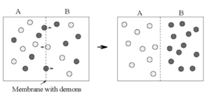


Figure 4. Maxwell's demons and a mixed gas.

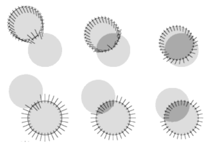
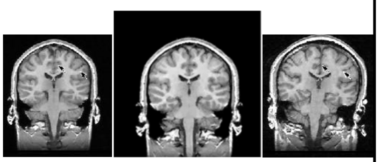


Figure 5. Three iterations of a model based on attraction (top row) and a rigid diffusing model (bottom row). These examples are produced by actual implementations.



Patient 1 Patient 2

Demons' algorithm (MRCAS 95, CVPR96, Media98)

- Inspired by Christensen & Miller's work
- Algorithm in O(N)
- 2 alternated steps
  - Image forces create a displacement field u_n (normalized optical flow)
  - Regularization of u_n by Gaussian filtering

J.P. Thirion: Image Matching as a diffusion process: an analogy with Maxwell's demons. Medical Image Analysis 2(3), 242-260, 1998.

Demons' algorithm (MRCAS 95, CVPR96, Media98)

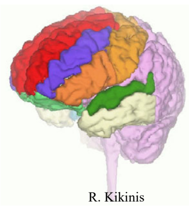
- T_0= Identity
- Correction field $$C_{n+1} = \frac{I - J \circ T_n}{\|\nabla I\|^2 + (I - J \circ T_n)^2} \nabla I$$
- Regularization by Gaussian filtering

Elastic
$$\hat{C}_{n+1} = U_n \circ C_{n+1}$$
$$U_{n+1} = G_\sigma * \hat{C}_{n+1}$$

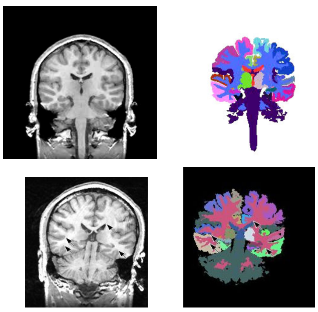
Fluid
$$\tilde{C}_{n+1} = G_\sigma * C_{n+1}$$
$$U_{n+1} = U_n \circ \tilde{C}_{n+1}$$

J.P. Thirion: Image Matching as a diffusion process: an analogy with Maxwell's demons. Medical Image Analysis 2(3), 242-260, 1998.

Demons' algorithm (MRCAS 95, CVPR96, Media98)



R. Kikinis
Harvard Medical School



Intensity-based deformable registration

Demons algorithm: why does it work?

- \+ Fast, efficient
- \- Do not minimize an energy
  - Difficult to analyze
  - Convergence?
  - Why does that work?
  - How to change the similarity measure?

Course overview

Feature-based registration

Multimodal Intensity-based Registration

Deformable intensity-based Registration

- A historical perspective
- A Pair and Smooth approach
- Morphing

Interpretation of demons

Why does that work? Convergence? Change the metric?

A variational framework to minimize a global energy

- Pennec-Cachier:

$$E = SSD + \int \|\nabla u\|_*^2$$

X. Pennec, P. Cachier and N. A. : Understanding the Demons Algorithm : 3D non rigid registration by gradient descent, MICCAI 1999, Springer-Verlag.

- Modersitzki: Min E with Neumann boundary conditions

$$E = SSD* + \int \|\nabla u\|^2$$

J. Modersitzki : Numerical Methods for Image Registration, Oxford University Press,2004.

PASHA: Pair-And-Smooth, Hybrid energy based Algorithm

$$E(C,T) = \frac{1}{\sigma_i^2} SSD(I,J,C) + \frac{1}{\sigma_x^2} \| C - T \|^2 + \mathrm{Reg}(T)$$

- *SSD* : measures the similarity of intensities
- *Reg* : regularization energy (quadratic)
- σ_x , σ_i : smoothing and noise parameters
- *C* : correspondences between points (vectors field)
- *T* : transformation (regularized vector field)

- Correspondences (matches) as an auxiliary variable

P. Cachier E. Bardinet, E. Dormont, X. Pennec and N. A.: *Iconic Feature Based Nonrigid Registration: the PASHA Algorithm*, Comp. Vision and Image Understanding (CVIU), Special Issue on Non Rigid Registration, 89 (2-3), 272-298, 2003.

PASHA: Pair-And-Smooth, Hybrid energy based Algorithm

$$E(C,T) = \frac{1}{\sigma_i^2} SSD(I,J,C) + \frac{1}{\sigma_x^2} \| C - T \|^2 + \mathrm{Reg}(T)$$

Alternated minimization

- Minimization with respect to *C* :
  - Find matches between points by optimizing E_S + **in the neighborhood of** ***T***
  - Gradient descent (1st, 2bd order, e.g. Gauss-Newton)
- Minimization with respect to *T* :
  - Find a smooth transformation that approximates *C*
  - Quadratic energy ⇒ convolution
- **Interest**: fast computation

Gauss-Newton optimization of the correspondences

$$E(C) = \int (I(x) - J(C(x))^2 .dx + \frac{\sigma_i^2}{\sigma_x^2} \int \| C(x) - T(x) \|^2 .dx$$

Newton optimization

- Second order Taylor expansion of E(C)
- Hessian matrix can be null or negative

Gauss-Newton

- 1st order Taylor expansion of error

$$[I\text{-}J \circ (T+u)(x)] = [I - J \circ T(x)] + (\nabla J \circ T)^T .u(x) + O(\|u(x)\|^2)$$

- Solve approximated SSD Criterion around C=T

$$\mathrm{E(C+u)} \approx SSD(T) + 2\int (J \circ T - I).(\nabla J \circ T)^t .u$$
$$+ \int u^t .(\nabla J \circ T).(\nabla J \circ T)^t .u + 2\frac{\sigma_i^2}{\sigma_x^2} \int (C - T)^t .u + \frac{\sigma_i^2}{\sigma_x^2} \|u\|^2$$

Gauss-Newton optimization of the correspondences

$$E(C) = \int (I(x) - J(C(x))^2 .dx + \frac{\sigma_i^2}{\sigma_x^2} \int \| C(x) - T(x) \|^2 .dx$$

Exact solution of the quadratic approximation of the SSD

- Solve $\left[(\nabla J \circ T).(\nabla J \circ T)^t + \frac{\sigma_i^2}{\sigma_x^2} Id \right] .u = (J \circ T - I).(\nabla J \circ T)$
- By inversion lemma: $u = \frac{(J \circ T - I).(\nabla J \circ T)}{\|\nabla J \circ T\|^2 + \sigma_i^2 / \sigma_x^2}$
- Local estimation of intensity variance: $\sigma_i^2 = (J \circ T - I)^2$
- Assuming isotropic voxel size: $\sigma_x^2 \approx 1$

$$u = \frac{I - J \circ T}{\|\nabla I\|^2 + (I - J \circ T)^2} \nabla I$$

Important Practical Remark

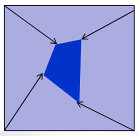
$$u = \frac{I - J \circ T}{\|\nabla I\|^2 + (I - J \circ T)^2} \nabla I$$

- Norm of update is bounded by construction

$$\left(\|\nabla I\| - (I - J \circ T)\right)^2 = \|\nabla I\|^2 + (I - J \circ T)^2 - 2(I - J \circ T)\|\nabla I\| > 0$$

$$\|u\| \le 1/2$$

- Update is diffeomorphic by tri-linear interpolation!



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Efficient Regularization

Quadratic regularizer $\mathrm{Reg}(T) = \int \sum_{k=1}^{\infty} \frac{\sum_{i_1 \ldots i_k} \|\partial_{i_1} \ldots \partial_{i_k} (T - Id)\|^2}{\sigma_d^{2k} . k!}$

Euler Lagrange optimization of $\mathrm{E(T)} = \int \|\mathrm{C} - \mathrm{T}\|^2 + \mathrm{Reg}(T)$

$$C - T + \sum_{k=1}^{\infty} \frac{(-1)^k \Delta^k (T - Id)}{\sigma_d^{2k} . k!} = 0$$

Solution: Gaussian smoothing $\mathrm{T_{opt}} = G_\sigma * C$ with $\sigma = 1/\sigma_d$

Pennec, Cachier, Ayache. Understanding the ``Demon's Algorithm'': 3D Non-Rigid registration by Gradient Descent. MICCAI 1999.

Extension to a family of quadratic filters

$$G_{\sigma,\kappa}(\mathbf{u}) = \frac{1}{(\sigma\sqrt{2\pi})^3 (1+\kappa)} \left(\mathrm{Id} + \frac{\kappa}{\sigma^2} \mathbf{u}\mathbf{u}^T\right) \exp\left(\frac{\mathbf{u}^T \mathbf{u}}{2\sigma^2}\right)$$

P. Cachier and N. Ayache. Isotropic energies, filters and splines for vectorial regularization. J. of Math. Imaging and Vision, 20(3):251-265, May 2004.

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Mixed Elastic / Fluid Regularization

$$E(C_n, T_n) = E_S(I, J, C_n) + \sigma \| C_n - T_n \|^2$$
$$+ \cancel{\sigma\lambda.\mathrm{Reg}(T_n)} + \sigma\lambda\left[\omega.\mathrm{Reg}(T_n - T_{n-1}) + (1-\omega)\mathrm{Reg}(T_n)\right]$$

- Result is still obtained by convolution:

$$T_n = (1-\omega).\, K * C_n + \omega.(T_n + K * (C_n - T_{n-1}))$$

- **Advantages**:
  - Mixes fluid and elastic
  - handles large displacements

P. Cachier N. A., *Isotropic Energies, Filters and Splines for Vector Field Regularization,* J. of Mathematical Imaging and Vision, 20: 251-265, 2004

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The Demons/PASHA Framework

Efficient energy minimization

$$E(C,T,\dot{T}) = \boxed{E_s(I,J,C)} + \sigma \boxed{\int \|C-T\|^2} + \boxed{\lambda\, Reg(T) + \mu\, Reg(\dot{T})}$$

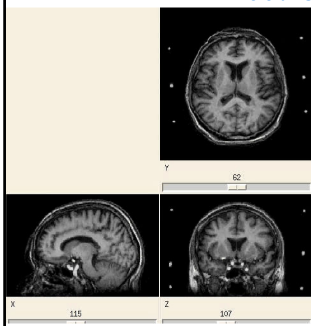
similarity — *Auxiliary* — *Elastic + Fluid Regularity*

Alternate Minimization

- on C, Correspondance Field (image forces)
  Gauss-Newton gradient descent: normalized optical flow
- on T, Deformation Field (regularization)
  Gaussian convolution

P. Cachier E. Bardinet, E. Dormont, X. Pennec and N. A.: Iconic Feature Based Nonrigid Registration: the PASHA Algorithm, Comp. Vision and Image Understanding (CVIU), 89 (2-3), 272-298, 2003.

Inter-subject registration
Affine transformation



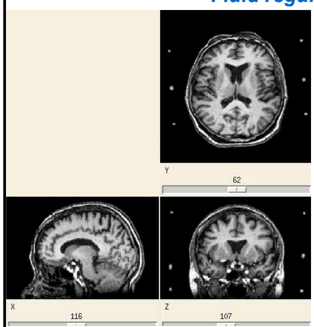
MR T1 Images

256x256x120 voxels

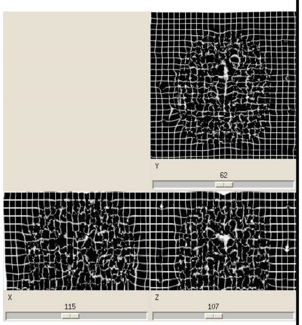
Atlas to patient registration for radiotherapy planning

Correct size and position but high remaining variability in cortex and deep structures

Inter-subject registration
Fluid regularization

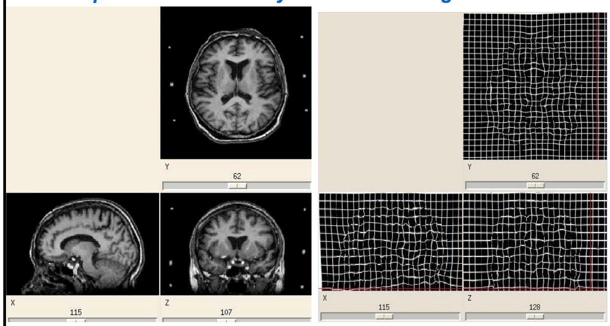


Very good image correspondence



But anatomically meaningless deformation
Jacobian [1/50;50]

Inter-subject registration
Adaptive non-stationary visco-elastic regularization



Registration in 5 min on 15 PCs

Anatomically more meaningful deformation
Jacobian [1/5;5]

Course overview

Feature-based registration

Multimodal Intensity-based Registration

Deformable intensity-based Registration

- A historical perspective
- A Pair and Smooth approach
- Morphing

Using registration for image interpolation and morphing

Interpolating motion and intensity changes

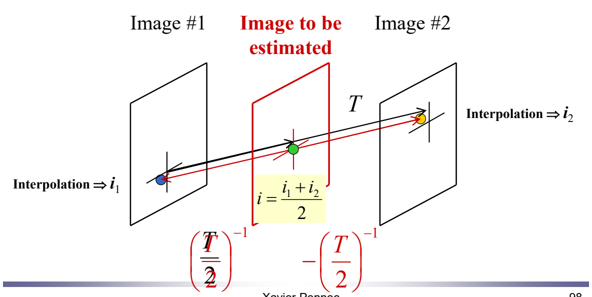
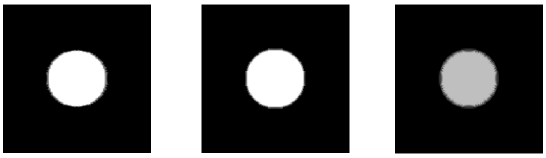


Image interpolation

Synthetic experiments

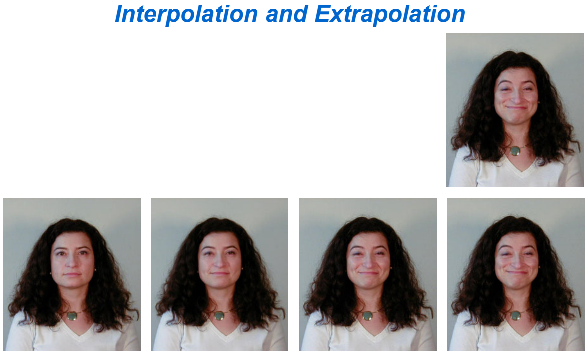
- Comparison with expected results



Translation | Shrinking | Shrinking + intensity variation

MORPHING

Interpolation and Extrapolation



t=0 neutral | t=0,5 attenuated expression | t=1 expression (smile) | t=1,5 exaggerated expression